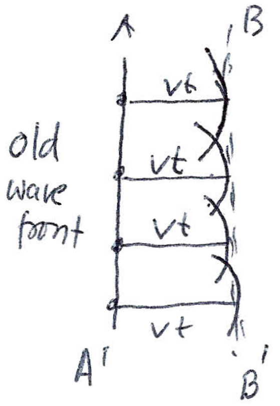
ture 8 Chapter 33 (Contd.)

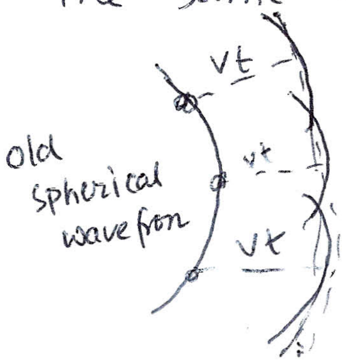
uygens's Principle

Huggens's principle is a geometrical model that allows us to determine the position of a wavefront from the knowledge of an earlier wavefront. All points on a wavefront behave as sources for spherical secondary waves, called wavelets, that propagate outwards with the speed of light in the medium.



As the figure shows, after time t the plane wavefront moves to the new location shown as $B'B$. The new wavefront is given by the envelope of the wavelets at time t.

The same is true for a spherical wavefront. Each point behaves as a source of wavelets. The tangent to the wavelets at time t gives the wavefront at time t.



Using the Huygens principle one can derive the laws of reflection and refraction, and can explain bending of light around corners (diffraction) as well as interference.

All the results obtained from Huygens's principle can also be obtained from Maxwell's equations. Thus it is not an independent principle, but convenient for calculations with wave phenomenon.

Chapter 35

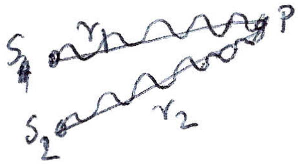
Interference

The properties of light that we have discussed so far (chapter 33) are based on the fact we can represent light as rays which move in straight lines that are bent at the reflecting or refracting surface. But there are other properties of light which can be explained only on the basis of wave nature of light. Interference and diffraction are two such properties which we will discuss now.

Interference is a phenomenon which occurs when light from two coherent sources combine. When waves and sources have a constant phase relationship, we call that wave coherent. A laser light can be coherent and monochromatic. Point sources on a light bulb filament oscillating randomly is an example of an incoherent source. Colors seen in oil films and soap bubbles are a result of interference between light reflected from the front and back surfaces of a thin film or soap solution.

Interference of two waves is based on the <u>principle of superposition</u> - When two or more waves overlap, the resultant displacement at any point is found by adding the displacements of the individual waves at that instant.

<u>Constructive interference</u> takes place when two waves arriving at the point are in phase. Destructive interference takes place when the two waves arriving at the point are out of phase, i.e. a crest of one wave arrives at the same time as a trough of the other wave.



It can be checked if $r_2 - r_1 = m\lambda$ $(m = 0, 1, 2 \ldots)$
constructive interference

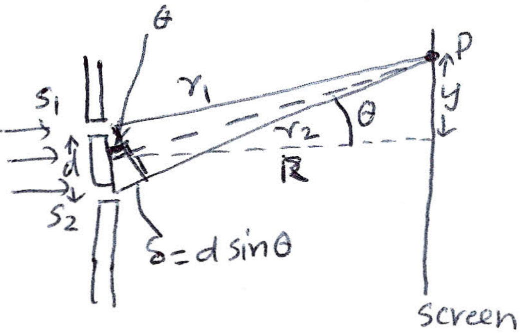
If $r_2 - r_1 = (m + \frac{1}{2})\lambda$ $(m = 0, 1, 2 \cdots)$ destructive interference.

[SEE FIGURE ON LAST PAGE]

(Demonstration with transparencies and overhead projector)

A classic ~~example~~ demonstration of interference effects is the so called Young's double slit experiment performed in 1800 by English scientist Thomas Young.

Young's Double Slit Experiment



Two coherent waves coming from slits S_1 and S_2 meet at the point P on the screen placed at a distance R from the two sources. All points on the screen are not uniformly illuminated. Some points have maximum intensity and some points have zero intensity. Assume that the two waves have same amplitude and same wave length.

The path difference between the two waves arriving at P is

$$\delta = r_2 - r_1 = d\sin\theta$$

E_1: $y_1(r_1, t) = A\cos(kr_1 - \omega t)$

E_2: $y_2(r_2 t) = A\cos[k(r_1 + \delta) - \omega t] = A\cos(kr_1 - \omega t + \phi)$

where $\phi = k\delta = \frac{2\pi}{\lambda} d\sin\theta$

The resultant wave at P

E: $\overset{E_1+E_2}{y = y_1 + y_2} = \underbrace{\left[2A\cos\frac{\phi}{2}\right]}_{A'} \cos(kr_1 - \omega t - \frac{\phi}{2})$

Since $I \propto |A'|^2$. The intensity will be maximum when

$\cos\frac{\phi}{2} = \pm 1 \Rightarrow \phi = 2m\pi \Rightarrow \frac{2\pi}{\lambda} d\sin\theta = 2m\pi \Rightarrow d\sin\theta = m\lambda$

where $m = 0, \pm 1, \pm 2 \cdots$

Destructive interference when $\cos\frac{\phi}{2} = 0$, $\phi = (2m+1)\pi \Rightarrow d\sin\theta = (m + \frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2 \cdots$

The pattern on the screen is a succession of bright and dark

fringes or interference fringes parallel to the slits S_1 and S_2
(Demonstration of double slit diffraction)

Distance between successive maxima

For small values of m ($m \leq 10$), the corresponding angles θ_m are small $\Rightarrow$

$$\sin\theta_m \approx \tan\theta_m = \frac{m\lambda}{d} \qquad (\text{maximum})$$

$$\frac{y_m}{R} = \frac{m\lambda}{d} \Rightarrow \boxed{y_m = m\lambda \frac{R}{d}}$$

Similarly $y_{m+1} = (m+1)\lambda \frac{R}{d}$

$$\Delta y = y_{m+1} - y_m = (m+1)\lambda\frac{R}{d} - m\lambda\frac{R}{d} = \lambda\frac{R}{d}$$

Example Let $d = 0.03$ mm and $R = 1.2$ m

Second order bright fringe is at $y_2 = 4.5$ cm from the center line. What is the wavelength

$$\lambda = \frac{d\, y_2}{Rm} = \frac{(3\times10^{-5}\text{m})(4.5\times10^{-2}\text{m})}{(1.2\text{m})(2)} = 5.6\times10^{-7}$$

$$= 560 \text{ nm}$$

Intensity in Interference Pattern

Since the average intensity at any point on the screen can be calculated from ~~$I \propto |A'|^2 = 4A^2\cos^2\frac{\phi}{2}$~~

~~$= 4$~~

The amplitude of the electric field at P

$$E_P = 2E\cos\frac{\phi}{2}$$

So the intensity at the point P is

$$I = S_{av} = \frac{E_P^2}{2\mu_0 c} = \frac{1}{2}\epsilon_0 c E_P^2, \qquad c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$I = 2\epsilon_0 c E^2 \cos^2\frac{\phi}{2}$$

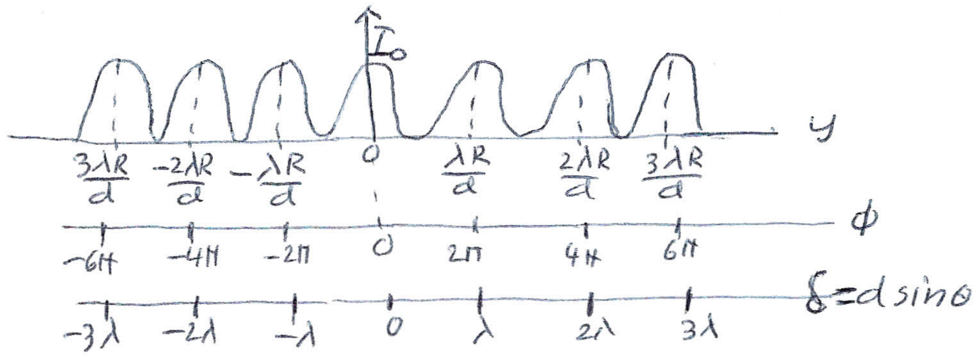
The maximum intensity I_0 occurs where the phase difference is zero ($\phi = 0$). So

$$I_0 = 2\epsilon_0 c E^2 \quad \text{(intensity at the central maximum)}$$

Note that this maximum intensity is 4 times (not twice) as large as the intensity $\frac{1}{2}\epsilon_0 c E^2$ from each individual wave. Substitution of I_0 in the expression for I gives

$$\boxed{I = I_0 \cos^2 \frac{\phi}{2}}$$

This intensity distribution pattern can be plotted as below



This result can be obtained by drawing phasor diagram as done in your text.

REVIEW FOR MIDTERM I

This is possible because waves can be added and subtracted.