Chapter 37

Relativity

Relativity consists of two rather different theories – special and general.

General theory is concerned with accelerated frames of reference and gravity. Needs sophisticated mathematics, has applications in areas of gravitation and cosmology, but not encountered in other areas of physics and engineering.

Special theory is concerned with comparison of measurements made in different inertial frames moving with constant velocity relative to each other. Applicable in a wide variety of situations encountered in physics and engineering. We will study this here.

The Principle of Newtonian Relativity and Mechanics

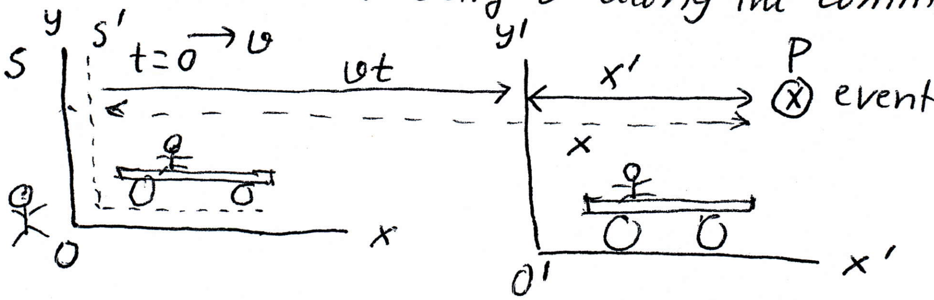
Before embarking on the study of special theory of relativity, let us briefly review the Newtonian relativity which worked very well for systems moving with low velocities.

In describing physical phenomena it is important to establish a frame of reference in which observations are made. A frame is an inertial frame if an object does not experience an acceleration unless it is subject to a force. A frame which moves with a constant velocity with respect to an inertial frame is also an inertial frame.

It turns out that measurements made in all inertial reference frames satisfy the following principle known as the Principle of Newtonian relativity:

The laws of mechanics must be the same in all inertial frames of reference.

Consider two inertial frames S and S′ with S′ moving with a constant velocity v along the common axes x and x′.



Suppose an event takes place at the point P at time t. The coordinates of this event in the two reference frames will be related by the following equations

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Galilean transformation

Time of the event in both reference will be the same because clocks in both reference frames is assumed to run at the same rate.

These equations of transformation from one set of inertial frame to another is called the Galilean transformation of coordinates

The differential displacement of the object at P in time dt in the two coordinates will be related by

$$dx' = dx - v\,dt$$

The speed of the object in the two reference frames can be calulated as

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \qquad dt' = dt$$

$$\boxed{u_x' = u_x - v}$$

The instantaneous speeds of the object in the two reference frames are related by this equation.

If we assume that the speed of the object changes with time

$$\frac{du_x'}{dt} = \frac{du_x}{dt} \Rightarrow \boxed{a_x' = a_x} \quad \text{since } v \text{ is constant.}$$

This shows that the acceleration of an object is invariant under transformation between Newtonian inertial frames of reference. Thus Newton's second law of motion will be the same in all inertial frames of reference.

Need for the Special Theory of Relativity

Allthough classical laws of mechanics are valid in all inertial frames moving with low velocities, it is found that laws of electricity and magnetism are not invariant under Galilean transformation. They depend on the reference frame used. It may be argued that these laws are wrong and need to be modified. But such an argument is difficult to make because these laws give results which are in complete agreement with experiment at all speeds.

This gave Einstein an idea that it is not the laws of electrodynamics that need to be changed but actually the laws of mechanics need modification.

Another very famous experiment carried out in 1887 by Michelson and Morley also indicated that the classical relativity as given by the Galilean transformation equations must be modified.

Michelson-Morley Experiment Revisited

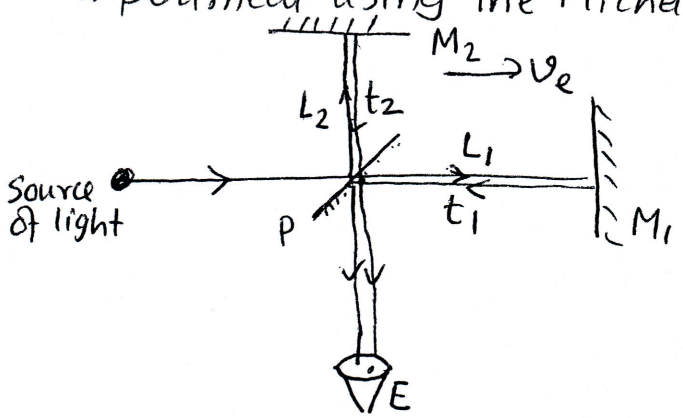
This experiment was actually carried out to prove or disprove the existence of ether which was believed to be a rigid medium through which electromagnetic wave can propagate. We have seen that all mechanical waves need a medium to propagate, so the electromagnetic wave needs a medium which must be all pervasive since light propagates not only through vacuum but also through material. They called this medium as ether which provided an inertial frame of reference. Ether was supposed to be massless and have elastic properties which can sustain electromagnetic waves. Earth's motion around the sun can be considered to be motion through the ether.

According to Galilean relativity

$$v_{\substack{\text{light} \\ \text{w.r.t. earth's frame}}} = v_{\substack{\text{light} \\ \text{in ether frame}}} \mp v_{\substack{\text{earth} \\ \text{w.r.t. ether}}}$$

or, $$v_{\substack{\text{light} \\ \text{w.r.to earth}}} = c \mp v_e$$ where v_e = speed of earth around the sun.

To test this equation Michelson-Morley did an interference experiment using the Michelson interferometer shown below



M_1, M_2 = mirrors

P = beam splitter (partially silvered mirror)

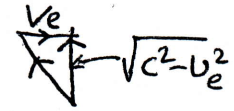
E = eye piece

V_e = velocity of earth w.r.t. ether

t_1 = time of flight moving along L_1 $= \frac{L_1}{c-v_e} + \frac{L_1}{c+v_e}$

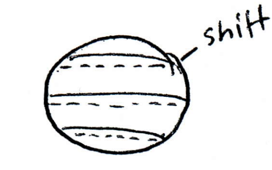
$$= \frac{2L_1}{c}\frac{1}{1-\frac{v_e^2}{c^2}} \cong \frac{2L_1}{c}\left(1+\frac{v_e^2}{c^2}\right)$$

t_2 = time of flight moving along L_2



$$= \frac{2L_2}{\sqrt{c^2-v_e^2}} = \frac{2L_2}{c}\left(1-\frac{v_e^2}{c^2}\right)^{-1/2} \approx \frac{2L_2}{c}\left(1+\frac{v_e^2}{2c^2}\right)$$

The time difference



$$\Delta t = t_2 - t_1 = -\frac{Lv^2}{c^3} \quad \text{if } L_1 \text{ and } L_2 = L$$

This time difference represents a phase difference between the two rays and an interference pattern in the eye piece.

If the apparatus is rotated by $90°$, the roles of L_1 and L_2 are interchanged, the time difference in this case is

$$\Delta t' = t_2' - t_1' \cong \frac{2}{c}\left[L_2\left(1+\frac{v_e^2}{c^2}\right) - L_1\left(1+\frac{1}{2}\frac{v_e^2}{c^2}\right)\right]$$

$$= \frac{Lv^2}{c^3} \quad \text{if } L_1 = L_2 = L$$

The difference $\Delta T = \Delta t' - \Delta t = \frac{2L}{c^2} \cdot \frac{v_e^2}{c^2} = \frac{L_1+L_2}{c}\frac{v_e^2}{c^2}$ represents a shift in the interference pattern as it corresponds to a path difference $\delta = c\,\Delta T$

The interference fringes observed in the first orientation should shift when the apparatus is rotated, by a number of fringes ΔN given by

$v_e = 2.97 \times 10^4$ m/s

$$\Delta N = \frac{c\,\Delta T}{\lambda} = \frac{L_1+L_2}{\lambda}\frac{v_e^2}{c^2} = \frac{22\text{m}}{5.5\times10^{-7}\text{m}}\left(10^{-4}\right)^2 = 0.4$$

No matter what the experiments they did, they alway got $\Delta N = 0$. This null result indicated that velocity of light is the same when measured along two perpendicular axes in a reference frame which, presumably, is moving relative to the ether frame.

$\Rightarrow$ The speed of light is not affected by the motion of the reference frame.

Michelson-Morley experiment contradicted the ether hypothesis and meant that it is impossible to measure the speed of the earth with respect to ether ⇒ it is impossible to find an absolute inertial frame. In later years the idea of ether was abandoned. Now it is understood that light is an electromagnetic wave which requires no medium for its propagation.

Einstein's Principle of Relativity

We have pointed out several difficulties with Newtonian Physics and Newtonian principle of relativity. To address these issues Einstein proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time. He based his relativity theory on just two postulates:

1. Principle of Relativity: The laws of physics are the same in all inertial frames of reference.

2. The Constancy of the speed of Light: The speed of light in vacuum has the same value in all inertial frames of reference, regardless of the velocity of the source or the velocity of the observer.

These two innocent-sounding postulates have far-reaching implications such as

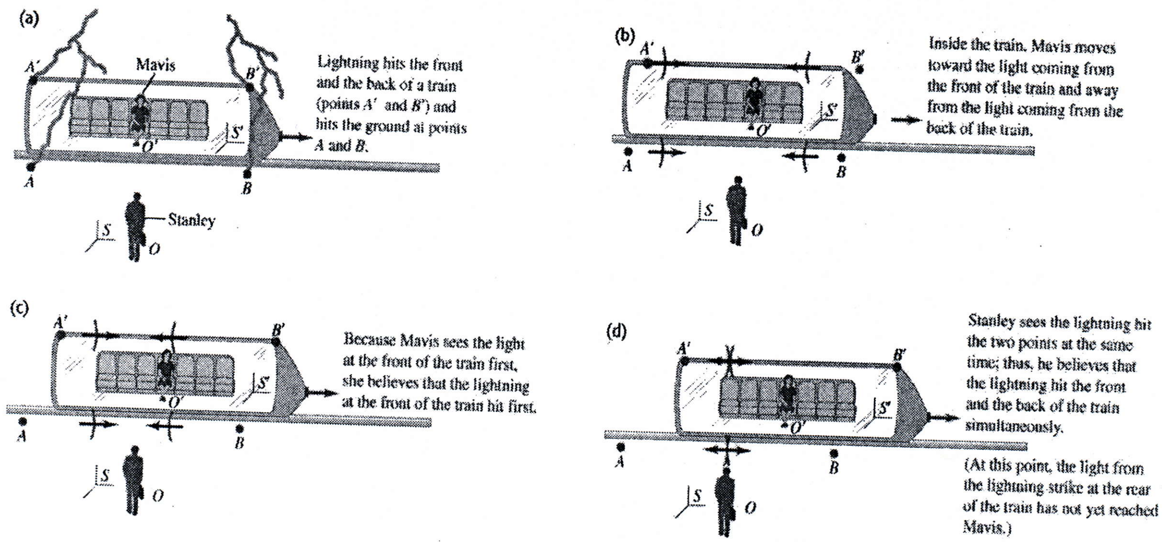
① Simultaneity: Events that are simultaneous for one observer may not be simultaneous for another observer

② Time dilation and length contraction: Two observers in relative motion measure different time interval and length.

③ Modification of Newton's second law and revision of expressions for kinetic energy and momentum of a particle.

Relativity of Simultaneity

According to Newtonian relativity if two events are simultaneous in one reference frame, they are simultaneous in another frame. According to Einstein a time measurement depends on the reference frame in which the measurement is made, as he showed by the following thought (Gedanken) experiment.

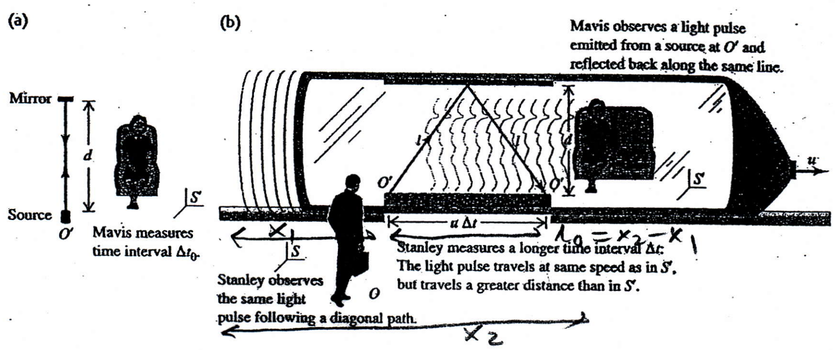


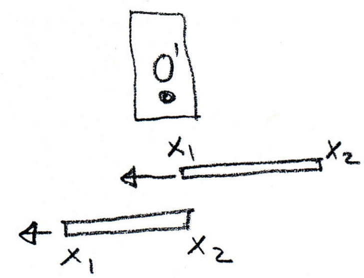
Two lightning strikes at two ends of a train car A' and B' as it moves uniformly with speed v. The lightnings make marks A and B on the ground. According to Stanley at the midpoint O on the ground, the two lightnings are simultaneous if the two light signals from the two bolts reach him at the same time. But for Mavis sitting at the midpoint O' of the train car, the left moving light will reach O' before it reaches O. The right moving light has not reached O' by the time it reaches O. So according to Mavis, the lightning at B' occurs before lightning at A' and thus they are not simutaneous. This shows simultaneity is not an absolute concept but one that depends on the state of motion of the observer.

A consequence of non-simultaneity is time ~~di~~ dilation as discussed below.

Length contraction

The length of an object measured in the reference frame in which the object is at rest is called the proper length. In a reference frame in which the object is moving, the measured length is shorter (along the direction of the relative motion) than its proper length.





We can see this from the previous example. Suppose x_1 and x_2 are the ends of a measuring rod of length $l_0 = x_2 - x_1$, measured in frame S in which the rod is at rest. Since the observer O' is moving relative to this frame with speed v, the distance moved in time Δt is $v\Delta t$. If O' moves from point x_1 to point x_2 in this time, the distance

$L_0 = x_2 - x_1 = v\Delta t.$

According to observer at O' in frame S', the measuring rod moves with speed v to the left and takes Δt_0 to move move past her. The length of the rod in this frame as measured by Mavi's is thus

$l' = v\Delta t_0$. Since $\Delta t_0 < \Delta t$, $l' < l_0$.

These lengths are related by

$$l' = v\frac{\Delta t}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}\, l_0 = \frac{l_0}{\gamma}$$

Such a contraction was first predicted by Lorentz and Fitzerald to explain the null effect in Michelson-Morley experimen

For this reason, the contraction is sometimes called the Lorentz-Fitzgerald contraction.

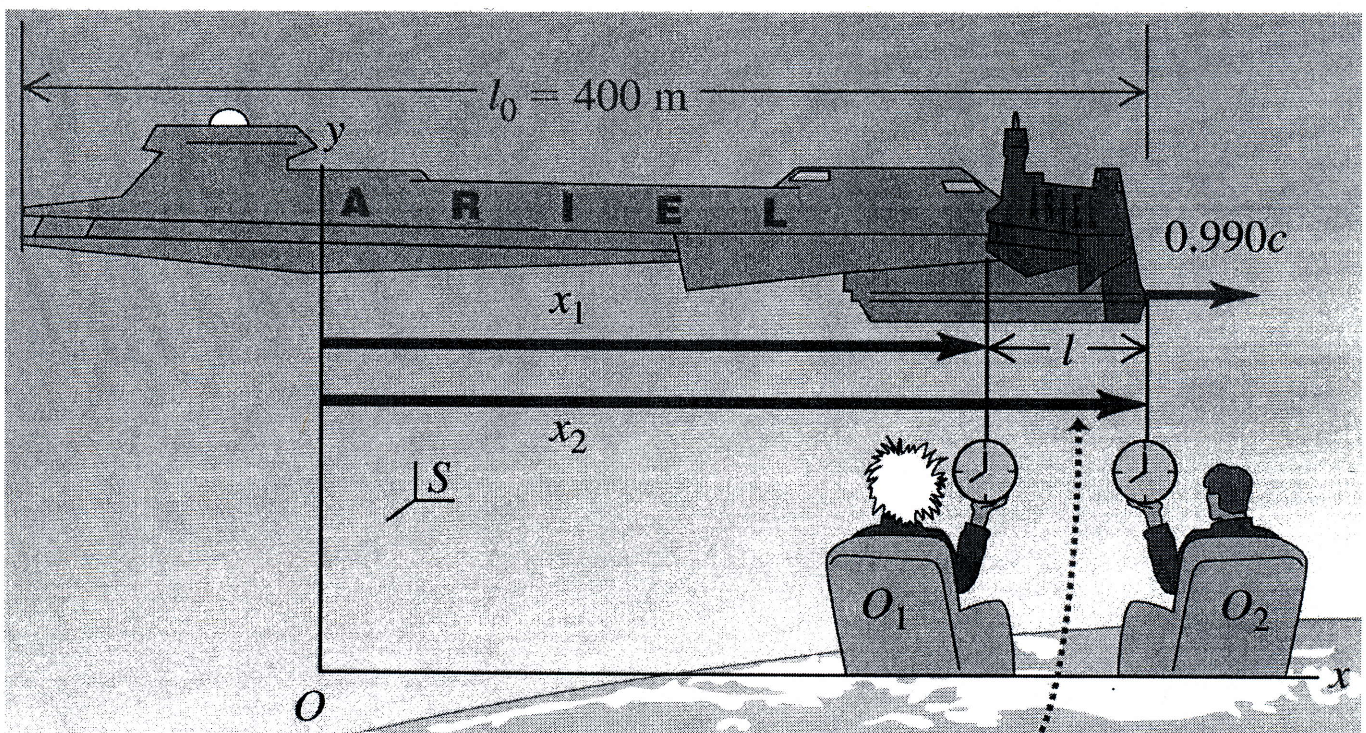
Example: Meterstick moves parallel to ~~the ground~~ its length with Speed $v = 0.6c$ relative to the ground.

a) Length of the meter stick by the ground observer

$$l = \frac{l_0}{\gamma} = \sqrt{1-0.6^2}\, l_0 = 0.8\, l_0 = 80\text{cm}$$

b) How much time will the meterstick take to pass you by?

$$\Delta t = \frac{l}{v} = \frac{0.8\text{ m}}{0.6c} = \frac{4\text{ m}}{3\times3\times10^8\text{ m/s}} = 4.4\times10^{-7}\text{s} = 4.44\text{ nsec}$$



The two observers on earth (S) must measure x_2 and x_1 simultaneously to obtain the correct length $l = x_2 - x_1$ in their frame of reference.