Chapter 38
Quantum Physics
Photons, Electrons and Atoms

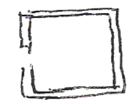
Work of Maxwell, Hertz and others established that light is an electromagnetic wave. Wave nature of light is also evidenced by the interference, diffraction and polarization phenomena. However, when one studies the emission, absorption and scattering of light one has to invoke the particle nature, albeit massless, of light. The energy of electromagnetic wave is quantized and is emitted or absorbed in particle like packages of definite energy called photons. The energy of a single photon is proportional to ~~the~~ frequency of the radiation $E = hf = \frac{hc}{\lambda}$. The momentum associated with a photon can be obtained from $pc = E \Rightarrow \boxed{p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}}$

In this chapter we will study certain phenomena involving electromagnetic waves which can be explained on the basis of the quantum nature of light.

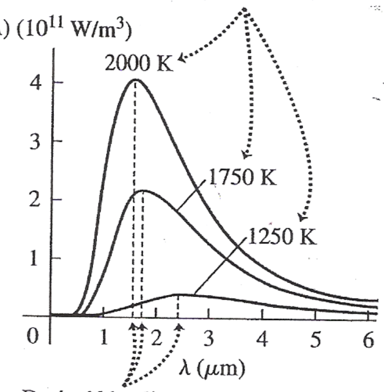
Blackbody Radiation (section 38.8)

An object emits radiation energy at any temperature which we call Thermal radiation. As the temperature is increased bodies start to glow red and at higher temperatures colors change and finally become white. Thermal radiation consists of a continuous distribution of wavelengths from all portions of the electromagnetic spectrum.

The classical view that thermal radiation is caused by accelerated charged particles is not adequate to explain the energy distribution coming out of a blackbody. A blackbody is an ideal object which absorbs all radiation falling on it. It is also the best possible emitter of electromagnetic radiation at any wavelength. A close approximation of a blackbody is a box with a small hole in it.



The experimental intensity of the radiation coming out of a blackbody depends on the temperature as well as the wavelength shown in the figure.



Dashed blue lines are values of λ_m in Eq. (38.30) for each temperature.

Two features of the blackbody radiation is noteworthy.

1. The total intensity I emitted at absolute temperature T is given by the <u>Stefan - Boltzmann law</u>:

$$I = \sigma T^4$$

where σ is a constant known as the <u>Stefan-Boltzmann</u> constant and is given as

$$\sigma = 5.6704 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

Example: If temperature is increased by a factor of 2, what will be the increase in intensity?

$$I_1 = \sigma T_1^4 , \quad I_2 = \sigma (2T_1)^4 = 16 \sigma T_1^4 = 16 I_1$$

2. The peak wavelength shifts to shorter wavelength with increase of temperature (<u>Wien's displacement law</u>)

$$\lambda_{max} T = 2.898 \times 10^{-3} \ m \cdot K$$

Example: If $\lambda_m = 700\ nm$, what is T?

$$T = \frac{2.898 \times 10^{-3} \ m \cdot K}{7 \times 10^{-7} m} = 4140 K$$

Classical theory of blackbody radiation as worked out by various authors (Rayleigh, Jeans) could not explain the wavelength dependence of the whole blackbody radiation spectrum, although it was successful in explaining the long wavelength part. Their theory based on the energy of

The normal modes of the radiation field of kT gave

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4}$$, where k is the Boltzmann constant.

Although this theory correctly explains the long wavelength behavior but at lower wavelength it goes to infinity which obviously is wrong and is known as - ultraviolet catastrophe.

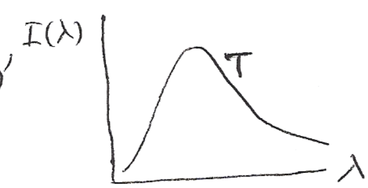
Planck and the Quantum Hypothesis (1900)

To explain the blackbody radiation spectrum Planck assumed that the oscillators which give out the electromagnetic waves can emit radiation of quantized or discrete energy. He suggested that the energy of an oscillator vibrating with a frequency f could have only certain values equal to

$E_n = nhf$, where $n = 0, 1, 2, 3 \cdots$, called a quantum number and h is a constant that bears Planck's name.

By using this expression, Planck calculated the intensity distribution of the radiation at a given temperature assuming Maxwell-Boltzmann distribution of the energy levels and found it to be

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)},$$



and good agreement was found with experimental data if one assumed the Planck constant as $h = 6.26 \times 10^{-34}$ J-s $= 4.136 \times 10^{-15}$ eV-sec

For large λ: $e^{hc/\lambda kT} - 1 \approx 1 + \frac{hc}{\lambda kT} + \cdots - 1 = \frac{hc}{\lambda kT}$

$$\Rightarrow \quad I(\lambda) = \frac{2\pi ckT}{\lambda^4}, \quad \text{Rayleigh-Jeans formula (classical result)}$$

For small λ $\quad I(\lambda) = \frac{2\pi hc^2}{\lambda^5} e^{-hc/\lambda kT} \to 0$ as $\lambda \to 0$

Planck's peculiar model regarding the quantized energy of radiation seemed to work.

By maximizing $I(\lambda)$ i.e. taking $\left.\frac{dI(\lambda)}{d\lambda}\right|_{\lambda_m} = 0$, calculate $\lambda_m = \frac{hc}{4.965\,kT}$

$\lambda_m T =$ const (Wien's displ. law)

By integrating $I(\lambda)$ one gets total intensity

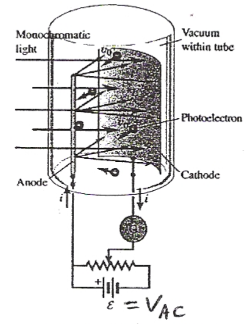
$$I = \int I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15 c^2 h^3} T^4 = \sigma T^4 \text{ (Stefan-Boltzmann law).}$$

The Photoelectric Effect (Section 38.2)

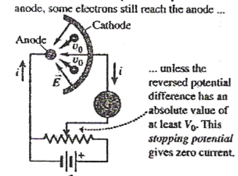
It was discovered by Hertz in 1887 that when electromagnetic waves are incident on a metal surface it can emit electrons. This is known as the photoelectric effect and the emitted electrons are called photoelectrons.

The modern day apparatus for the photoelectric effect is shown here. When the apparatus is dark, there is no electron emission and hence no current. When the electromagnetic wave is incident on the cathode, a current starts to flow provided the frequency of the electromagnetic wave is greater than a threshold frequency. For positive values of V_{AC} the current increases but it attains a saturated value.

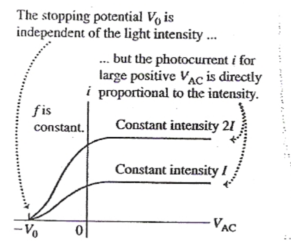
(a) Light causes the cathode to emit electrons, which are pushed toward the anode by the electric-field force.



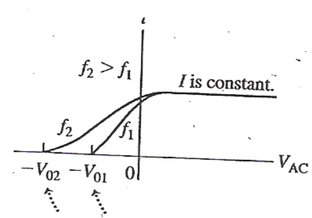
(b) Overhead view with $\vec{E}$ field reversed. Even when the direction of $\vec{E}$ field is reversed so that the electric-field force points away from the anode, some electrons still reach the anode ...



If the polarity is changed, that is V_{AC} is negative the current decreases as shown. Only those electrons which have enough kinetic energy to overcome the negative potential energy $e|V_{AC}|$ will be able to reach the collector anode. When V_{AC} reaches the critical value of $-V_0$, the current stops.



If the intensity of the electromagnetic wave is increased, the photoelectron current is higher as shown, but it is found that the stopping potential $-V_0$ is exactly the same as the case of low intensity electromagnetic wave of the same frequency. The stopping



potential $|V_0|$ is higher for electromagnetic wave of higher frequency. Electrons are ejected with various kinetic energy. Only those electrons which can overcome the negative potential can reach the collector plate. At the stopping potential $-V_0$, the electrons of maximum kinetic energy barely fail to reach the collector plate. Then $E_f = E_i \Rightarrow 0 + (-e)(-V_0) = K_{max} + 0$

or $\boxed{K_{max} = eV_0}$ This equation allows us to measure the maximum kinetic energy with which the electrons can be ejected.

Observations

(a) For incident light of the same frequency (wavelength), the current increases if the intensity increases, but the stopping potential remains the same. Stopping potential does not depend on intensity.

(b) If f is decreased or λ is increased, stopping potential $|V_0|$ decreases in magnitude.

(c) If $f < f_c$ or $\lambda > \lambda_c$, no electrons are emitted, no current flows, no matter what the intensity of the electromagnetic wave is.

Classical Theory

1. More intense light should transfer more energy to the electrons and they should come out with more kinetic energy
2. Electrons should be ejected by electromagnetic wave of any frequency as long as the intensity is high enough.
3. Ejection of electrons should not be instantaneous because it would take the electrons some time to absorb enough energy to overcome the potential barrier of the metal (work function) and get ejected
4. There should not be any relationship of the kinetic energy of the photoelectrons with the frequency of the em waves. Kinetic energy should depend on intensity only.

Thus we see that almost none of the experimental observations can be explained by classical theory.

A successful explanation of the photoelectric effect was provided by Einstein in 1905 for which he won the Nobel prize in 1921. In 1905 Einstein published two other epoch making work: theory of special relativity and theory of Brownian motion.

Einstein's Explanation

Extending Planck's idea Einstein assumed that light energy consists of discrete quanta of energy hf.

$\boxed{E = hf = \frac{hc}{\lambda}}$ ⇒ Light can be considered to be particles of zero mass moving with a velocity c, called photons.

According to Einstein model a photon incident on a metal surface can give all its energy to an electron instantaneously. These electrons can then come out of the metal if the energy it receives can overcome the barrier energy or the work function ϕ of the metal. So the maximum kinetic energy with which the electron can be ejected is

$$K_{max} = \frac{1}{2} m V_{max}^2 = hf - \phi$$

Substituting $K_{max} = eV_0$, we have

$$\boxed{eV_0 = hf - \phi = \frac{hc}{\lambda} - \phi}$$

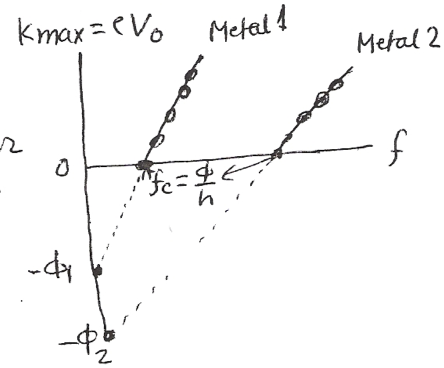
which is known as the Einstein equation for the photoelectric effect.

This theory can explain all aspects of the photoemission experiments:

(a) Stopping potential V_0 depends on f or λ and not on intensity.

(b) If $hf < \phi$, there is no photoemission. $f_c = \frac{\phi}{h}$

(c) There is no time lag of electron ejection, since each photon has enough energy to eject an electron.



If we plot K_{max} versus f we should get a straight line for a given metal. Experimental observation of such a linear relationship has confirmed Einstein's theory.

The value of f when it crosses the horizontal axis gives the critical frequency below which there is no Photoemission. The intercept on the vertical axis gives a measure of the work function.

The cut off wavelength is $\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{\phi}{h}} = \frac{hc}{\phi}$

Light with $\lambda > \lambda_c$ cannot cause photoemission.

The combination $hc = 1240$ eV-nm will appear many times.

Example: Na surface illuminated with $\lambda = 300$ nm. $\phi = 2.46$ eV

a) $K_{max} = ?$ $\Rightarrow$ $K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi$

$$= \frac{1240 \text{ ev-nm}}{300 \text{ nm}} - 2.46 \text{ eV} = 1.67 \text{ eV}.$$

b) $\lambda_c = ?$ for Na.

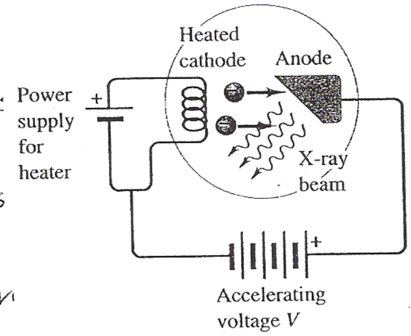
$$\lambda_c = \frac{hc}{\phi} = \frac{1240 \text{ ev-nm}}{2.46 \text{ ev}} = 504 \text{ nm}$$

Photoelectric effect has many important applications.

X-Ray Production and Compton Effect (Section 38.7)

X-rays were discovered in 1895 by Wilhelm Roentzen when he was working with cathode-ray tubes. His apparatus was similar to the one shown in the figure. He found that the cathode rays (electrons) originating from the heated cathode produce some mysterious rays when they hit the metallic anode. He found that these rays could pass through material opaque to light and activate fluorescent screens or photographic plates. He found all materials are transparent to these rays to some degree and the transparency decreased with density incr. This fact led to medical use of x-rays with months of Roentzen's paper.

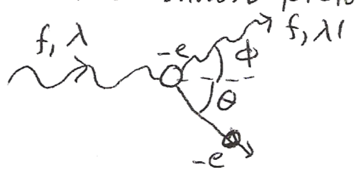
Electrons are emitted thermionically from the heated cathode and are accelerated toward the anode; when they strike it, x rays are produced.



Roentzen was unable to deflect these rays by magnetic fields, nor was he able to observe refraction or interference phenomena associated with waves. So he called these mysterious rays as x-rays. Now, of course, we know that x-rays are electromagnetic waves of very short wavelengths 0.01 to 0.05 nm. They can be diffracted by the atoms in a solid. They are used to find crystal structures of unknown materials.

Compton Scattering (1923, Nobel prize 1927)

Arthur H. Compton studied scattering of x-rays by free electrons or loosely bound electrons. His investigations lent further evidence to the idea that in interaction with matter electromagnetic waves behave like massless particles with a quantum of energy hf and momentum $\frac{hf}{c}$. The scattering process can be shown pictorially as follows:



According to classical wave theory electron will oscillate with the frequency of the wave and will reradiate with same frequency. Compton observed a change in wavelength.

After collision of the photon, the electron gets scattered through an angle θ gaining some energy from the photon and the photon gets scattered in the direction ϕ and acquires a lower frequency f' and longer wavelength λ'. The energy and momenta of the photons before and after collision can be written as

$$E = pc = hf \Rightarrow p = \frac{hf}{c} = \frac{h}{\lambda}$$

and $$E' = p'c = hf' \Rightarrow p' = \frac{hf'}{c} = \frac{h}{\lambda'}$$

Momentum conservation during collision gives

$$\vec{p} = \vec{p}' + \vec{p}_e$$

$$p_e^2 = (\vec{p}' - \vec{p})^2 = p^2 + p'^2 - 2pp'\cos\phi \qquad (1)$$

Conservation of energy gives

$$pc + E_R^e = p'c + \sqrt{E_R^{e2} + p_e^2c^2} \quad \text{where } E_R = m_ec^2$$

Transposing $p'c$ to left hand side and squaring

$$\cancel{E_R^{e2}} + (p-p')^2c^2 + 2E_R^ec(p-p') = \cancel{E_R^{e2}} + p_e^2c^2$$

$$\Rightarrow \quad p_e^2 = p^2 + p'^2 - 2pp' + \frac{2E_R^e}{c}(p-p') \qquad (2)$$

Equating the right hand sides of Eqs. (1) and (2)

$$\cancel{2}pp'(1-\cos\phi) = \frac{\cancel{2}E_R^e}{c}(p-p')$$

$$\frac{h^2}{\lambda\lambda'}(1-\cos\phi) = m_ech\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{m_ech}{\lambda\lambda'}(\lambda'-\lambda)$$

$$\Rightarrow \quad \boxed{\lambda' - \lambda = \frac{h}{m_ec}(1-\cos\phi)} \quad \text{Compton shift equation}$$

$$\Delta\lambda = \lambda_c(1-\cos\phi) \quad \text{where } \boxed{\lambda_c = \frac{h}{m_ec} = 0.00243\,nm.}$$

λ_c is called the Compton wavelength.

Compton's measurements were in excellent agreement with this prediction. They were the first experimental results to convince most physicists of the time of the fundamental validity of the quantum theory.

Note that since the Compton wavelength is very small compared to the incident wavelength, the scattered wave represents 0.001% change of wavelength of red light. So it will not be significant in the visible, but it is a significant part of the incident wavelength in the X-ray region. Compton verified his prediction using characteristic x-ray line of wavelength 0.0711 nm from Molybdenum (Molybdenum K_α line). The energy of this photon is $E = hf = \frac{hc}{\lambda} = \frac{1240\ eV\text{-}nm}{0.0711\,nm} = 17.4\ KeV >>$ binding energy of the valence electrons in carbon. Thus these electrons can be treated essentially free.

Example: Compton used Mo K_α line, $\lambda = 0.0711\,nm$. Calculate

(a) Wavelength of the photon scattered at 180°

$$\lambda' = \lambda + \lambda_c\ (1 - \cos 180°) = \lambda_0 + 2\lambda_c$$
$$= 0.0711\,nm + 2(0.00243)\,nm$$
$$= 0.07596\,nm > \lambda$$

(b) Energy of incident and scattered photons

$$E = \frac{hc}{\lambda} = \frac{1240\ eV\text{-}nm}{0.0711\,nm} = 1.744 \times 10^4\ eV$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\ eV\text{-}nm}{0.07596\,nm} = 1.63 \times 10^4\ eV$$

c) Energy of recoil electron

$$E_e = E - E' = 1116\ eV$$