Lecture 16



# Chapter 39
# Wave Nature of Particles

## de Broglie Waves (1923, Nobel Prize 1929)

Einstein's wave-particle dualism is an absolute phenomenon extending to all physical nature: All forms of matter have wave as well as particle properties:

Frequency + wavelength of matter wave

$$f = \frac{E}{h} \quad \text{and} \quad \lambda = \frac{h}{p}$$

$$E = \sqrt{p^2c^2 + m^2c^4} - mc^2 \approx \frac{1}{2}mv^2$$

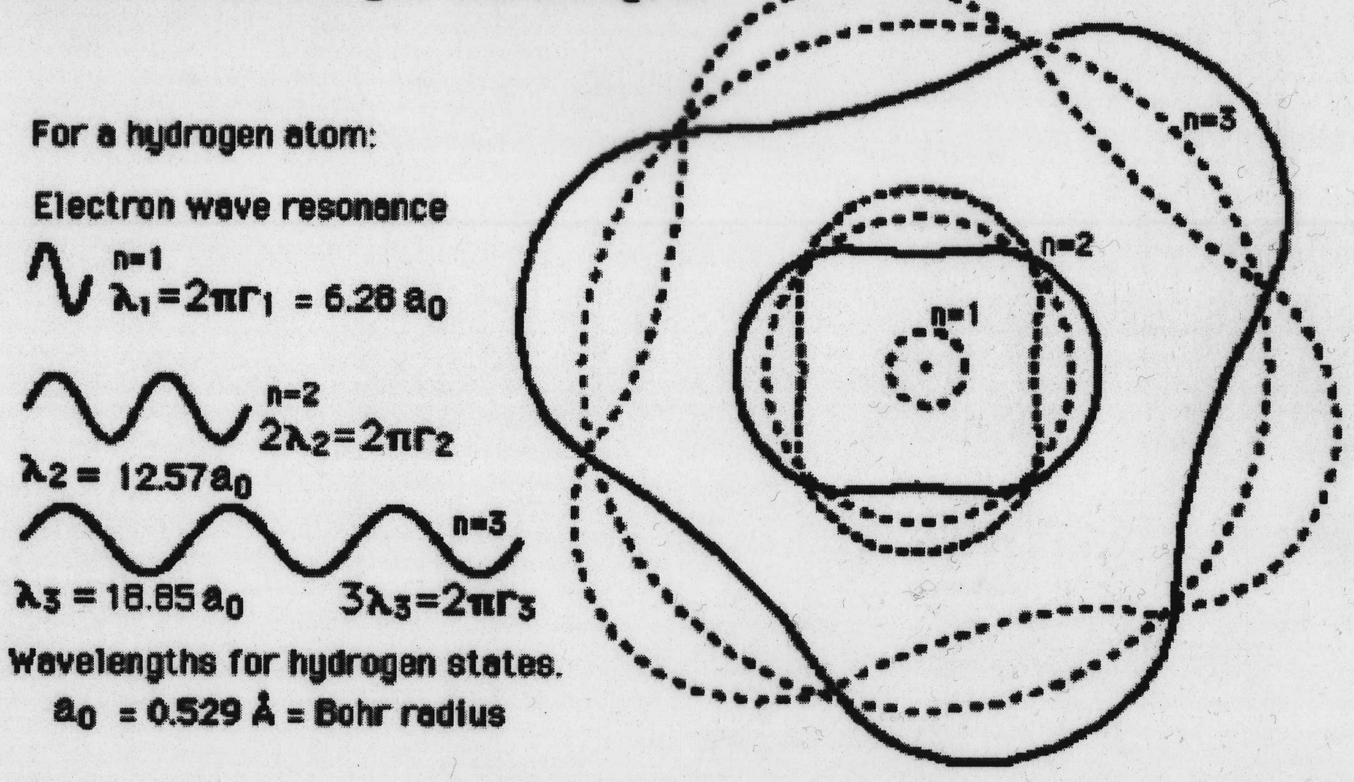
$$p = \gamma m v \cong mv$$

Orbiting electron has a matter wave associated with it.

## Bohr Model and de Broglie Waves

Quantization of angular momentum of electrons in Bohr's theory in terms of standing matter waves associated with orbiting electrons.

The only way for a wave along the electron's orbit to survive is if it is a standing wave. This impies that the circumference of a circular orbit Is an integral number of de Broglie wavelengths.

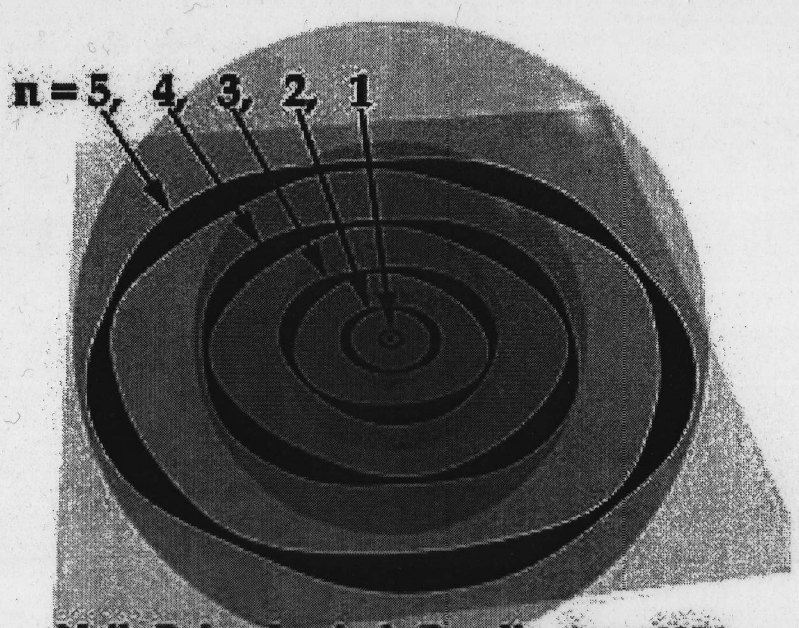



$2\pi r = n\lambda = nh/p = nh/mv$ or

L = $\rightarrow$ $mvr = nh/2\pi = n\hbar$ (Bohr's quantization rule)

This is the explanation for Bohr's strange quantization rule !

If an electron orbiting a nucleus was a matter wave, how would it look like ?




Neils Bohr - Louis de Broglie atom, 1924

# Example: <u>Electronic Wavelength</u>


Electron accelerated from rest to a pot. diff $\Delta V = 10\ \text{KV}$

$$\left.\begin{array}{l}\text{Final energy} = \frac{1}{2}mv^2 - |e|\Delta V \\ \text{Initial energy} = 0\end{array}\right\} \Rightarrow \frac{1}{2}mv^2 = |e|\Delta V$$

$$v = \sqrt{\frac{2|e|\Delta V}{m_e}}\ , \quad \lambda = ?$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2|e|\Delta V m}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\,(1.6\times10^{-19})(1\times10^{4})(9.11\times10^{-31})}} = 1.23\times10^{-11}\,m$$

$$= 0.0123\,nm$$

$\lambda \sim$ X-ray wavelength, thus shows diffraction in crystals.

# Experimental Evidence of Matter Waves

## Electron Diffraction

1. Davisson and Germer experiment (1927) of scattering of electrons by nickel single crystal showed diffraction pattern
   ⇒ Electrons behave like waves

See next page for experimental set up + results

2. G. P. Thomson performed electron diffraction expt (1928) using polycrystalline metallic foil as target.

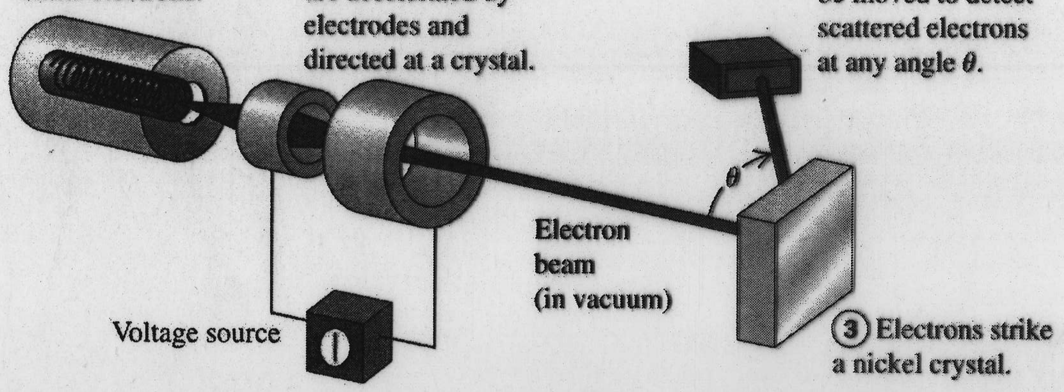
Davisson and Thomson Nobel prize in 1937.

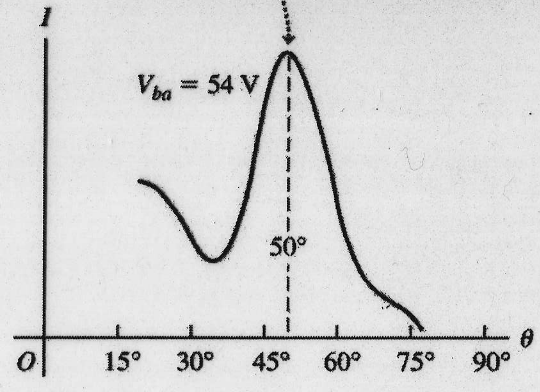
3. Diffraction of hydrogen, helium, other light atoms as well as neutron has been observed.

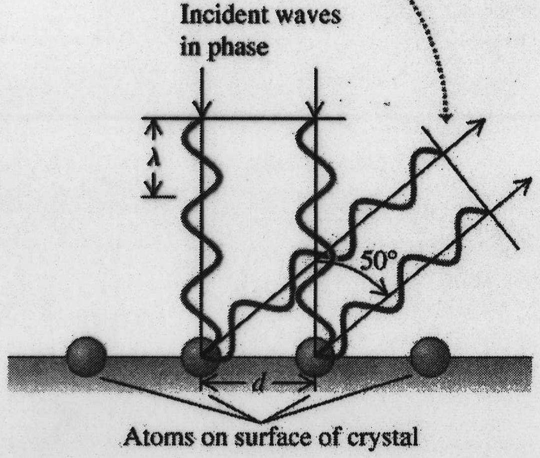
4. Double slit expt. with electrons in 1961 by Claus Johnson proved interference of electrons passing through two slits.

# Electron diffraction sets a foundation for microscopy

- Heated filaments and electrostatic lenses can create and manipulate beams of electrons for experimentation. Refer to Figure 39.3 at the bottom of the slide.
- Graphs of the results for electron scattering are shown in Figure 39.4 at right. Follow Example 39.2.



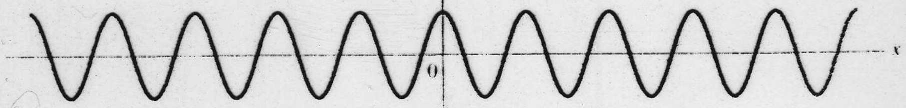



# Wave Packet and Matter Wave

Sinusoidal wave: $y = A\cos(kx - \omega t)$
extends to all space

FIGURE 28.15 (a) An idealized wave of an exact single frequency is the same throughout space and time. (b) If two ideal waves with slightly different frequencies are combined, beats result (Section 14.6). The regions of space at which there is constructive interference are different from those at which there is destructive interference.



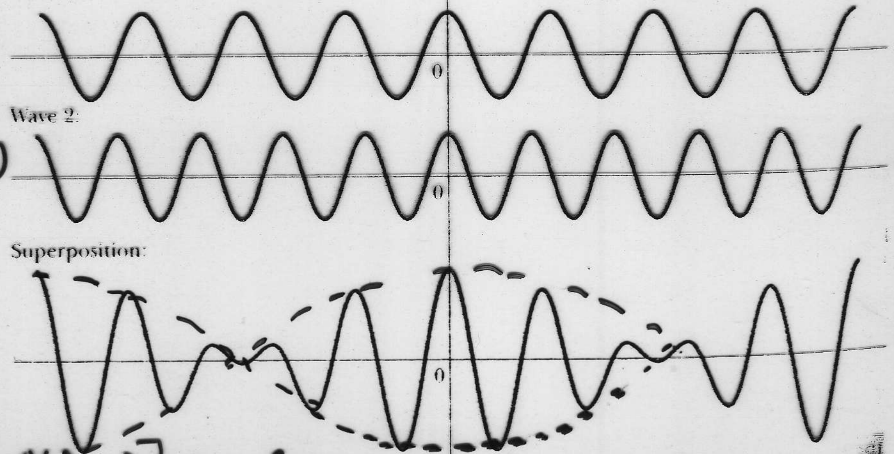

cannot represent matter wave $\quad v_p = \frac{\omega}{k}$

(a)

$y_1 = A\cos(k_1 x - \omega_1 t)$

$y_2 = A \cos(k_2 x - \omega_2 t)$

$y = y_1 + y_2$

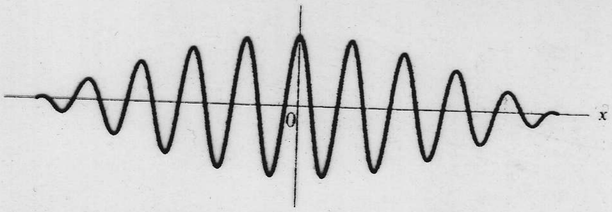



$$= \left[2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right]\cos\left\{\left[\frac{k_1+k_2}{2}\right]x - \left(\frac{\omega_1+\omega_2}{2}\right)t\right\}$$

(b)

where $\Delta k = (k_2 - k_1)$ and $\Delta\omega = \omega_2 - \omega_1$

Envelope shows signs of localization and travels with speed different from $v = \omega/k \Rightarrow v_g = \frac{\Delta\omega}{2} / \frac{\Delta k}{2} = \frac{\Delta\omega}{\Delta k}$, group velocity




ACTIVE FIGURE 28.16 If a large number of waves are combined, the result is a wave packet, which represents a particle.

Physics Now™ Choose the number of waves to add together and observe the resulting wave packet by logging into PhysicsNow at www.pop4e.com and going to Active Figure 28.16.

Localization grows when more and more waves with their crest at $x=0$ are added to form a wave packet or pulse as shown above

$$y(x,t) = \sum_i A_i \cos(k_i x - \omega_i t) = \int dk\, A(k)\cos(kx - \omega(k)t)$$

Such a pulse or wave packet can represent the wave nature of a particle.

## Group Velocity and Particle Velocity

It can be shown the group velocity of the pulse is the same as the velocity of the particle which it represents.

Superposition of a large number of waves gives

$$\omega = \omega(k)$$

Group speed of the wave packet (group of waves)

$$V_g = \frac{d\omega}{dk} = \frac{d(2\pi f)}{d\left(\frac{2\pi}{\lambda}\right)} = \frac{d(hf)}{d(h/\lambda)}$$

$$= \frac{dE}{dp} \quad \text{since } E = hf \text{ and } \lambda = \frac{h}{p}$$

But $E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow \frac{dE}{dp} = \frac{2p}{2m} = \frac{p}{m} = v_{particle}$

$p = mv$

$$\Rightarrow \boxed{v_g = \frac{dE}{dp} = v_{particle}}$$

$\Rightarrow$ group speed of wave packet = speed of particle

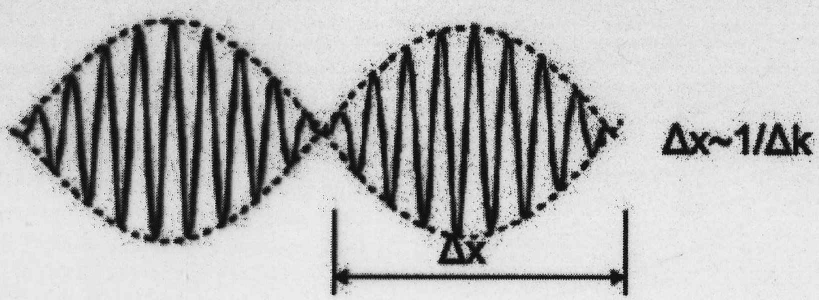
$\Rightarrow$ wave packet is a reasonable way to represent a particle.

**Note relativistic calculation of $\frac{dE}{dp}$ will give the same result.**

# Classical Uncertainty Relations

The range of wavelengths or frequencies of harmonic waves needed to form a wave packet depends on the extent in space and duration in time of the pulse. If it is very localized in space (small $\Delta x$), the range of wavenumbers ($\Delta k$) must be large. If it is not localized at all in space, the range of wavenumbers will be small.

$$\Delta k \Delta x \sim 1$$




Similarly

$$\Delta \omega \Delta t \sim 1$$

If the duration in time t is small, the range of frequencies $\Delta\omega$ must be large [redacted].

*These uncertainty relations arise because of the wave nature of the phenomena being studied (e.g.an electronic signal). It has nothing to do with quantum mechanics. This is classical wave physics.*

| ***Classical Uncertainty Relations*** | $p = \frac{h}{\lambda} = \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \hbar k$ | ***Quantum Uncertainty Relations*** |
|---|---|---|
| $\Delta x\, \Delta k \sim 1$ | **multiply by ħ** → | $\Delta x\, \Delta p \sim \hbar$ |
| | $E = hf = \frac{h}{2\pi}\cdot 2\pi f = \hbar\omega$ | |
| $\Delta\omega\, \Delta t \sim 1$ | Use $p = \hbar k$<br>$E = \hbar\omega$ | $\Delta E\, \Delta t \sim \hbar$ |

$x$, $p_x$ are canonically-conjugate to each other

Observations:

The wave nature of matter implies that we cannot know (measure) both position and momentum of an object to an arbitrary degree of accuracy.

- sets a fundamental limit to the accuracy of physical measurement that is not instrument-dependent.

## The Uncertainty Principle ( Heisenberg 1927, Nobel Prize 1932)

Because of wave-particle duality, it is fundamentally impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy

$$\Delta x \, \Delta p_x \geq \hbar \quad \text{where } \hbar = \frac{h}{2\pi}$$

uncertainty in position coord. $x$

uncertainty in momentum component $p_x$

→ Uncertainties not related to instumental uncertainty

→ Uncertainty due to <u>intrinsic</u> quantum nature of matter.

To understand uncertainty, consider wavelength of particle known exactly.

Since $p = \frac{h}{\lambda}$ ⇒ $p$ known exactly

⇒ $\Delta p_x = 0$

⇒ single wavelength wave will exist throughout space

⇒ no location in space could be identified with position particle.

⇒ no knowledge implies infinite uncertainty in position of particle

Some uncertainty in momentum $\Rightarrow$
range of wavelengths allowed according to de Broglie
$\Rightarrow$ particle represented by a range of waves of different wavelengths forming a wave packet
Wave packet represents particle

This implies particle is some where within the wave packet and not outside.
$\Rightarrow$ Particle's position is not exactly known but known within a certain range of space.

Losing some information about momentum correspond to gaining information about position.
$\Rightarrow \quad \Delta x \, \Delta p_x \geq \hbar$
similarly $\quad \Delta y \, \Delta p_y \geq \hbar$ and $\Delta z \, \Delta p_z \geq \hbar$

Considering wave as a function of time


Can make same arguments about knowledge of frequency and time. Since $E = hf$, uncertainty becomes $\Delta E \, \Delta t \geq \hbar$
$\Rightarrow$ knowledge of energy uncertainty leads to determination of lifetime of a state.

Example: An electron ($m_e = 9.11 \times 10^{-31}$ kg) and a bullet ($m = 0.02$ kg) each having a velocity $v = 500$ m/s, accuracy 0.01%. What is the uncertainty in the position of the particles?

$$\frac{\Delta v}{v} = 0.01\% = 0.0001, \quad \Delta v = (0.0001)v$$

$$= (0.0001)500 \, m/s$$

$$= 0.05 \, m/s$$

$$\Delta p_e = m\Delta v = (9.11 \times 10^{-31} kg)(0.05 m/s)$$

$$= 4.55 \times 10^{-32} \, \frac{kg.m}{s}$$

$$\Delta x_e = \frac{\hbar}{\Delta p_x} = \frac{1.05 \times 10^{-34} \, J.S}{4.55 \times 10^{-32} \, \frac{kg.m}{s}} = 2.32 \, mm$$

$$\Delta p_b = m_b \Delta v = (0.02)(0.05) = 1 \times 10^{-3} \, \frac{kg.m}{s}$$

$$\Delta x = \frac{\hbar}{\Delta p_x} = \frac{1.05 \times 10^{-34}}{1 \times 10^{-3}} = 1.05 \times 10^{-31} \, m$$

$\Rightarrow$ Uncertainty in position of heavy particles is very small.

<u>Example</u>: Unstable particle produced in high energy collision has $m = 3m_{proton}$ and uncertainty in mass is 1% of m. Assuming mass and energy are related, estimate lifetime of particle.

$$\Delta m = 0.01m = (0.01)(3m_p)$$

$$= (0.01)(3)(1.6726 \times 10^{-27} kg)$$

$$\tau = \Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{\Delta m c^2}$$

$$= \frac{1.055 \times 10^{-34}}{(0.03)(1.6726 \times 10^{-27})(3 \times 10^8)^2} = 2.34 \times 10^{-23} s$$