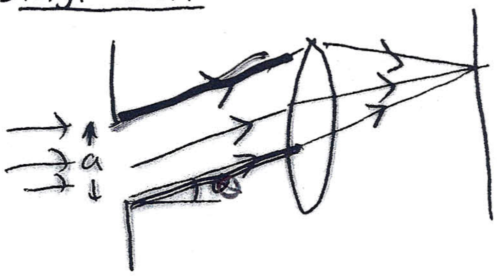
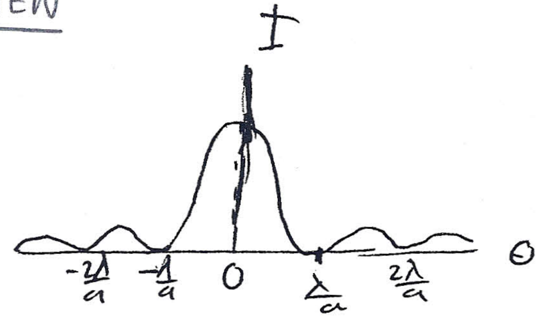
Lecture 11

REVIEW

Fraunhofer Diffraction

Single slit



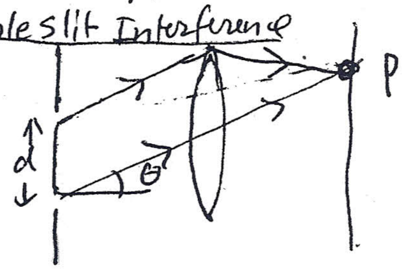


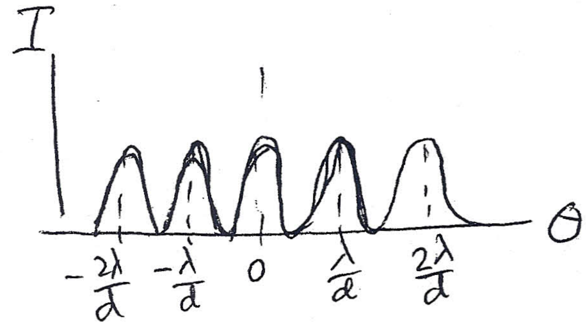
Minima : $a \sin\theta = m\lambda$

$$I(\theta) = I_0 \left[\frac{\sin\frac{\beta}{2}}{\beta/2} \right]^2$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Double slit Interference



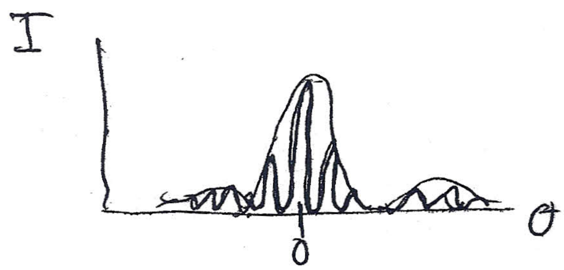


Maxima $d \sin\theta = m\lambda$

$$I(\theta) = I_0 \cos^2\frac{\phi}{2}$$

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

Combined Interference and Diffraction



$$I(\theta) = I_0 \cos^2\frac{\varphi}{2} \left[\frac{\sin\frac{\beta}{2}}{\beta/2} \right]^2$$

If the slits are narrow

$$I(\theta) = I_0 \cos^2\frac{\varphi}{2} \left[\frac{\frac{\beta}{2}}{\beta/2} \right]^2$$

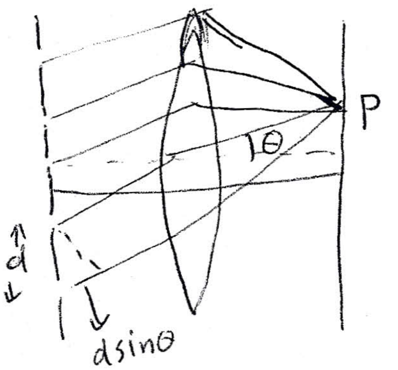
$$= I_0 \cos^2\frac{\varphi}{2}$$

The diffraction becomes that of the interference pattern of a double slit

Demo: Double slit
Multiple slits
Diffraction grating
Single aperture

Several Very Narrow Slits

If each slit is narrow in comparison to the wavelength, its diffraction pattern would spread out nearly uniformly.



constructive interference occurs for rays at angle θ ~~to~~ the normal with a path difference between adjacent slits satisfying the condition

$$d\sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2 \ldots)$$

This was the condition for interference maximum for a two-slit system. These are called principal maxima

As we increase the number of slits keeping d fixed, it turns out that the principal maxima width decreases as $(N-1)$ minima develop between two successive ~~maxim~~ principal maxima. The height of the principal maxima increases by a factor of N^2 and the width of the principal maxima decreases by a factor $\frac{1}{N}$ as shown in the following figure.

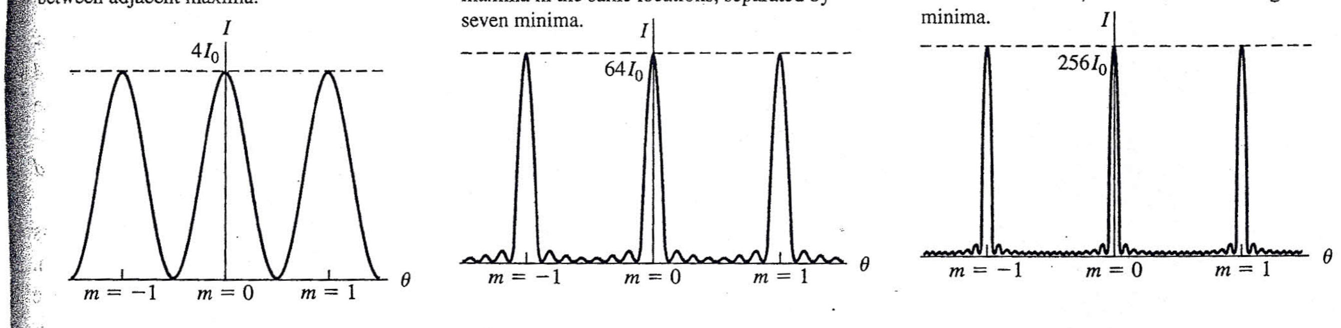
The narrowness of the principal maxima in a multiple-slit pattern has great practical applications as we will see later.

36.15 Interference patterns for N equally spaced, very narrow slits. (a) Two slits. (b) Eight slits. (c) Sixteen slits. The vertical scales are different for each graph; I_0 is the maximum intensity for a single slit, and the maximum intensity for N slits is $N^2 I_0$. The width of each peak is proportional to $1/N$.

(a) $N = 2$: two slits produce one minimum between adjacent maxima.

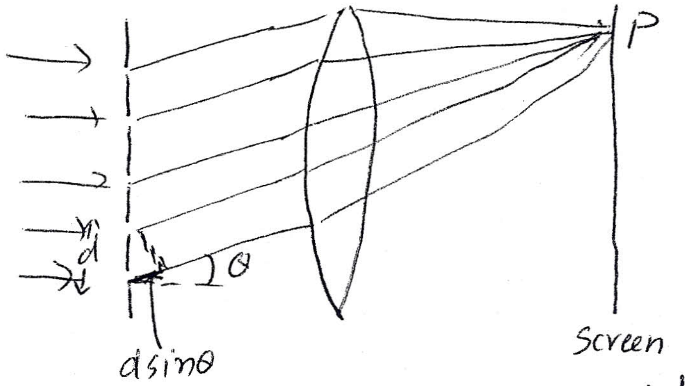
(b) $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.

(c) $N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



Diffraction Grating

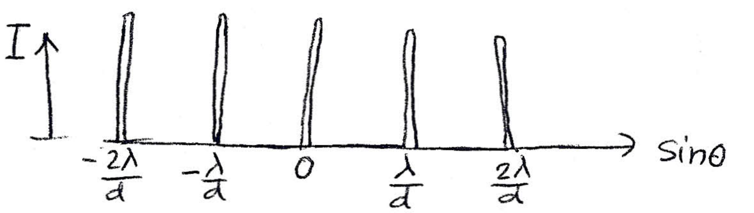
A diffraction grating consists of a large number of parallel slits all of the same width and spaced at regular intervals. Typically if 10000 lines are etched per inch on a thin glass plate, we would have a diffraction grating.



We study Fraunhofer diffraction by such a grating. Coherent light passing normally through a grating will get diffracted in all directions and parallel diffracted rays will be brought to focus on a screen by a converging lens and will produce a diffraction pattern. The condition for the mth maximum is that when light from two adjacent slits arrive at the plate in phase, i.e., the path difference is a multiple of λ

$$d \sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2 \cdots)$$

where d = grating spacing, the distance between two successive slits $= \frac{1}{n}$ where n = number of slits per unit length



If the grating is illuminated by light consisting of a mixture of wavelengths, the diffraction maxima should occur at different points on the screen for different wavelengths, i.e. the colors should be resolved by a grating.

In a reflection grating we have an array of equally spaced ridges or grooves on a reflective surface. The condition for the reflected light maximum is the shown above. The rainbow-colored reflections that we see from the surface of a compact disc are a reflection grating effect. The iridescent colors of certain butterflies arise from ridges on butterflies wings that form a reflection grating.

Resolving Power

Diffraction gratings are widely used to measure the spectrum of light emitted by a source. The process is called Spectroscopy or spectrometry. In spectroscopy it is often important to distinguish slightly different wavelengths. In order that two such wavelengths can be resolved, there must at least be one minimum between the maxima of two different wavelengths.

The chromatic resolving power of a grating is defined by

$$R = \frac{\lambda_{ave}}{\Delta\lambda} \quad \text{where } \lambda_{ave} = \frac{\lambda_1+\lambda_2}{2}, \quad \Delta\lambda = |\lambda_1-\lambda_2|$$

The greater the R of a grating, the closer two emission lines can be and still be resolved.

It can be shown (see text) that for the mth maximum the resolving power of a grating is

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

where N is the total number of rulings of the grating. The greater the number of slits N, the better the resolution. Also higher the order m of the diffraction pattern maximum, the better the resolution.

Example: A diffraction grating has 1.26×10^4 rulings uniformly spread over width $W = 25.4$ mm. It is illuminated by yellow light from a sodium vapor lamp at normal incidence. This contains two closely spaced emission lines (known as sodium doublet) of wavelengths 589.00 nm and 589.59 nm.

a) At what angle does the first order maxima occur for $\lambda = 589$ nm?

$$d = \frac{W}{N} = \frac{2.54\times10^{-2}\,m}{1.26\times10^4} = 2.016\times10^{-6}\,m = 2016\,nm$$

For first order maximum $m=1 \Rightarrow \theta = \sin^{-1}\frac{\lambda}{d} = \sin^{-1}\frac{589}{2016} = 16.99^\circ$

b) What is the minimum number of rulings a grating can have and still be able to resolve the sodium doublet in the 1st order

$$N = \frac{R}{m} = \frac{\lambda_{ave}}{\Delta\lambda\, m} = \frac{589.30\,nm}{(1)(0.59\,nm)} = 999 \text{ rulings.}$$

Diffraction by Circular Apertures and Resolving Power

Consider diffraction by a circular opening such as a circular lens through which light can pass. Figure below (next page) shows the image of a distant point source of light (a star, for instance) formed on a screen placed in the focal plane of a converging lens. The image is a circular disk surrounded by several progressively fainter secondary rings.

The (complex) analysis of the pattern shows that the first minimum for the diffraction pattern of a circular aperture of diameter d is located by

$$\sin\theta_1 = 1.22\frac{\lambda}{d}$$

Comparing this with the 1st minimum of a single slit of width a

$\sin\theta_1 = \frac{\lambda}{a}$ we notice that the main difference is the factor 1.22 which enters because of the circular nature of the aperture.

Resolving Power

Since on many occasions we would like to resolve (distinguish) two distant point objects whose angular separation is small, we define a criterion called the Rayleigh criterion for resolvability. Two point sources are said to be resolved if the central maximum of the diffraction pattern of one source is centered on the first minimum of that of the other. Thus two objects that are barely resolvable by this criterion must have angular separation θ_R of

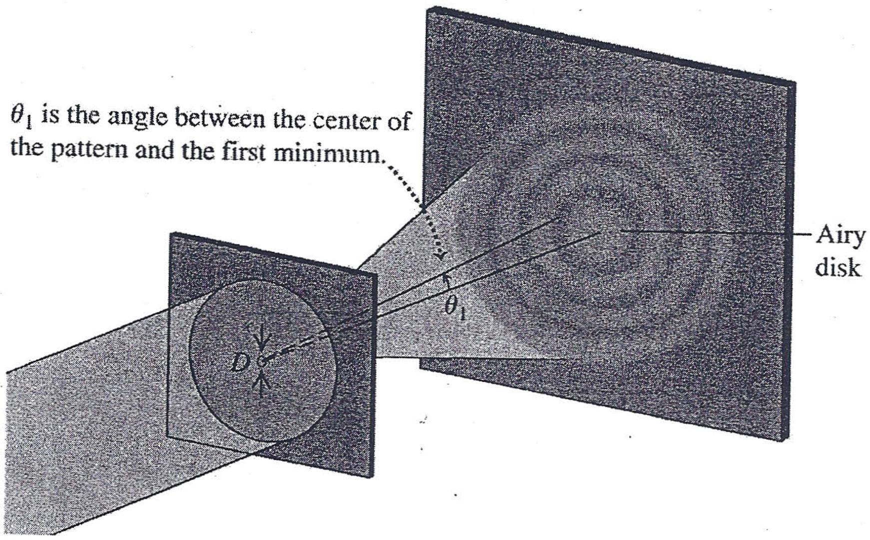
$\theta_R = \sin^{-1}\frac{1.22\lambda}{d}$. Since the angles are small we can express it as $\theta_R = 1.22\frac{\lambda}{d}$ in radians. (Rayleigh criterion)

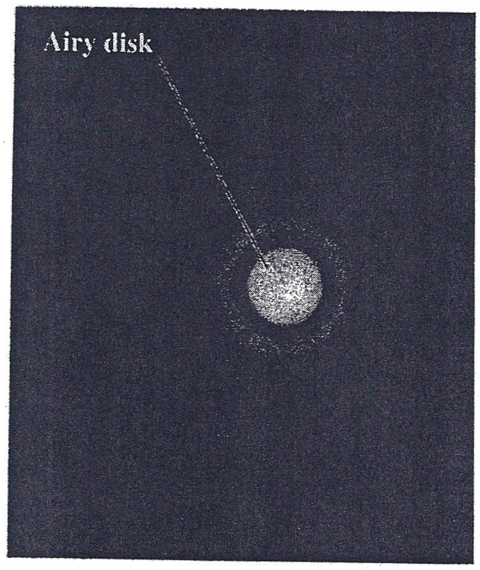
Figures Here: Higher the d better the resolution.

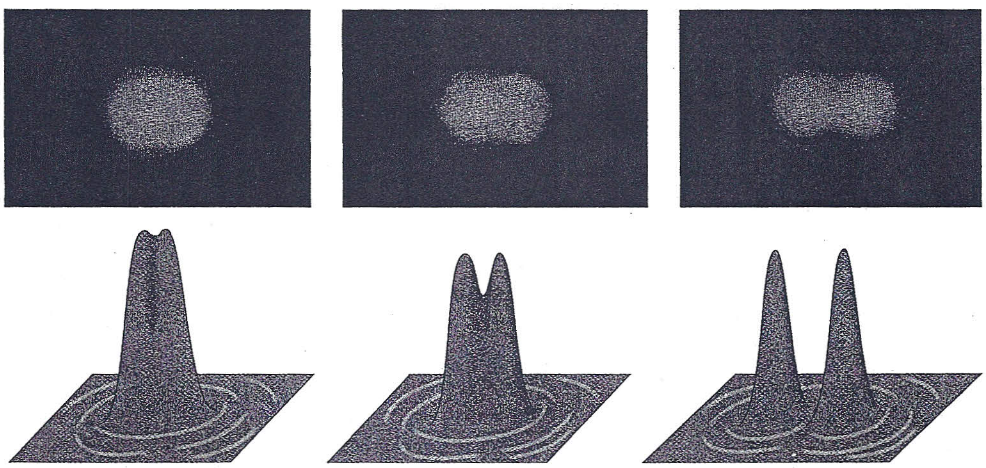
Example: The Keck telescope at Mauna Kea, Hawai, has an effective diameter of 10 m. What is its limiting angle of resolution for 600-nm light?

$$\theta_R = 1.22\frac{\lambda}{d} = 1.22\,\frac{6.00\times10^{-7}\,m}{10\,m} = 7.3\,8\times10^{-8}\ rad$$

The second figure next page shows the central image is resolved.







(a) unresolved

(b) Rayleigh Criterion satisfied

(c) Fully resolved