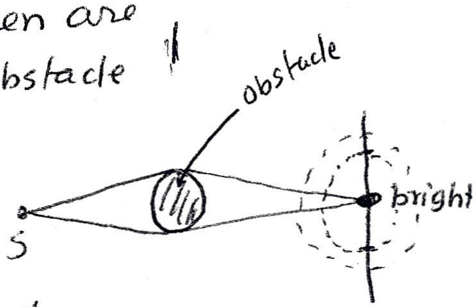
Difference between Interference and Diffraction

Interference: Superposition of waves originating from two different sources

Diffraction: Superposition of wavelets originating from a single wavefront. See next page figure.

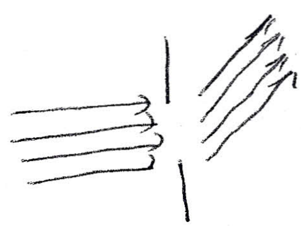
Two kinds of Diffraction

Fresnel Diffraction: Source and screen are at large but finite distances from the obstacle forming the diffraction pattern.



Fresnel

Fraunhofer Diffraction: Both source and screen are at infinite distance from the obstacle, i.e., the incident rays are a parallel and so are the diffracted rays.

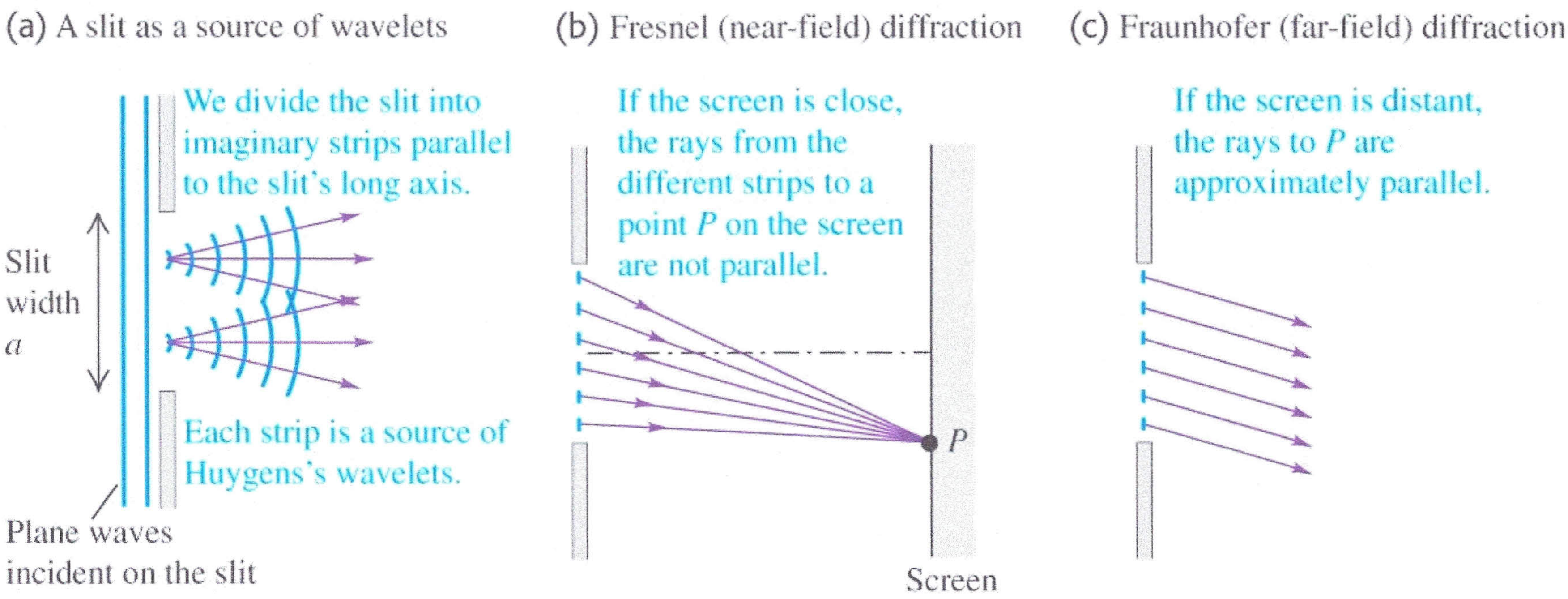


Fraunhofer

Mathematical description of Fresnel ~~Fraunhofer~~ diffraction is difficult and is beyond the scope of this course. We discuss, therefore, only Fraunhofer diffraction.

If we place the screen and the source at infinite distances, the intensity of light would become too low for measurement. However, we can bring the infinite to a finite distance with the help of converging lenses.

Diffraction: Origins and Classes

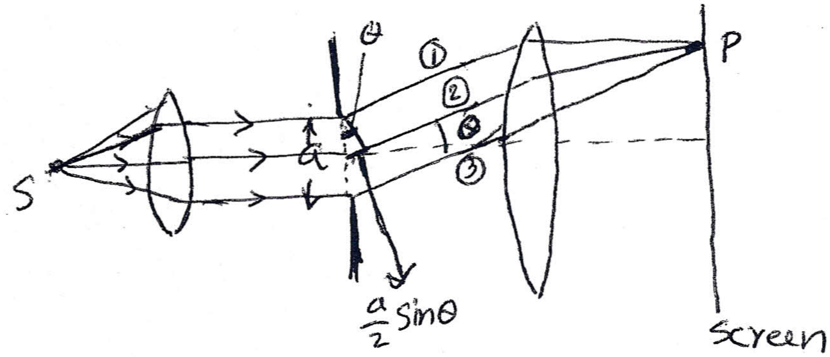


Single-slit Diffraction – produced by interference from secondary wavelets from point sources within the slit width.

Fresnel diffraction – "near-field"

Fraunhofer diffraction – "far-field" (this is simpler to analyze!)

Fraunhofer Diffraction from a Single Slit



To analyze the diffraction pattern, it is convenient to divide the slit into two halves. All the waves at the slit are in phase.

The path difference between the two rays, one diffracted from the upper edge and the other from the middle of the opening is $\delta = \frac{a}{2}\sin\theta$.

The point P will be a minimum because of the two rays if

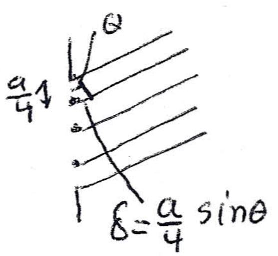
$$\delta = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or $$a\sin\theta = n\lambda$$

Similarly the path difference between the middle ray and the bottom parallel ray is $\delta = \frac{a}{2}\sin\theta$ and point P will be minimum because of the two rays if

$$\delta = \frac{a}{2}\sin\theta = \frac{\lambda}{2} \Rightarrow a\sin\theta = \lambda.$$

If we divide the slit into four parts, instead of two



and consider two adjacent rays, the path difference will be $\frac{a}{4}\sin\theta$. So ~~a will be a minimum~~ the point on the screen where these two rays meet will be a minimum if

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \quad \text{or} \quad a\sin\theta = 2\lambda$$

If we divided the slits into six parts and considered two diffracted rays from adjacent points, the path difference will be $\delta = \frac{a}{6}\sin\theta$. For minimum $\frac{a}{6}\sin\theta = \frac{\lambda}{2}$

$\Rightarrow a\sin\theta = 3\lambda$

So the general condition for destructive interference for a single slit is $\boxed{\sin\theta = \frac{m\lambda}{a}}$ where $m = \pm 1, \pm 2, \pm 3 \ldots$

This equation gives us the position of the minima on the screen. In between two successive minima somewhere near the middle there will be a maximum. The central maximum with $\theta = 0$ has the largest intensity as the light from the entire slit arrives here in phase. The central bright fringe is wider than the other bright fringes. The successive maxima are of much lower intensity.

Example: a) If for red light $\lambda = 650\text{ nm}$, the first minimum occurs at $\theta = 15°$, what is the slit width?

$$a\sin\theta = \lambda \Rightarrow a = \frac{\lambda}{\sin\theta} = \frac{650\text{nm}}{\sin 15°} = 2511\text{ nm} \simeq 2.5\,\mu m$$

b) what is λ' whose first side maximum is at $\theta = 15°$.

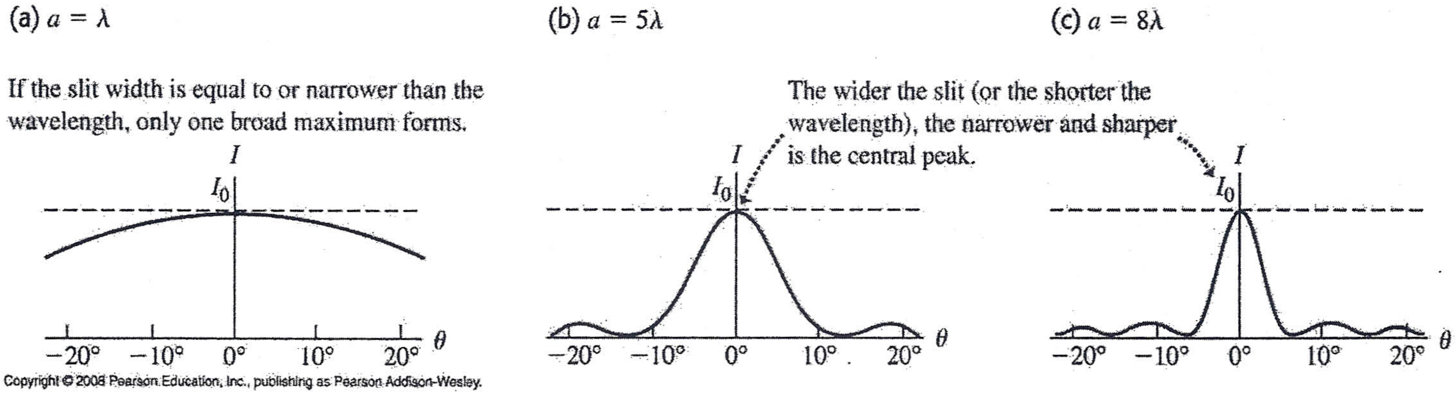
We know that the first side maximum will occur at

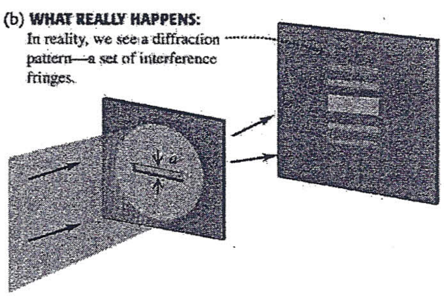
$$a\sin\theta \equiv 1.5\lambda' \Rightarrow \lambda' = \frac{a\sin\theta}{1.5} = \frac{(2511\text{nm})\sin 15°}{1.5} = 430\text{nm}$$

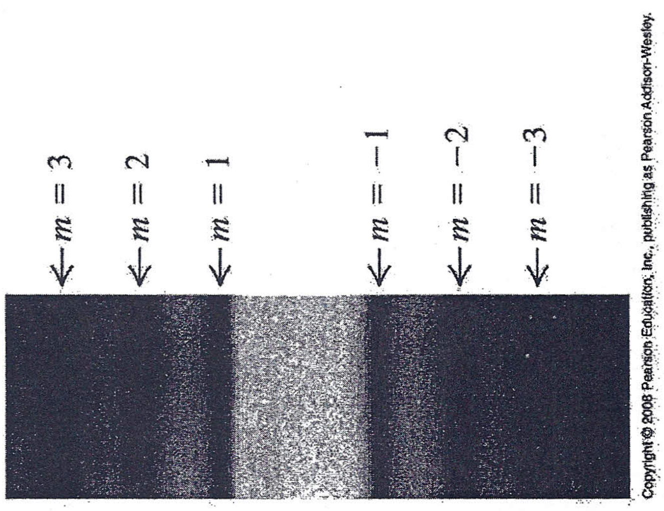
This light is in violet.

The equation $\sin\theta = m\frac{\lambda}{a}$ ($m = \pm 1, \pm 2, \pm 3 \ldots$) gives us the position of the minima on the screen. In between two successive minima somewhere near the middle there will be a maximum. The central maximum with $\theta = 0$ has the largest intensity and widest, and the successive maxima are of much lower intensity a shown below. Note that the width of the maxima decrease with increasing width of the slit a.

Minima (dark fringe): $\sin\theta = +/- (m\lambda/a)$







Intensity in the Single-Slit Pattern

The intensity pattern on the screen can be shown to be

$$I(\theta) = I_0 \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta} \right]^2$$

This equation which will be derived below using phasor diagram shows that there will be minima when

$$\frac{\pi a}{\lambda}\sin\theta = m\pi \quad \text{or} \quad a\sin\theta = m\lambda \quad (m = \pm 1, \pm 2 \cdots)$$

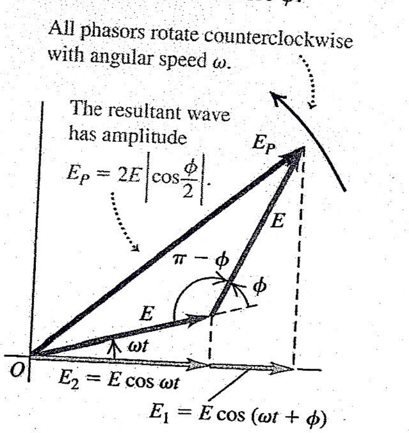
The actual position of the maxima can be obtained by maximizing the above equation $\frac{dI(\theta)}{d\theta} = 0$. While this will give a maximum position not quite in the middle of two successive minima, we will approximate it to be so, i.e, we will take maxima occurring at $a\sin\theta \simeq (m+\frac{1}{2})\lambda$. (*) example next page 6a

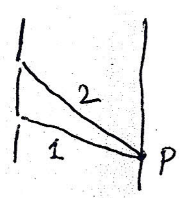
Next page shows the derivation of the intensity $I(\theta)$. using the phasor diagram. We start with the phasor diagram of two waves

$$E_1(t) = E\cos(\omega t + \phi)$$
$$E_2(t) = E\cos\omega t$$

which can be applied to the case of interference of waves from a double slit.

35.9 Phasor diagram for the superposition at a point P of two waves of equal amplitude E with a phase difference ϕ.





Each sinusoidal function is represented by a rotating vector (phasor) whose projection on the horizontal axis at any instant represents the instantaneous value of the sinusoidal function. Each phasor rotates counterclockwise with angular speed. The vector sum of the two phasors gives us the resultant vector E_P, which is calculated as

$$E_P^2 = E^2 + E^2 - 2E^2\cos(\pi - \phi)$$
$$= 2E^2 + 2E^2\cos\phi = 2E(1+\cos\phi)$$
$$= 4E^2\cos^2\frac{\phi}{2} \implies E_P = 2E\left|\cos\frac{\phi}{2}\right|$$
$$I = |E_P|^2 = 4E^2\cos^2\frac{\phi}{2} = 4I_0\cos^2$$

(*) Intensity of Secondary Maxima

Secondary maxima lie approximately halfway between the minima and are found form

$$a \sin\theta \simeq (m+\tfrac{1}{2})\lambda$$

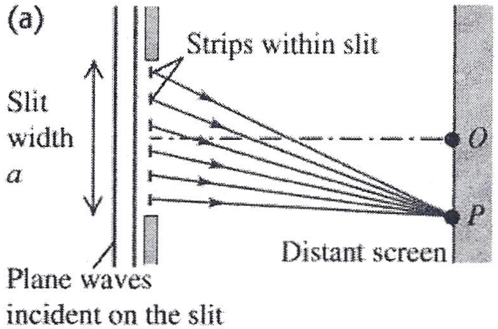
$$I_\theta = I_0 \left[\frac{\sin (m+\frac{1}{2})\pi}{(m+\frac{1}{2})\pi} \right]^2 = \frac{1}{(m+\frac{1}{2})^2\pi^2}$$

$$\frac{I_\theta}{I_0} = 0.045, \quad 0.016, \quad 0.0085$$

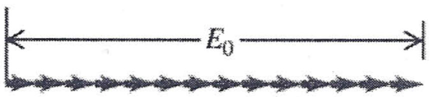
$$m = \pm 1, \quad m = \pm 2, \quad m = \pm 3$$

Calculating the Intensity of Single Slit Diffraction

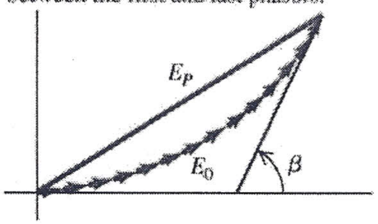
(a)



(b) At the center of the diffraction pattern (point O), the phasors from all strips within the slit are in phase.

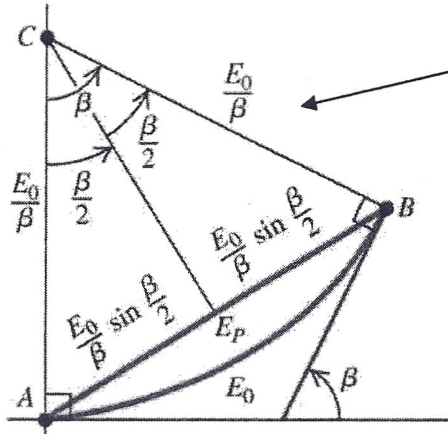


(c) Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.



E_p = amplitude of the resultant E field at point P
= geometrically the "chord" of a trail of phasors

(d) As in (c), but in the limit that the slit is subdivided into infinitely many strips



Radius = arclength/β

In the limit of many sources,
trail of phasors → circular sector
can get the "chord" of circular sector

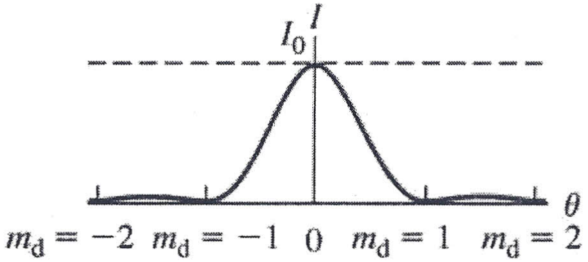
$$E_p = E_o \frac{\sin(\beta/2)}{(\beta/2)}$$

$$I = I_o \left| \frac{\sin(\beta/2)}{(\beta/2)} \right|^2$$

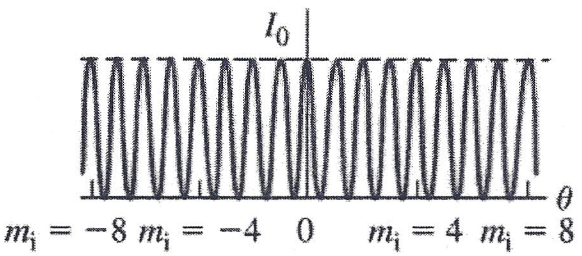
$$\beta = (2\pi/\lambda)\, a \sin\theta$$

The Double Slit Interference Pattern with Diffraction

(a) Single-slit diffraction pattern for a slit width a



(b) Two-slit interference pattern for narrow slits whose separation d is four times the width of the slit in **(a)**



Slit separation = 4 x slit width

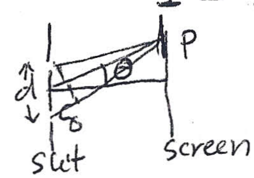
d = 4a i.e. slit is "narrow"

→ This is the familiar interference fringes for Young's double slit experiment.

We had always assumed narrow slits.

$I = I_0 \cos^2\frac{\phi}{2}$ where $\phi = k\delta$

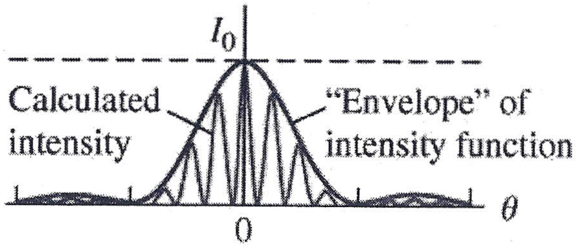
$= \frac{2\pi}{\lambda} d\sin\theta$



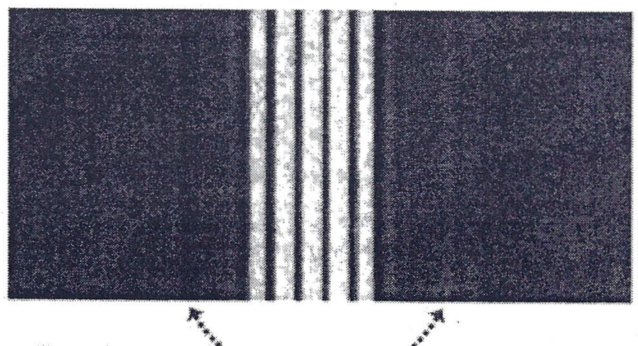
But what if slits are not narrow ?

Then we must factor in diffraction !

(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



(d) Actual photograph of the pattern calculated in (c)



For $d = 4a$, every fourth interference maximum at the sides ($m_i = \pm 4, \pm 8, \ldots$) is missing.

Diffraction "modulates" the two-slit interference pattern.

$$I = \underbrace{\boxed{I_o \cos^2(\varphi/2)}}_{\text{two-slit interference intensity}} \underbrace{\boxed{\left|\frac{\sin(\beta/2)}{(\beta/2)}\right|^2}}_{\text{diffraction factor (envelope)}}$$

$$\varphi = (2\pi/\lambda)\, d \sin\theta$$

$$\beta = (2\pi/\lambda)\, a \sin\theta$$