Constants

$\sigma = 5.6703x10^{-8} \frac{W}{m^2K^4}$, h=6.626x10^{-34} J.s = 4.136x10^{-15} eV.s,
1eV=1.602x10^{-19} J, c=3x10^8m/s, hc=1240 eV.nm,
m_e=9.11x10^{-31} kg = 0.511 MeV/c^2
e=-1.602x10^{-19} C
ε_0=8.85x10^{-12} C^2/N.m^2, μ_0=4πx10^{-7} Wb/A.m, c=3x10^8 m/s

Simple Harmonic Motion

$F = -kx$, $x(t) = A\cos(\omega t + \phi)$, $\omega = \sqrt{\frac{k}{m}}$

$f = \frac{1}{T} = \frac{\omega}{2\pi}$ $v(t) = -\omega A\sin(\omega t + \phi)$ $a(t) = -\omega^2 A\cos(\omega t + \phi)$

$k = \frac{mg}{\Delta l}$, for a hanging spring

$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$, $\phi = \cos^{-1}\left(\frac{x_0}{A}\right)$ and $\sin^{-1}\left(-\frac{v_0}{\omega A}\right)$

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$\omega = \sqrt{\frac{g}{l}}$ $\omega = \sqrt{\frac{mgd}{I}}$

$$x(t) = \left[Ae^{-\frac{b}{2m}t}\right]\cos(\omega' t + \phi) \text{ where } \omega' = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

Mechanical Waves

$y(x,t) = A\cos(kx - \omega t)$, $k = \frac{2\pi}{\lambda}$, $\omega = \frac{2\pi}{T} = 2\pi f$

$v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$, $v = \sqrt{\frac{F}{\mu}}$, where $\mu = \frac{m}{l}$

$v_y = \frac{\partial y}{\partial t} = \omega A\sin(kx - \omega t)$, $a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(kx - \omega t)$

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = \frac{1}{2}\mu\omega^2 A^2 v$$

$$I = \frac{P}{4\pi r^2}$$

Superposition Principle

$$y(x,t) = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(kx - \omega t + \phi) = [2A\cos\frac{\phi}{2}]\sin(kx - \omega t + \frac{\phi}{2})$$

Normal modes:
$$y(x,t) = y_1 + y_2 = A\cos(kx - \omega t) + A\cos(kx + \omega t) = [2A\sin kx]\sin \omega t$$

Amplitude maximum or antinode when
$\sin kx = \pm 1, \Rightarrow kx = \frac{2n+1}{2}\pi$, $\quad x = \frac{2n+1}{4}\lambda$, $\quad$ n=0,1,2,3 …

Amplitude=0 or node when
$\sin kx = 0, \Rightarrow kx = n\pi$, $\quad x = \frac{n}{2}\lambda$ $\quad$ n=1,2,3…

Separation between two successive nodes =$\lambda/2$

Standing waves on a stretched string fixed at both ends
$$y(0,t) = y(L,t) = 0$$
$$\Rightarrow 2A\sin kL = 0,$$
$$\Rightarrow kL = n\pi,$$
$$\Rightarrow \lambda_n = \frac{2L}{n}$$

Allowed frequencies: $f_n = \frac{v}{\lambda_n} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = nf_1$

Electromagnetic waves

$\vec{E} = \vec{E}_{\max}\cos(kx - \omega t)$, $\quad \vec{B} = \vec{B}_{\max}\cos(kx - \omega t)$, $\quad E = cB$, $\quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$n = \frac{c}{v} \approx \sqrt{K}$, $\quad \lambda = \frac{\lambda_0}{n}$

$u = \varepsilon_0 E^2$

Poynting vector $\vec{S} = \frac{\vec{E}x\vec{B}}{\mu_0}$, $\quad S_{av} = I = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$

$P = \frac{I}{c}$ (complete absorption), $\quad P = \frac{2I}{c}$ (total reflection)

Standing waves similar to standing waves on a stretched string.

Nature of Light:

Laws of reflection and refraction: $\theta_r = \theta_a$, $\quad n_a \sin\theta_a = n_b \sin\theta_b$

$$\sin\theta_{crit} = \frac{n_b}{n_a}$$

Polarization $I = \frac{1}{2}I_0$ for unpolarized wave passing through one polarizing sheet.

$I = I_{\max}\cos^2\phi$ for plane polarized wave passing though one polarizing sheet.

Brewster's angle: $\tan\theta_p = \frac{n_b}{n_a}$

Interference and Diffraction

Young's double slit experiment

$$E_{tot} = (2A\cos\frac{\phi}{2})\sin(kr_1 - \omega t + \frac{\phi}{2})$$

$$\phi = k\delta\,,\quad k = \frac{2\pi}{\lambda} \quad \delta = d\sin\theta\,,\quad \sin\theta \approx \frac{y}{R}$$

Constructive interference: $\cos\frac{\phi}{2} = \pm 1$, $\phi = 2m\pi$, $\delta = m\lambda$;

Destructive interference: $\cos\frac{\phi}{2} = 0$, $\phi = (2m+1)\pi$, $\delta = (m+\frac{1}{2})\lambda$

$$I(\theta) = I_0\cos^2(\frac{\phi}{2})$$

Thin film Interference

$\lambda = \frac{\lambda_0}{n}$, Path difference in a medium of refractive index n: $\delta = 2t$

Additional phase difference of π by reflection from denser medium

Total phase difference $\phi = \frac{2\pi}{\lambda}\delta - \pi$

Constructive Interference: $\phi = 2m\pi$, Destructive Interference: $\phi = (2m+1)\pi$

Diffraction

$$a\sin\theta = m\lambda\,, \qquad I(\theta) = I_0\left[\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{(\frac{\pi a}{\lambda}\sin\theta)}\right]^2$$

$$I(\theta) = I_0\cos^2\frac{\varphi}{2}\left[\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{(\frac{\pi a}{\lambda}\sin\theta)}\right]^2 \text{ where } \phi = \frac{2\pi d}{\lambda}\sin\theta$$

$$d\sin\theta = m\lambda\,, \quad R = \frac{\lambda_{avg}}{\Delta\lambda} = mN$$

$$\sin\theta_1 = 1.22\frac{\lambda}{d}$$

Relativity

$$\Delta t = \gamma \Delta t_0 , \qquad L = \frac{L_0}{\gamma}, \qquad \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$x' = \gamma(x - ut), \qquad t' = \gamma(t - \frac{ux}{c^2})$$

$$v_x' = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}, \qquad v_y' = \frac{v_y}{\gamma(1 - \frac{uv_x}{c^2})}, \qquad v_z' = \frac{v_z}{\gamma(1 - \frac{uv_x}{c^2})}$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m \vec{v}, \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

$$K = mc^2(\gamma - 1), \qquad E_R = mc^2, \qquad E = \gamma mc^2 = \sqrt{(pc)^2 + (mc^2)^2}$$

$$m = 0 \Rightarrow E = pc \Rightarrow p = \frac{E}{c}$$

Photons, Electrons and Atoms

$$I = \sigma T^4, \qquad I = \frac{P}{area}$$

$$\lambda_m T = 2.898x10^{-3}\, m.K$$

$$I(\lambda) = \frac{2\pi hc^2 / \lambda^5}{e^{hc/\lambda kt} - 1},$$

$$E = hf = \frac{hc}{\lambda}, \qquad p = \frac{E}{c} = \frac{h}{\lambda},$$

$$K_{\max} = eV_0 = hf - \phi = \frac{hc}{\lambda} - \phi, \qquad hf_c = \frac{hc}{\lambda_c} = \phi$$

$$\lambda' - \lambda_0 = \lambda_c(1 - \cos\phi), \quad \lambda_c = \frac{h}{mc} = 0.00243 nm$$

$$L = mvr = n\hbar$$

$$r_n = \varepsilon_0 \frac{h^2 n^2}{\pi m e^2} = n^2 a_0, \qquad E_n = -\frac{1}{\varepsilon_0^2} \frac{me^4}{8h^2 n^2} = -\frac{E_0}{n^2}, \qquad E_0 = 13.6\ eV$$

$$E_{ion} = E_\infty - E_1 = 13.6\ eV$$

$$f = \frac{E_i - E_f}{h} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \qquad n_i > n_f$$

$$R_H = \frac{mk_e^2 e^4}{4\pi c\hbar^3} \qquad =1.097\text{x}10^7\text{m}^{-1}$$

The Wave Properties of Particles

$$\lambda = \frac{h}{p} = \frac{h}{mv}, \quad f = \frac{E}{h}, \qquad \frac{1}{2}mv^2 = |e|\Delta V$$

The Quantum Particle $\quad v_{phase} = \frac{\omega}{k}, \quad v_g = \frac{d\omega}{dk}$

The Uncertainty Principle $\quad \Delta x \Delta p_x \geq \hbar, \quad \Delta E \Delta t \geq \hbar$

Matter Wave: $\quad P(x)dx = |\psi(x)|^2 dx$

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi(x)}{dx^2} + U(x)\varphi(x) = E\varphi(x)$$

Particle in an infinite potential well

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}, \qquad E_n = \frac{h^2}{8mL^2}n^2,$$

Tunneling: $T \cong e^{-2\kappa L}$ where $\kappa = \frac{\sqrt{2m(U-E)}}{\hbar}$

Harmonic oscillator: $E_n = (n + \frac{1}{2})\hbar\omega$