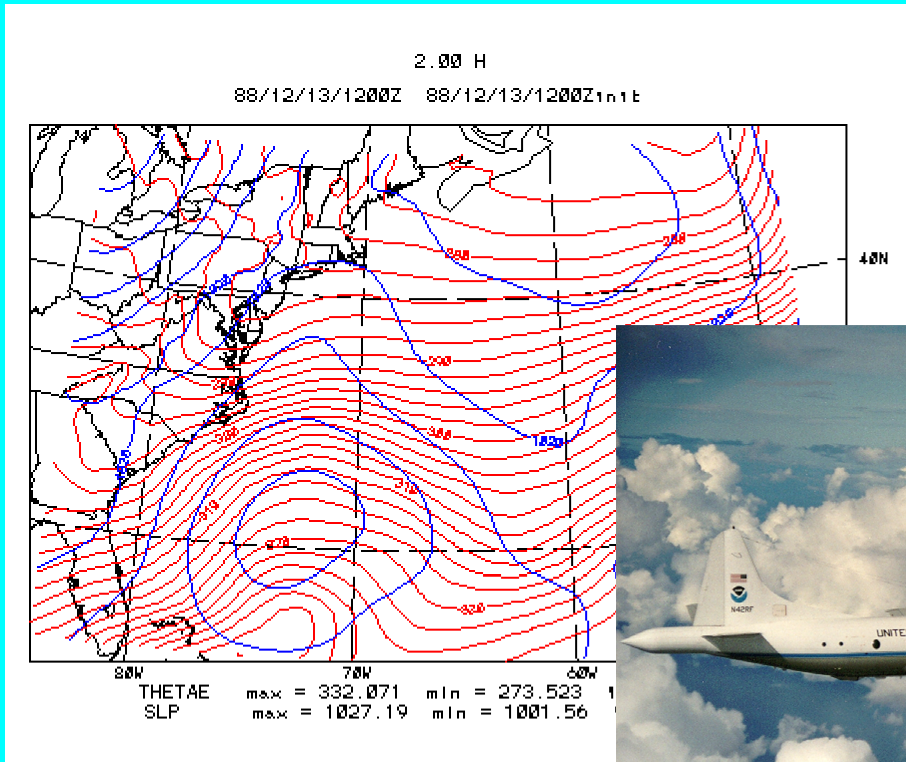

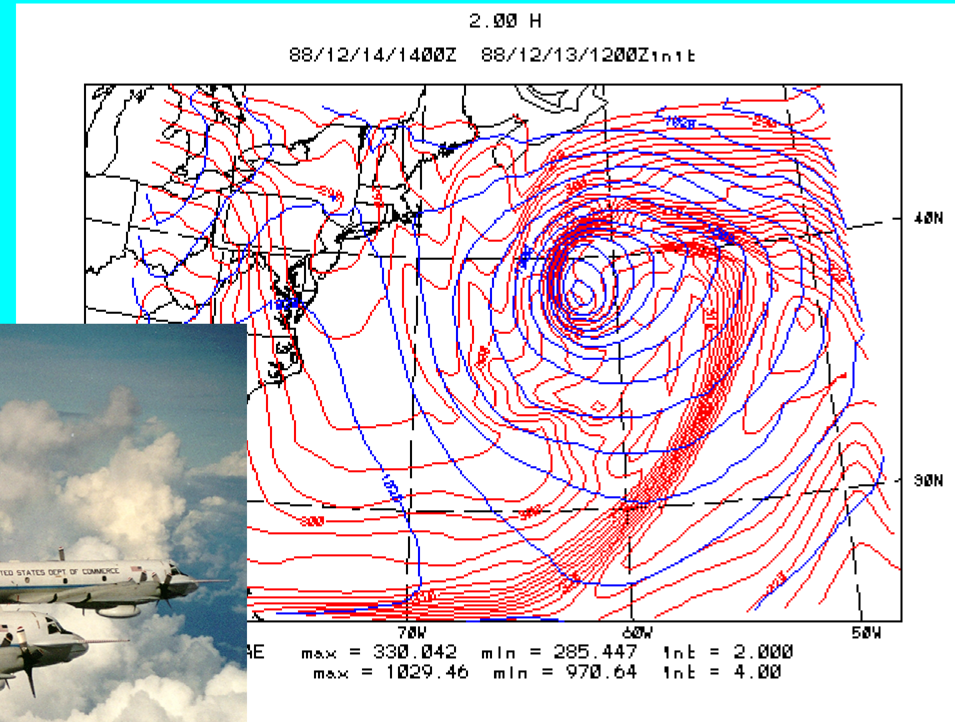

Atmospheric Physics
Computational Models and Data Analysis

A Presentation to the Physics Department
at the Richard Stockton State College of New Jersey
Wednesday, 2 March 2011

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Drexel University
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610-348-6495

I. A Short Course on Atmospheric Physics
II. ERICA Aircraft Data
III. Cyclone Models
IV. LAMPS90 NWP Model Simulations
V. Data Analysis - Wavelets

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A Short Course on Atmospheric Physics

Terminology:

Front:

A front may be defined as a sloping zone of pronounced transition in the thermal and wind fields. Consequently, they are characterized by a combination of:

A. Large Static Stability
B. Large Horizontal Temperature Gradient .
C. Absolute Vorticity (horizontal wind shear).
D. Vertical Wind Shear.

When depicted on a quasi-horizontal surfaces, fronts appear as long, narrow features in which the the along front scale is typically an order of magnitude greater than the across front scale. (1000-2000 km versus 100-200 km). (Keyser, 1986)

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$$10\,km \approx 6.21\,miles$$

Front:

A front is an "elongated" zone of "strong" temperature gradient and relatively large static stability and cyclonic vorticity.
(Bluestein, 1986)

$$10\,km \approx 6.21\,miles$$

Terminology:

Front:

A front is an "elongated" zone of "strong" temperature gradient and relatively large static stability and cyclonic vorticity.
(Bluestein, 1986)

- **Strong - At least an order of magnitude greater than the synoptic scale value of 10 K/ 1000 km.**
- **Elongated - A zone whose length is roughly half an order of magnitude greater than its width.**
- **Static Stability - Buoyancy forces and gravitational forces lead to decelerating a parcel's upward motion**
- **Cyclonic Vorticity - The curl (turning) of the wind with counter clockwise cyclonic circulation around a low in N. H. being positive.**

$$10\,km \approx 6.21\,miles$$

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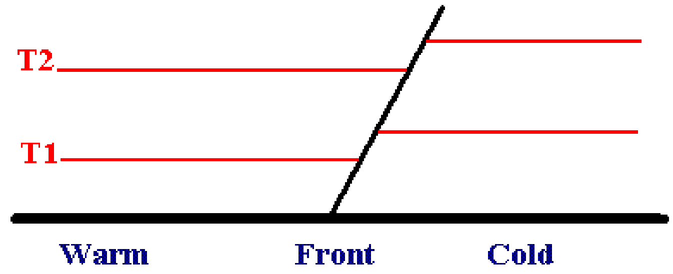
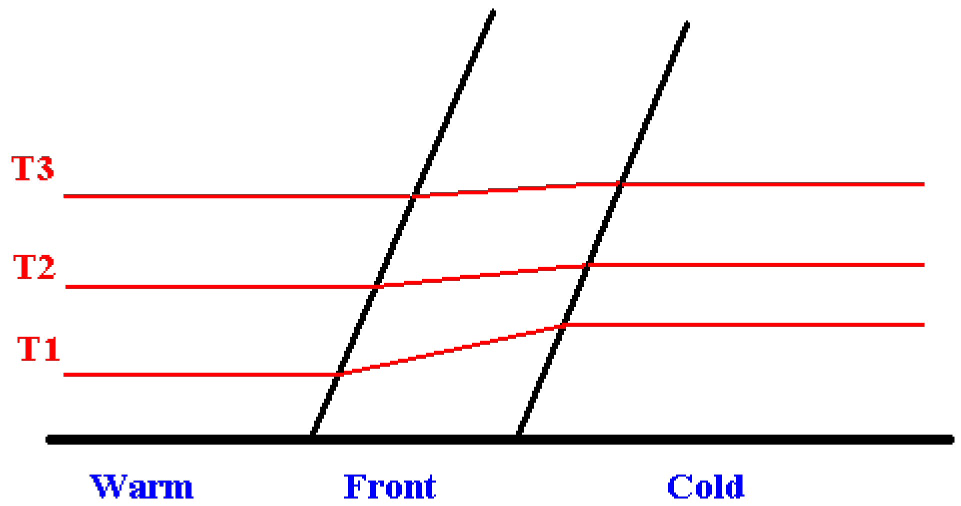
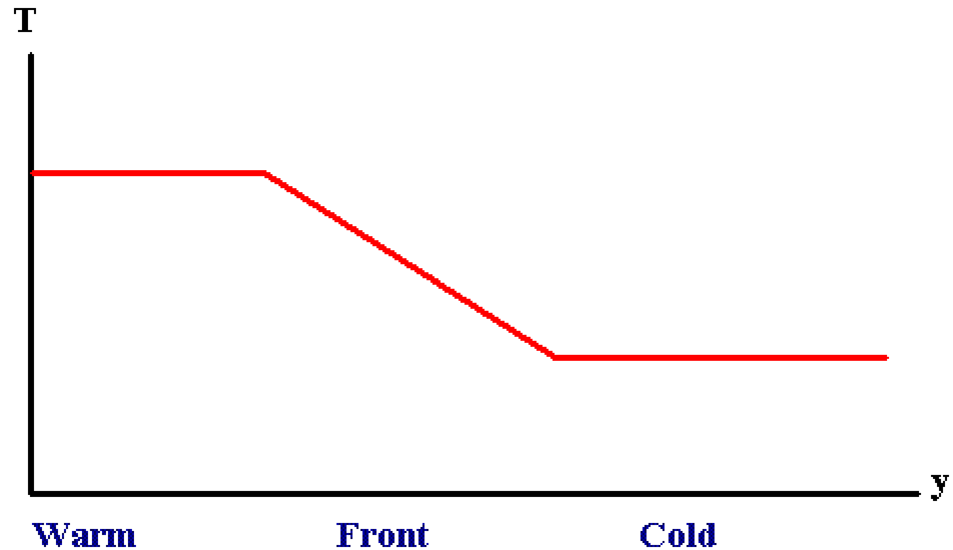
Zero Order Model



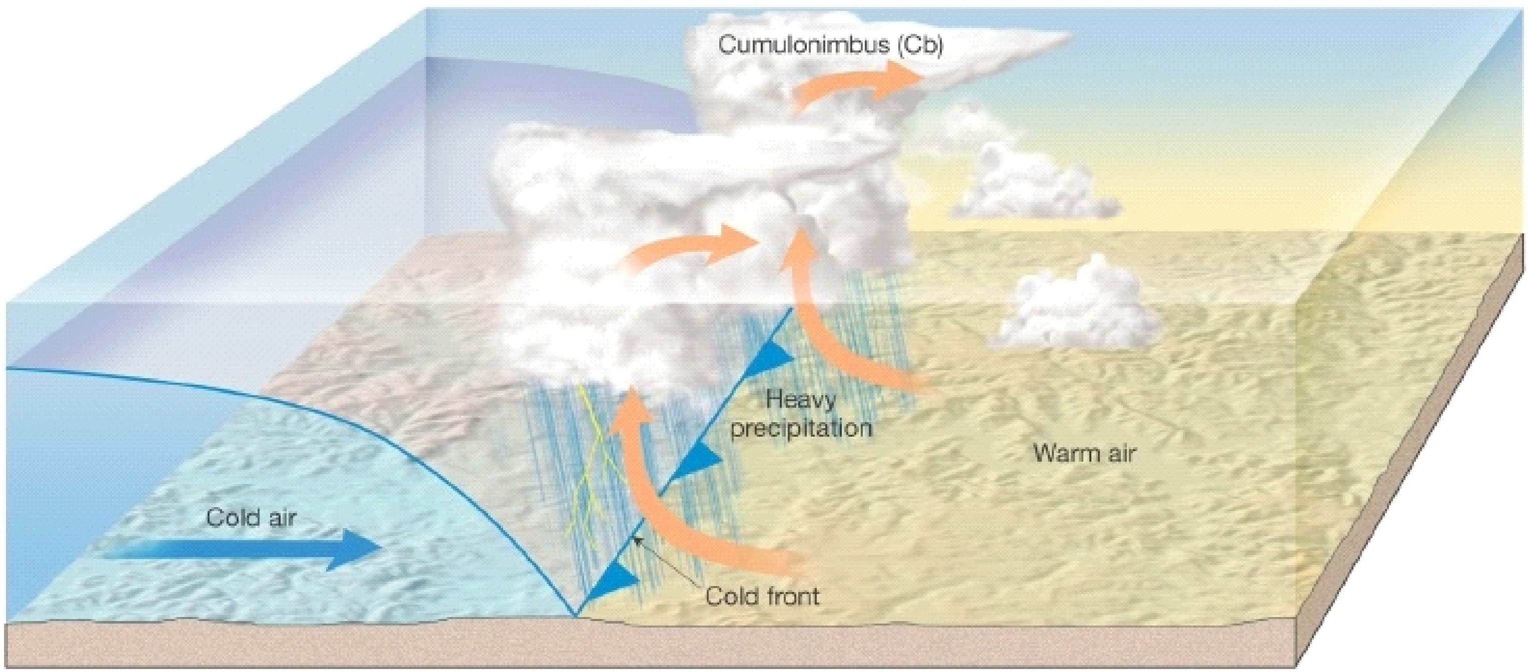
First Order Model

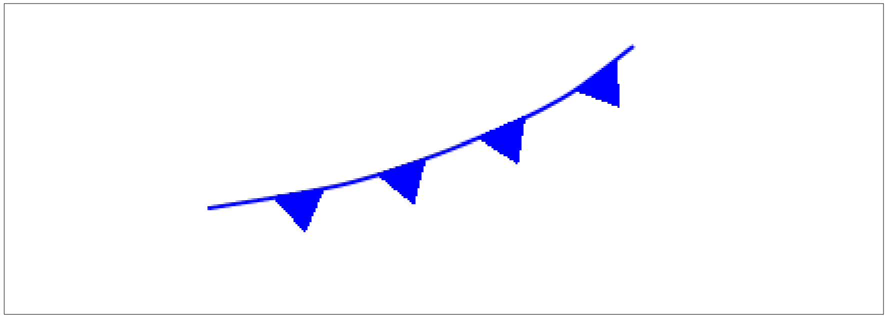




Terminology:

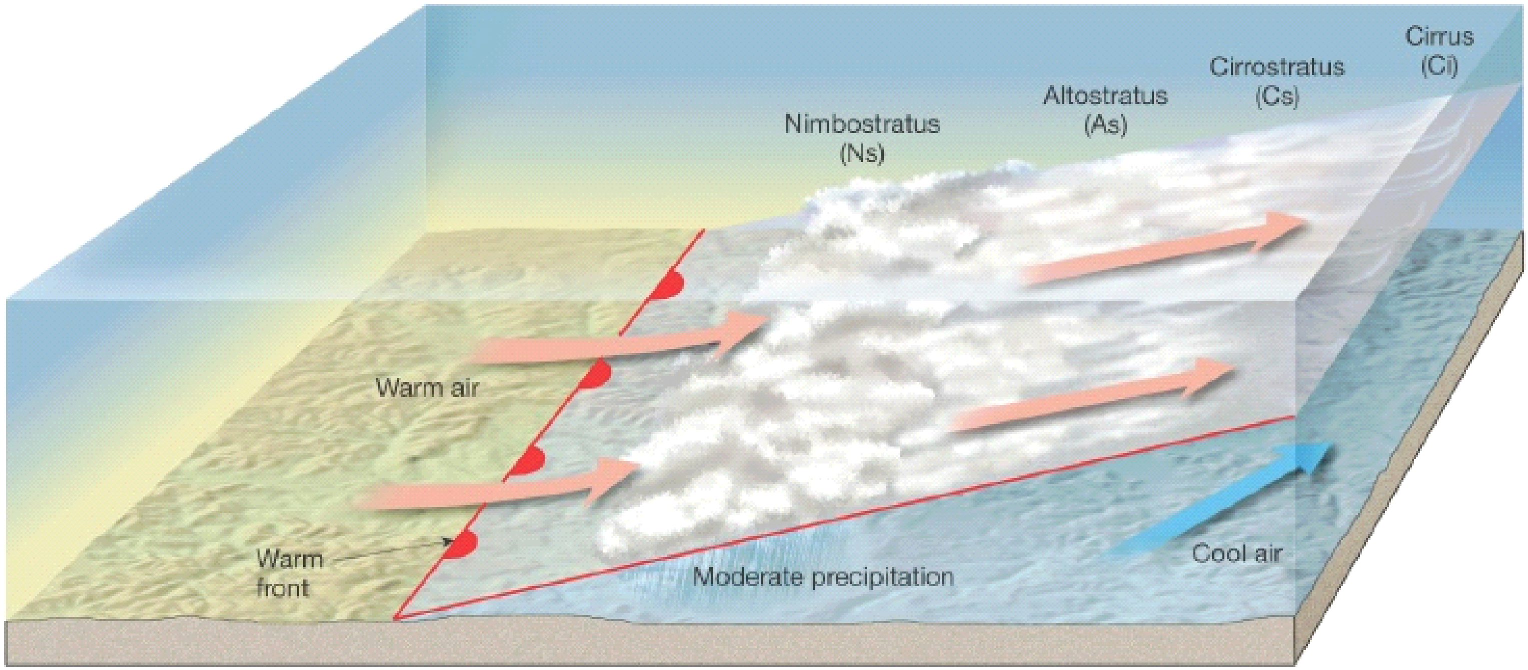
Cold Front – "Cold" Air Mass Replaces "Warm" Air Mass.

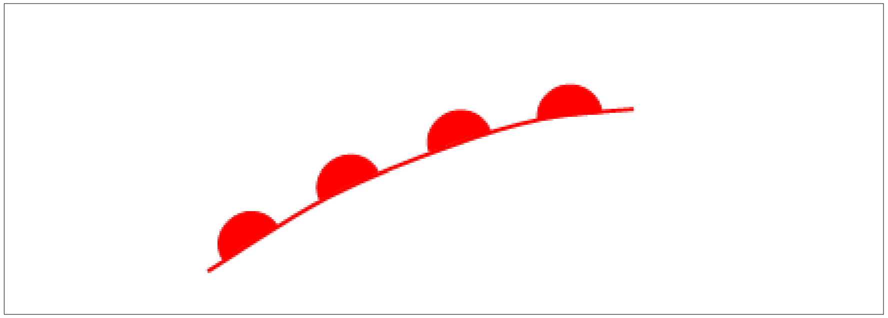




Terminology:

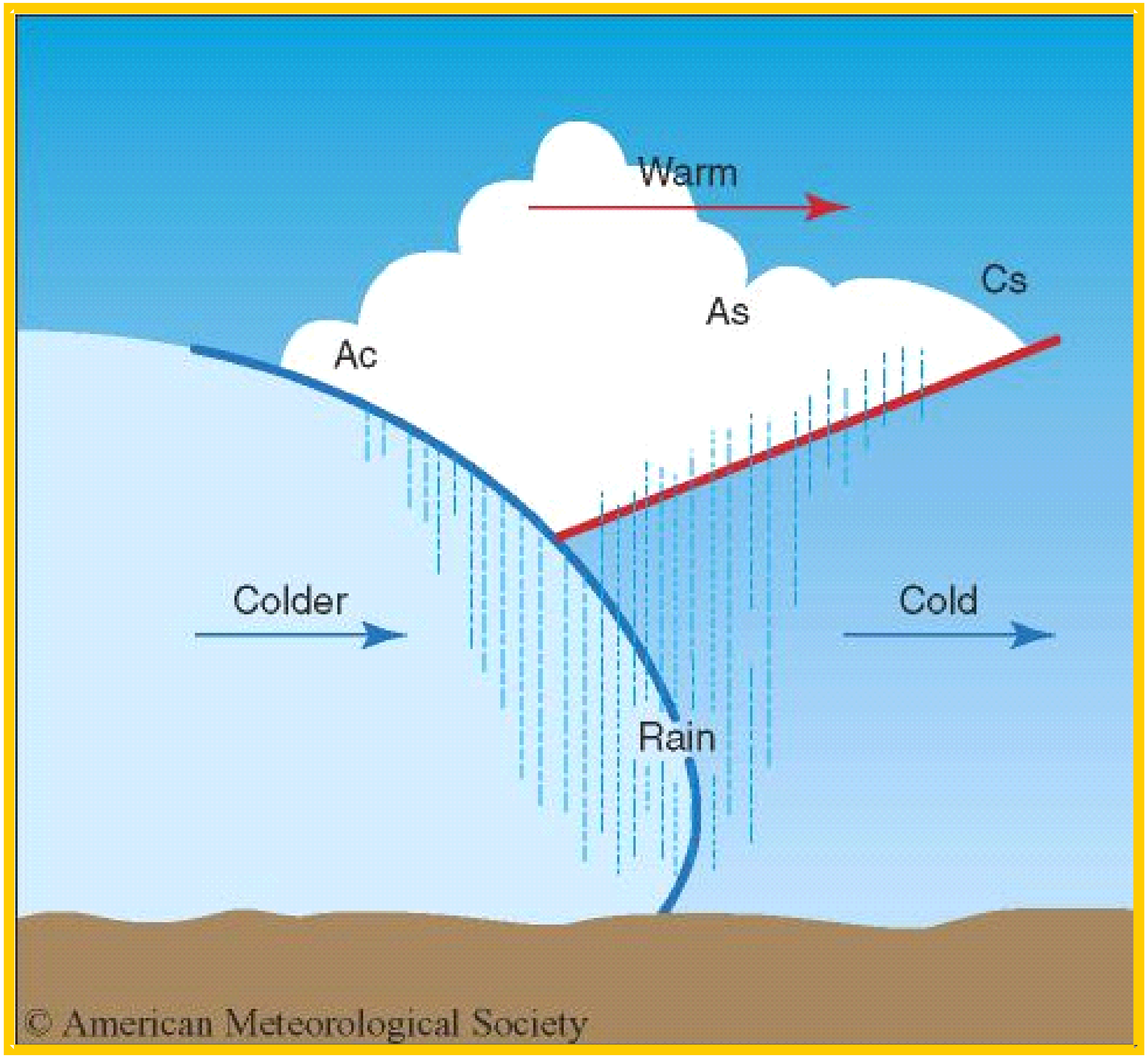
Cold Front – "Cold" Air Mass Replaces "Warm" Air Mass.





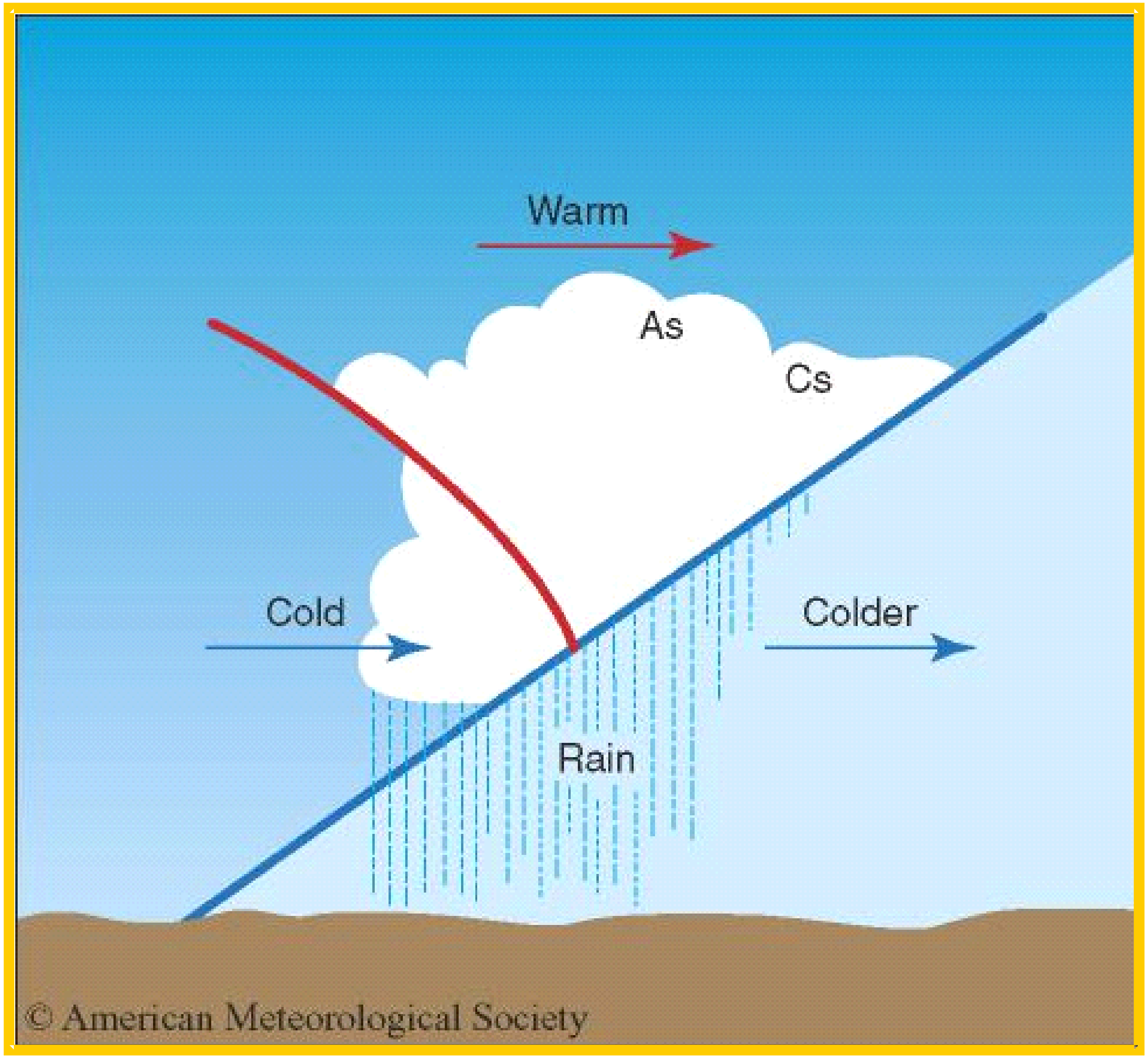
Terminology:

Cold-Type Occlusion – Air behind advancing cold front colder than cool air ahead of warm front

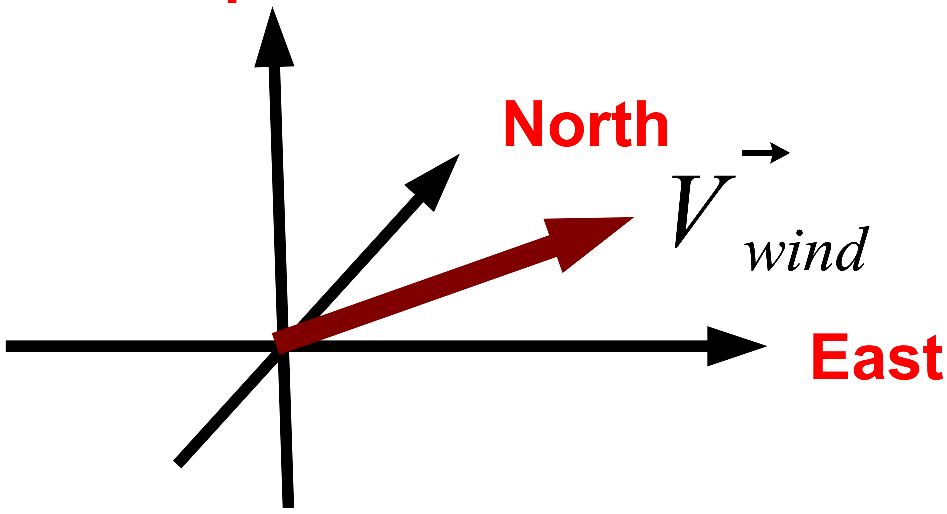


Terminology:

Warm-Type Occlusion – Air behind the advancing cold front is not as cold as the air ahead of the warm front.



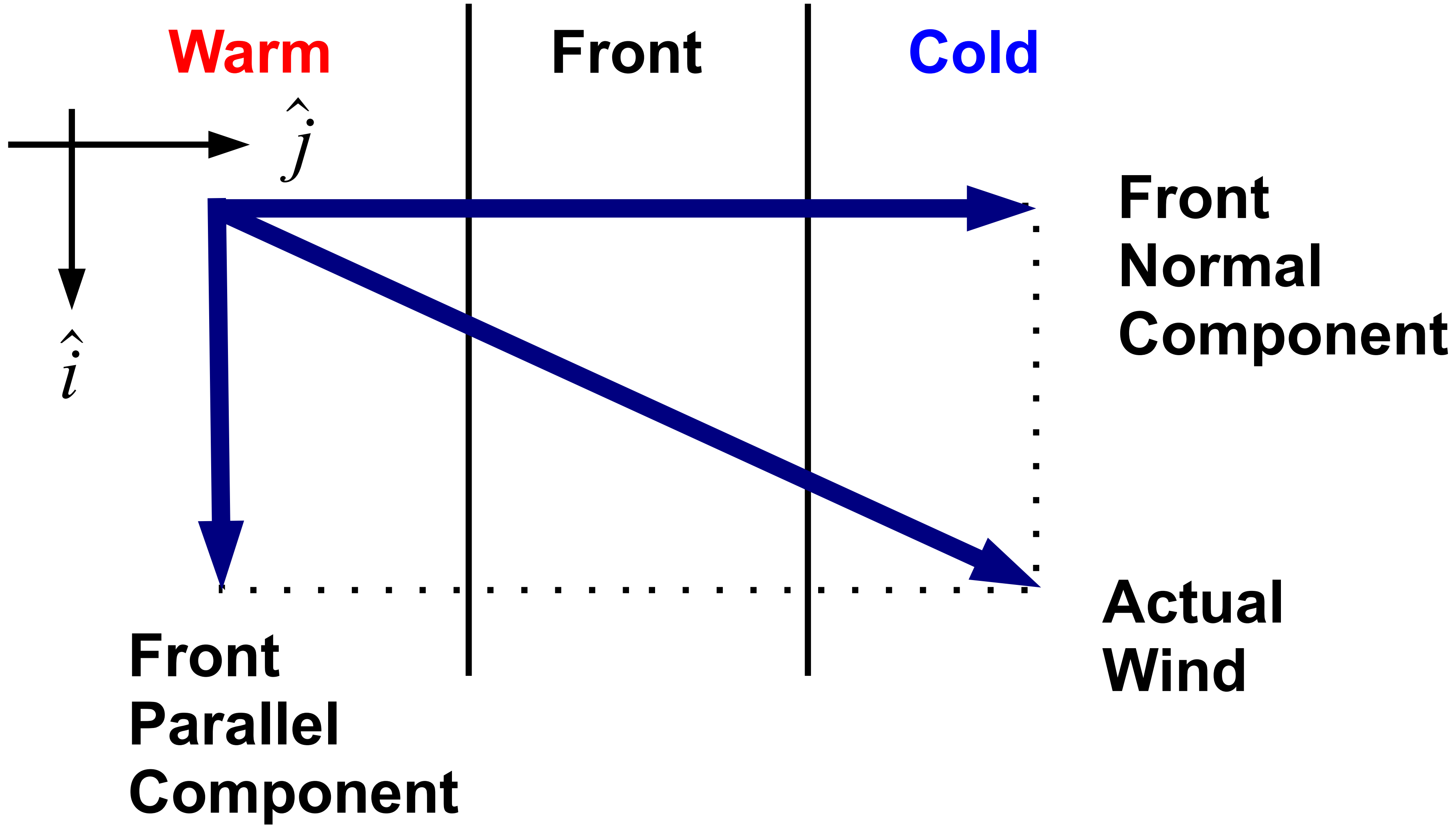
Terminology:



$$\vec{V}_{wind} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$

$$\vec{V}_{wind} = u\,\hat{i} + v\,\hat{j} + \omega\,\hat{k}$$

Terminology:



Terminology:

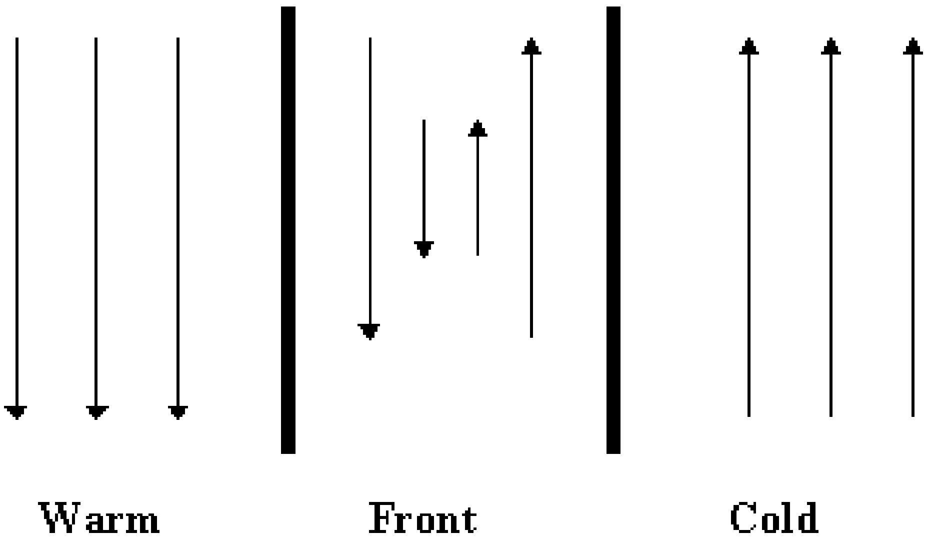
Vorticity :

Defined as the curl of a vector. It is the rotation of air around a vertical axis.

$$\vec{\nabla} X \vec{V} = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial y} \right) X \left(u \hat{i} + v \hat{j} + \omega \hat{k} \right)$$

$$\vec{\nabla} X \vec{V} = \xi \hat{i} + \eta \hat{j} + \zeta \hat{k}$$

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \approx - \frac{\partial u}{\partial y}$$



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Terminology:

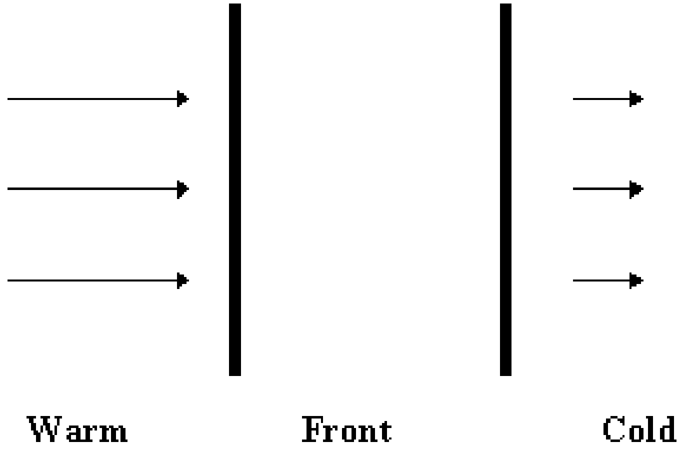
Convergence (Divergence) :

Convergence - The winds result in a net inflow of air. Generally associate this type of convergence with low-pressure areas, where convergence of winds toward the center of the low results in an increase of mass into the low and an upward motion.

Divergence – The winds produce a net flow of air outward. Generally associated with high-pressure cells, where the flow of air is directed outward from the center, causing a downward motion.

$$\vec{\nabla}_H \cdot \vec{V} = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} \right) \cdot \left(u \hat{i} + v \hat{j} \right)$$

$$\vec{\nabla}_H \cdot \vec{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx \frac{\partial v}{\partial y}$$



Terminology:

Virtual Temperature:

Virtual Temperature takes into account the moisture dependence of an air parcel. Virtual Temperature is the temperature that makes the ideal gas equation correct when considering moist air.

$$T_v = (1 + 0.6\,q)\,T$$

$T_v =$ Virtual Temperature

$q =$ Mixing Ratio $=$ Ratio of mass of water vapor to mass of dry air.

$T =$ Temperature

$PV = nRT$ Ideal Gas Law

Terminology:

Potential Temperature:

Potential Temperature is the temperature an air parcel would have if it were bought adiabatically to a reference pressure of 1000mb.

$$\theta \approx T\left(\frac{P_o}{P}\right)^{R/cp}$$

$\theta =$ Potential Temperature
$T =$ Temperature of the Air Parcel
$P_o =$ Reference Pressure $(1000\,mb)$
$P =$ Pressure of the Air Parcel
$R =$ Universal Gas Constant
$C_P =$ Specific Heat at Constant Pressure

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Terminology:

Equivalent Potential Temperature:

Equivalent Potential Temperature is the temperature an air parcel would have if it were bought adiabatically to a reference pressure of 1000mb and all the moisture condensed out and the latent heat used to warm the parcel.

$$\theta_e \approx \theta e^{\left(\frac{L_C q_s}{C_P T}\right)}$$

$\theta_e =$ Equivalent Potential Temperature

$\theta =$ Potential Temperature

$T =$ Temperature of the Air Parcel

$R =$ Universal Gas Constant

$C_P =$ Specific Heat at Constant Pressure

$q_s =$ Saturation Mixing Ratio $= 621.97\left(\frac{e_s}{P - e_s}\right)$

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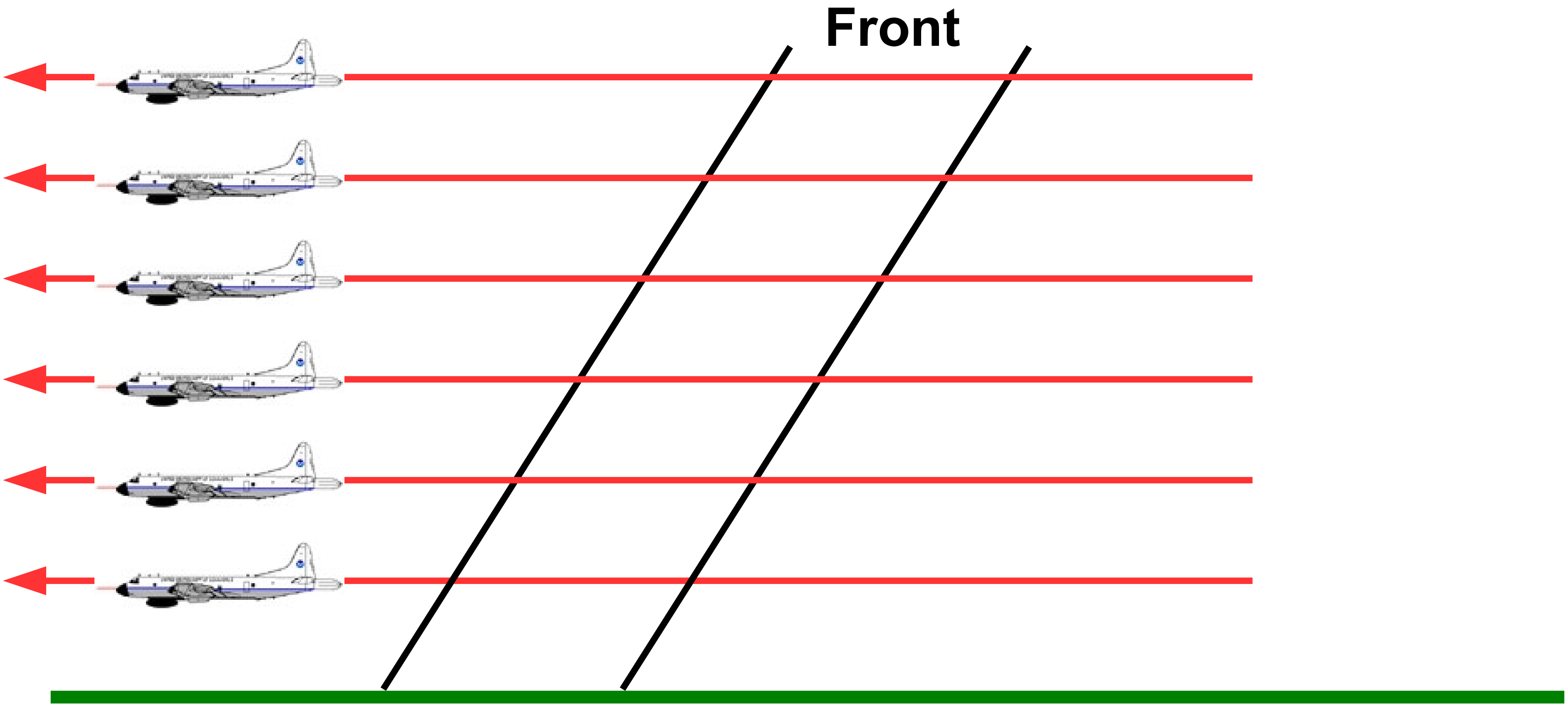
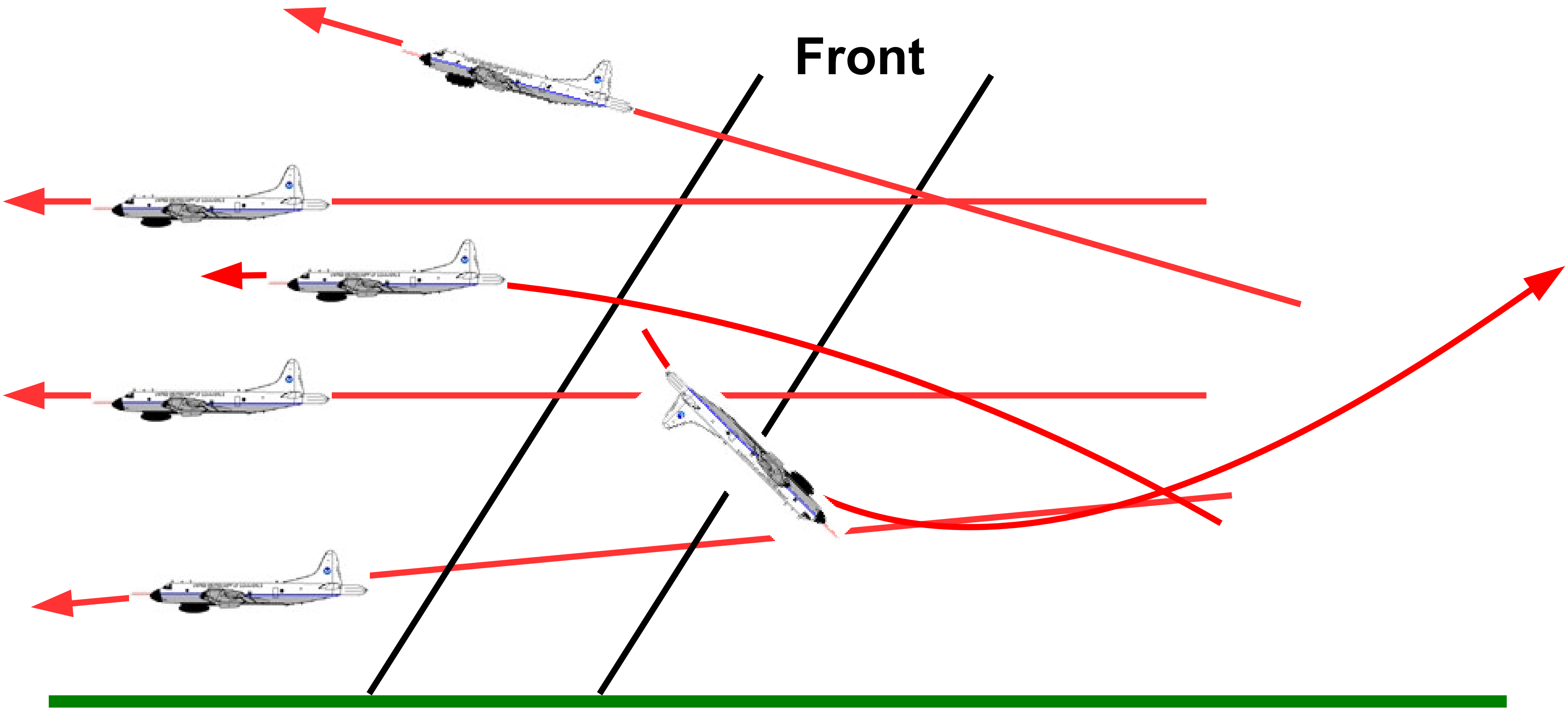
Experiment on Rapidly Intensifying Cyclones over the Atlantic (ERICA)

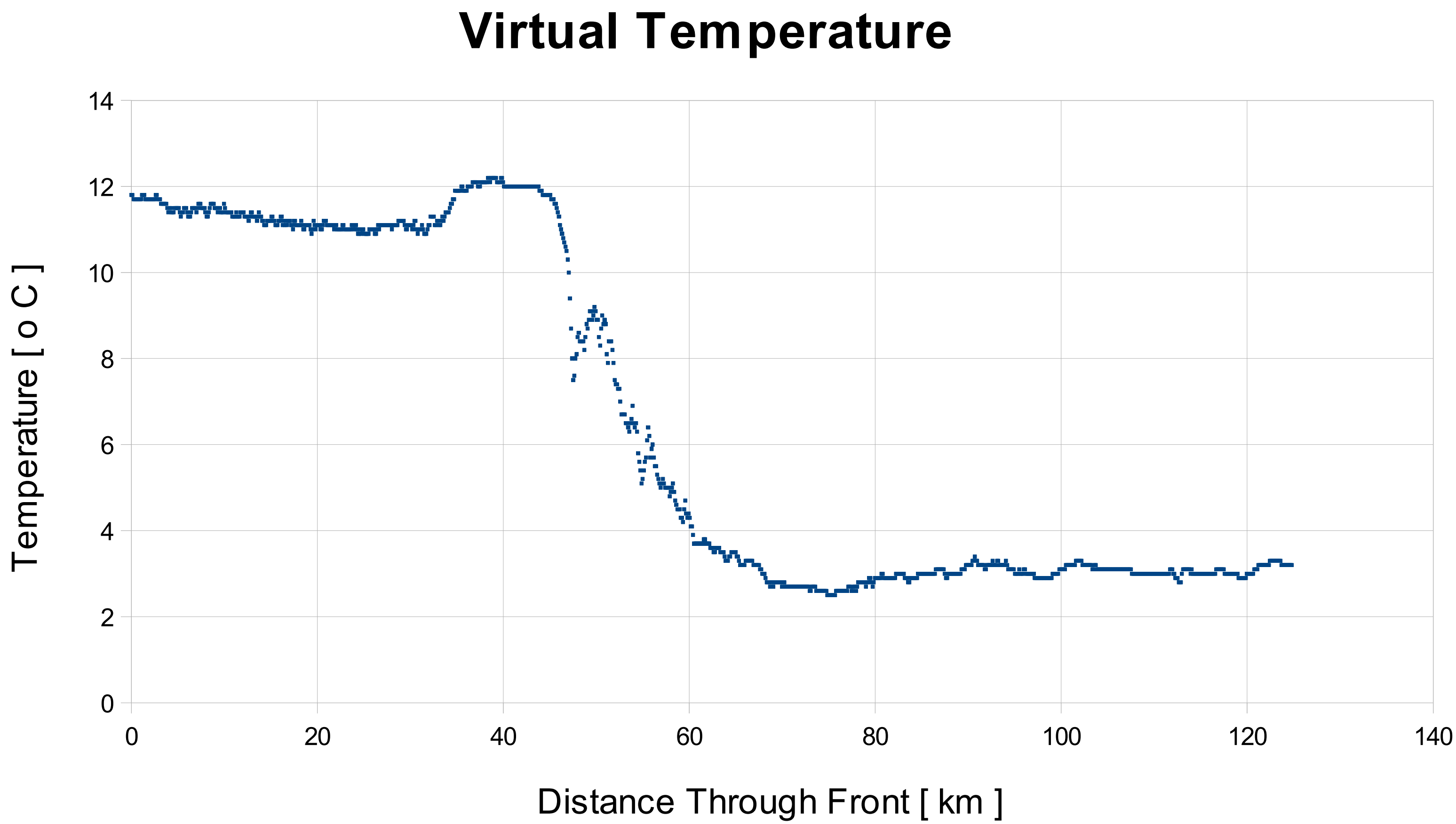
Data collected by aircraft.

This is what I want.

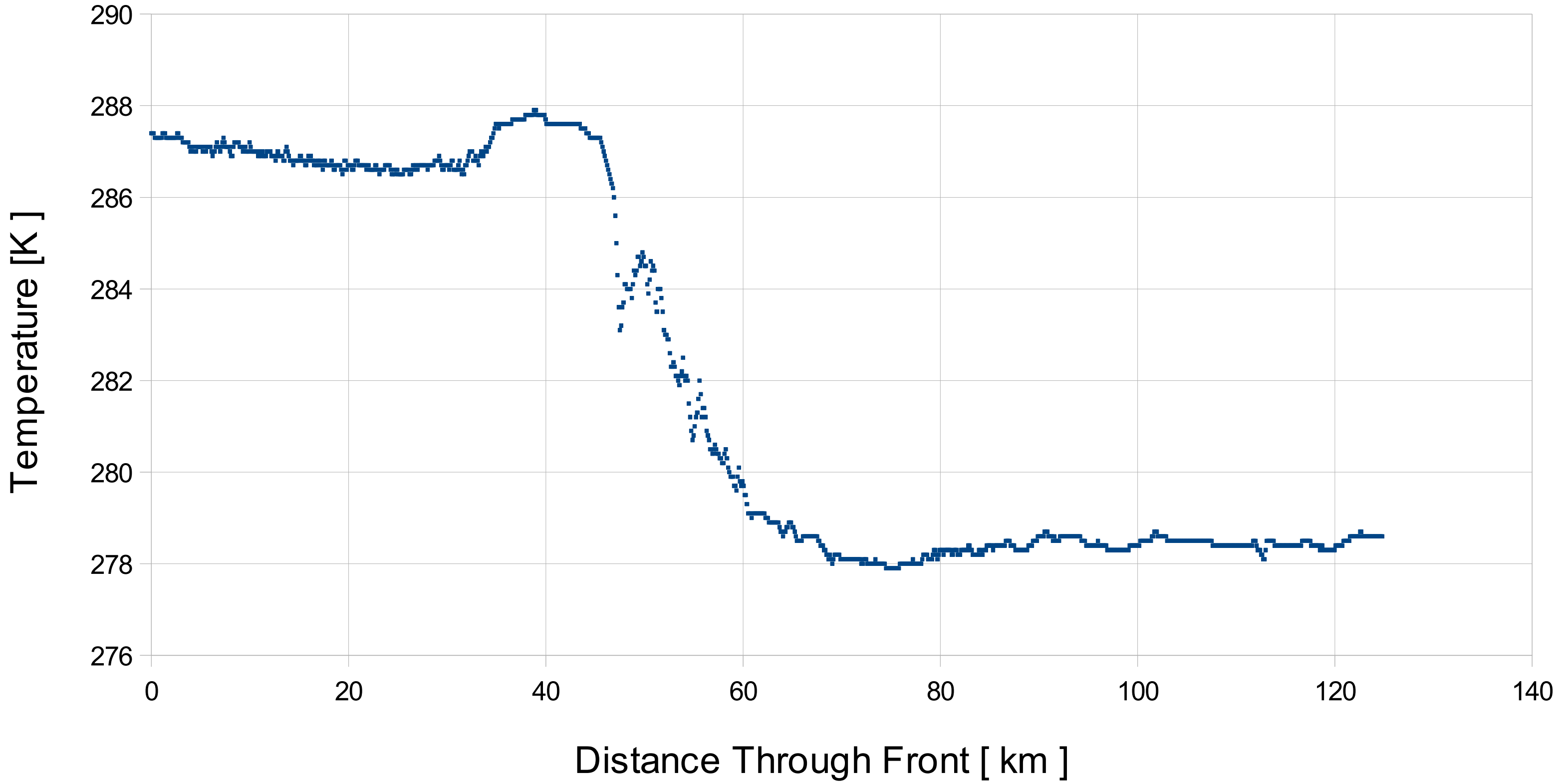


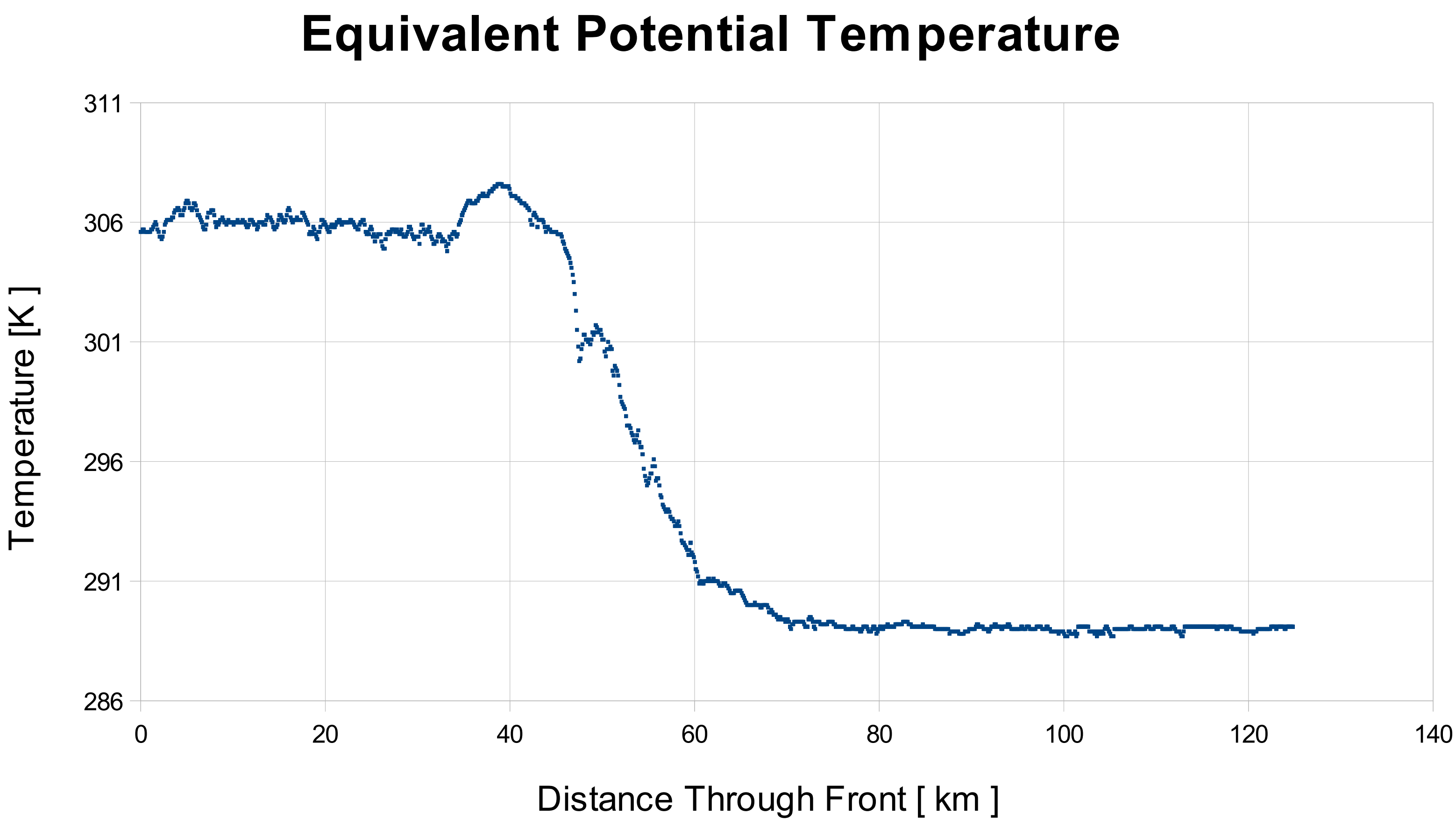
This is what I received.

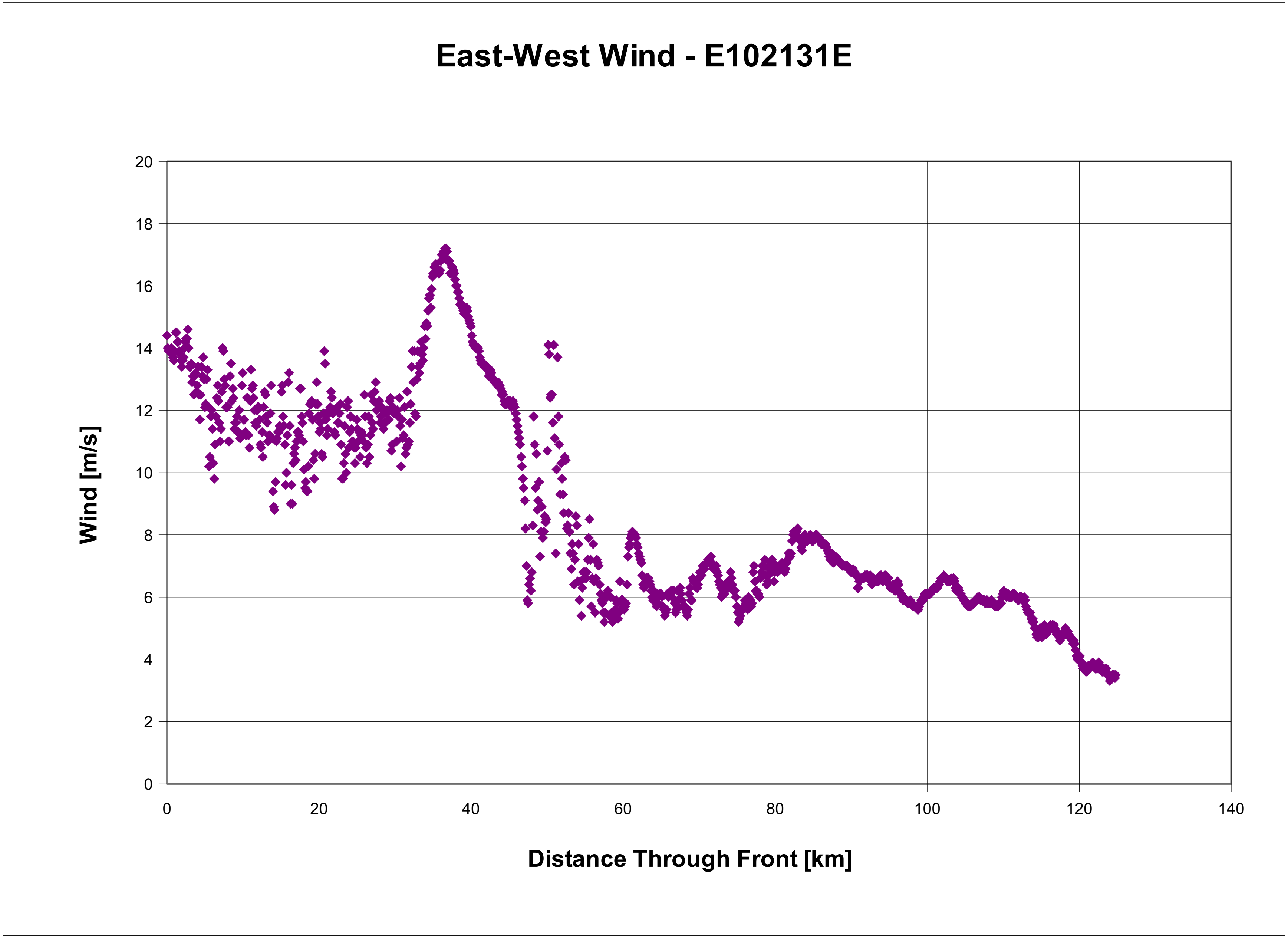


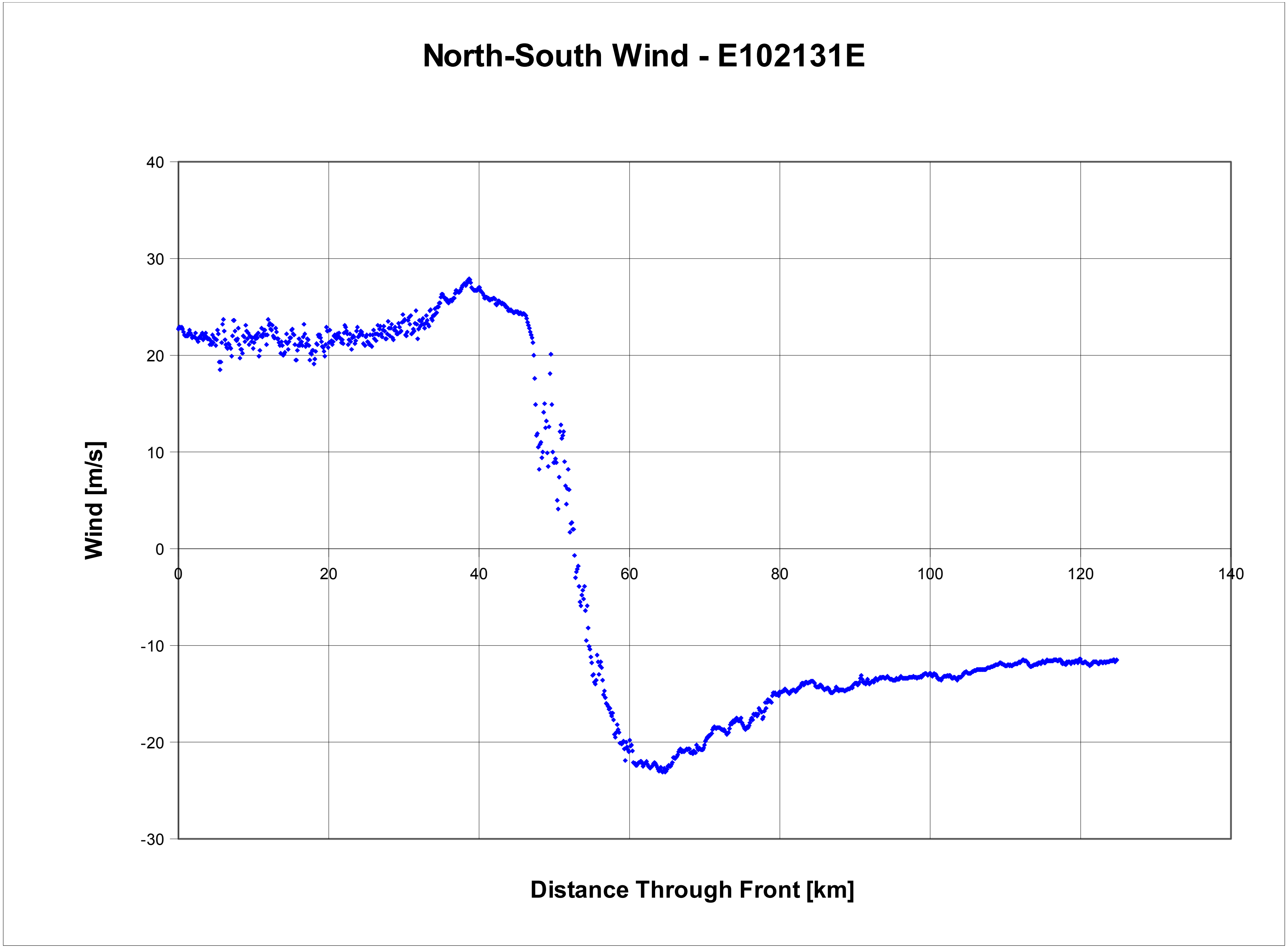

Virtual Temperature
Temperature [o C]
Distance Through Front [km]
0
2
4
6
8
10
12
14
0
20
40
60
80
100
120
140

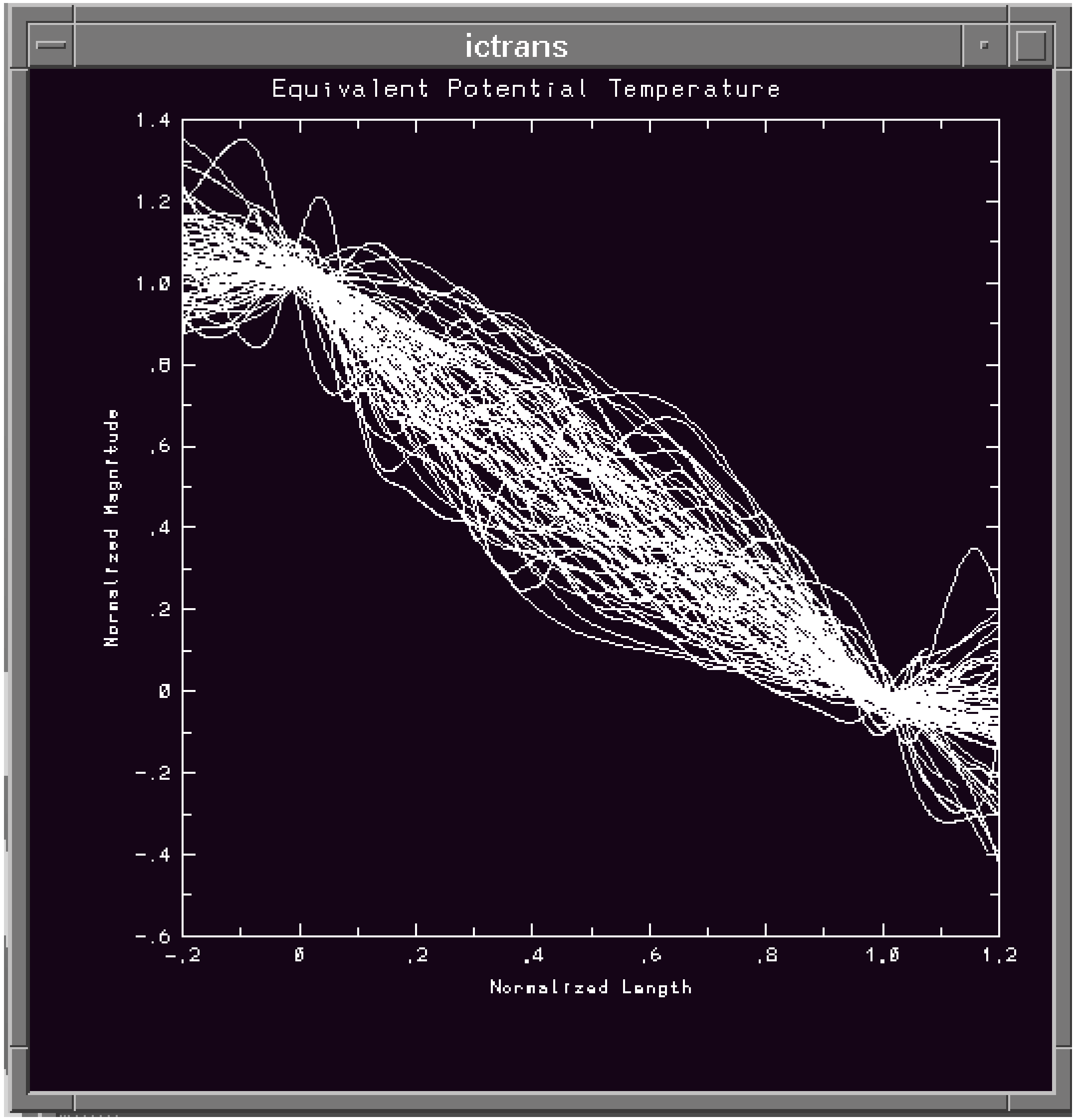
Potential Temperature




Equivalent Potential Temperature
Temperature [K]
311
306
301
296
291
286
0
20
40
60
80
100
120
140
Distance Through Front [km]

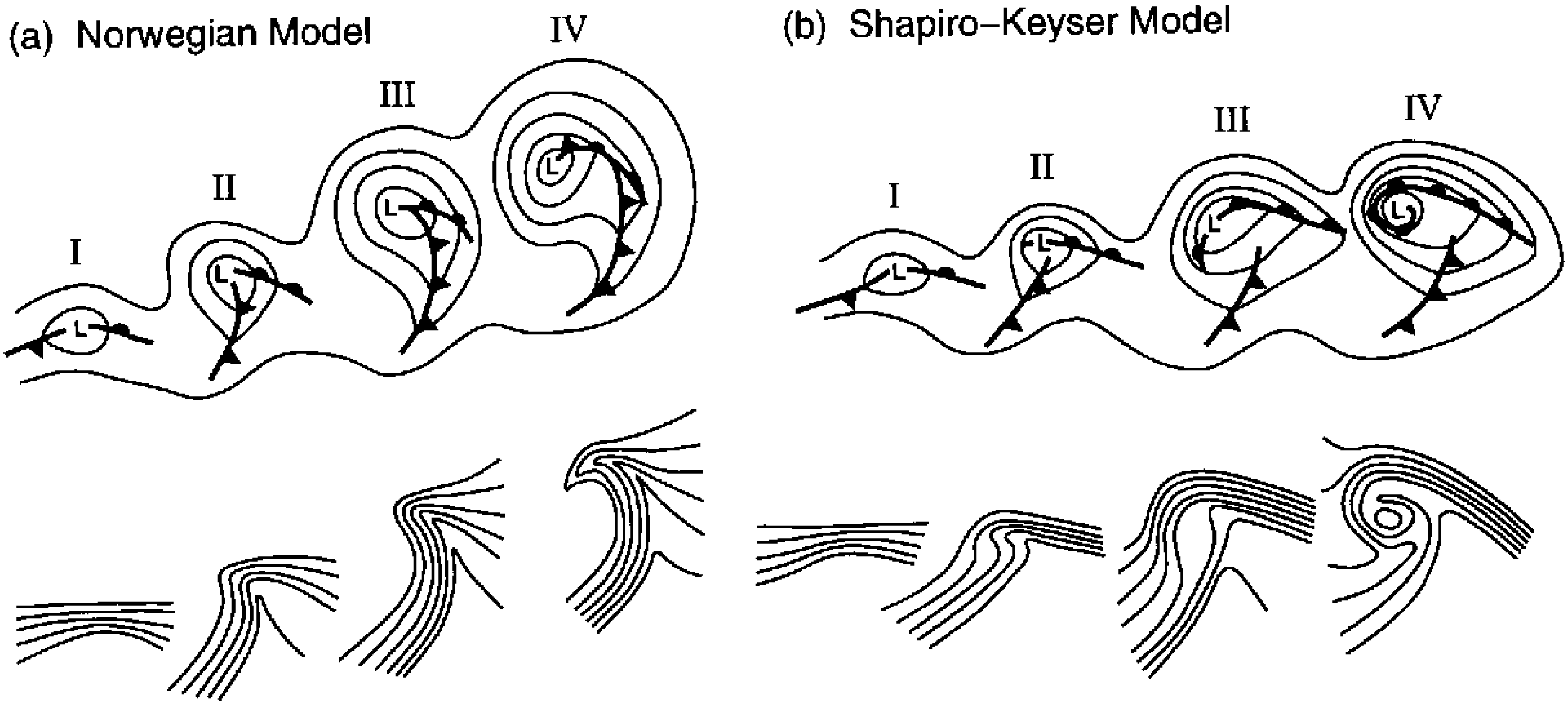

East-West Wind - E102131E
Wind [m/s]
Distance Through Front [km]
0
2
4
6
8
10
12
14
16
18
20
0
20
40
60
80
100
120
140

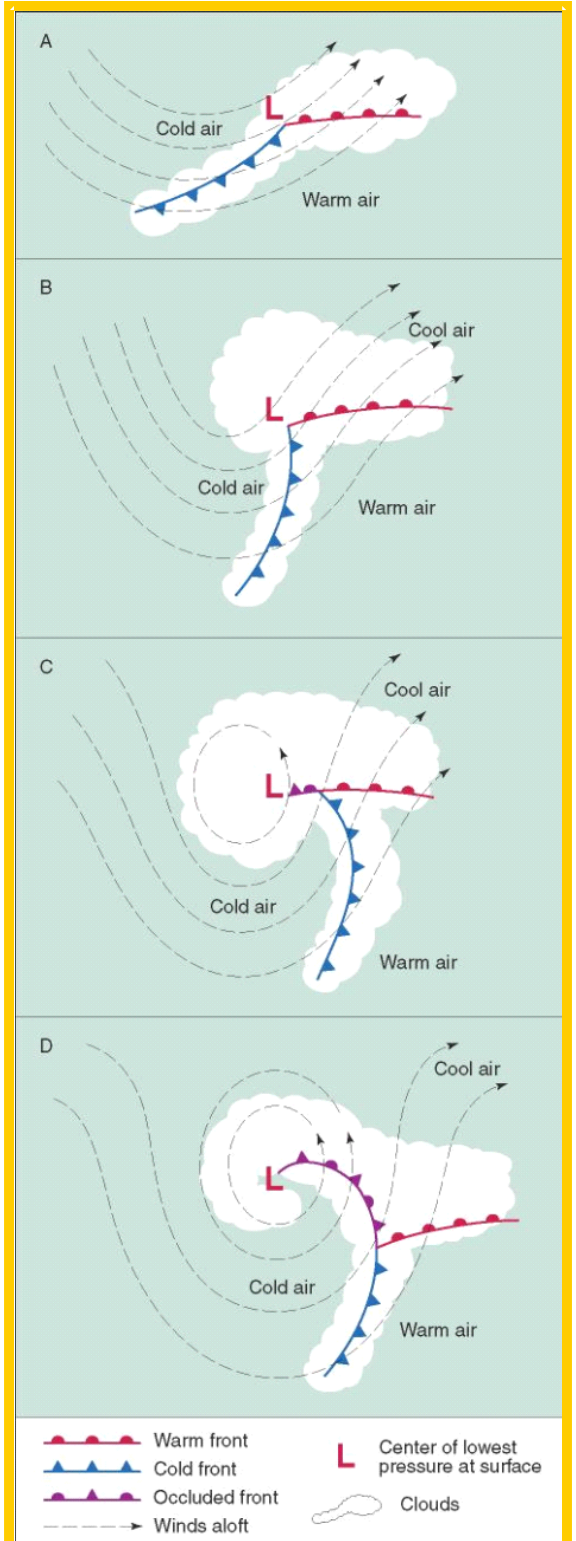

North-South Wind - E102131E
40
30
20
10
0
-10
-20
-30
Wind [m/s]
0
20
40
60
80
100
120
140
Distance Through Front [km]



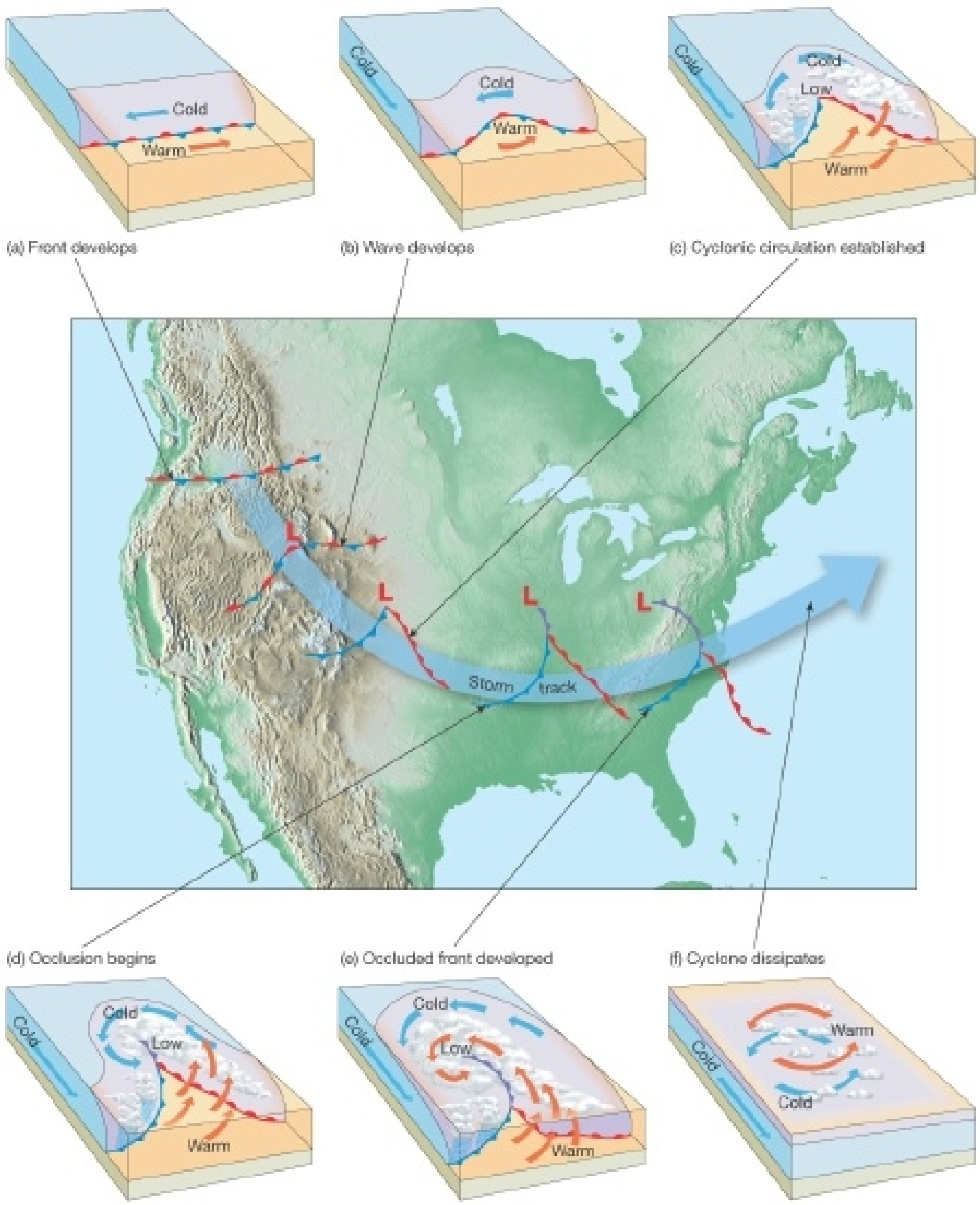
ictrans
Equivalent Potential Temperature
1.4
1.2
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
Normalized Magnitude
-.2
0
.2
.4
.6
.8
1.0
1.2
Normalized Length

Cyclone Models

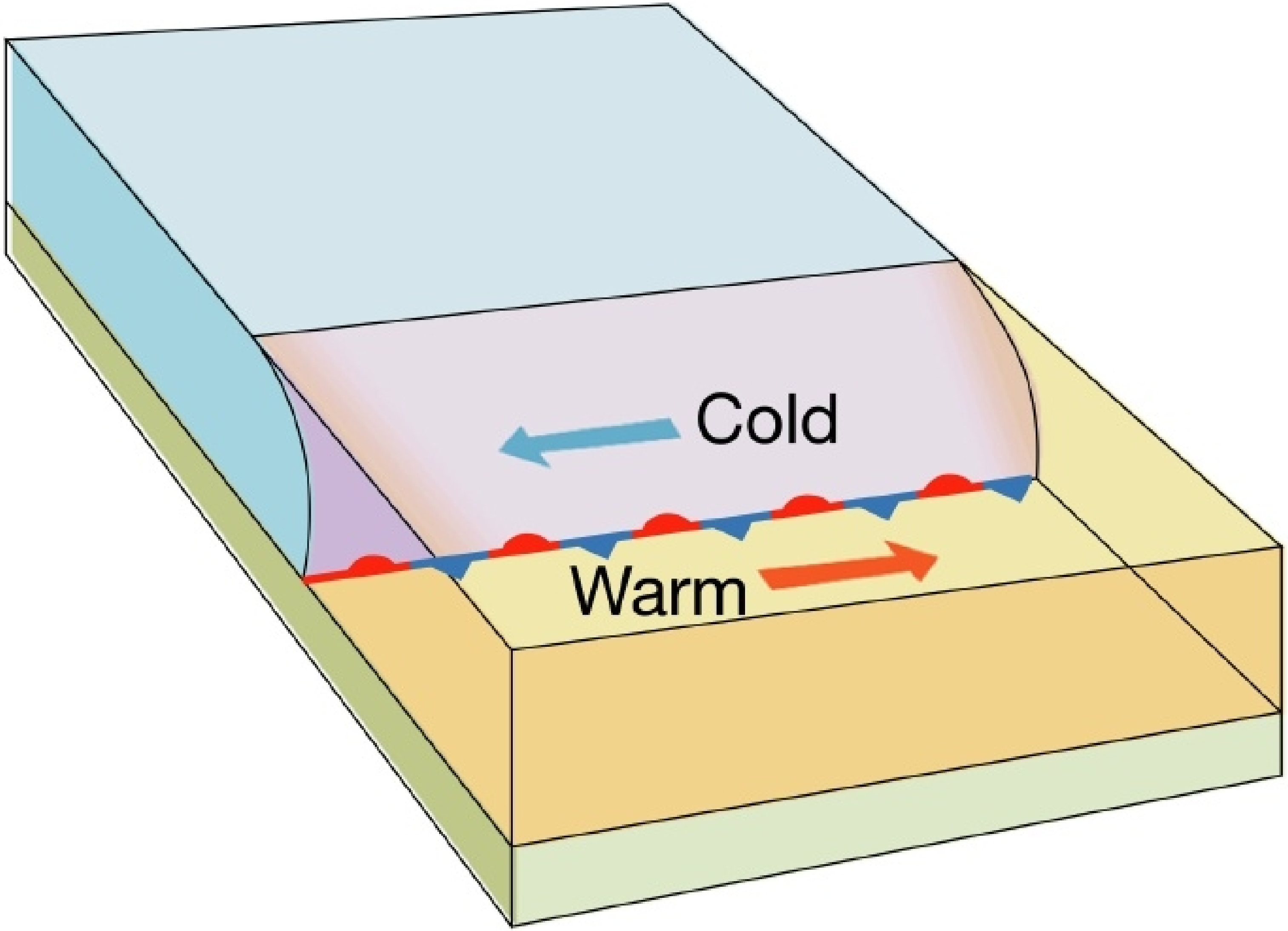



A
Cold air
Warm air
B
Cool air
Cold air
Warm air
C
Cool air
Cold air
Warm air
D
Cool air
Cold air
Warm air
Warm front
Cold front
Occluded front
Winds aloft
L Center of lowest pressure at surface
Clouds

Stages in the Life Cycle of a Mid-latitude Wave Cyclone

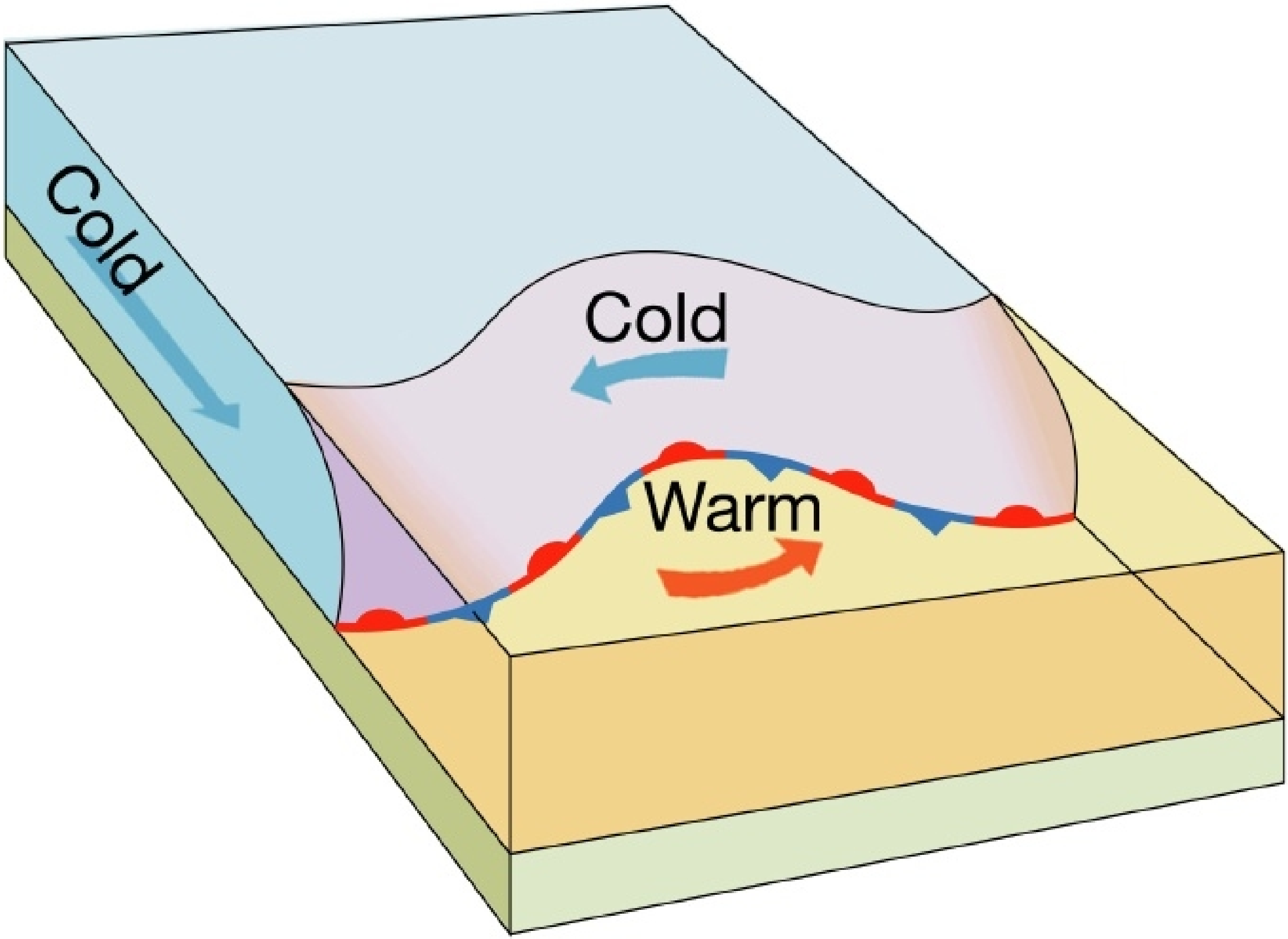


Life Cycle of a Wave Cyclone



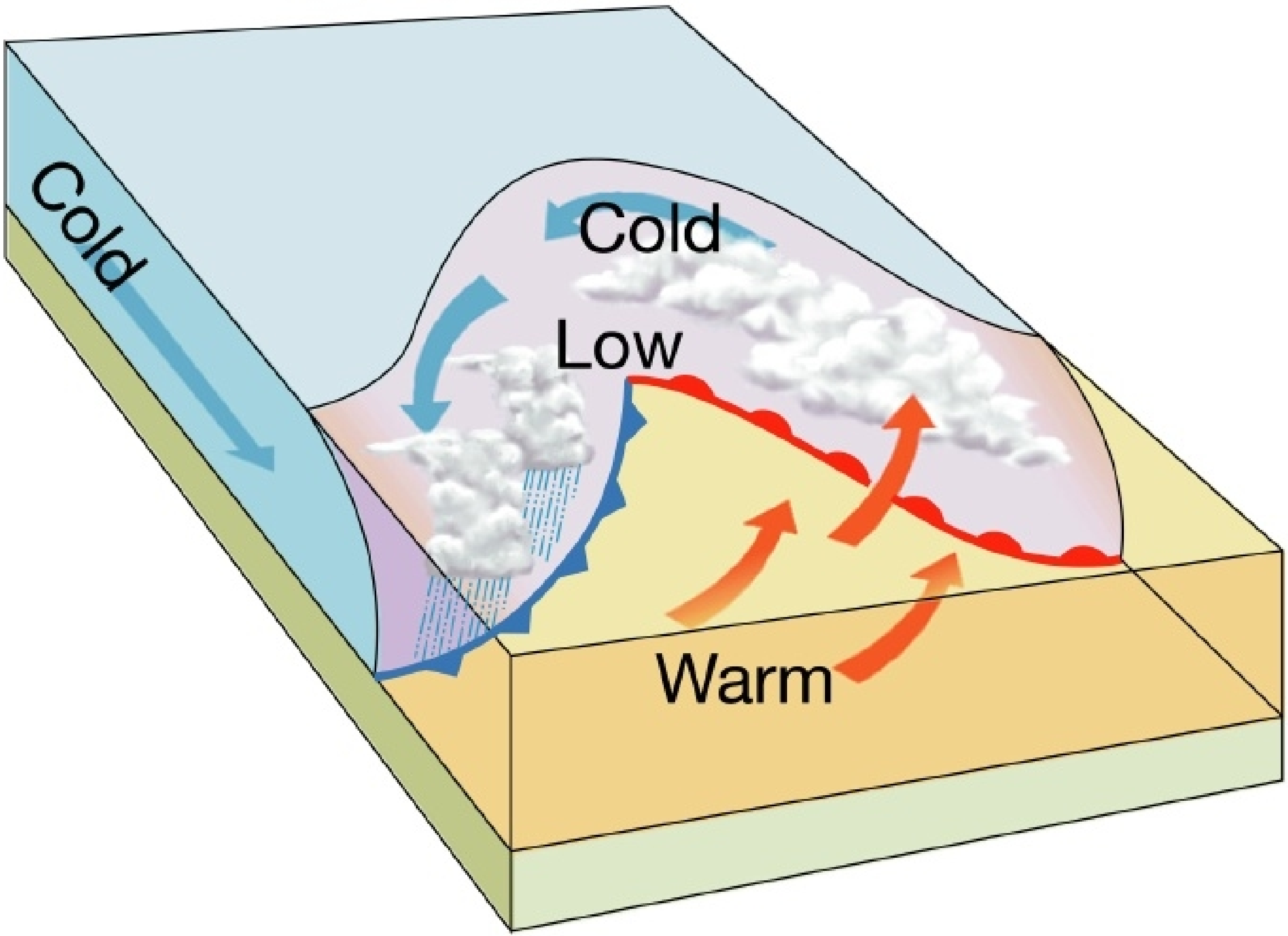
(a) Front develops

Life Cycle of a Wave Cyclone



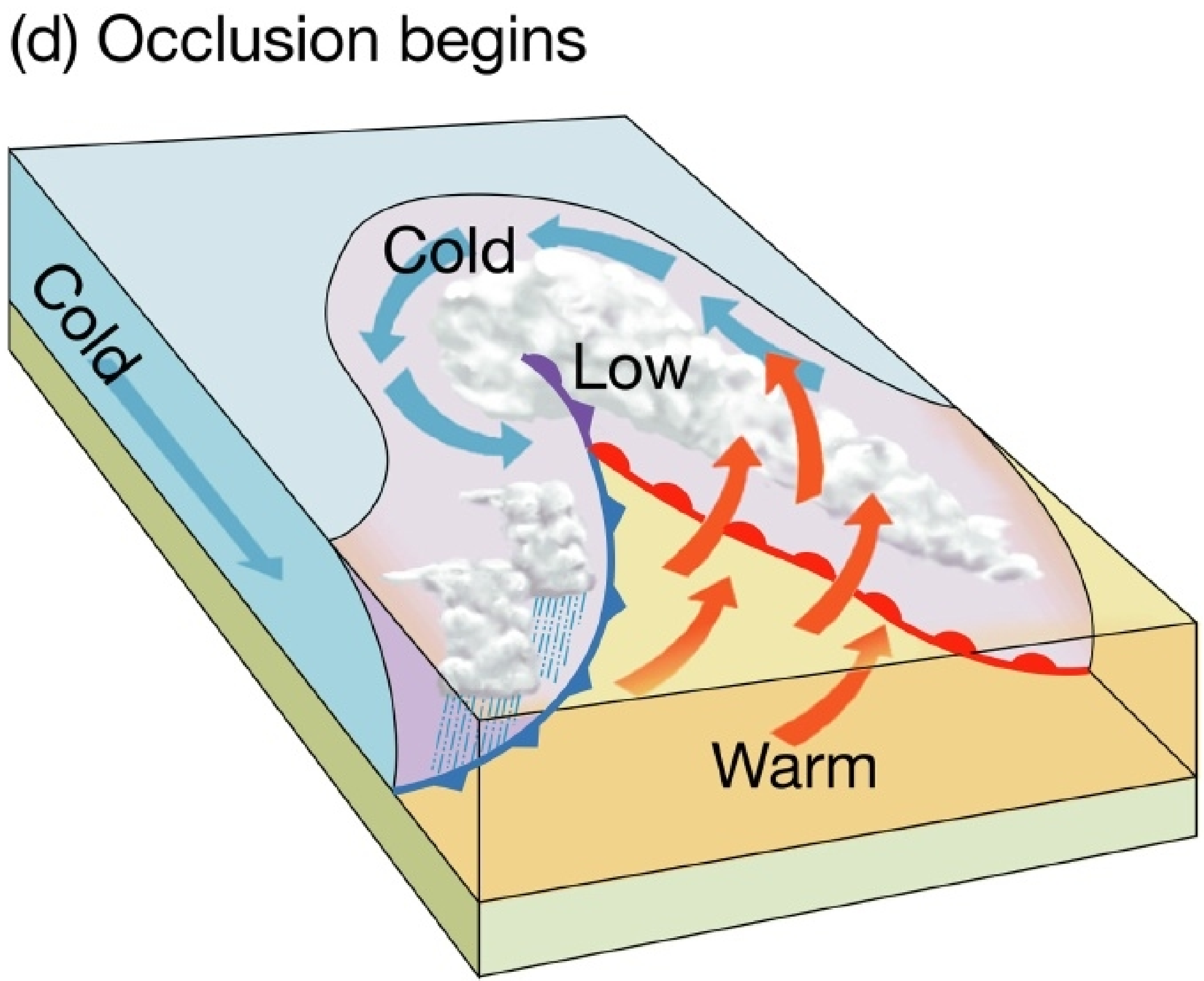
(b) Wave develops

Life Cycle of a Wave Cyclone



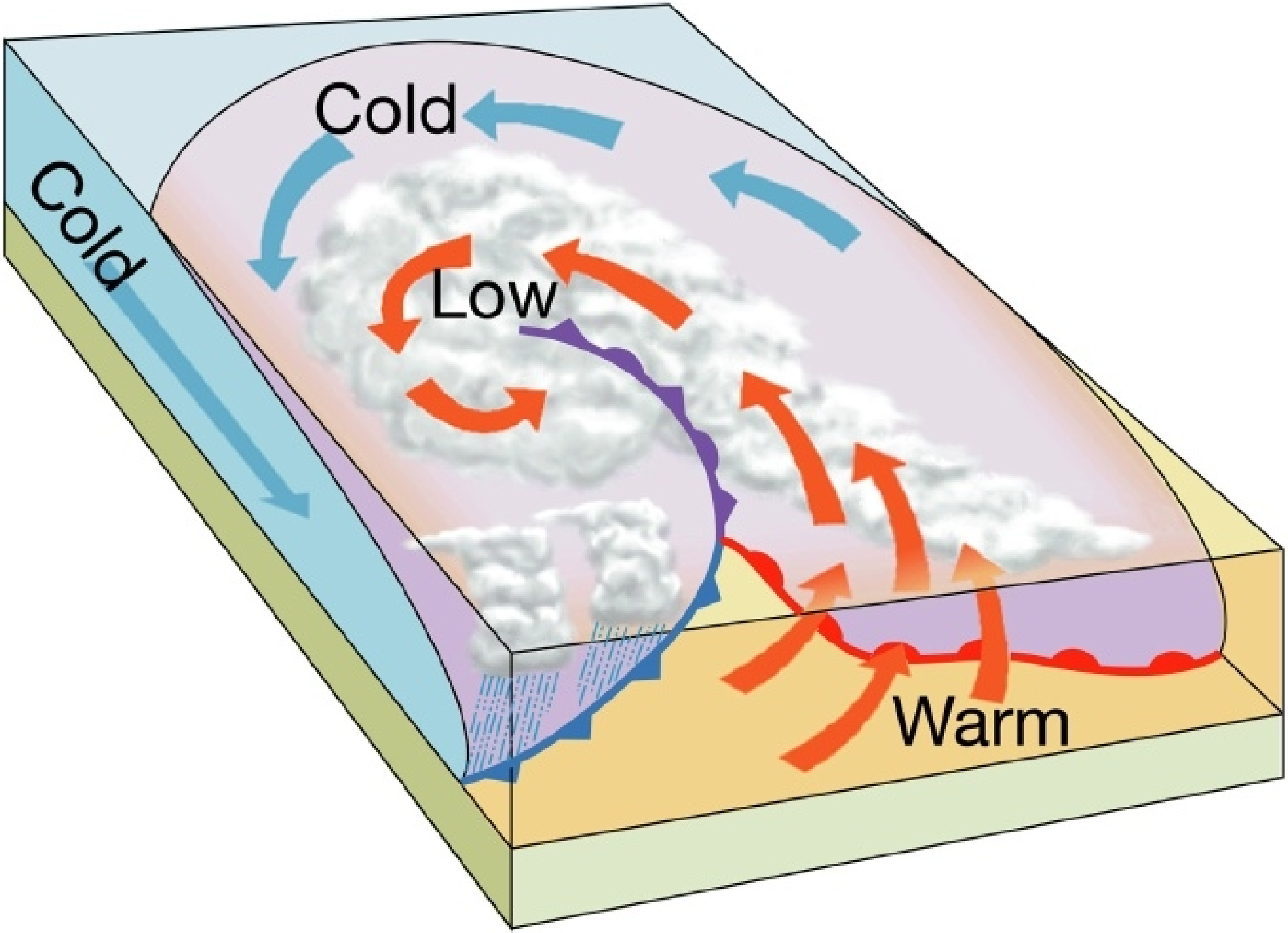
(c) Cyclonic circulation established

Life Cycle of a Wave Cyclone



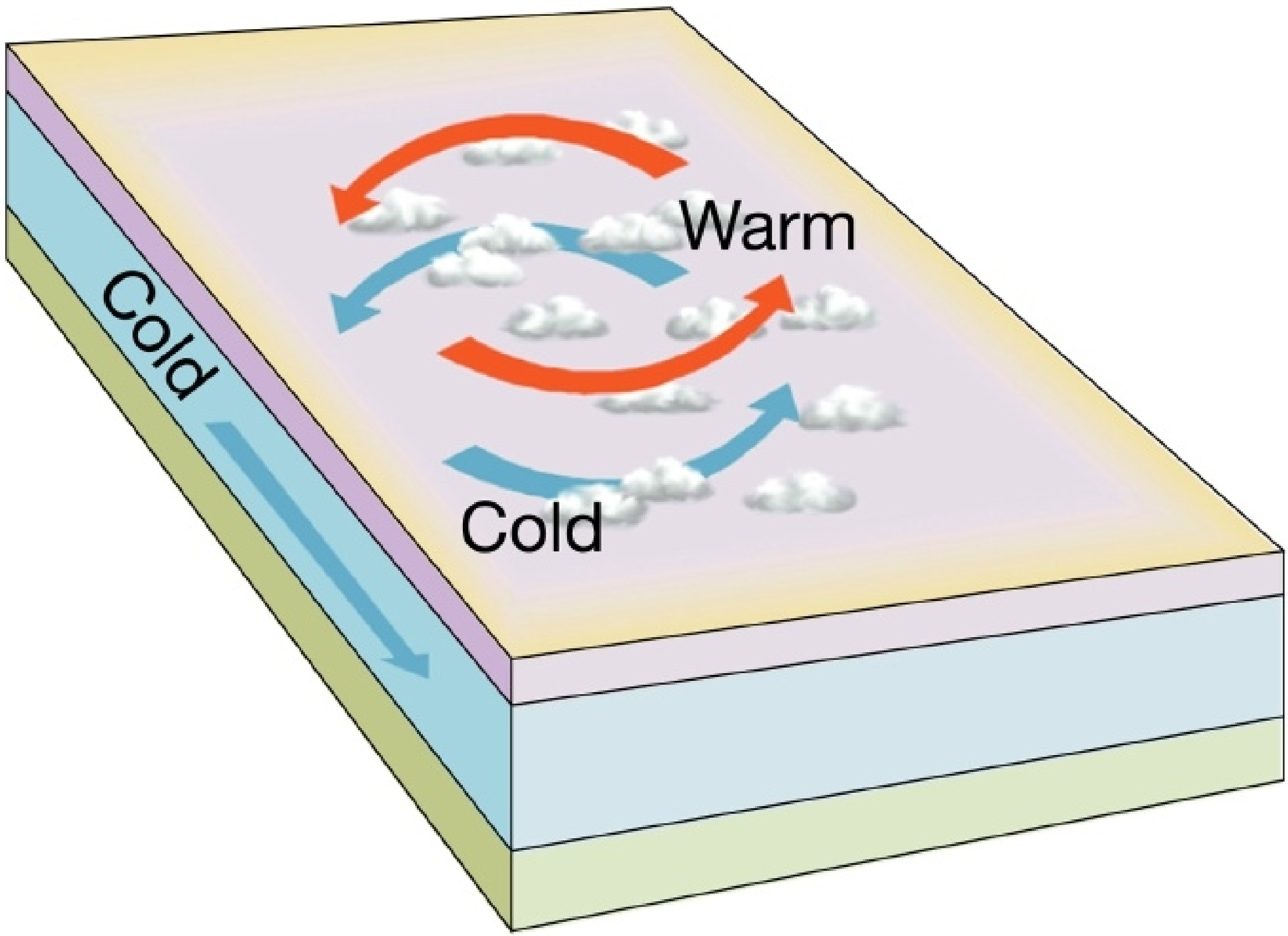
Life Cycle of a Wave Cyclone

(e) Occluded front developed

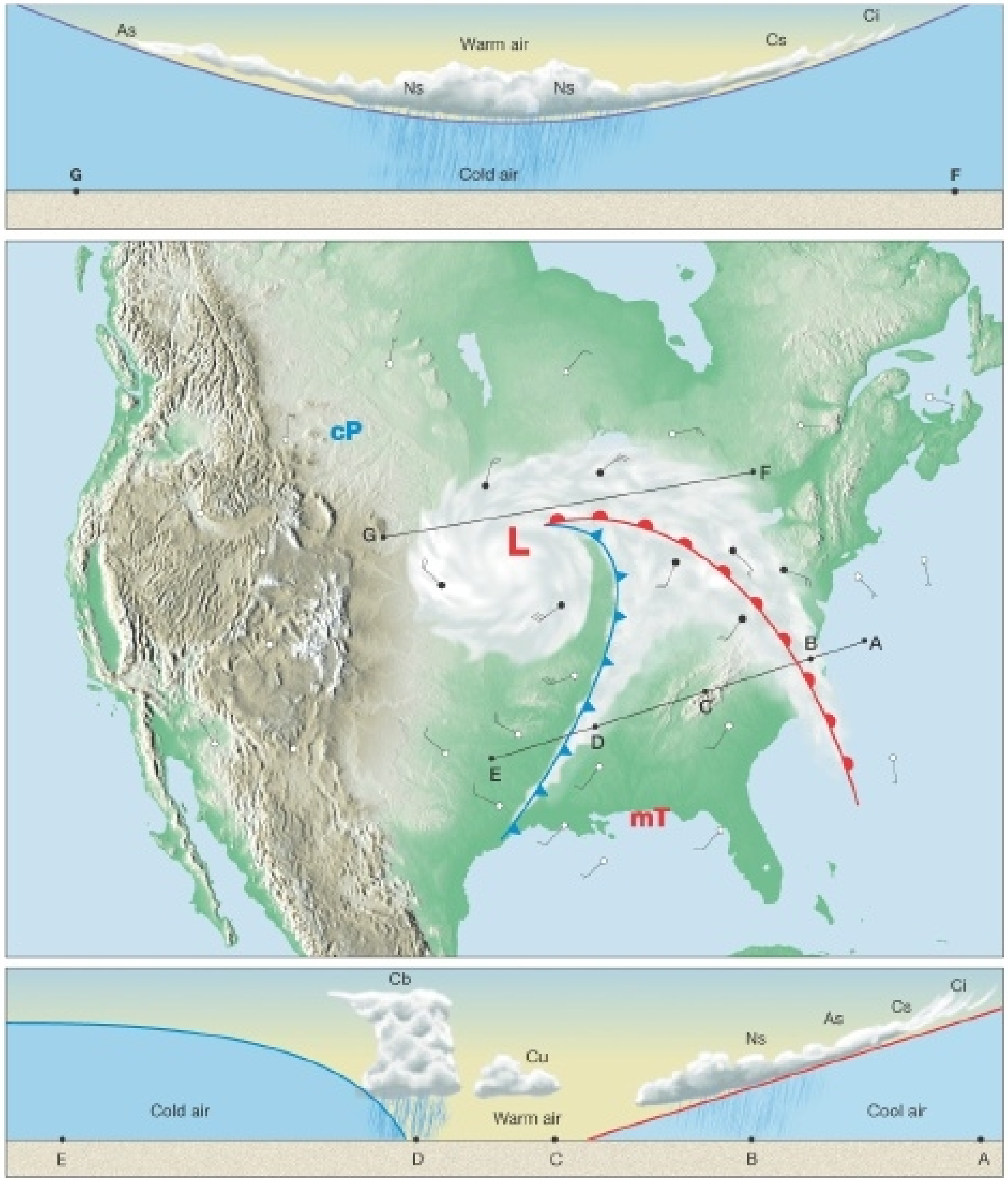


Life Cycle of a Wave Cyclone

(f) Cyclone dissipates



Cloud Patterns – Mature Wave Cyclone



Limited Area Mesoscale Prediction System

(LAMPS 90)

Conservation of Horizontal Momentum

$$\frac{\partial u}{\partial t} = \left(-u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} \right) - \frac{1}{\rho}\frac{\partial P}{\partial y} + fv + \frac{uv}{r_e}\tan\alpha + \frac{\partial u}{\partial t_{conv}} + \frac{\partial u}{\partial t_s}$$

$$\frac{\partial v}{\partial t} = \left(-u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} \right) - \frac{1}{\rho}\frac{\partial P}{\partial y} + fu + \frac{u^2}{r_e}\tan\alpha + \frac{\partial v}{\partial t_{conv}} + \frac{\partial v}{\partial t_s}$$

i . **ii .** **iii .** **iv .** **v .** **vi .** **vii .**

Conservation of Horizontal Momentum

$$\frac{\partial u}{\partial t}=\left(-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}-w\frac{\partial u}{\partial z}\right)-\frac{1}{\rho}\frac{\partial P}{\partial y}+fv+\frac{uv}{r_e}\tan\alpha+\frac{\partial u}{\partial t}_{conv}+\frac{\partial u}{\partial t}_{s}$$

$$\frac{\partial v}{\partial t}=\left(-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}-w\frac{\partial v}{\partial z}\right)-\frac{1}{\rho}\frac{\partial P}{\partial y}+fu+\frac{u^2}{r_e}\tan\alpha+\frac{\partial v}{\partial t}_{conv}+\frac{\partial v}{\partial t}_{s}$$

i . ii . iii . iv . v . vi . vii .

i. local time change
ii. advection
iii. pressure gradient force
iv. Coriolis force
v. curvature term
vi. convective transport
vii. eddy transport

Conservation of Vertical Momentum

$$\frac{\partial w}{\partial t} = \left(-u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} \right) - \frac{1}{\rho}\frac{\partial P}{\partial z} - g + \frac{u^2 + v^2}{r_e} + \frac{\partial w}{\partial t_{conv}} + \frac{\partial w}{\partial t_s}$$

i . ii . iii . iv . v . vi . vii .

i. local time change
ii. advection
iii. pressure gradient force
iv. gravity
v. curvature term
vi. convective transport
vii. eddy transport

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Conservation of Thermal Energy

$$\frac{\partial T}{\partial t} = \left(-u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z} \right) - \frac{1}{c_P} Q$$

i . ii . iii .

$$+\frac{1}{\rho c_P}\left(\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y} - w\frac{\partial P}{\partial z} \right) + \frac{\partial T}{\partial t_{conv}} + \frac{\partial T}{\partial t_{rad}} + \frac{\partial T}{\partial t_s}$$

iv . v . vi . vii .

i. local time change
ii. advection
iii. latent heat (condensation/evaporation)
iv. compressional warming
v. convective transport
vi. radiation (heating/cooling)
vii. eddy transport

Conservation of Mass

$$\frac{\partial \rho}{\partial t} = \left(-u\frac{\partial \rho}{\partial x} - v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z} \right) + \left(-\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} \right)$$

i . **ii .** **iii .**

i. local time change
ii. advection
iii. divergence

Ideal Gas Law

$$PV = nRt$$

$$P = \rho R T$$

Incoming and Outgoing Long and Short Wave Radiation

$$R_N = K\uparrow + K\downarrow + I\downarrow + I\uparrow - G = Q_* - G$$

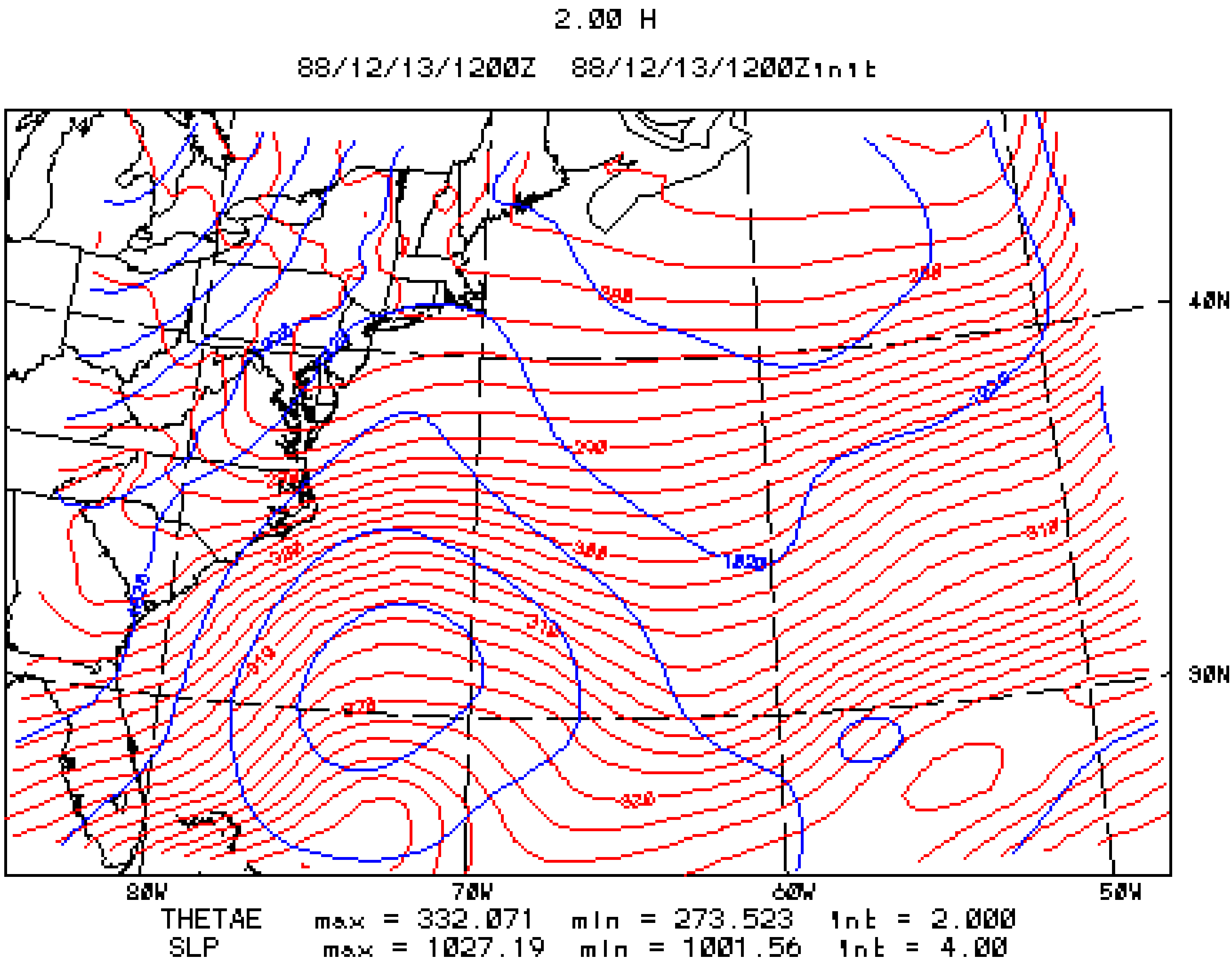
$$K\downarrow = \begin{Bmatrix} ST_k \sin\Psi & daytime, & \sin\Psi > 0 \\ 0 & nighttime, & \sin\Psi \le 0 \end{Bmatrix}$$

$$elevation\ angle \quad \sin\Psi = \sin\phi \sin\delta_s - \cos\phi \cos\delta_s \cos\left[\left(\frac{\pi\, t_{UTC}}{12}\right) + \lambda\right]$$

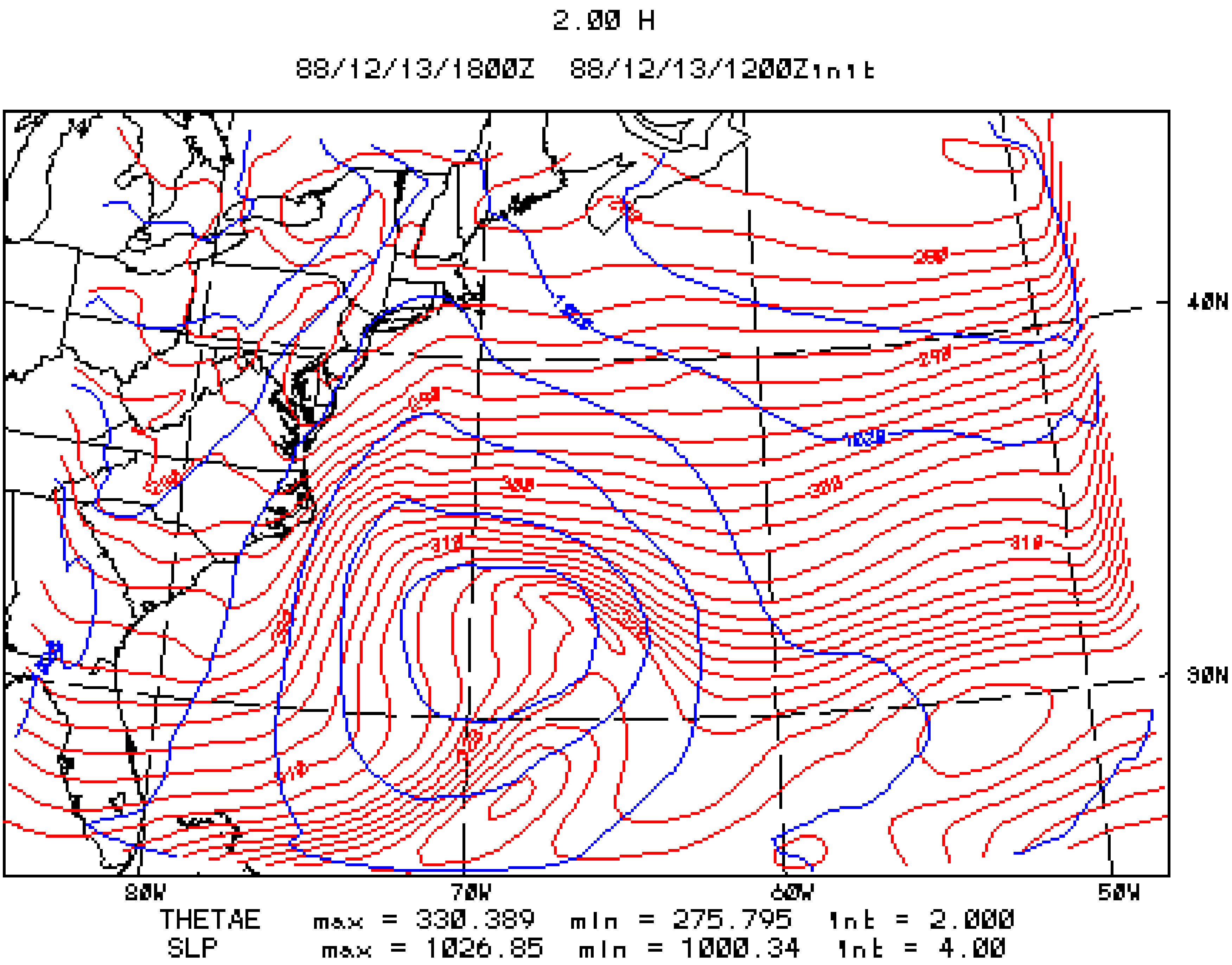
$$\left(\begin{array}{ccc} & \phi \quad latitude & \\ & \lambda \quad \mathrm{log}\,itude & \\ \delta_s = \tan^{-1}\left(\frac{\sin t_l \sin o_b}{\sqrt{1 - x^2}}\right) & solar\ declination & \begin{array}{c} t_l\ celestial\ longitude \\ o_b = .409095 \end{array} \end{array} \right)$$

There are eight variables required to start the model:

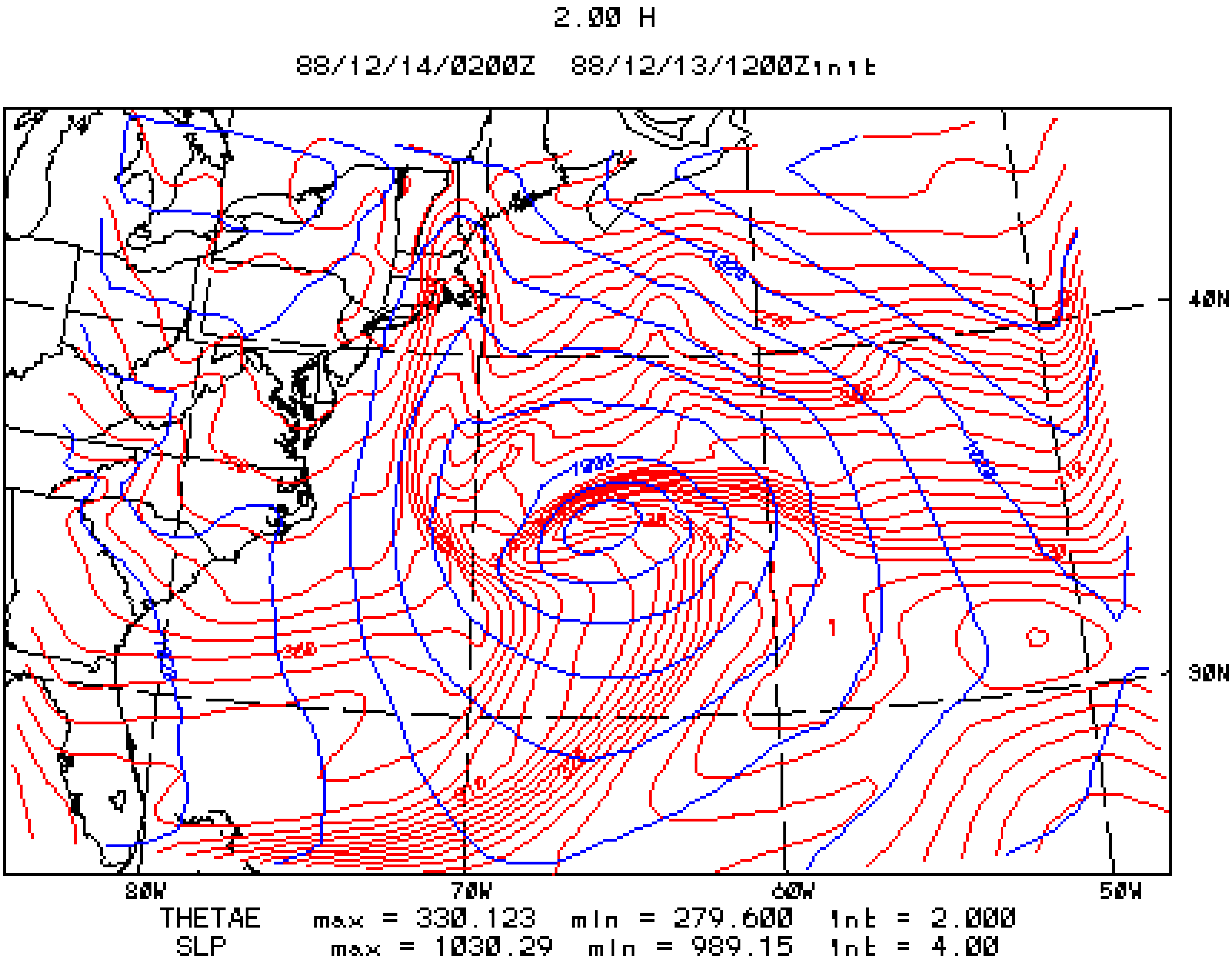
1. East-West Components of the Horizontal Wind
2. North-South Components of the Horizontal Wind
3. Temperature
4. Moisture
5. Pressure
6. Terrain
7. Surface Water Coverage
8. Surface Temperature



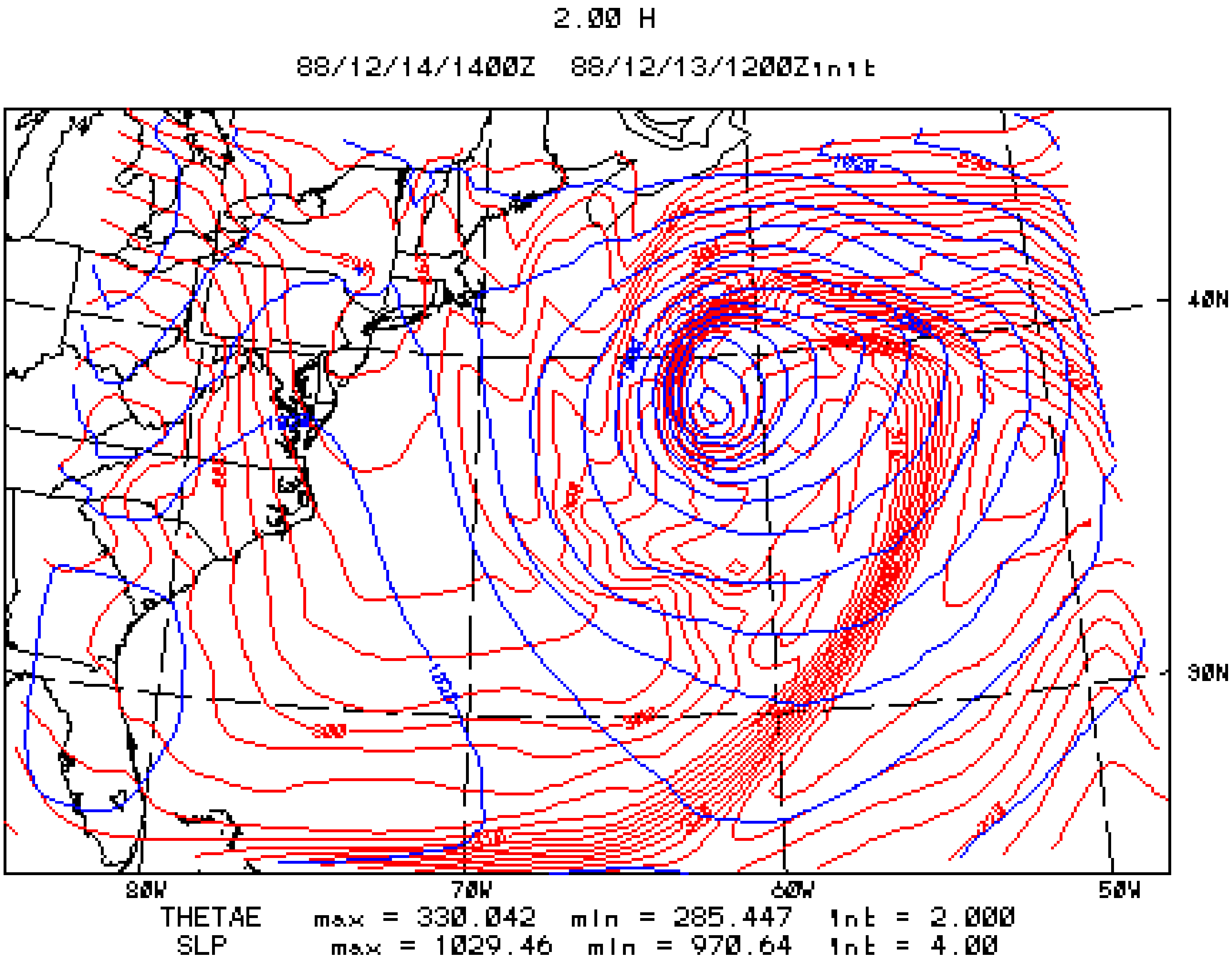
Incipient Frontal Cyclone



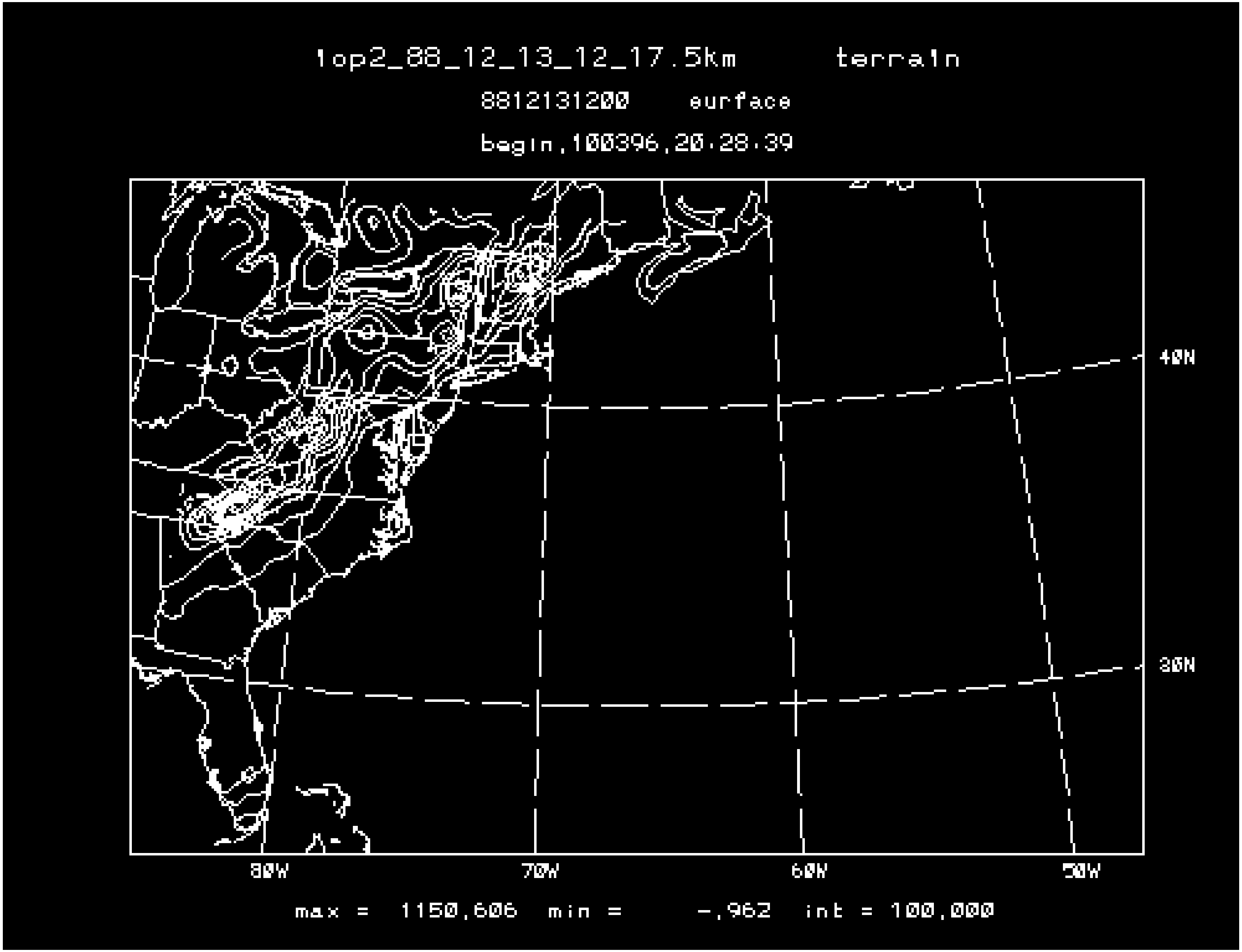
Frontal Fracture

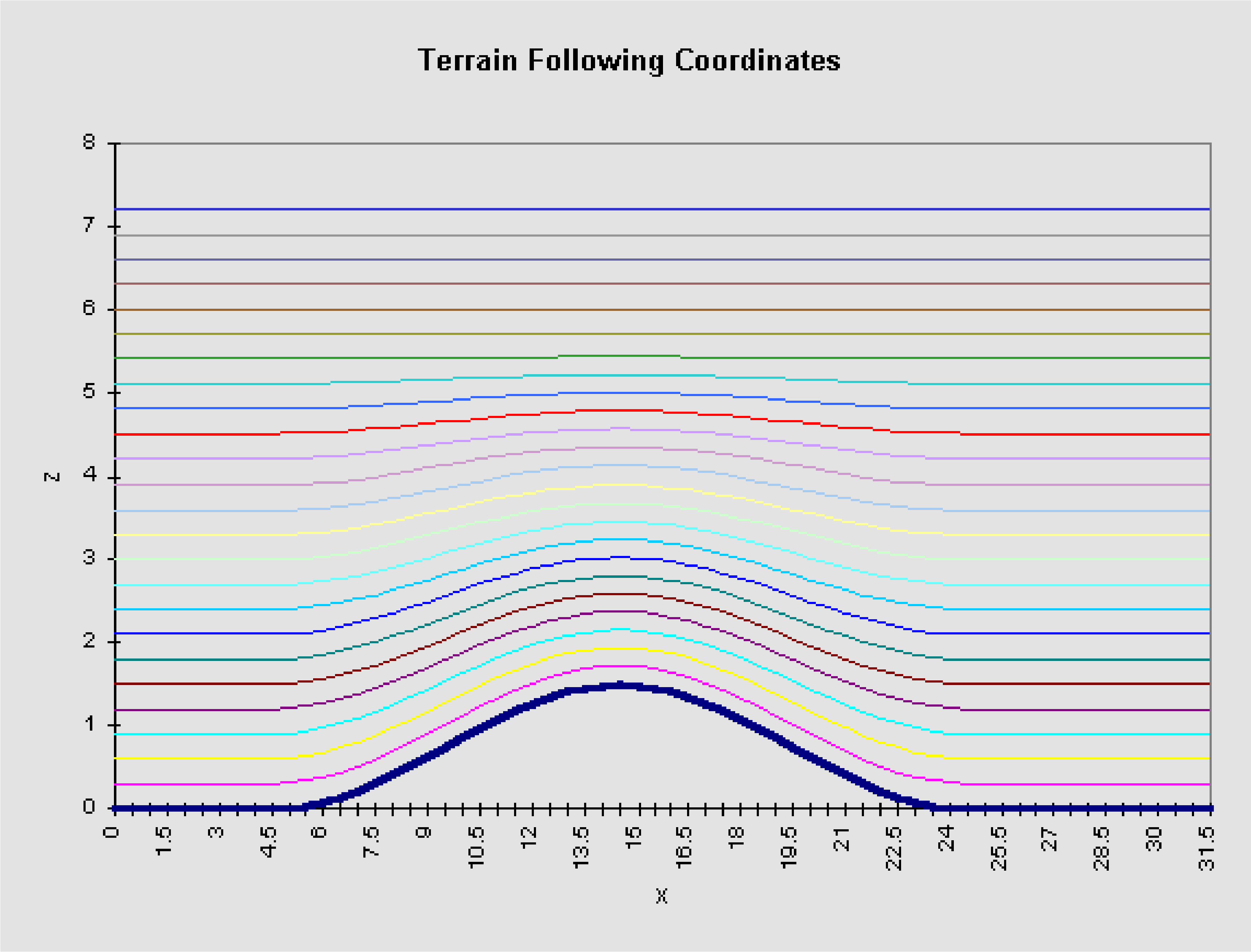


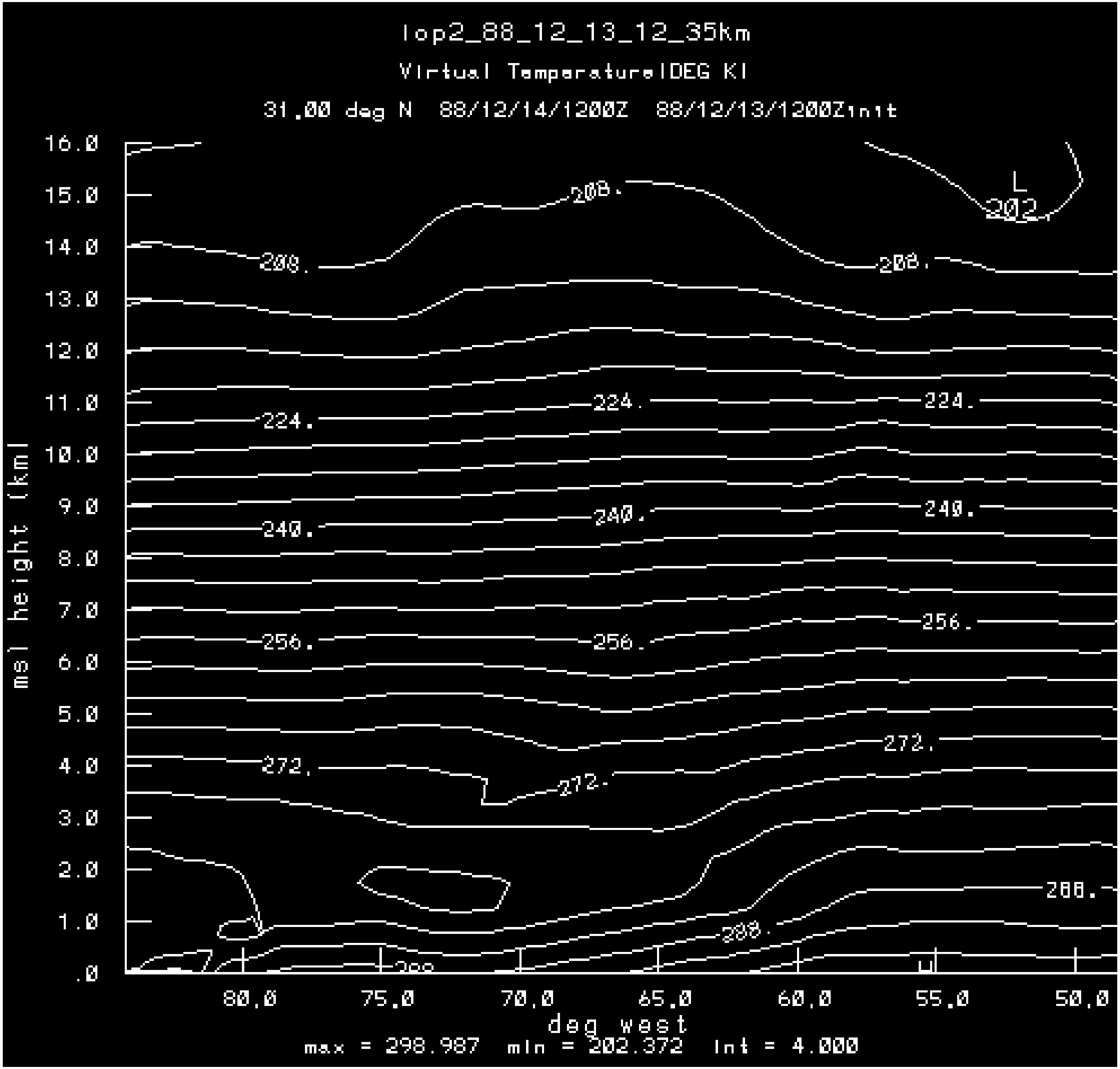
Bent Back and T- Bone Phase



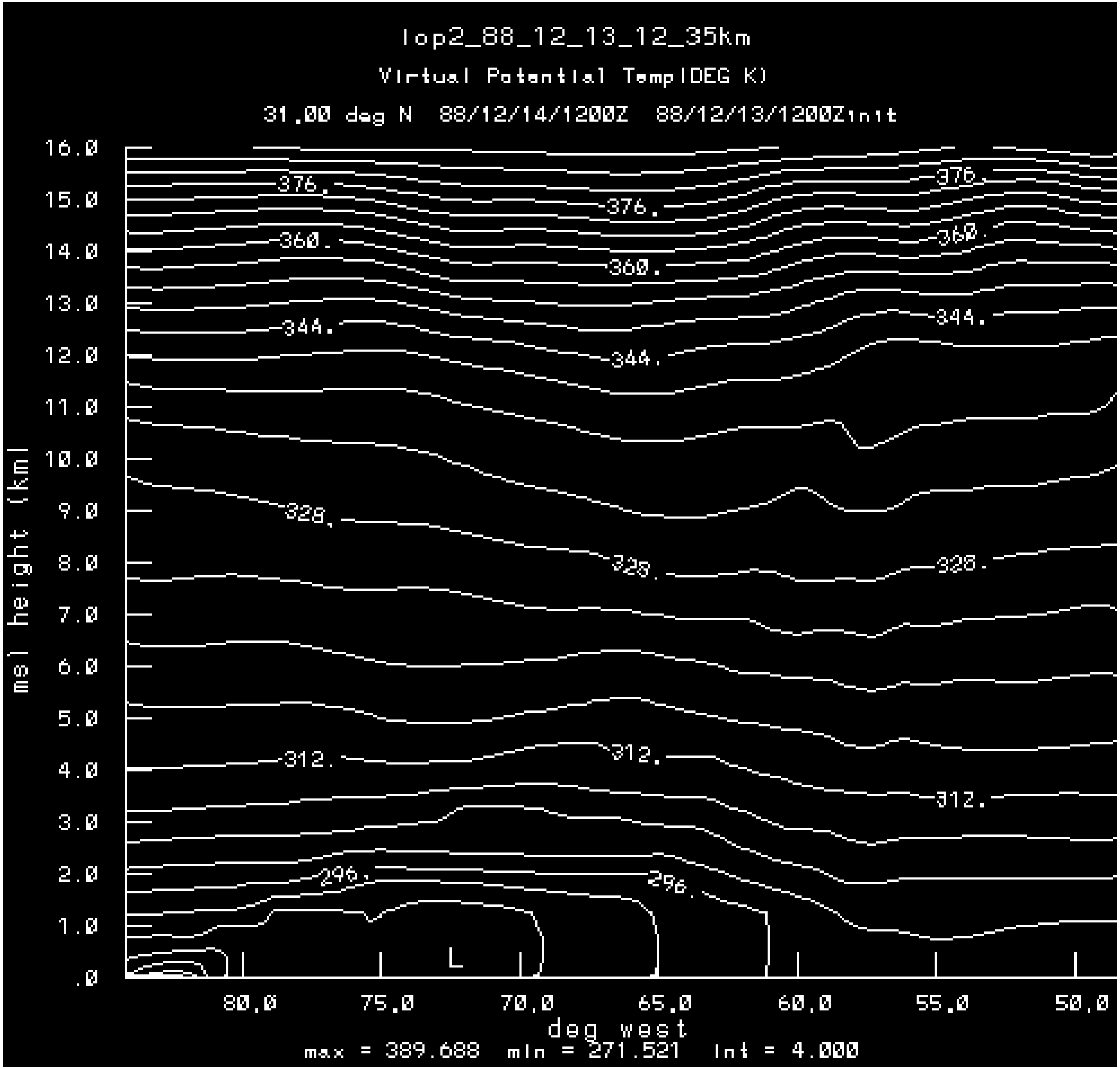
Warm Core Frontal Seclusion

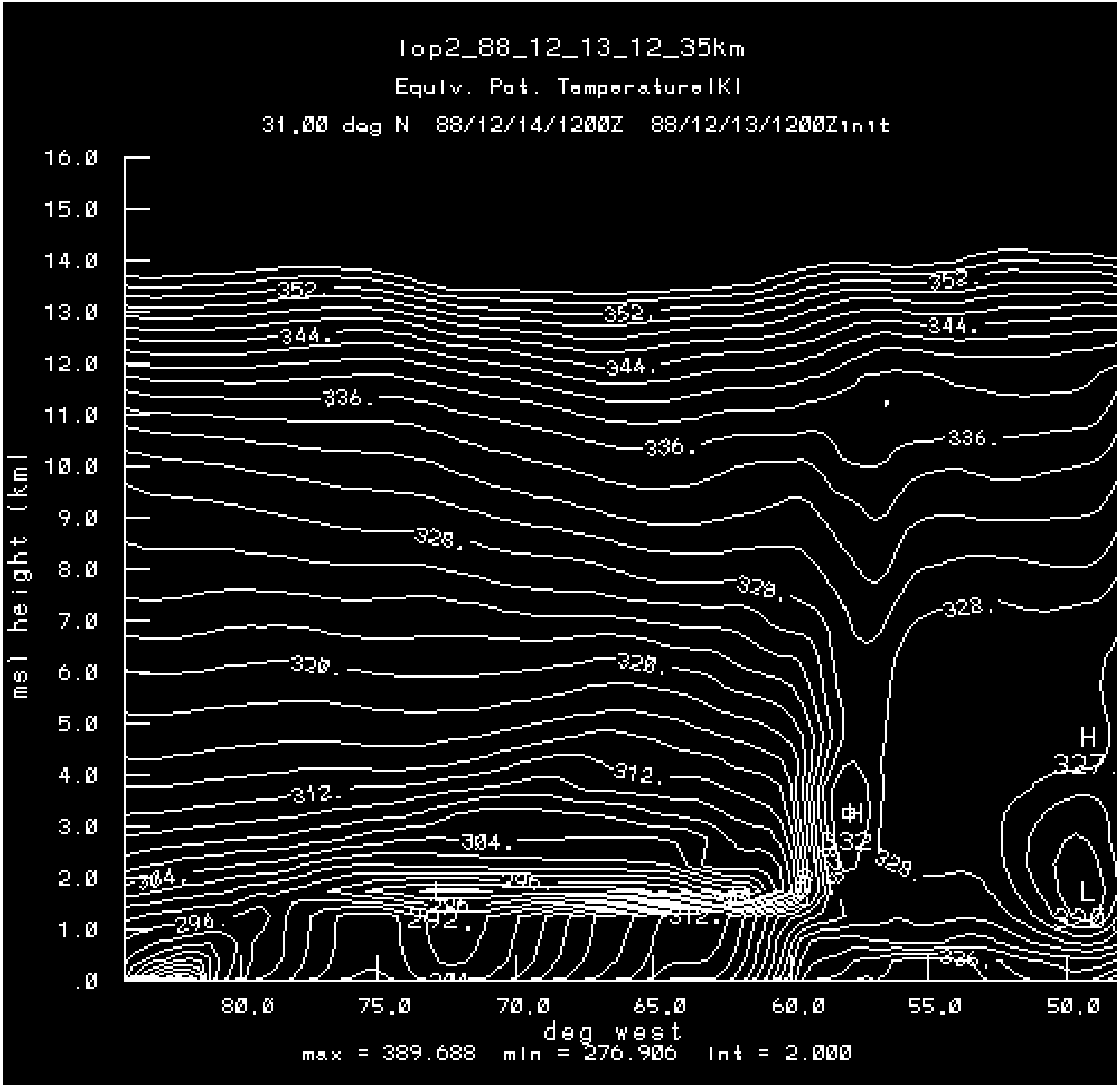

1op2_88_12_13_12_17.5km terrain
8812131200 surface
begin,100396,20:28:39
40N
30N
80W
70W
60W
50W
max = 1150.606 min = -.962 int = 100.000

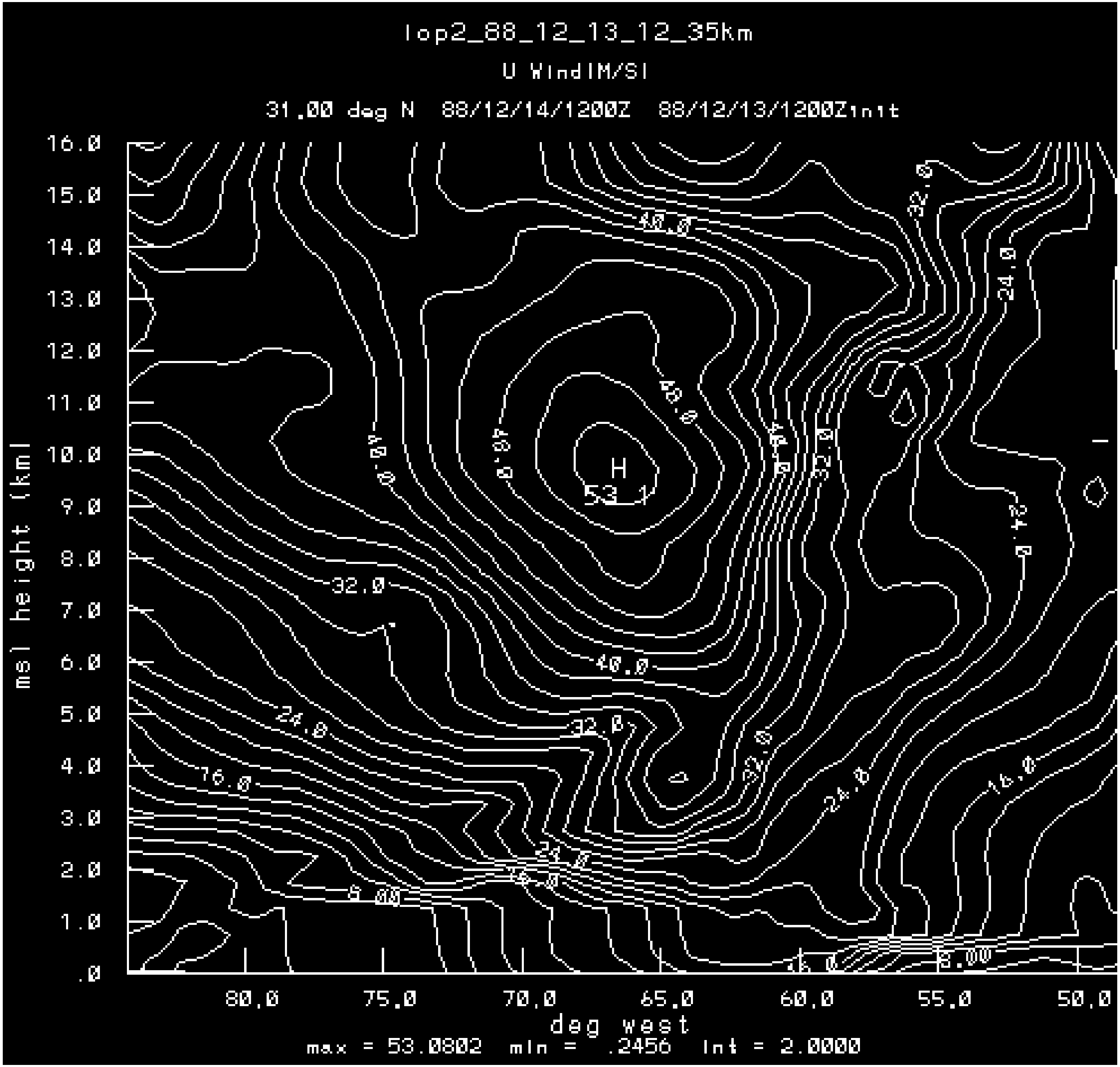

Terrain Following Coordinates
8
7
6
5
4
3
2
1
0
z
0
1.5
3
4.5
6
7.5
9
10.5
12
13.5
15
16.5
18
19.5
21
22.5
24
25.5
27
28.5
30
31.5
x



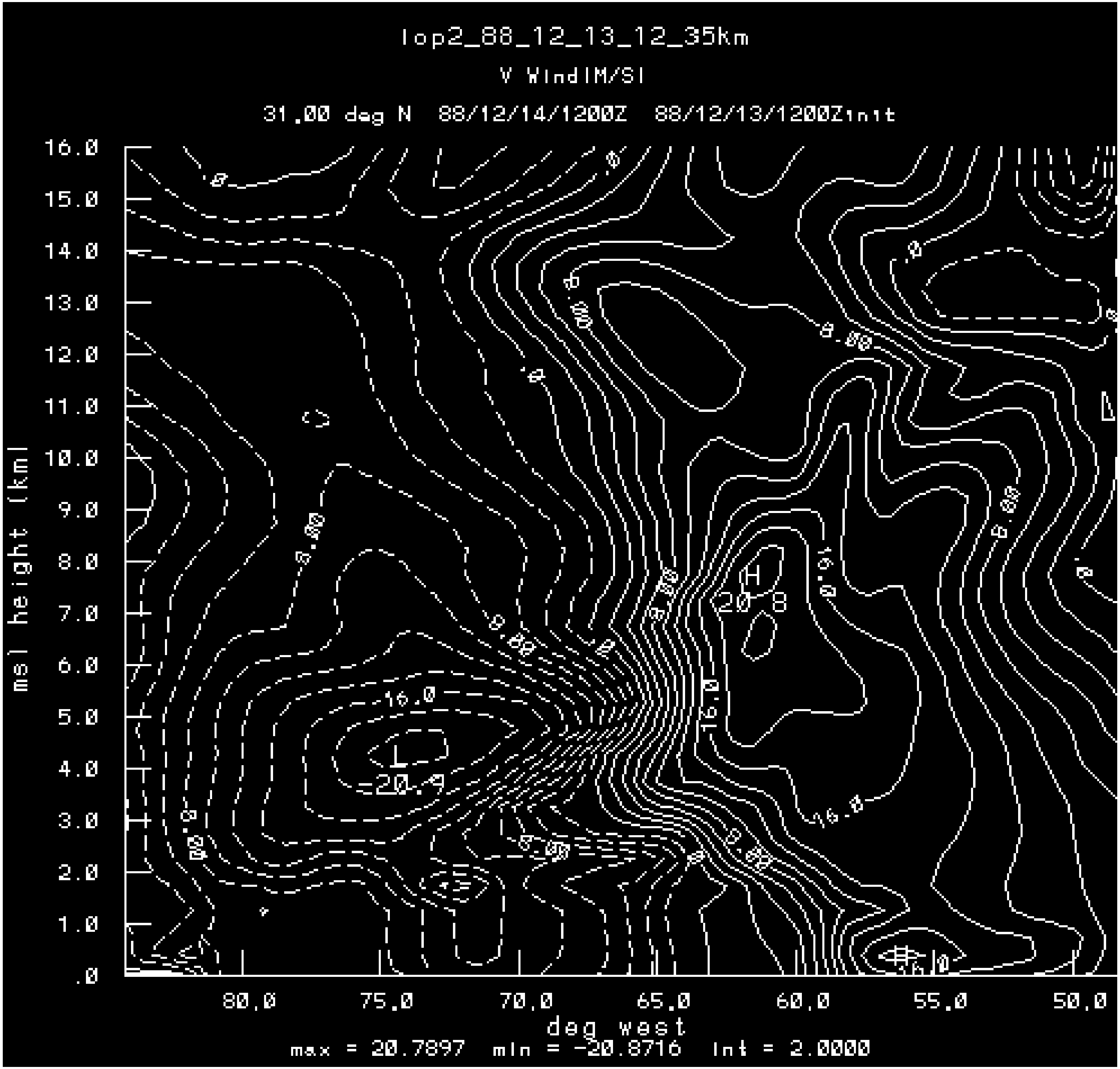
lop2_88_12_13_12_35km
Virtual Temperature[DEG K]
31.00 deg N 88/12/14/1200Z 88/12/13/1200Zinit
msl height (km)
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
.0
208.
L
202.
200.
208.
224.
224.
224.
240.
240.
240.
256.
256.
256.
272.
272.
272.
288.
288.
80.0
75.0
70.0
65.0
60.0
55.0
50.0
deg west
max = 298.987 min = 202.372 int = 4.000


iop2_88_12_13_12_35km
Virtual Potential Temp(DEG K)
31.00 deg N 88/12/14/1200Z 88/12/13/1200Zinit
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
.0
msl height (km)
376.
376.
376.
360.
360.
360.
344.
344.
344.
328.
328.
328.
312.
312.
312.
296.
296.
L
80.0
75.0
70.0
65.0
60.0
55.0
50.0
deg west
max = 389.688 min = 271.521 int = 4.000

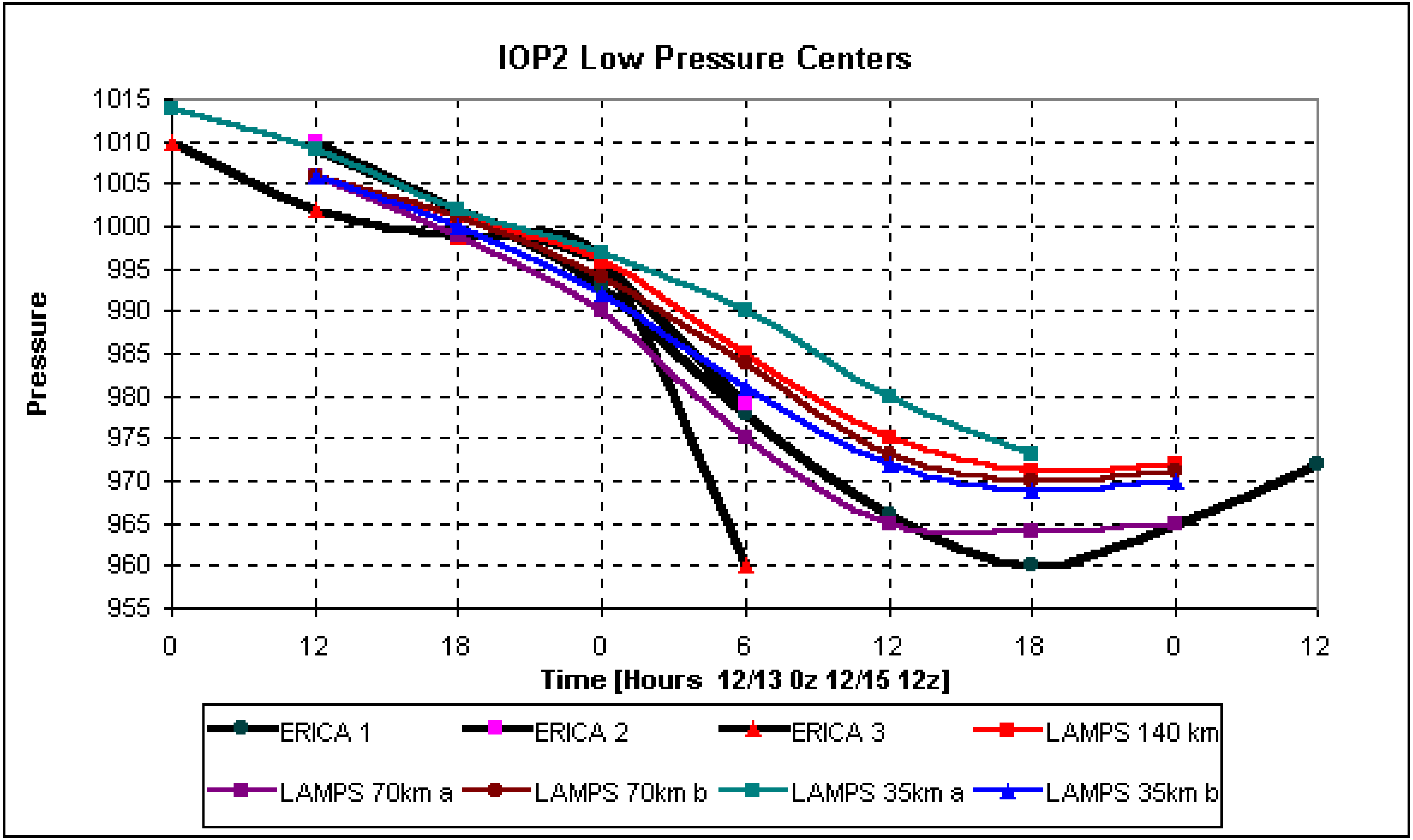

lop2_88_12_13_12_35km
Equlv. Pot. Temperature[K]
31.00 deg N 88/12/14/1200Z 88/12/13/1200Zinit
msl height (km)
deg west
max = 389.688 min = 276.906 int = 2.000



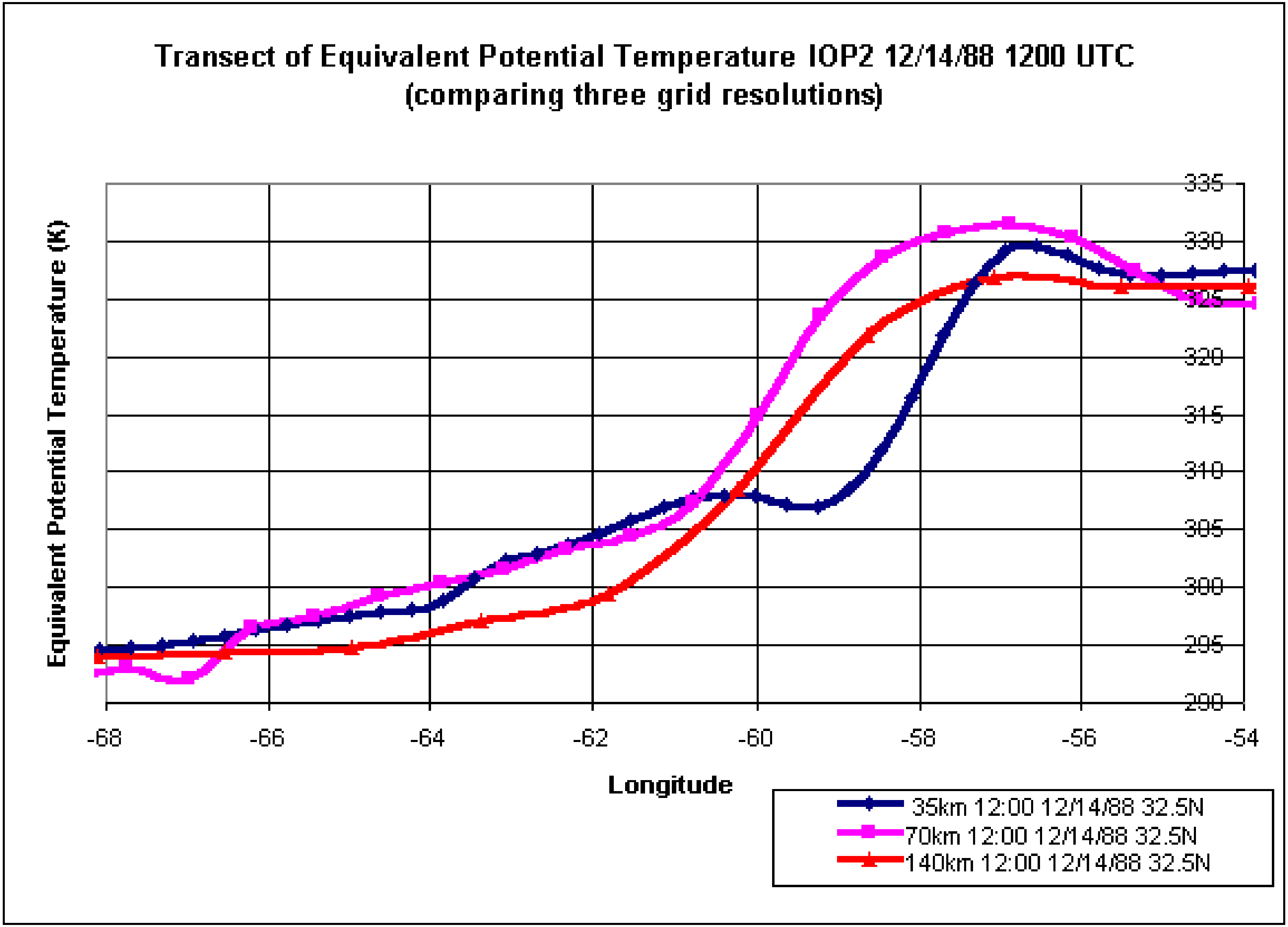
lop2_88_12_13_12_35km
U Wind(M/S)
31.00 deg N 88/12/14/1200Z 88/12/13/1200Zinit
msl height (km)
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0
deg west
H
53.1
max = 53.0802 min = .2456 int = 2.0000



lop2_88_12_13_12_35km
V Wind[M/S]
31.00 deg N 88/12/14/1200Z 88/12/13/1200Zinit
msl height (km)
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
.0
80.0
75.0
70.0
65.0
60.0
55.0
50.0
deg west
H
20.8
L
-20.9
16.0
8.00
max = 20.7897 min = -20.8716 int = 2.0000



IOP2 Low Pressure Centers
Pressure
1015
1010
1005
1000
995
990
985
980
975
970
965
960
955
0
12
18
0
6
12
18
0
12
Time [Hours 12/13 0z 12/15 12z]
ERICA 1
ERICA 2
ERICA 3
LAMPS 140 km
LAMPS 70km a
LAMPS 70km b
LAMPS 35km a
LAMPS 35km b


Transect of Equivalent Potential Temperature IOP2 12/14/88 1200 UTC
(comparing three grid resolutions)
Equivalent Potential Temperature (K)
335
330
325
320
315
310
305
300
295
290
-68
-66
-64
-62
-60
-58
-56
-54
Longitude
35km 12:00 12/14/88 32.5N
70km 12:00 12/14/88 32.5N
140km 12:00 12/14/88 32.5N

Data Analysis

Fourier versus Wavelet

Computation of Fourier Series on Interval -π <= x <= +π

$$f(x) = a_o + \sum_{k=1}^{\infty} \left[a_k \cos(kx) + b_k \sin(kx) \right]$$

Fourier Coefficients of the Function f(x):

$$a_0 = \frac{1}{2\pi} \int_{-a}^{a} f(x) dt$$

$$a_k = \frac{1}{\pi} \int_{-a}^{a} f(x) \cos(k\, x) dt$$

$$b_k = \frac{1}{\pi} \int_{-a}^{a} f(x) \sin(k\, x) dt$$

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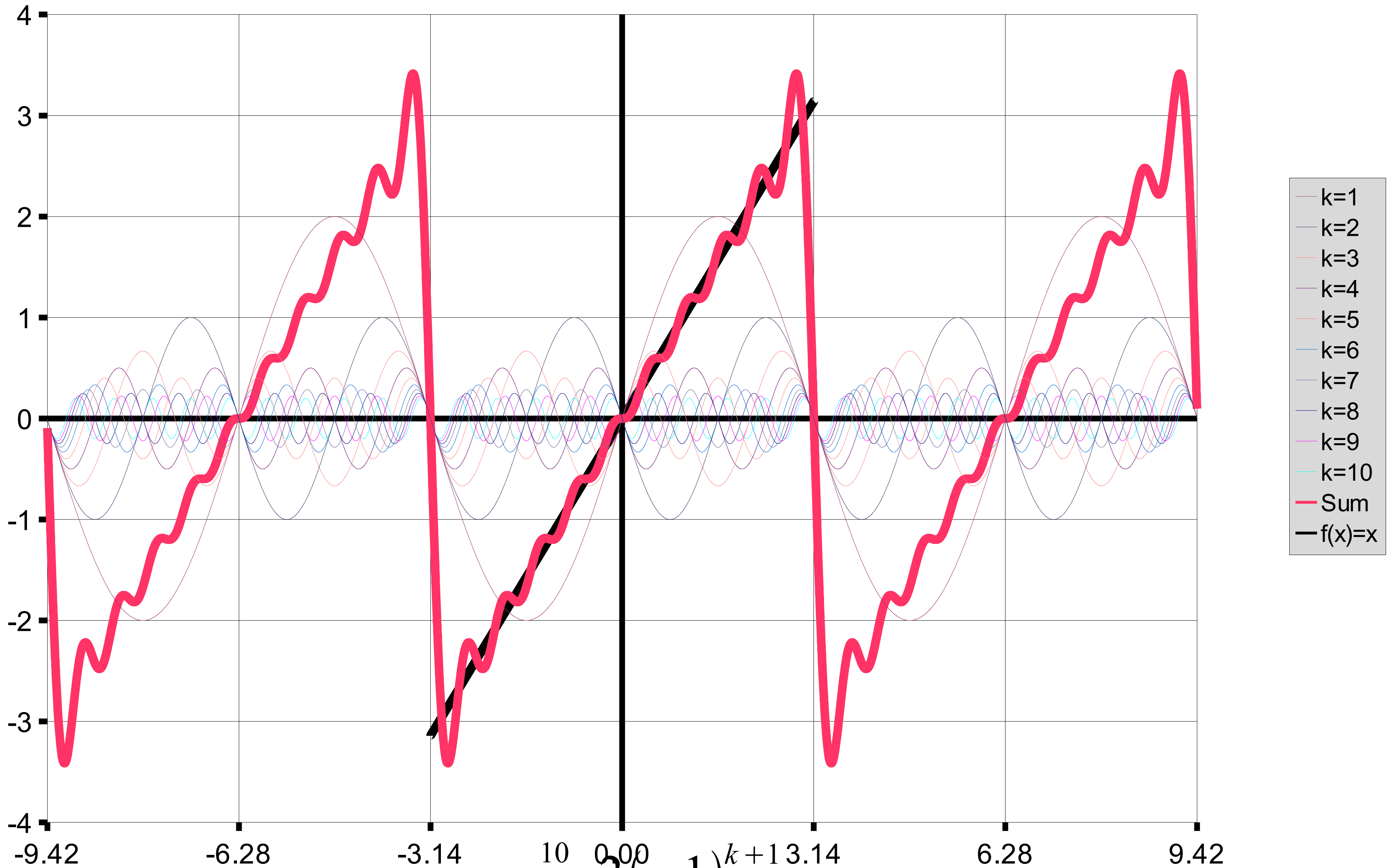
Computation of Fourier Series on Interval $-\pi <= x <= \pi$



$$S_{10}(x) = \sum_{k=1}^{10} \frac{2(-1)^{k+1}}{k} \sin(kx)$$

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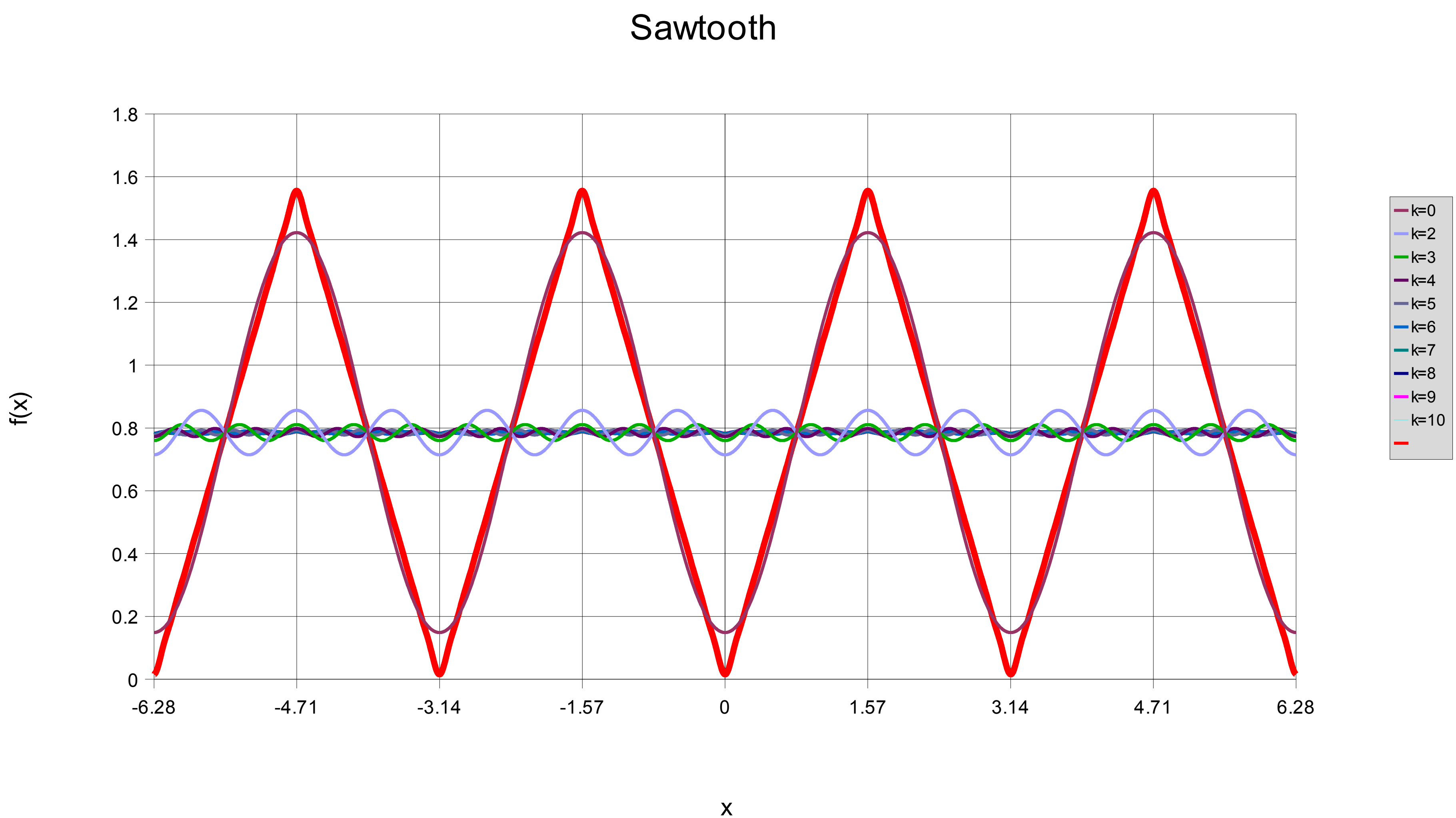
Computation of Fourier Series on Interval -a <= x <= a

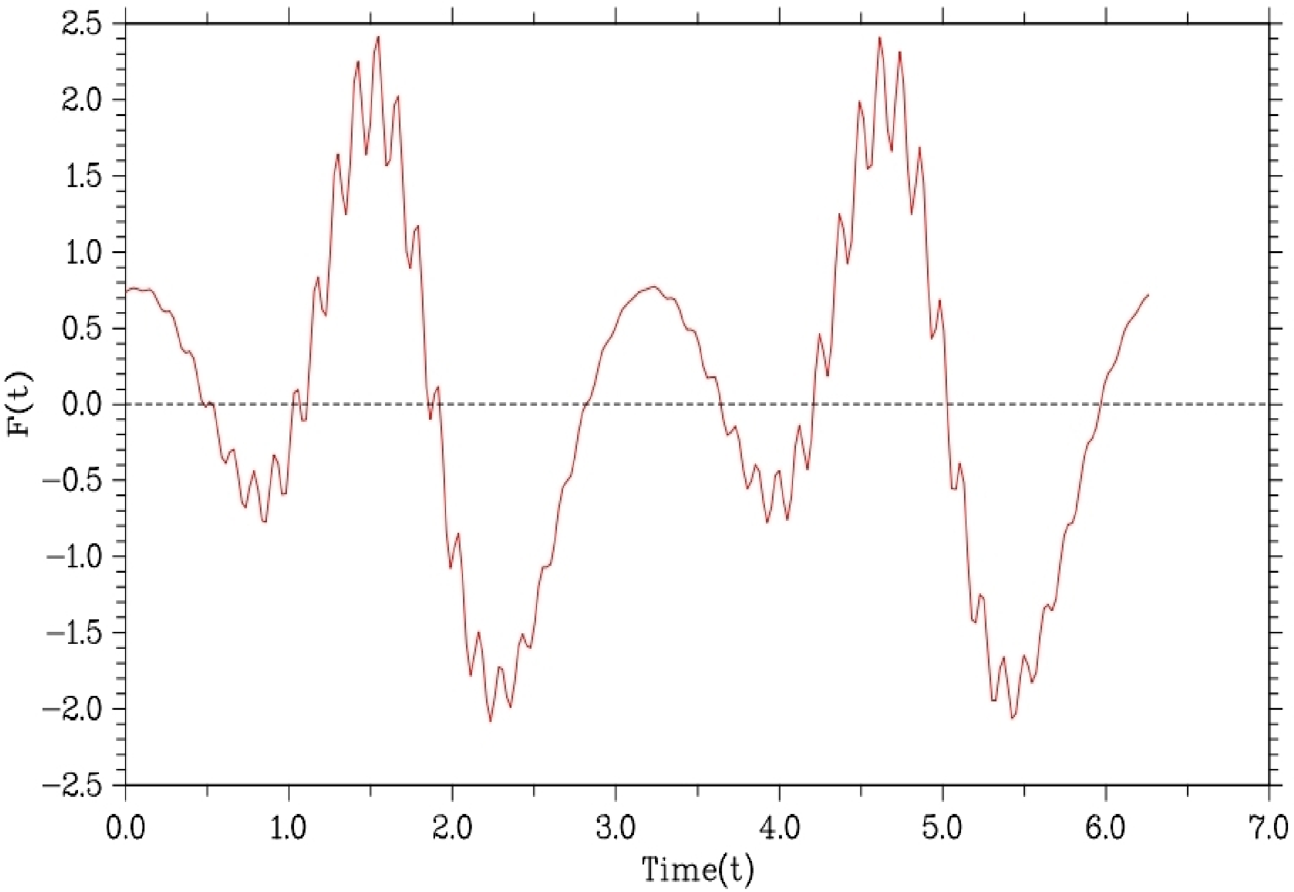
Even



$$S_{10}(x) = \frac{\pi}{4} - \frac{2}{\pi}\sum_{k=0} \frac{1}{(2k+1)^2}\cos\left((4k+2)x\right)$$

Fast Fourier Transform (FFT)

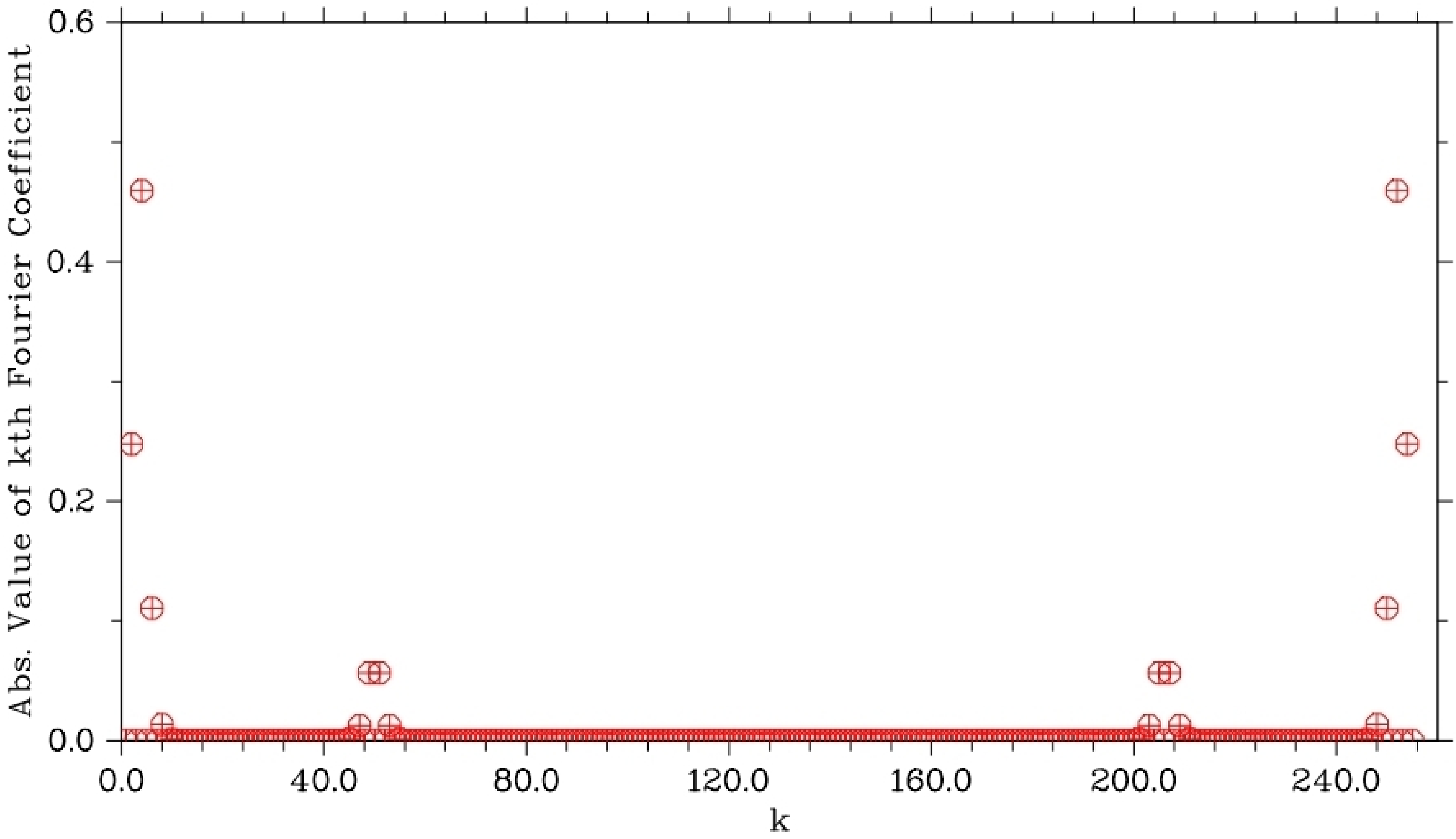
Function: $y(t)=e^{-(\cos t)^2}\left(\sin 2t+2\cos 4t+0.4\sin t\sin 50t\right)$



Original Signal discretized with $2^8=256$ points. $0\leq t\leq 2\pi$

Fast Fourier Transform (FFT)

Function: $y(t) = e^{-(\cos t)^2}\left(\sin 2t + 2\cos 4t + 0.4\sin t \sin 50t\right)$



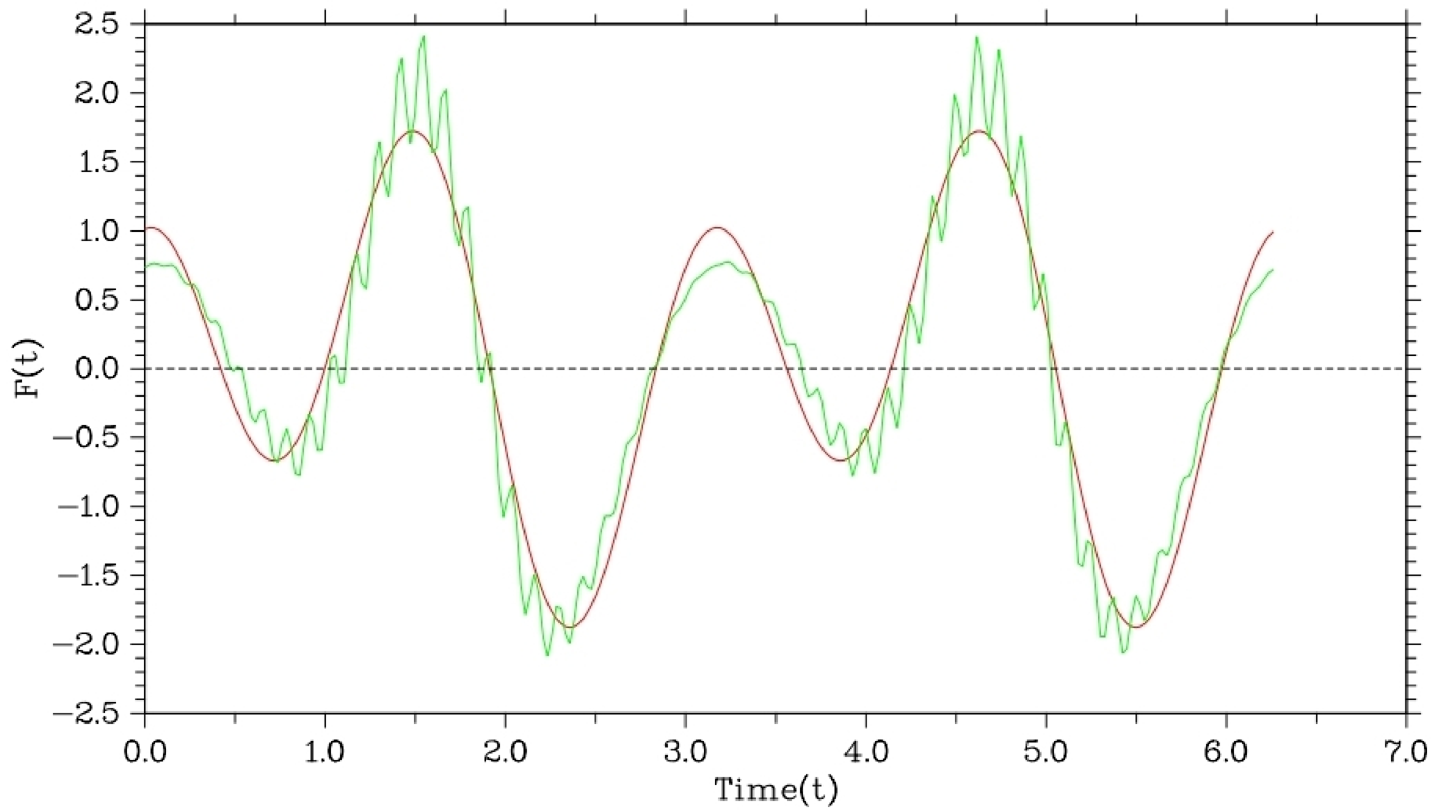
Original Signal discretized with $2^8 = 256$ points. $0 \le t \le 2\pi$

Sun Performance Library FFT used to generate the Discrete Fourier Coefficients $\hat{y}_k$, $k = 0, \ldots, 255$.

Noise has a frequency that is larger than 5 cycles per 2π interval.

Fast Fourier Transform (FFT) – Use to Filter out Noise.

Function: $y(t) = e^{-(\cos t)^2}\left(\sin 2t + 2\cos 4t + 0.4\sin t \sin 50t\right)$



Filtered by keeping first five coefficients and completing inverse FFT.

$$Error = \frac{\|y - y_c\|_{l^2}}{\|y\|_{l^2}} = 0.26$$

Original Signal discretized with $2^8 = 256$ points. $0 \le t \le 2\pi$

Sun Performance Library FFT used to generate the Discrete Fourier Coefficients $\hat{y}_k$, $k = 0,\dots,255$.

Noise has a frequency that is larger than 5 cycles per 2π interval.

Keep only $\hat{y}_k$ for $0 \le k \le 5$ and set $\hat{y}_k = 0.0$ for $6 \le k \le 128$.

By Theorem, $\hat{y}_k = 0$ for $128 \le k \le 250$. Applying FFT to filter $\hat{y}_k$

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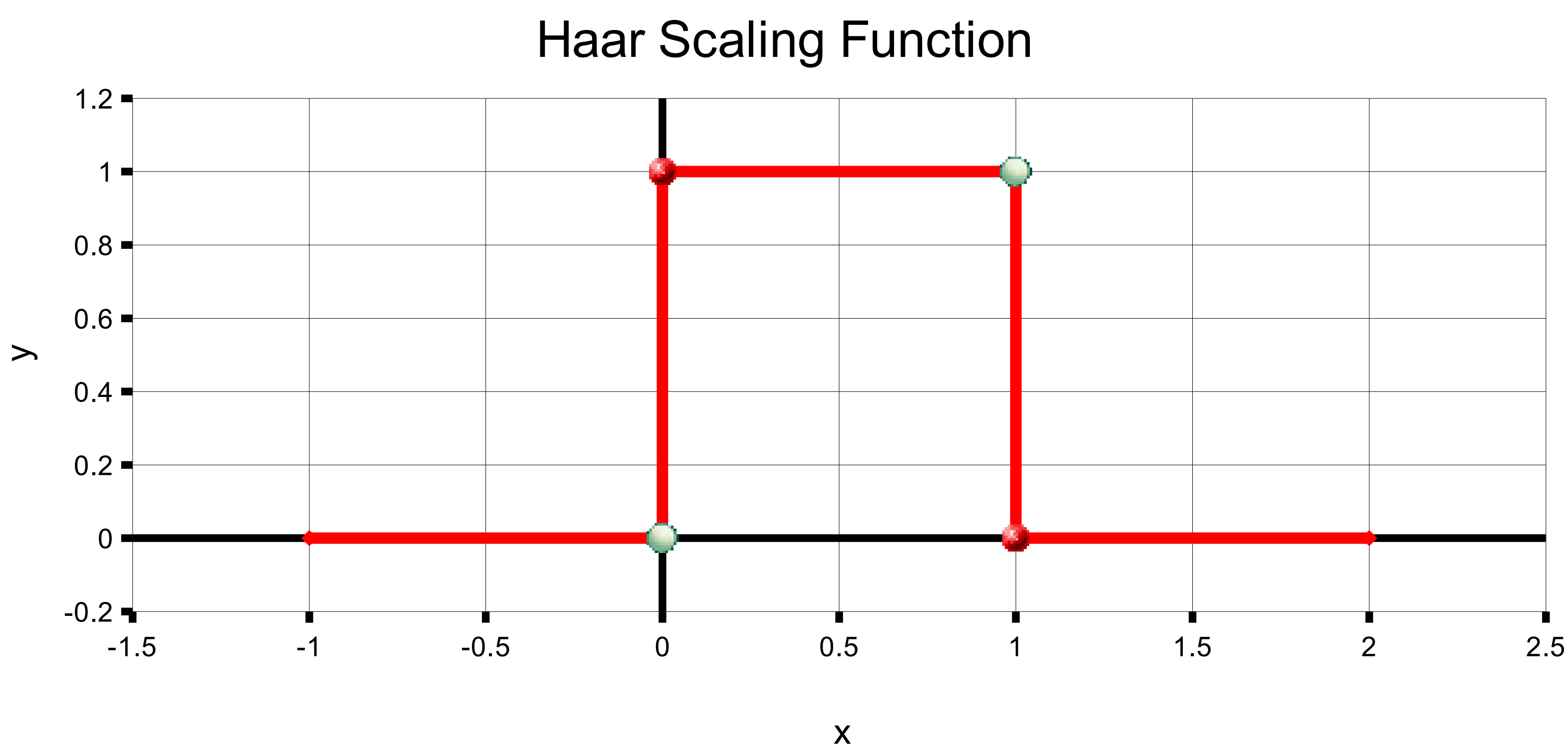
III. What is a wavelet?

Haar Wavelet Analysis

Scaling Function: ϕ Sometimes called the Father Wavelet.

Wavelet : ψ Sometimes called the Mother Wavelet.



$$\phi(x)\begin{cases} =1 & \text{if } 0 \le x < 1 \\ =0 & \text{Elsewhere.} \end{cases}$$

Let V_0 be the space of all functions of the form:

$$\sum_{k \in Z} a_k \phi(x-k) \qquad a_k \in R$$

Where k can range over any finite set of positive and negative integers.

White dots are discontinuities.

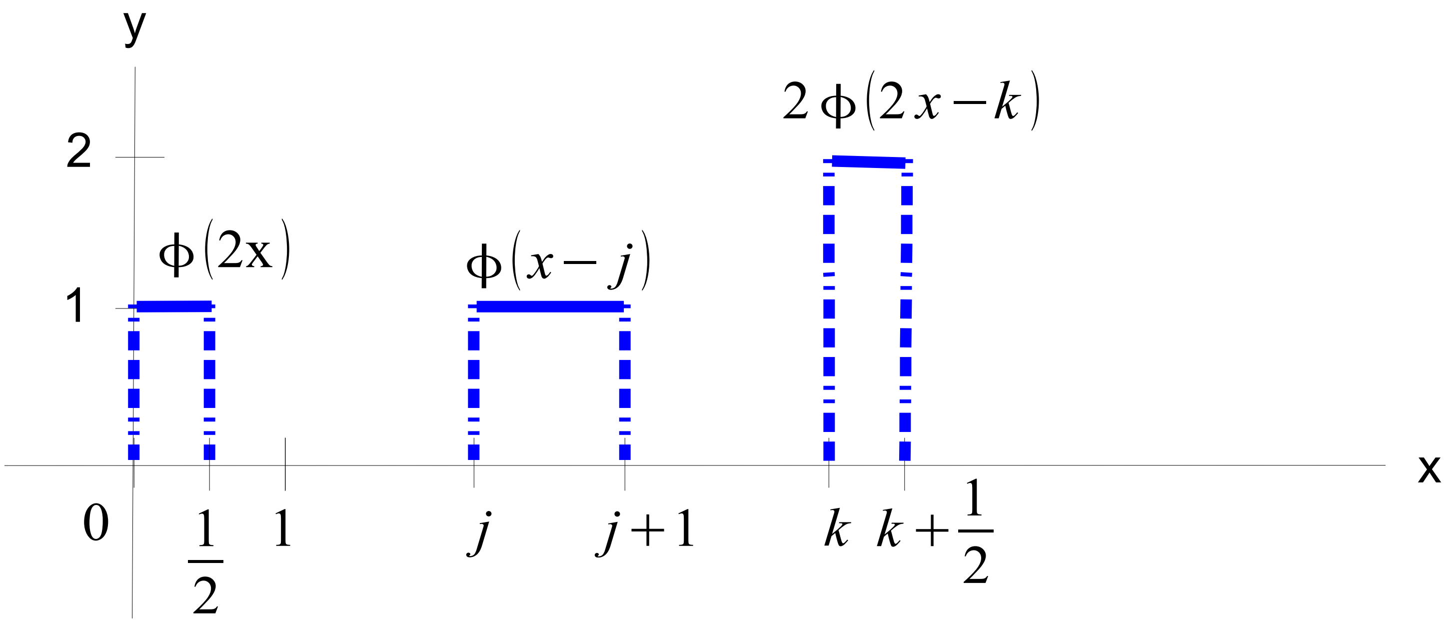
Haar Wavelet Analysis

Scaling Function: ϕ Sometimes called the Father Wavelet.

Wavelet : ψ Sometimes called the Mother Wavelet.

Haar Scalir

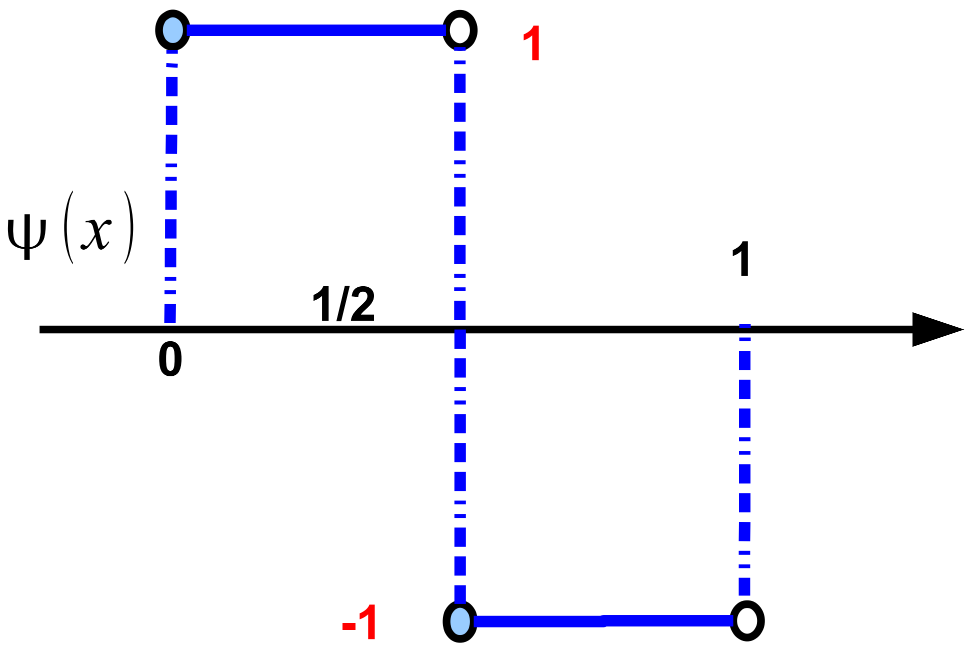
$$\phi(x) \begin{cases} =1 & \text{if } 0 \le x < 1 \\ =0 & \text{Elsewhere.} \end{cases}$$



Haar Wavelet Analysis

Scaling Function: ϕ Sometimes called the Father Wavelet.

Wavelet : ψ Sometimes called the Mother Wavelet.

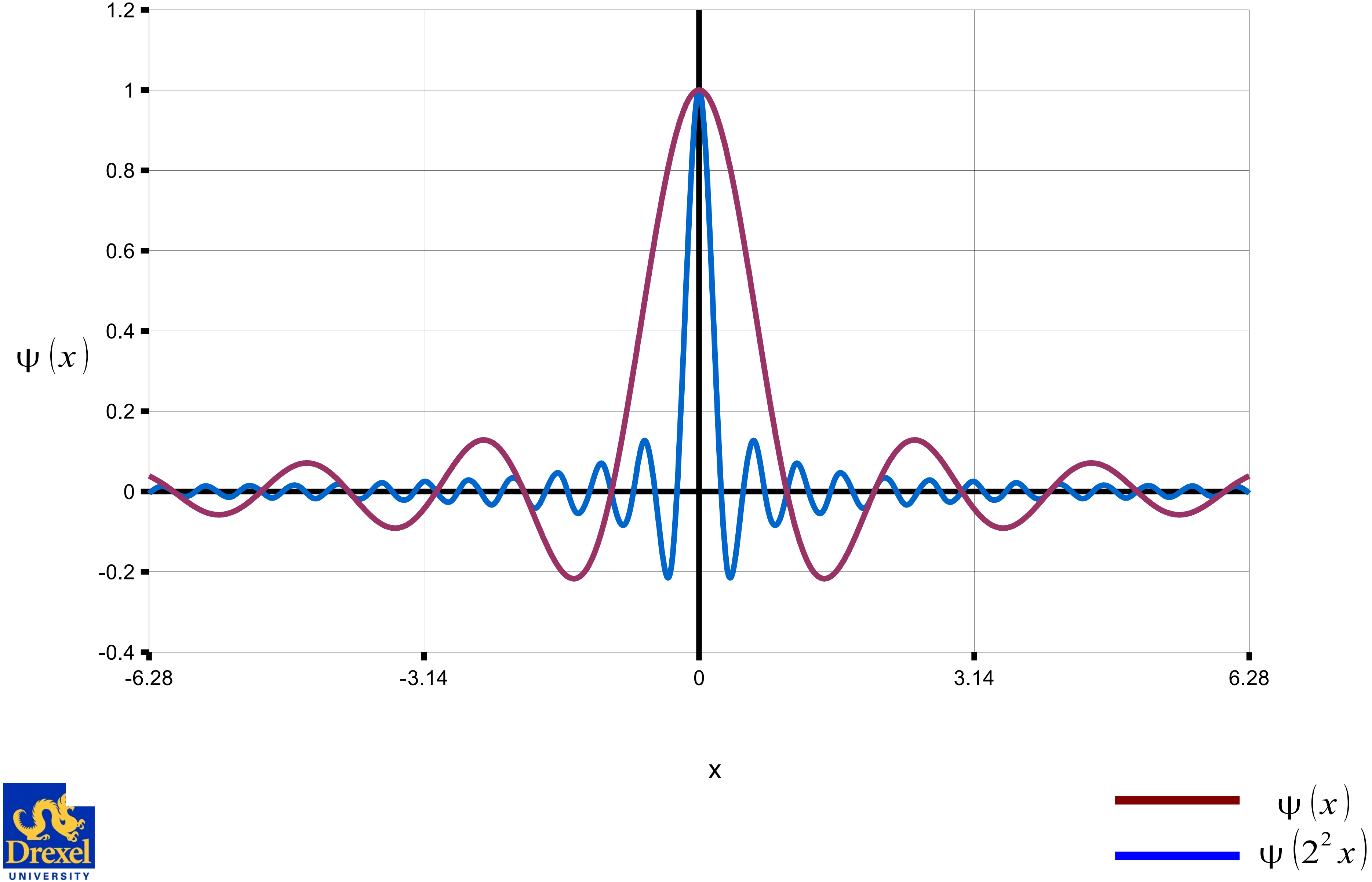


$$\psi(x) = \phi(2x) - \phi(2(x - 1/2)) = \phi(2x) - \phi(2x - 1)$$

White dots are discontinuities.

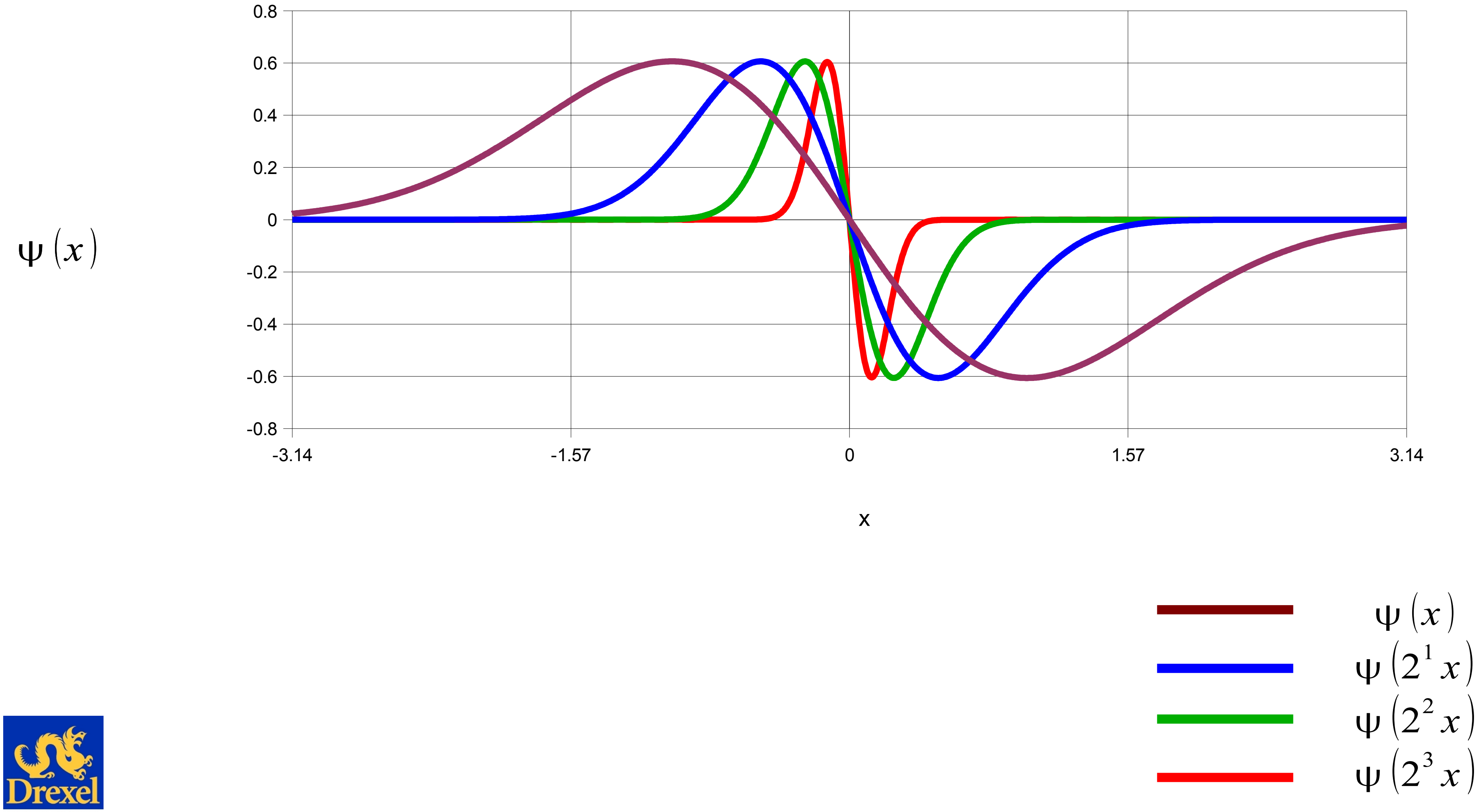
Other Wavelets - Shannon

$$\psi(x) = \frac{\sin(\pi x)}{\pi x}$$



Other Wavelets – Linear Spline

$$\psi(x) = -x\, e^{\frac{-x^2}{2}}$$



Other Wavelets

1) **Haar**
   - Compact Support
   - Discontinuous
2) **Shannon**
   - Very Smooth
   - Extend throughout the whole real line
   - Decay at infinity very slowly
3) **Linear Spline**
   - Continuous
   - Have infinite support
   - Decay rapidly at infinity

Finite or *Compact Support*

Let V_0 be the space of all functions of the form: $\sum_k a_k \phi(x-k) \quad a_k \in R$

where k range over set of positive or negative integers. Since k ranges over a finite set, each element of V_0 is zero outside a bounded set.

Other Wavelets - Daubechies

Enter Ingrid Daubechies:

•Simplest is Haar wavelet, only discontinuous wavelet.

•Others are continuous and compactly supported.

•As you move up hierarchy, become increasingly smooth (have prescribed number of continuous derivative).

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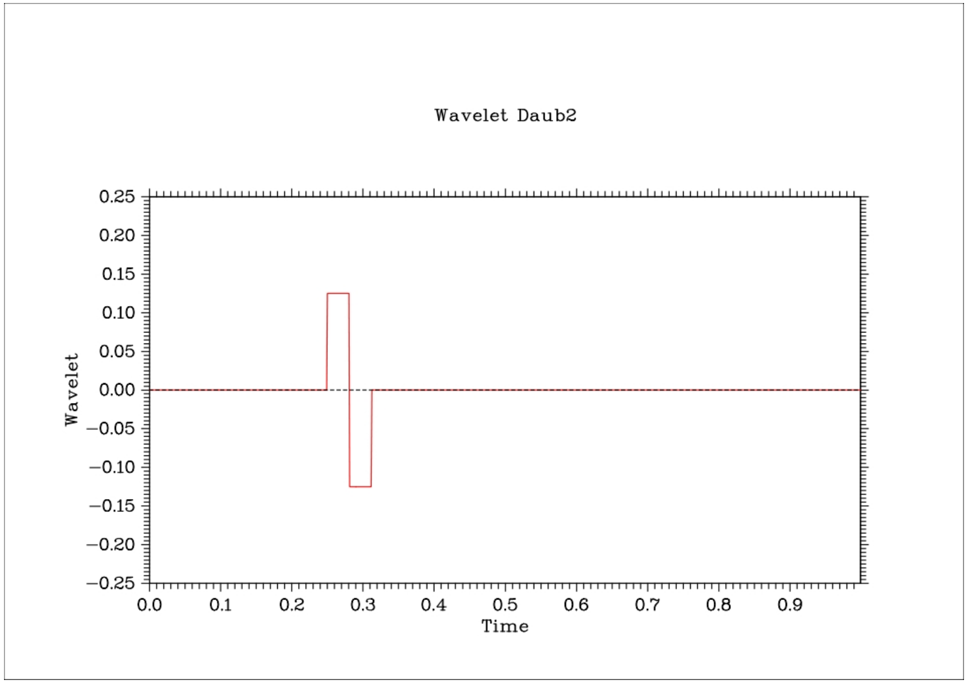
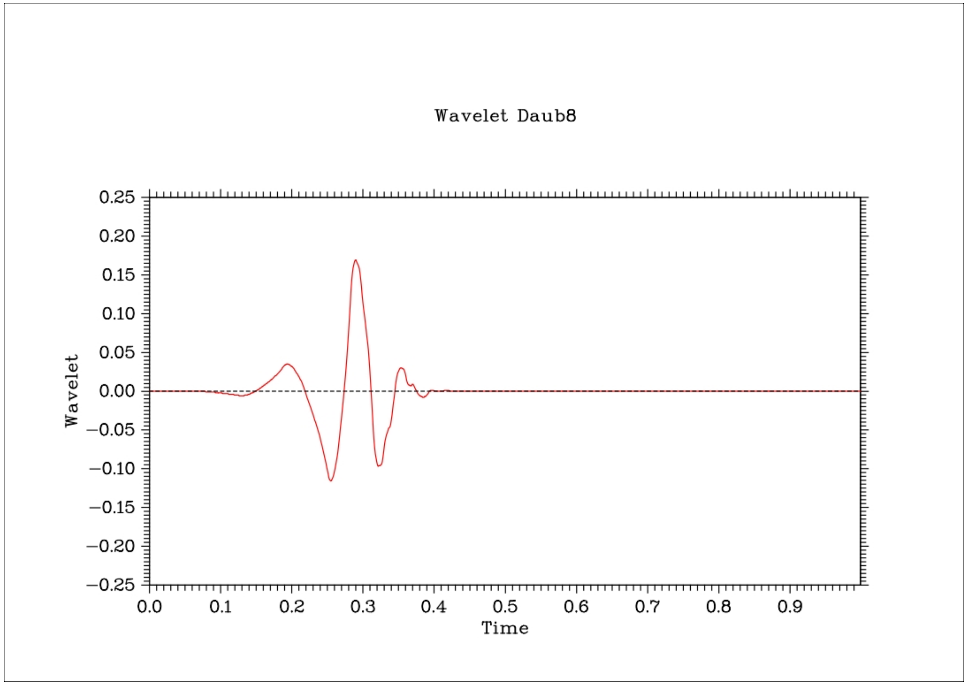
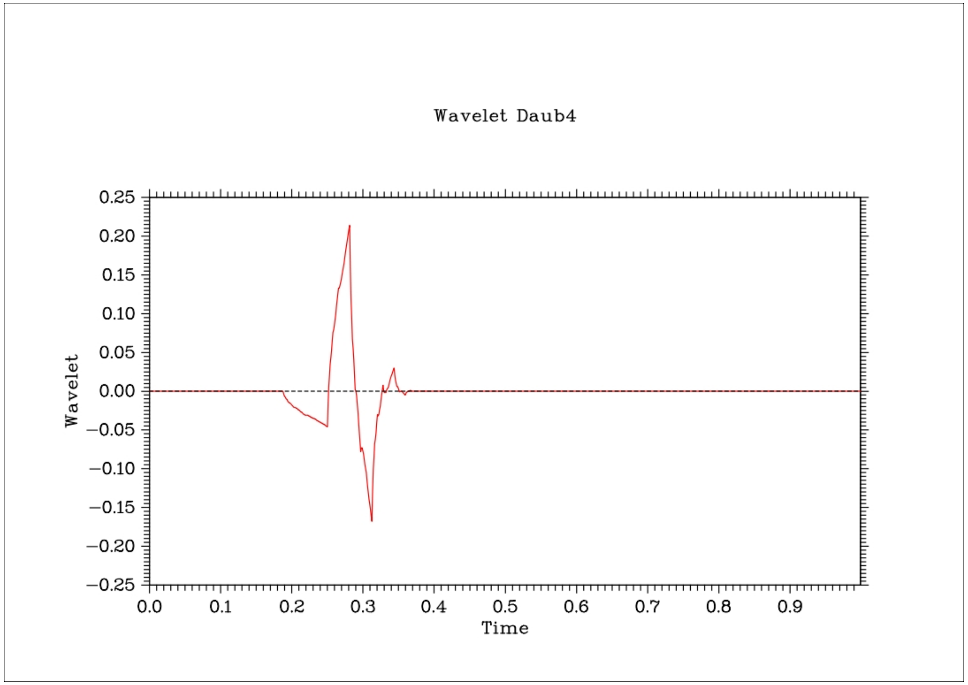
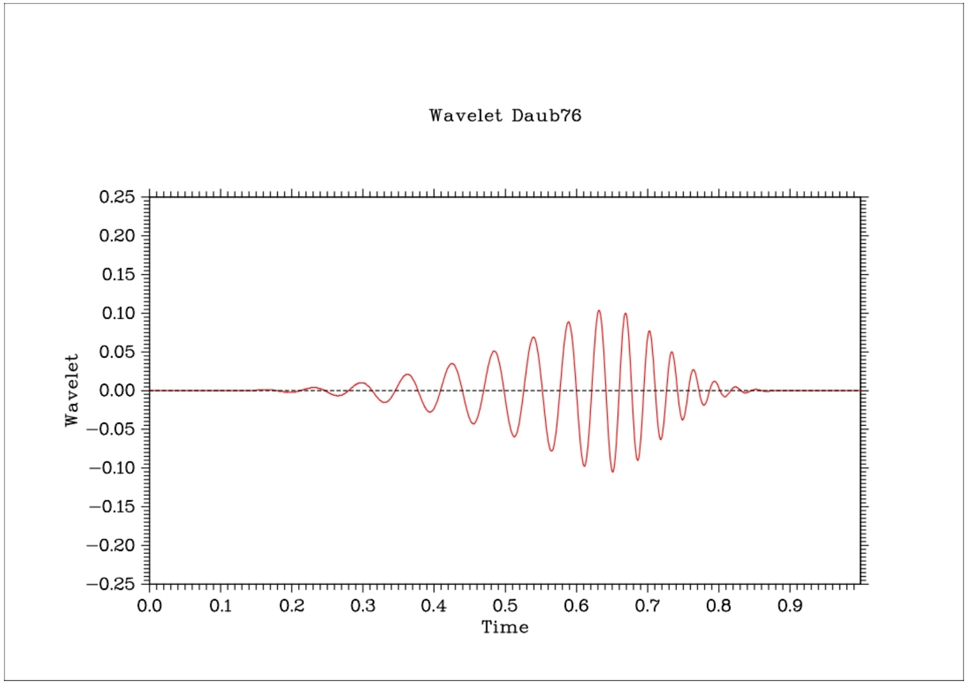
Other Wavelets - Daubechies









Other Wavelets - Daubechies

DAUB4 Initial Values:

$$\phi(0) \equiv 0$$

$$\phi(1) \equiv \frac{1+\sqrt{3}}{2}$$

$$\phi(2) \equiv \frac{1-\sqrt{3}}{2}$$

$$\phi(3) \equiv 0$$

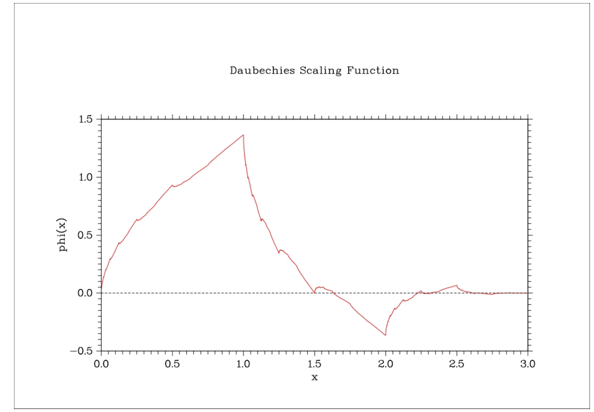
DAUB4 Abreviations:

$$h_0 = \frac{1+\sqrt{3}}{4}$$

$$h_1 = \frac{3+\sqrt{3}}{4}$$

$$h_2 = \frac{3-\sqrt{3}}{4}$$

$$h_3 = \frac{1-\sqrt{3}}{4}$$

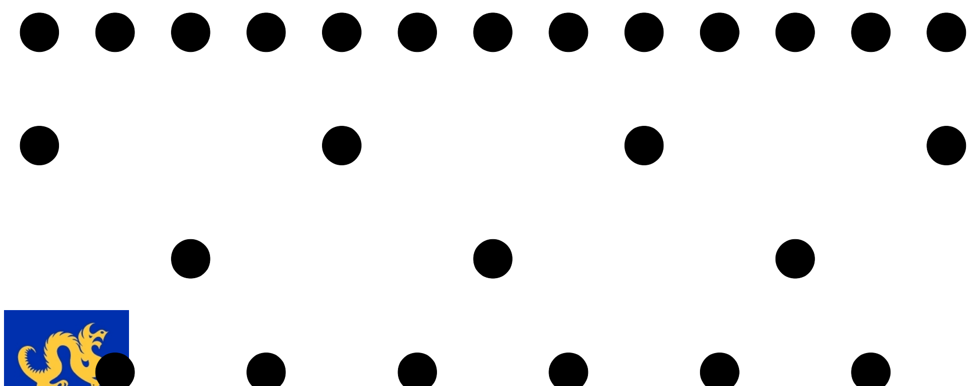


DAUB4

DAUB4 Recurrence Relation:

$$\phi(r) = h_0 \phi(2r) + h_1 \phi(2r-1) + h_2 \phi(2r-2) + h_3 \phi(2r-3)$$

Say you want to approximate 12+1 points.



Initial Values : $r = 0,1,2,3$

First Iteration : $r = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$

Second Iteration : $r = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}$

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Other Wavelets - Daubechies

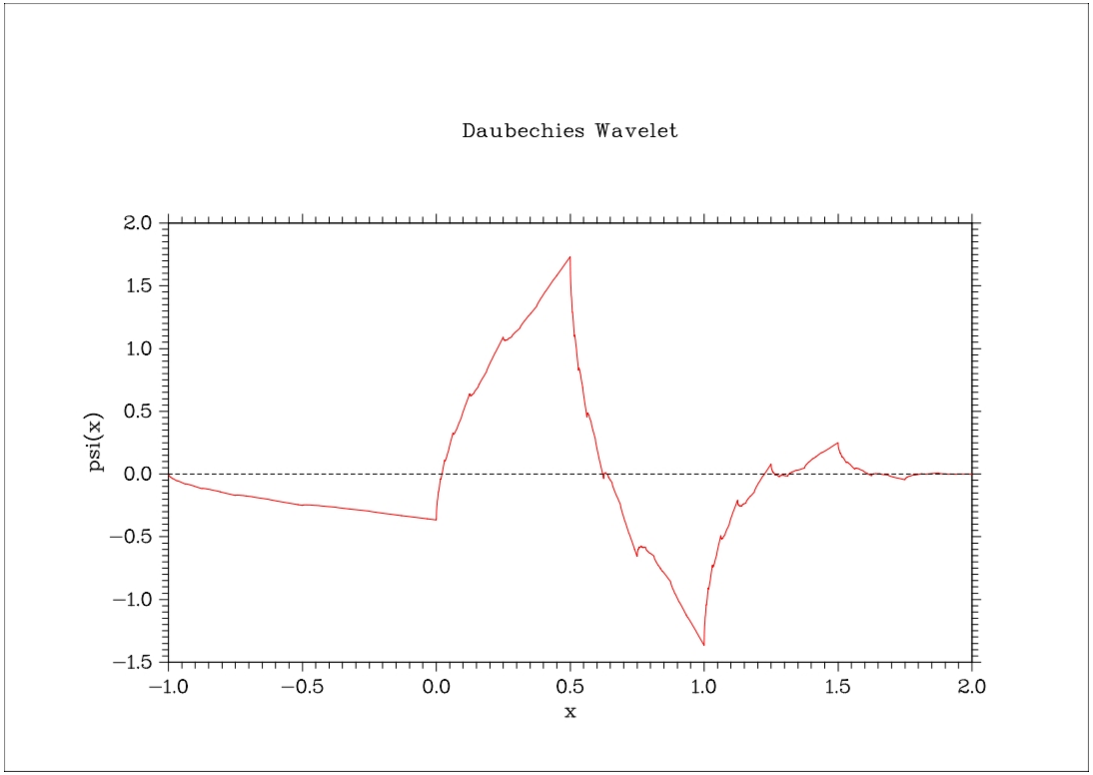
The function ϕ serves as the basic building block for its associated wavelet, denoted by ψ, and defined by the following recursion:

$$\psi(r) \equiv -\left(\frac{1+\sqrt{3}}{4}\right)\phi(2r-1)+\left(\frac{3+\sqrt{3}}{4}\right)\phi(2r)-\left(\frac{3-\sqrt{3}}{4}\right)\phi(2r+1)+\left(\frac{1-\sqrt{3}}{4}\right)\phi(2r+2)$$

$$\psi(r) \equiv -h_0\phi(2r-1)+h_1\phi(2r)-h_2\phi(2r+1)+h_3\phi(2r+2)$$

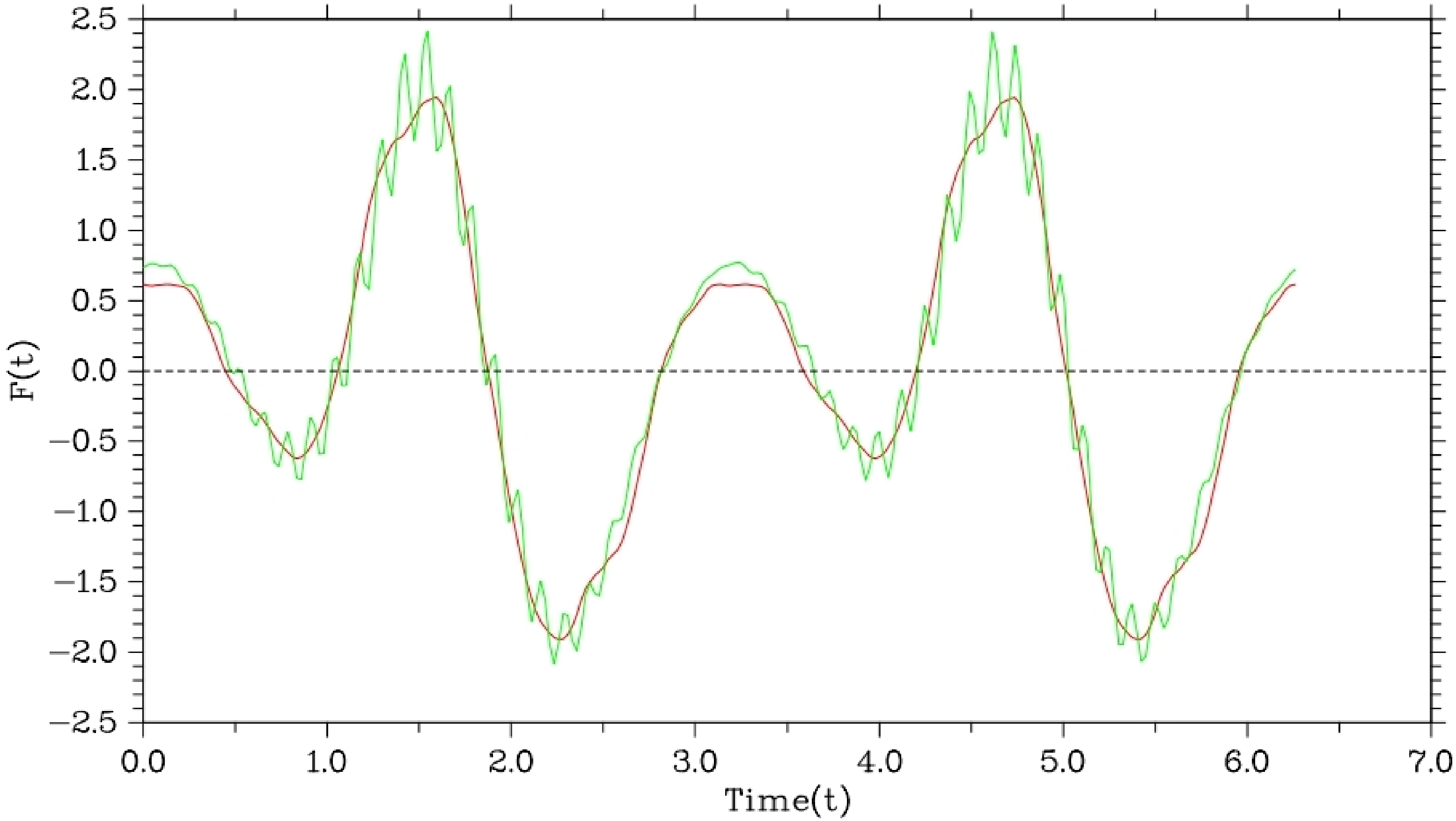
$$\psi(r)=(-1)^{(1)}h_{1-1}\phi(2r-1)+(-1)^{(0)}h_{1-0}\phi(2r-0)+(-1)^{(-1)}h_{1-[-1]}\phi(2r-[-1])+(-1)^{(-2)}h_{1-[-2]}\phi(2r-[-2])$$

Because $\phi(r)=0$ if $r\leq 0$ or $r\geq 3$, it follows that $\psi(r)=0$ if $2r+2\leq 0$ or $2r-1\geq 3$, or, equivalently, if, $r\leq -1$ or $r\geq 2$. For values r such that $-1<r<2$, the recursion yields $\psi(r)$.



Using Wavelets to Filter Data

Function: $y(t) = e^{-(\cos t)^2}(\sin 2t + 2\cos 4t + 0.4 \sin t \sin 50t)$



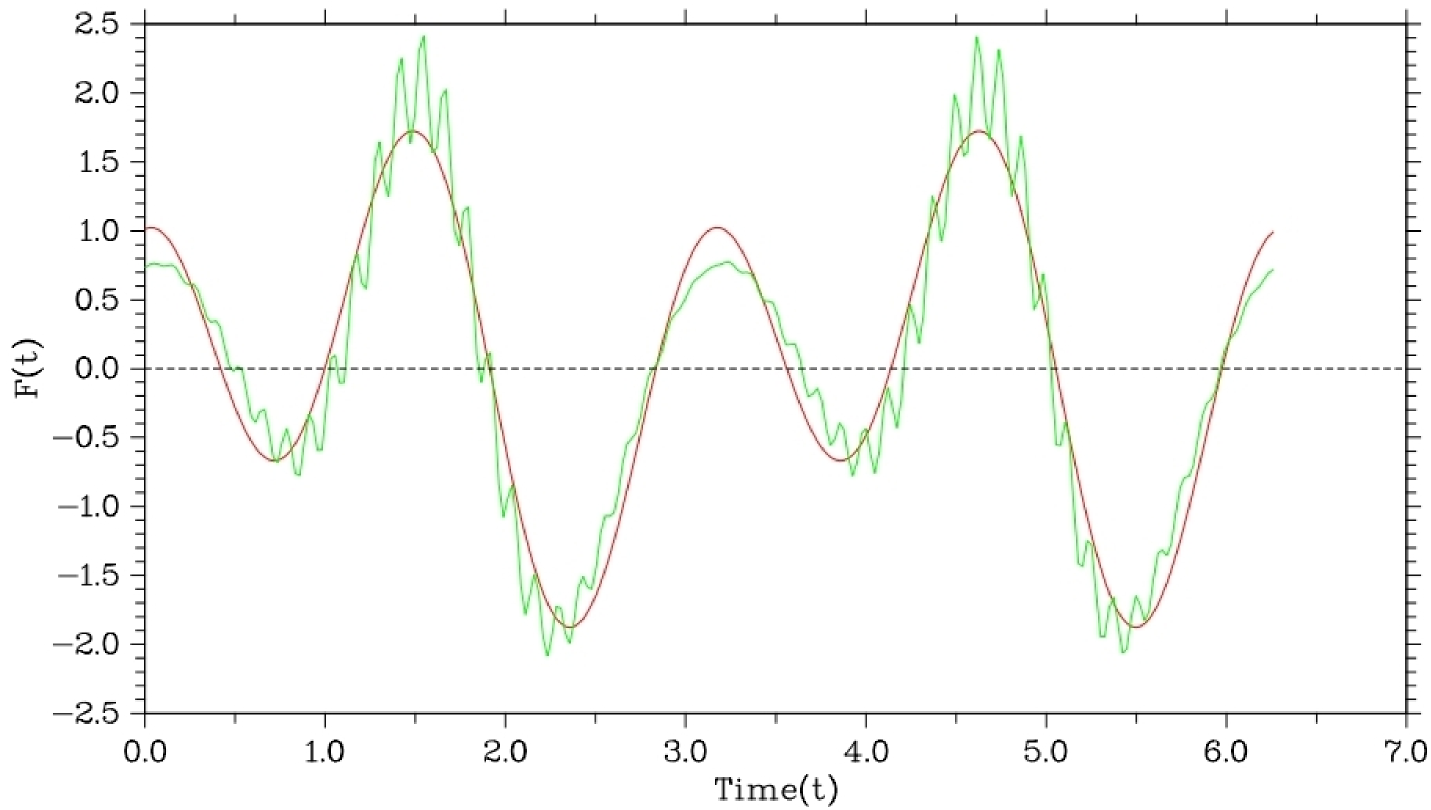
Original Signal discretized with $2^8 = 256$ points. $0 \le t \le 2\pi$

Numerical Recipes Daub 8 used to generate wavelet coefficients $\hat{y}_k$, $k = 0, \ldots, 255$.

Keep only wavelet coefficients with a magnitude > 1 .

Fast Fourier Transform (FFT) – Use to Filter out Noise.

Function: $y(t) = e^{-(\cos t)^2} \left(\sin 2t + 2\cos 4t + 0.4 \sin t \sin 50t \right)$



Filtered by keeping first five coefficients and completing inverse FFT.

$$Error = \frac{\| y - y_c \|_{l^2}}{\| y \|_{l^2}} = 0.26$$

Original Signal discretized with $2^8 = 256$ points. $0 \le t \le 2\pi$

Sun Performance Library FFT used to generate the Discrete Fourier Coefficients $\hat{y}_k$, $k = 0, \ldots, 255$.

Noise has a frequency that is larger than 5 cycles per 2π interval.

Keep only $\hat{y}_k$ for $0 \le k \le 5$ and set $\hat{y}_k = 0.0$ for $6 \le k \le 128$.

By Theorem, $\hat{y}_k = 0$ for $128 \le k \le 250$. Applying FFT to filter $\hat{y}_k$

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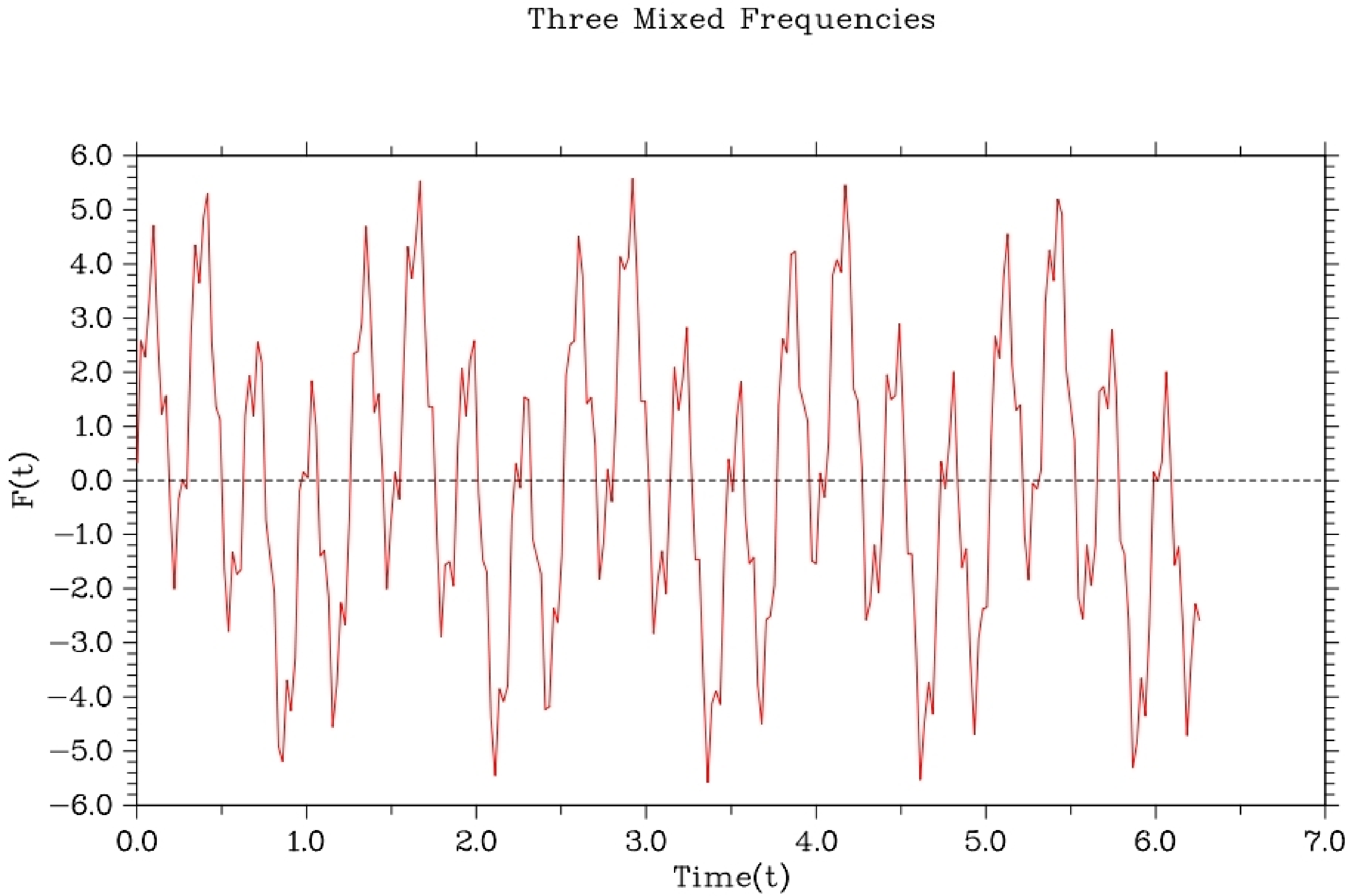
IV. Comparison of DWT to FFT

Comparison of DWT to FFT

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t) + \sin(2\pi f_3 t)$$

$f_1 = 5\ Hz$
$f_2 = 20\ Hz$
$f_3 = 80\ Hz$



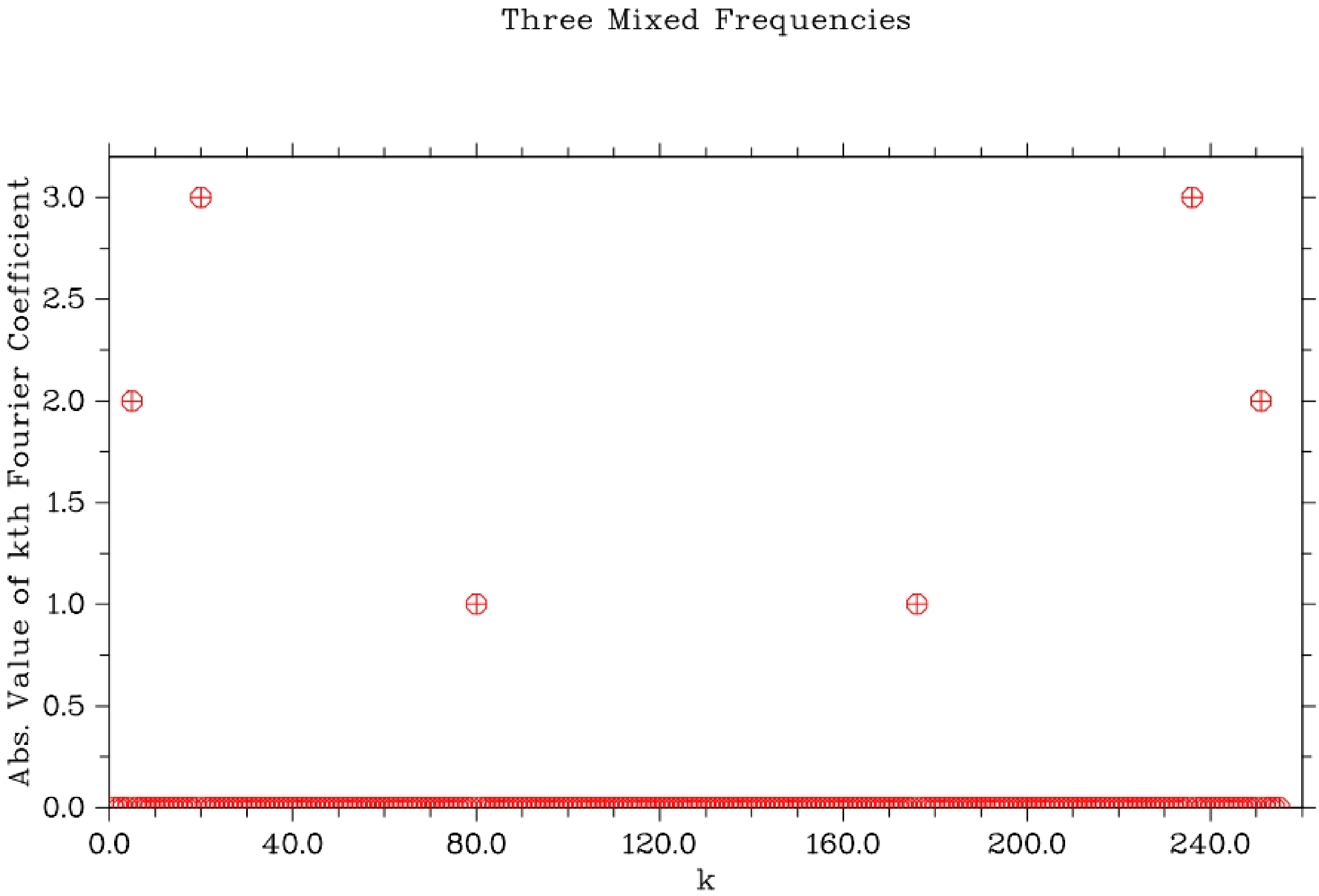
Comparison of DWT to FFT

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t) + \sin(2\pi f_3 t)$$

$f_1 = 5\,Hz$

$f_2 = 20\,Hz$

$f_3 = 80\,Hz$



Comparison of DWT to FFT

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t) + \sin(2\pi f_3 t)$$

$f_1 = 5\ Hz$

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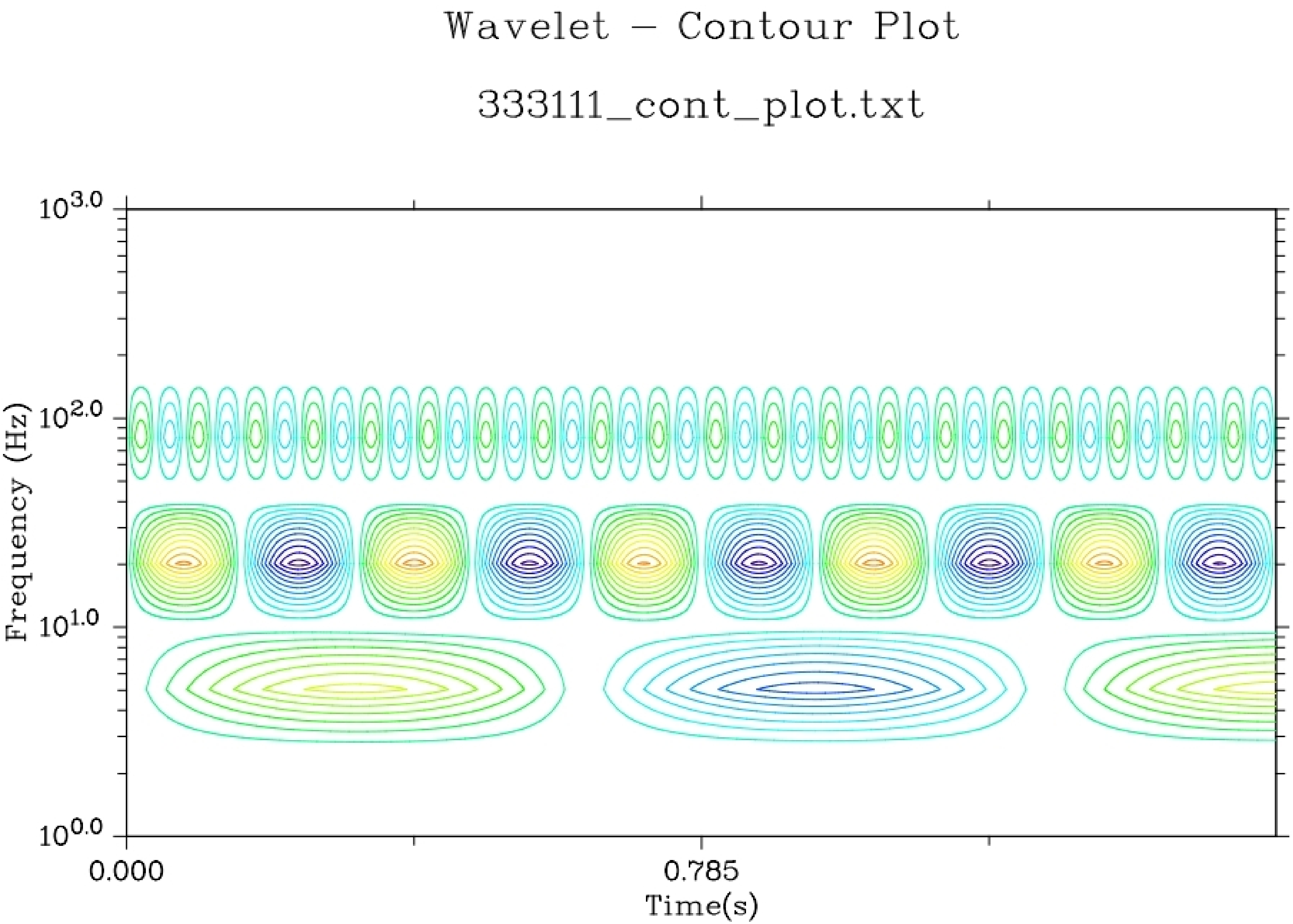

Comparison of DWT to FFT

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

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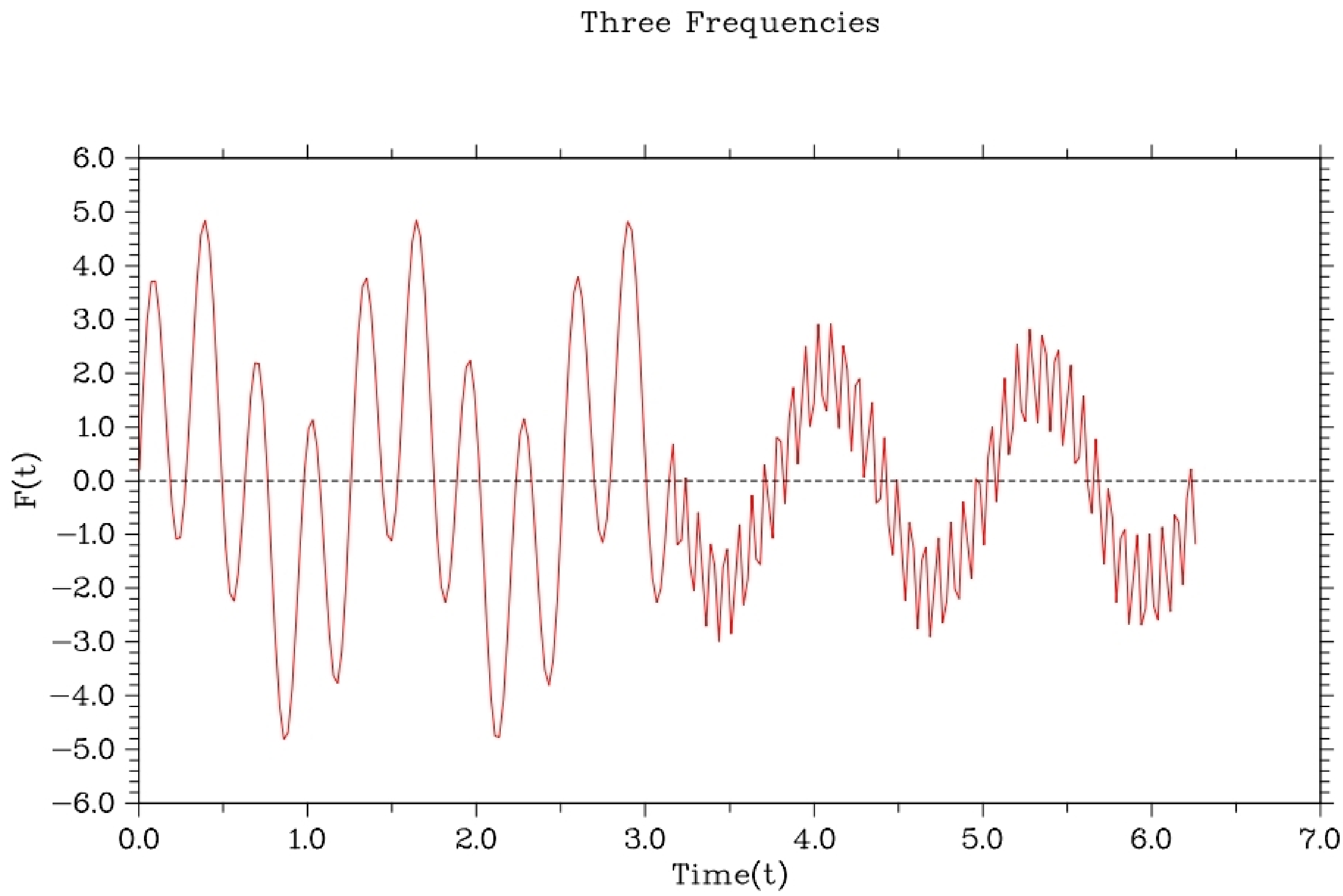

Comparison of DWT to FFT

$$f_1 = 5\ Hz$$
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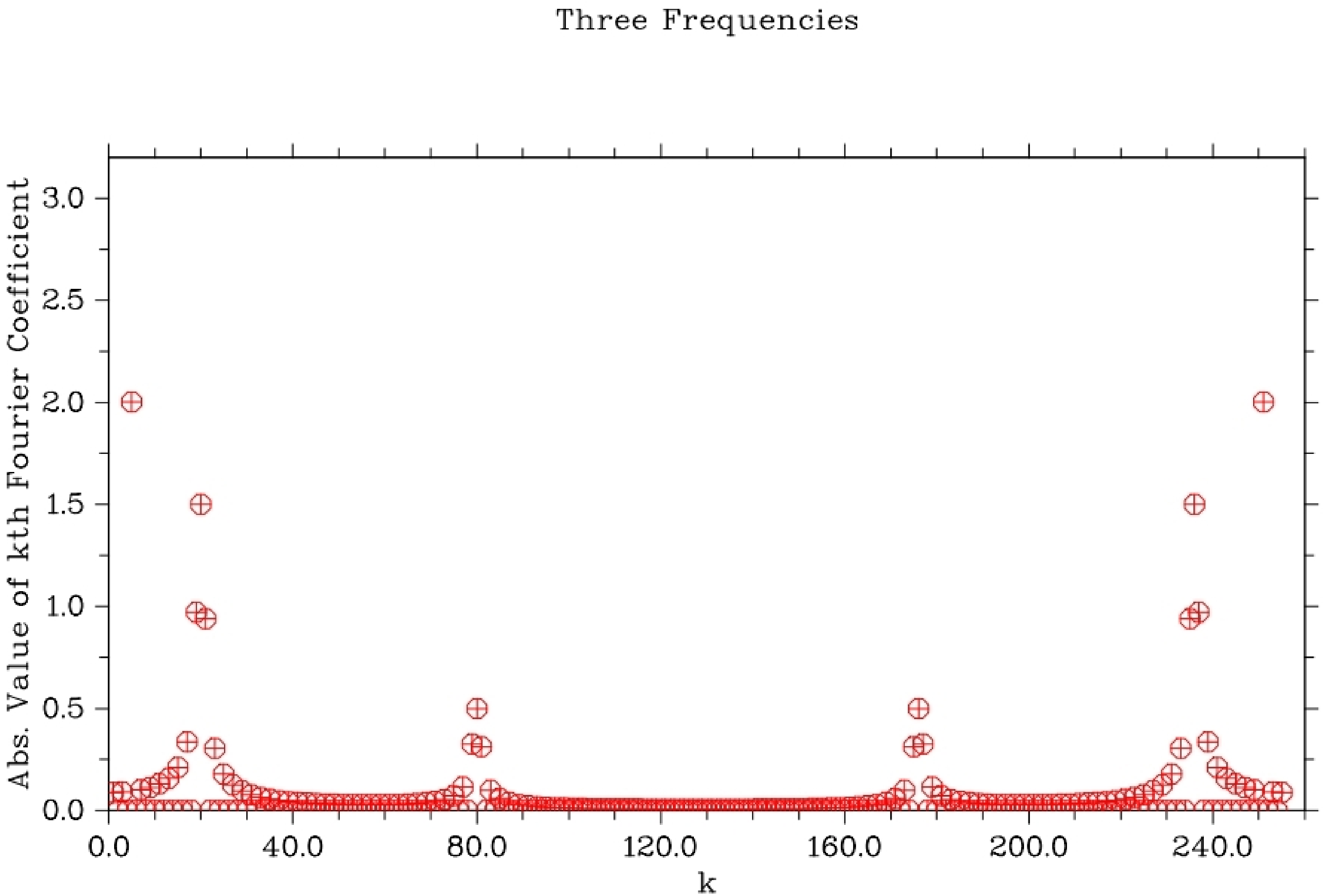
$$f_3 = 80\ Hz$$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

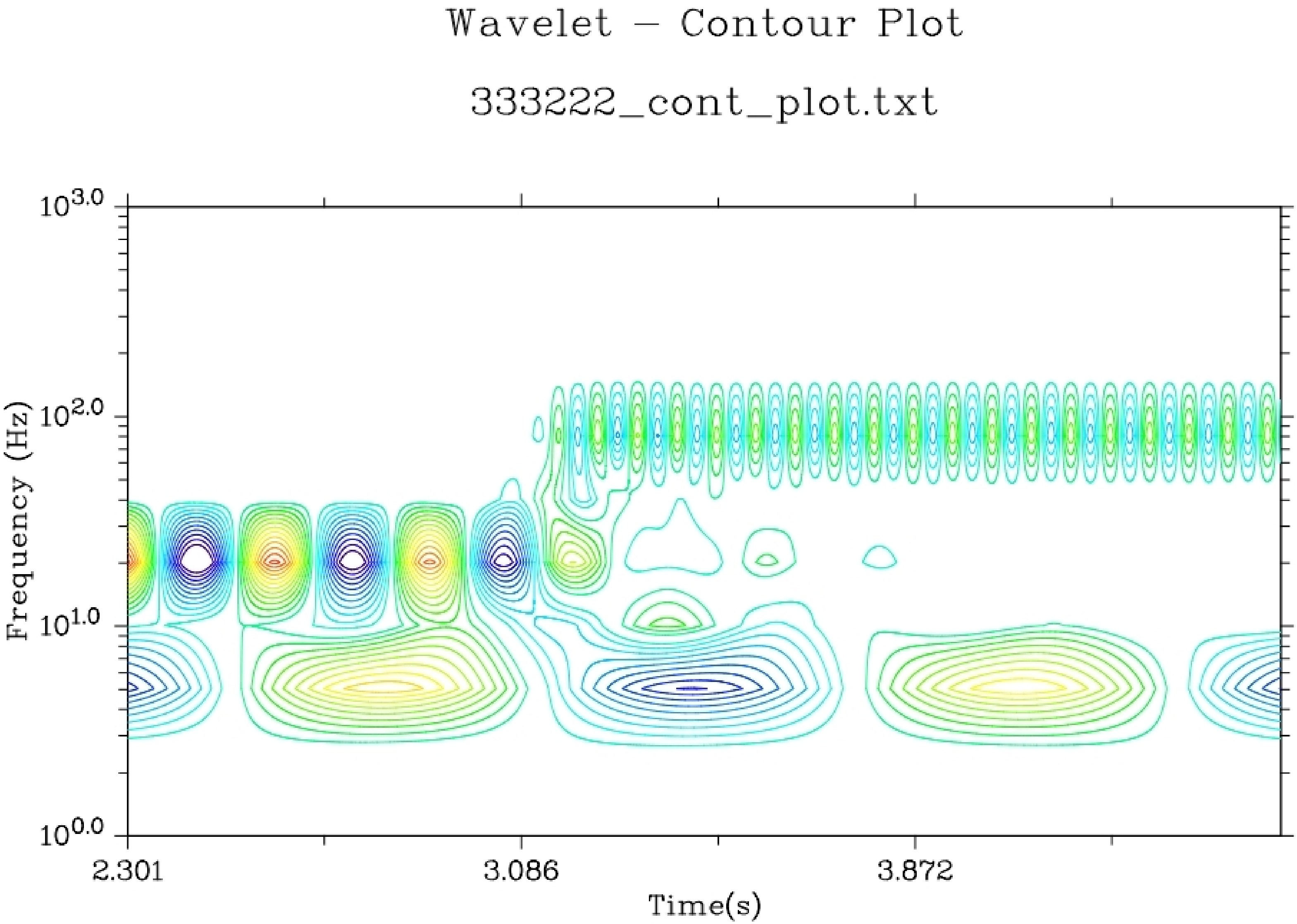


Comparison of DWT to FFT

$f_1 = 5\ Hz$
$f_2 = 20\ Hz$
$f_3 = 80\ Hz$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

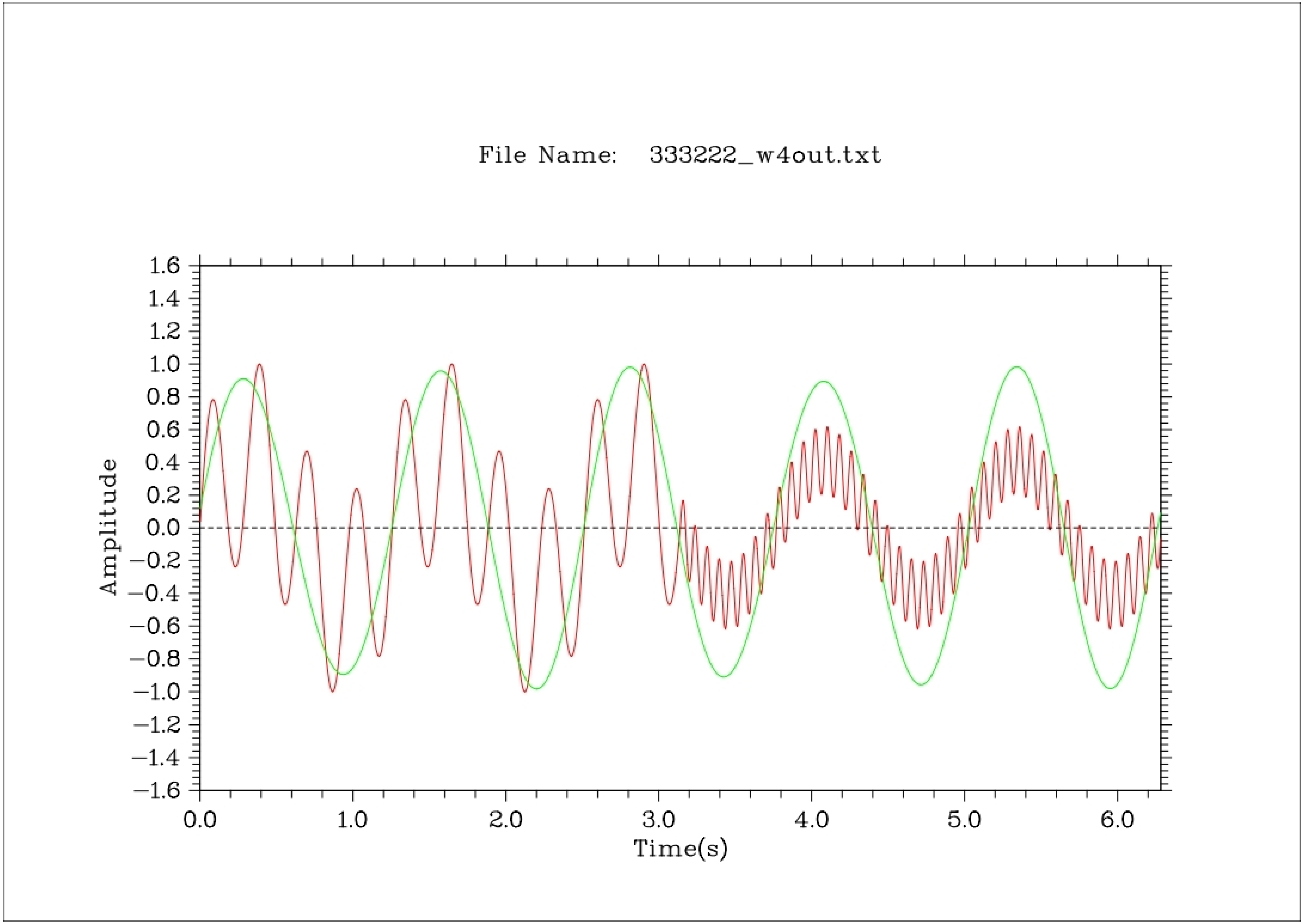


Comparison of DWT to FFT

$f_1 = 5\ Hz$
$f_2 = 20\ Hz$
$f_3 = 80\ Hz$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

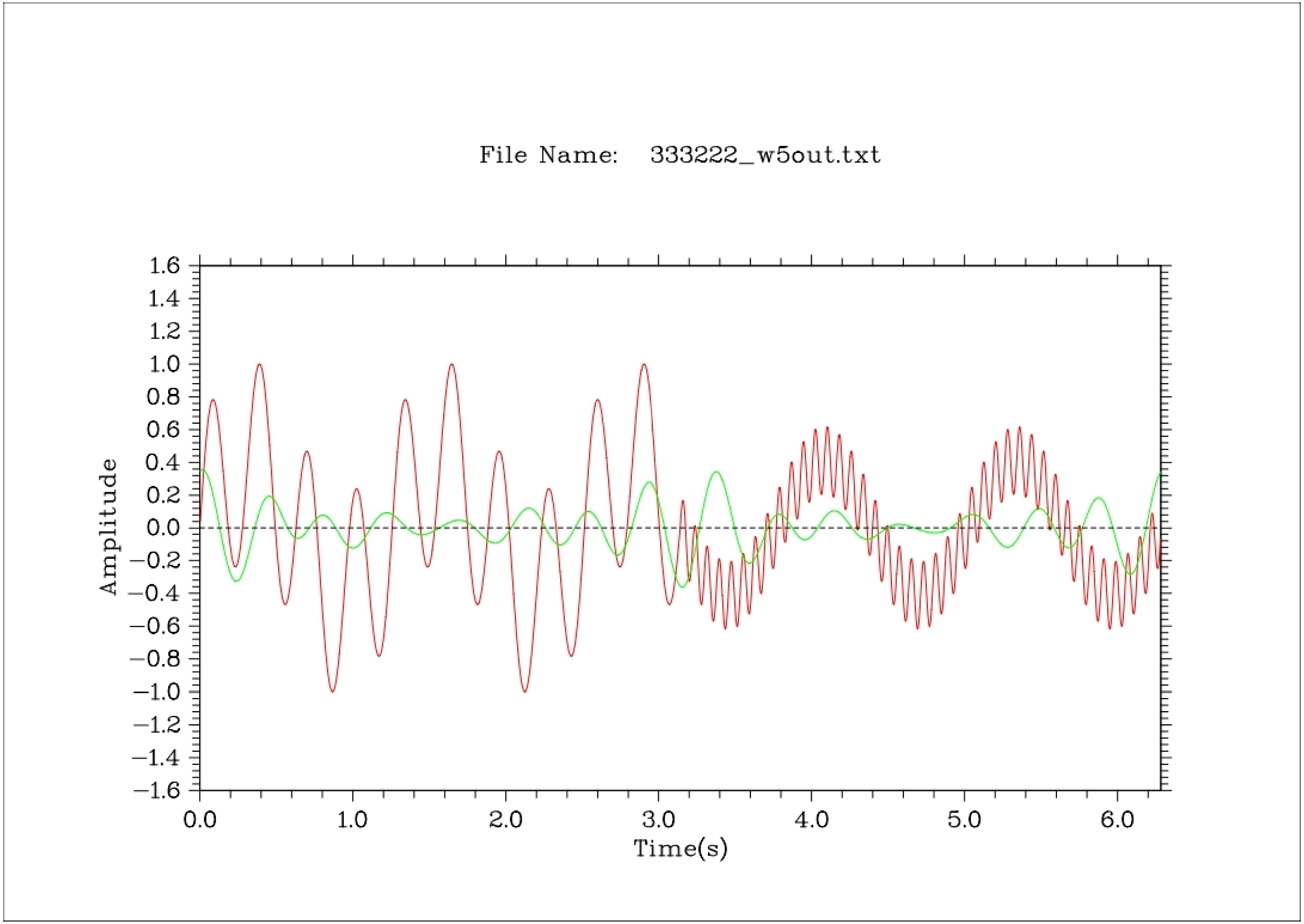


Comparison of DWT to FFT

$$f_1 = 5\ Hz$$
$$f_2 = 20\ Hz$$
$$f_3 = 80\ Hz$$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

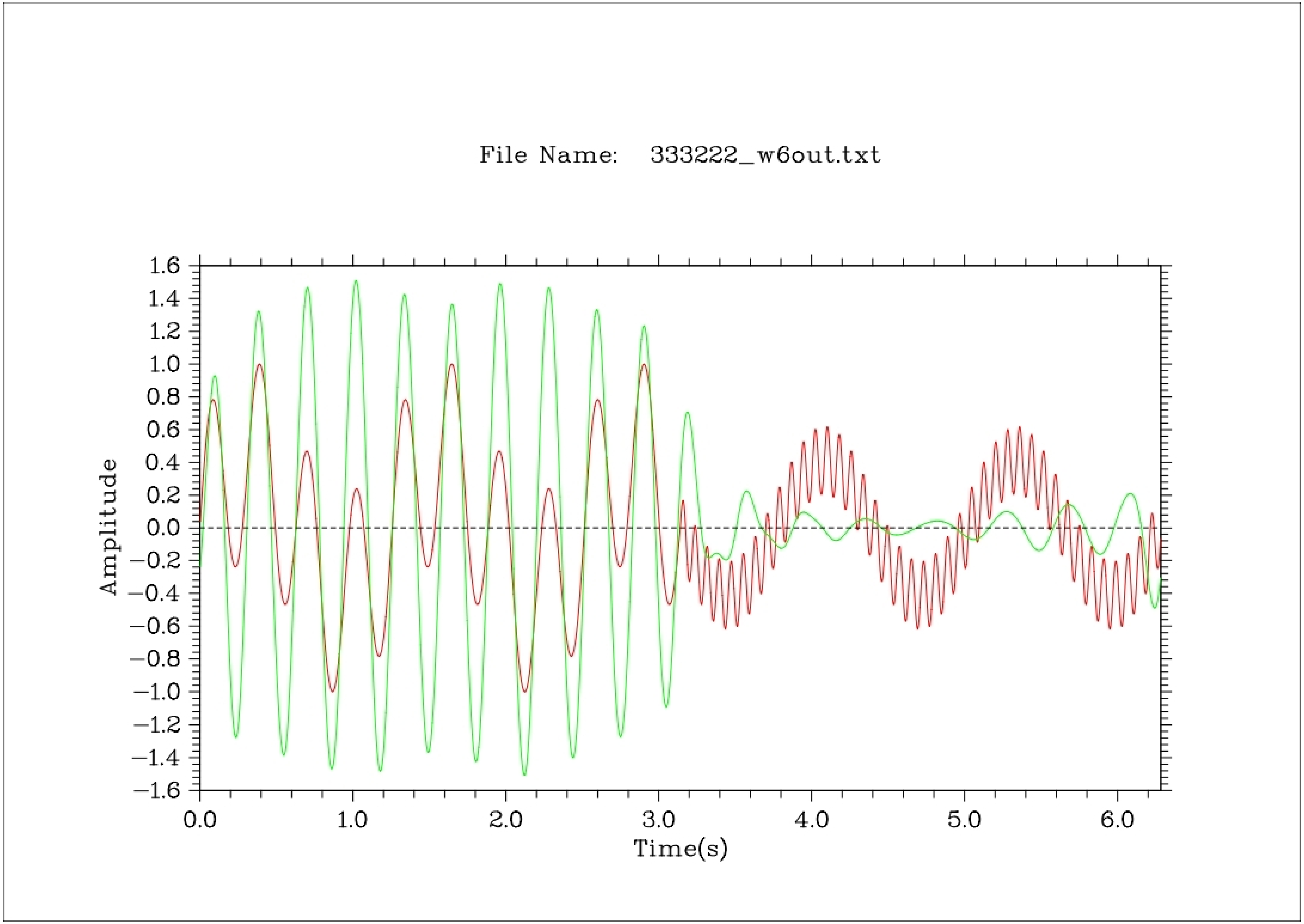


Comparison of DWT to FFT

$$f_1 = 5\ Hz$$
$$f_2 = 20\ Hz$$
$$f_3 = 80\ Hz$$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$

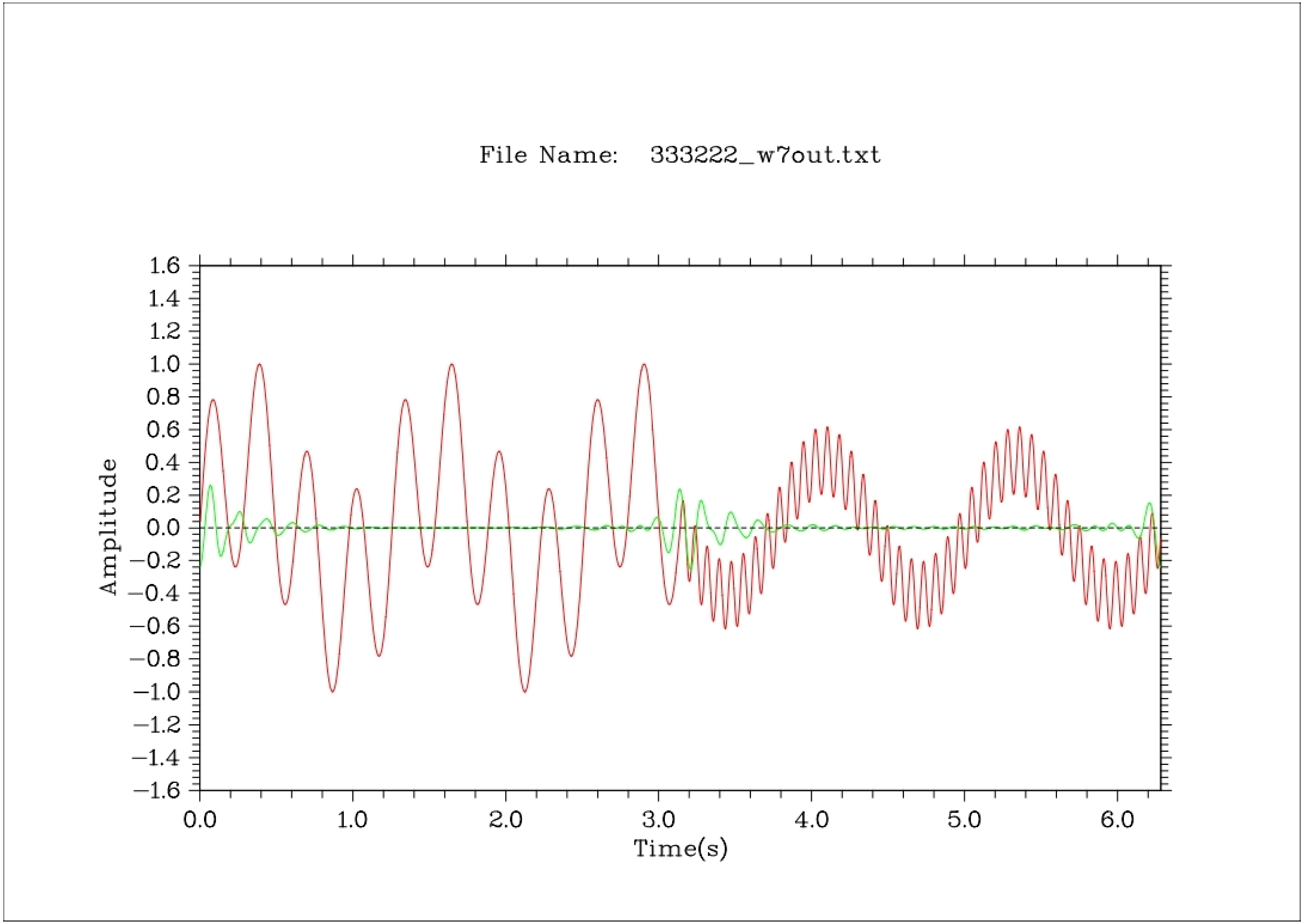


Comparison of DWT to FFT

$$f_1 = 5\ Hz$$
$$f_2 = 20\ Hz$$
$$f_3 = 80\ Hz$$

$$f(t) = 2\sin(2\pi f_1 t) + 3\sin(2\pi f_2 t)$$

$$f(t) = 2\sin(2\pi f_1 t) + \sin(2\pi f_3 t)$$



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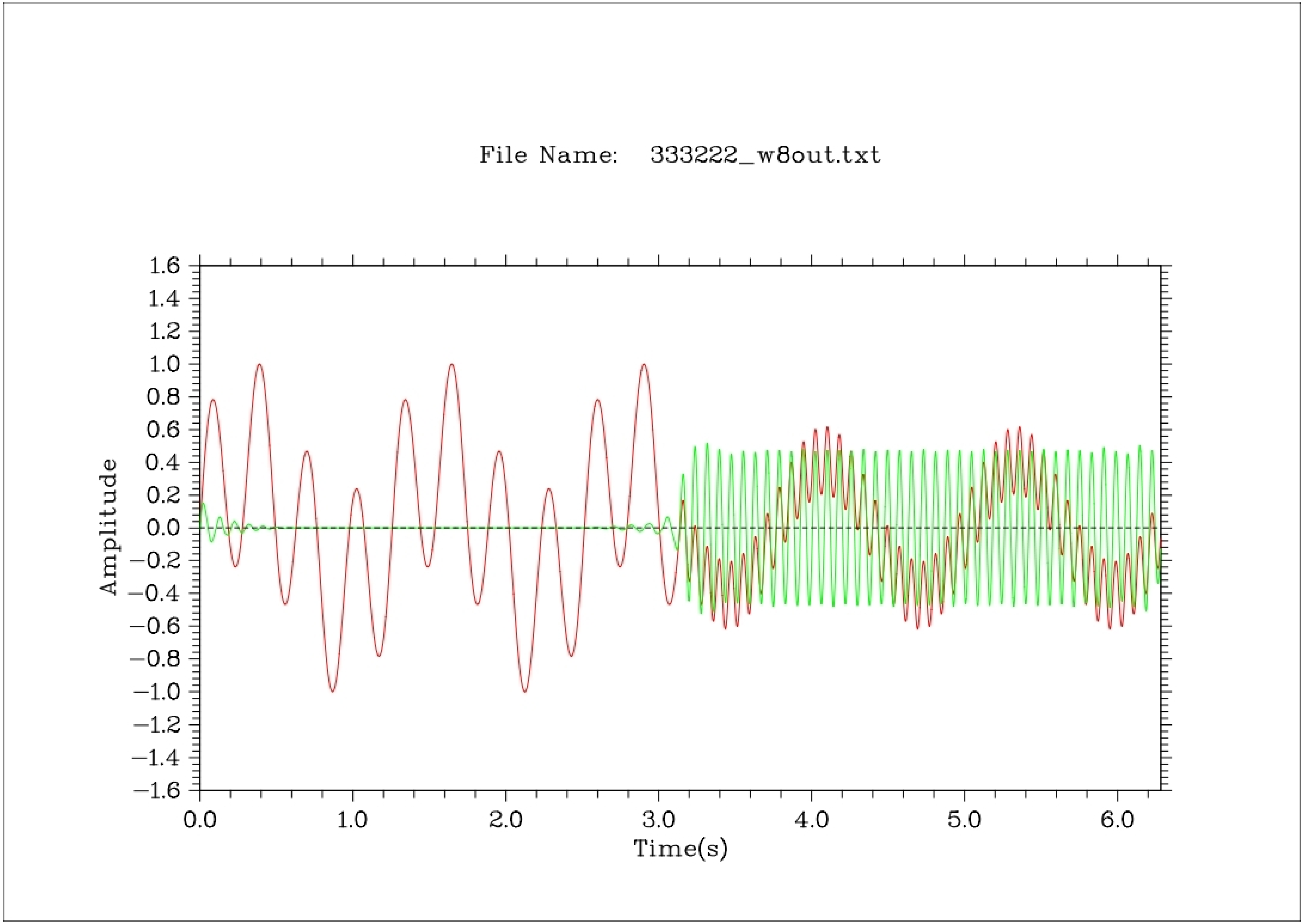
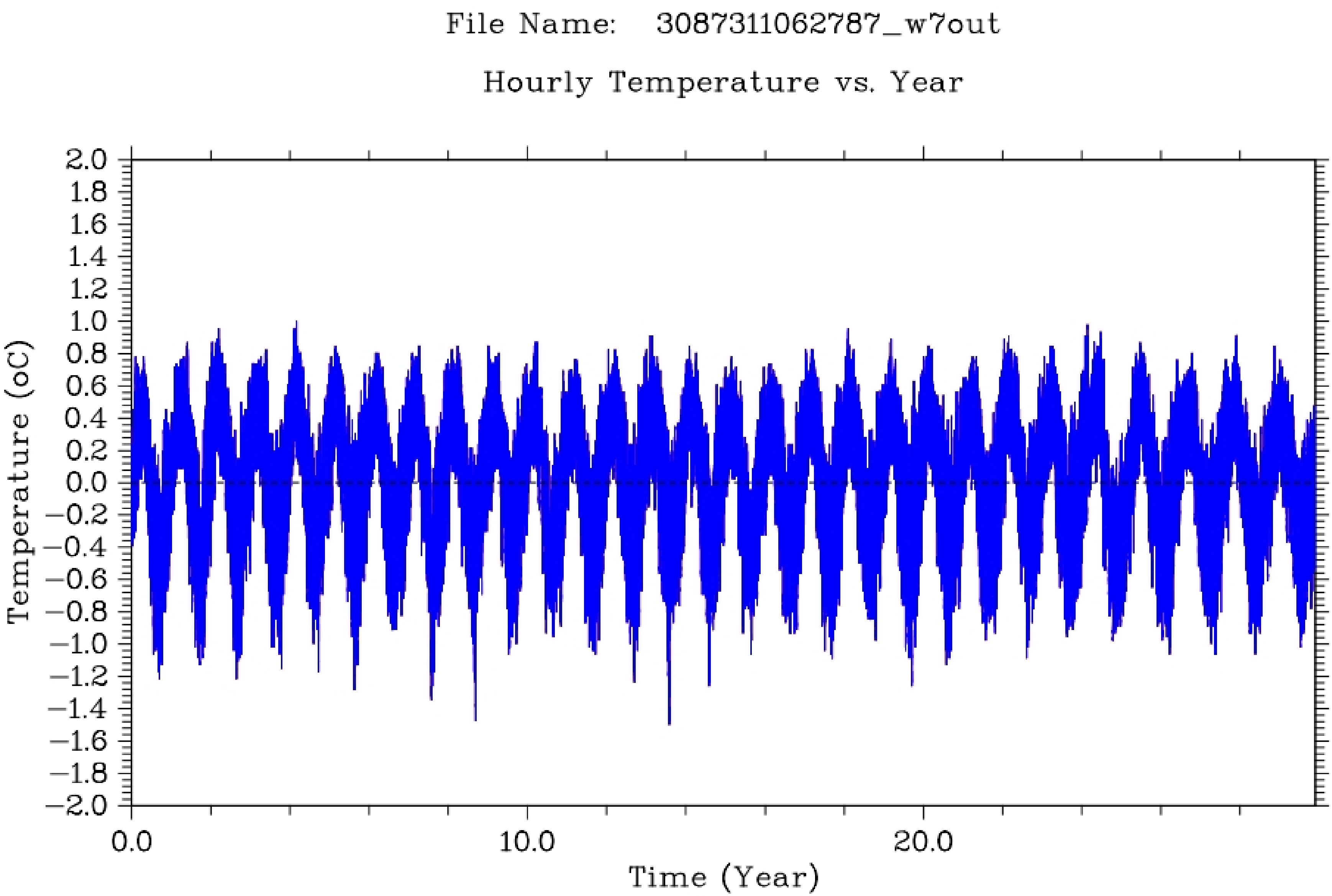


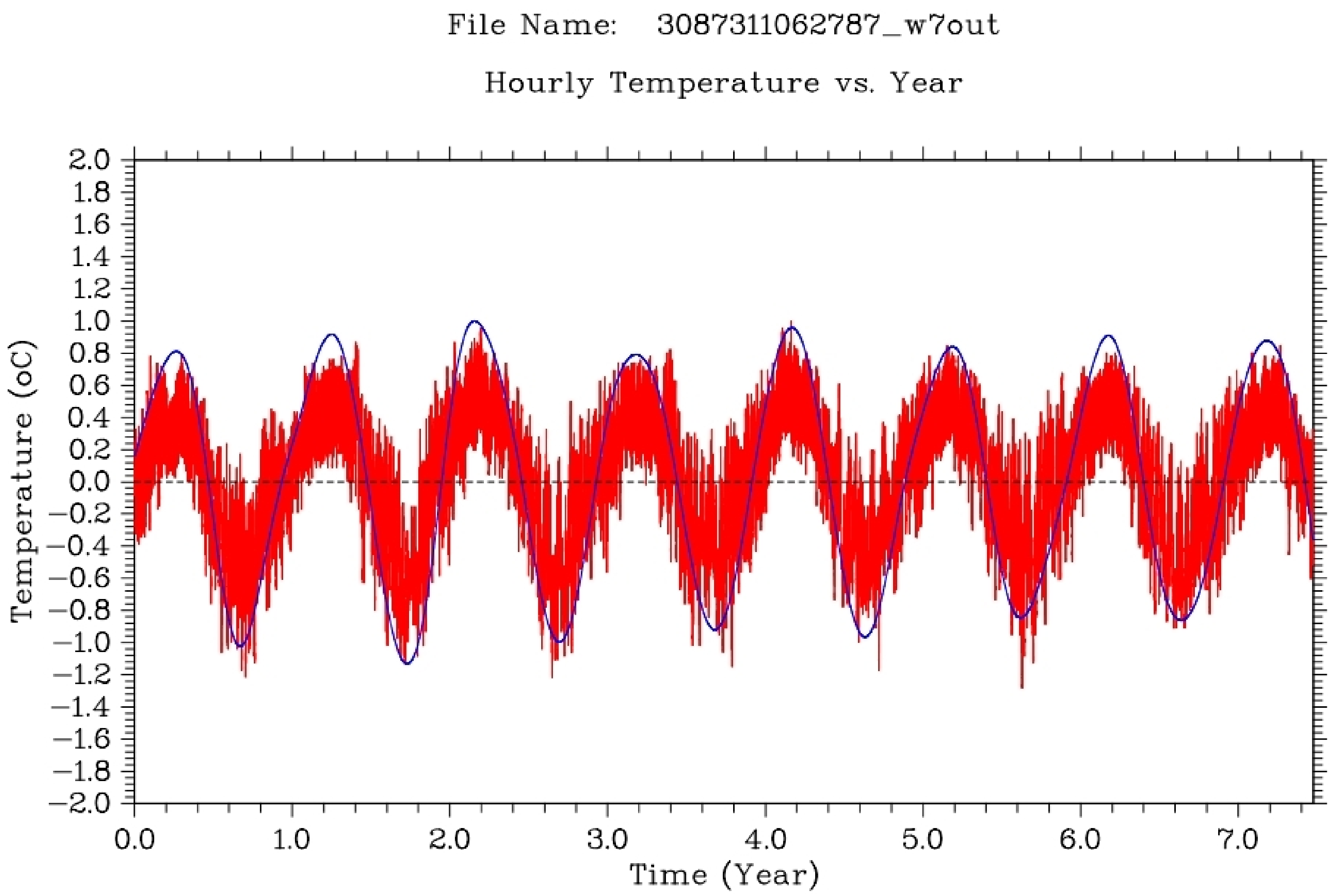
V. Wavelet Analysis for Temperature Data

Wavelet Analysis for Temperature Data



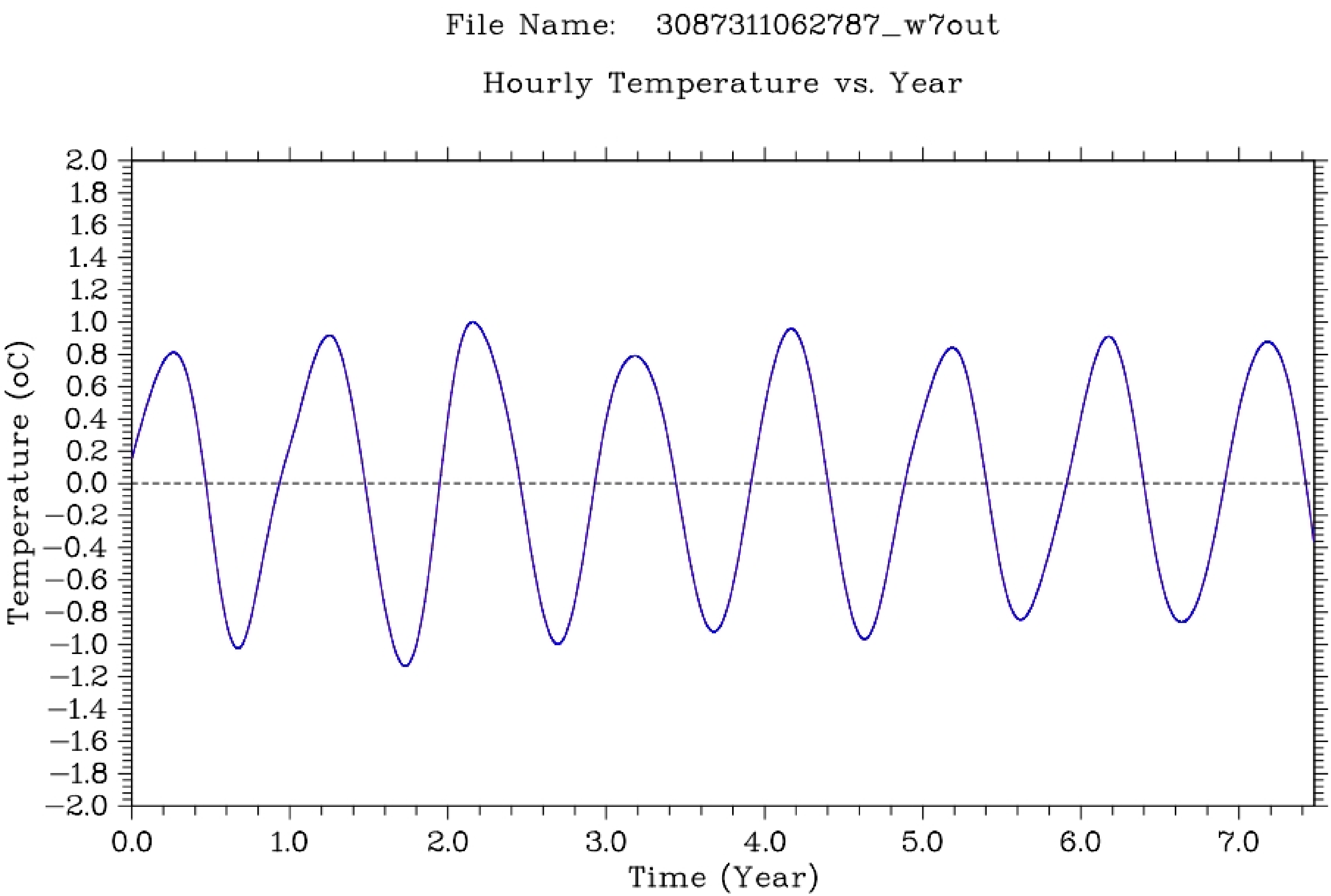
Original Signal

Wavelet Analysis for Temperature Data



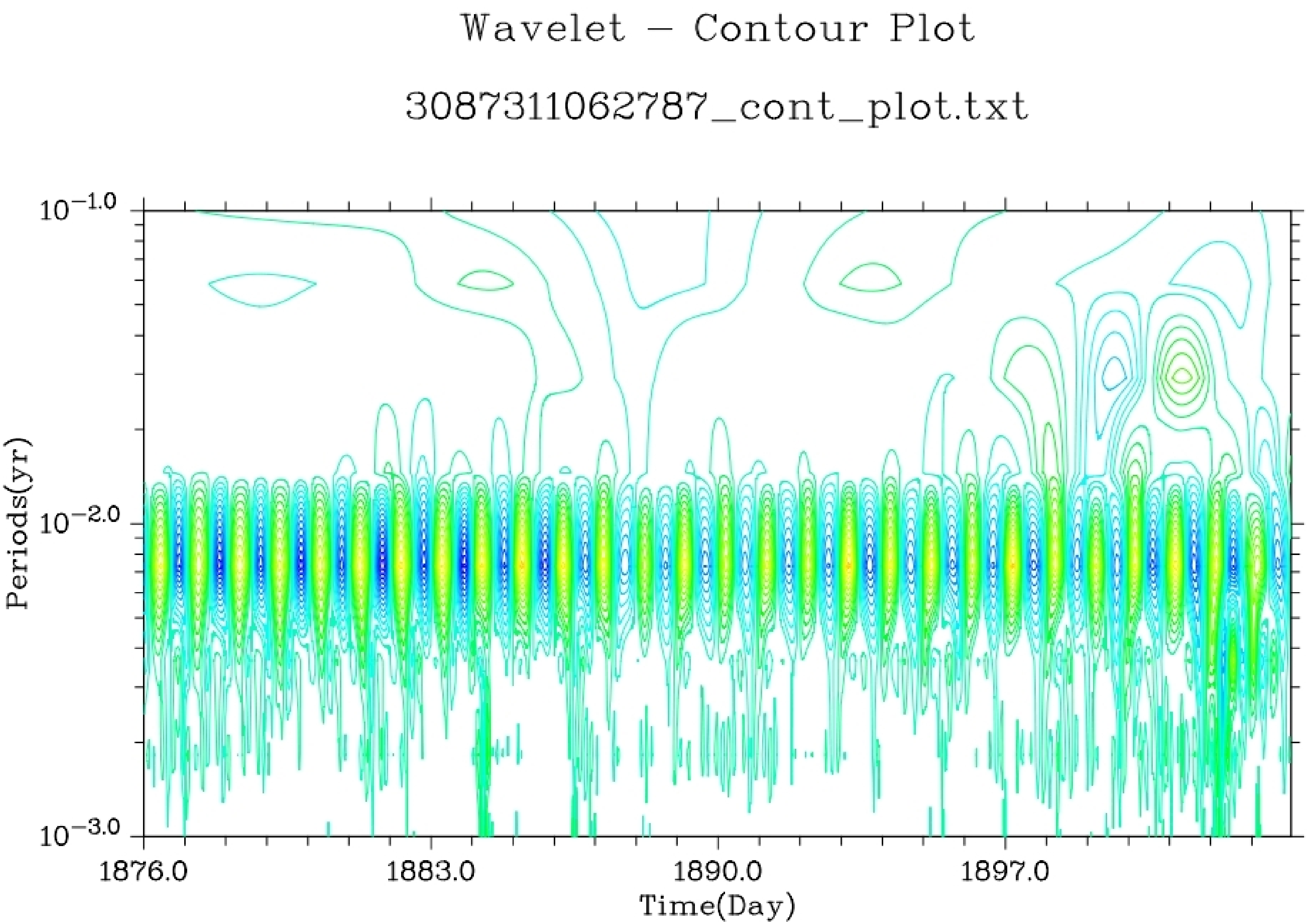
Original and Filtered Signal

Wavelet Analysis for Temperature Data

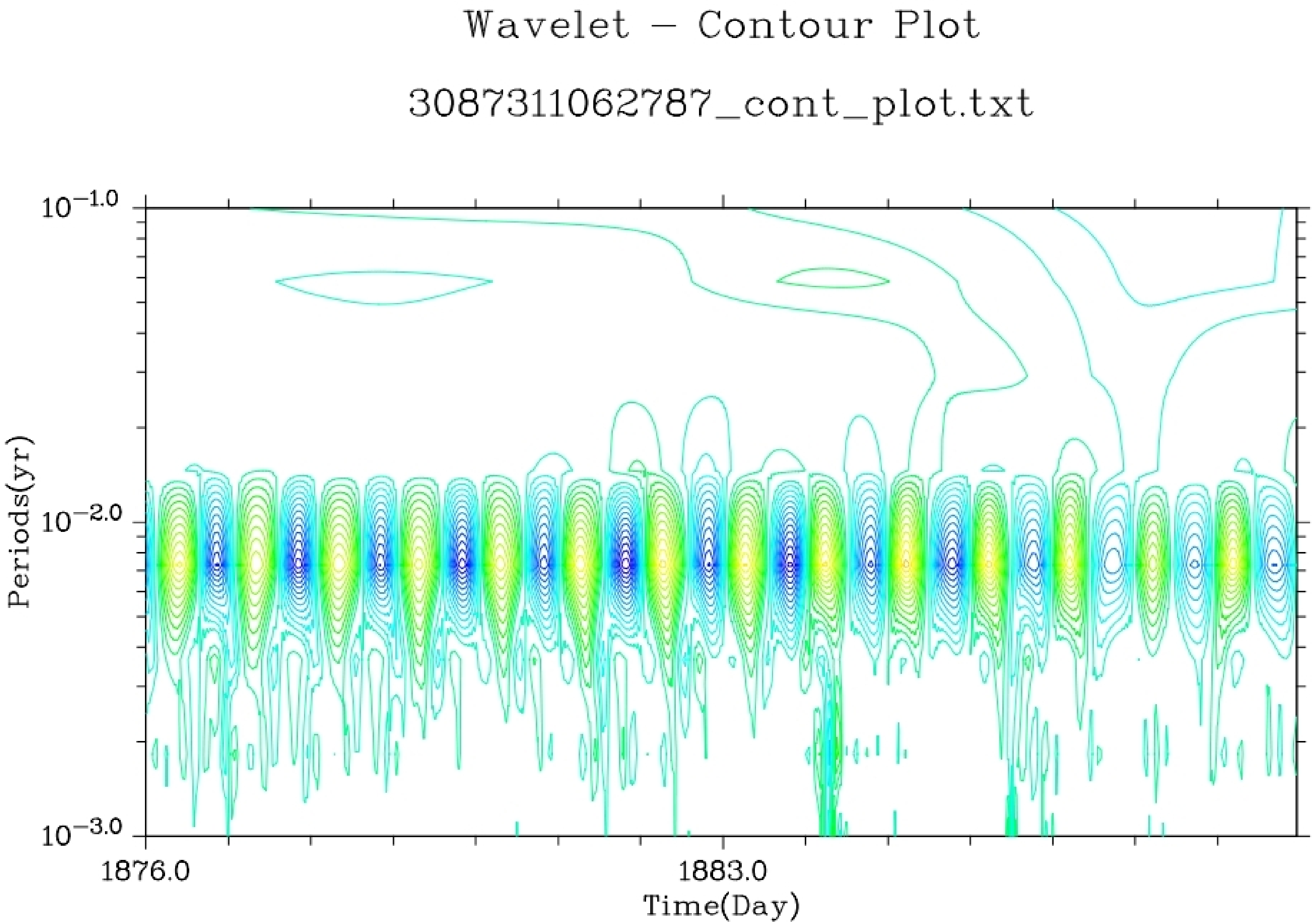


Filtered Signal

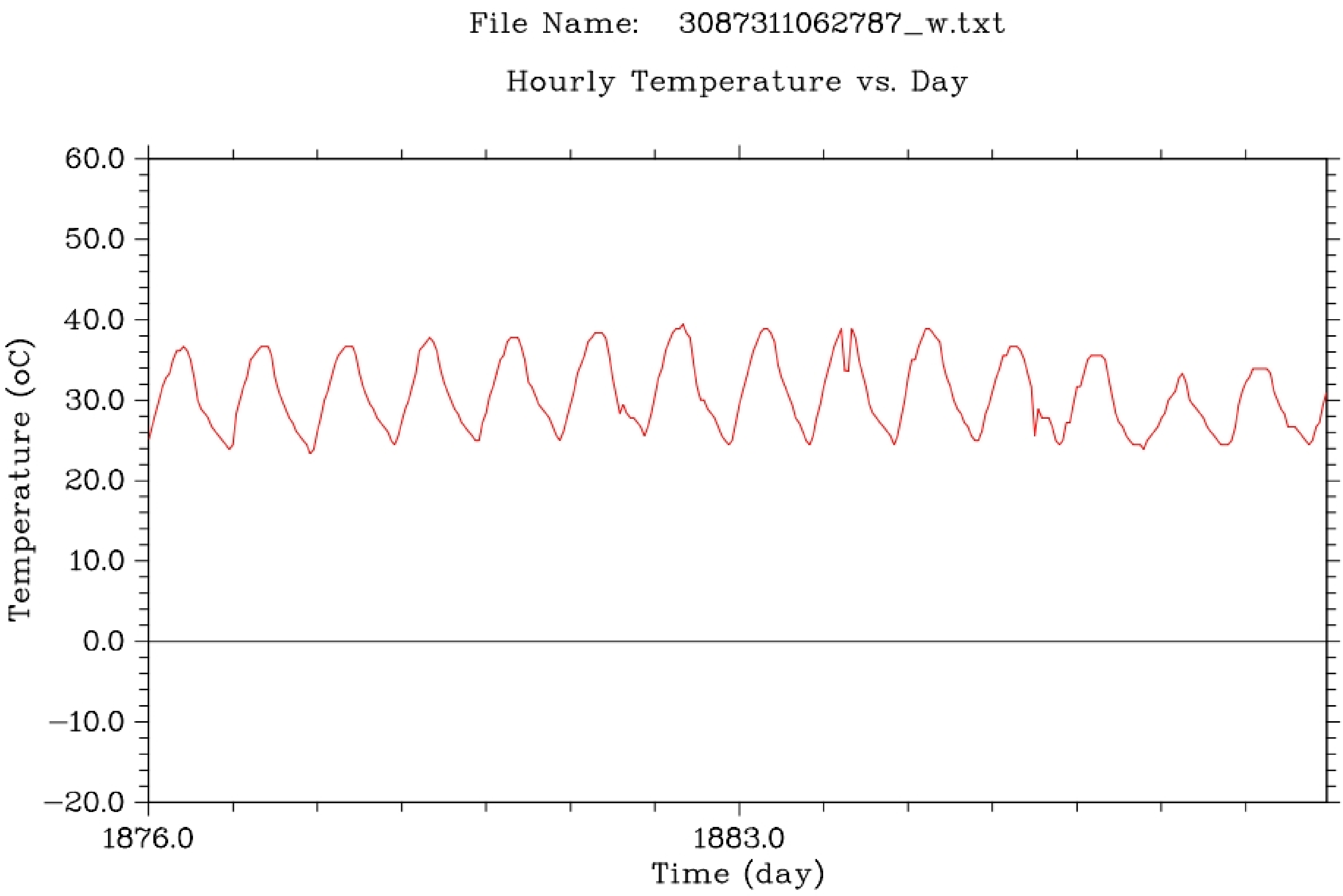
Wavelet Analysis for Temperature Data



Wavelet Analysis for Temperature Data

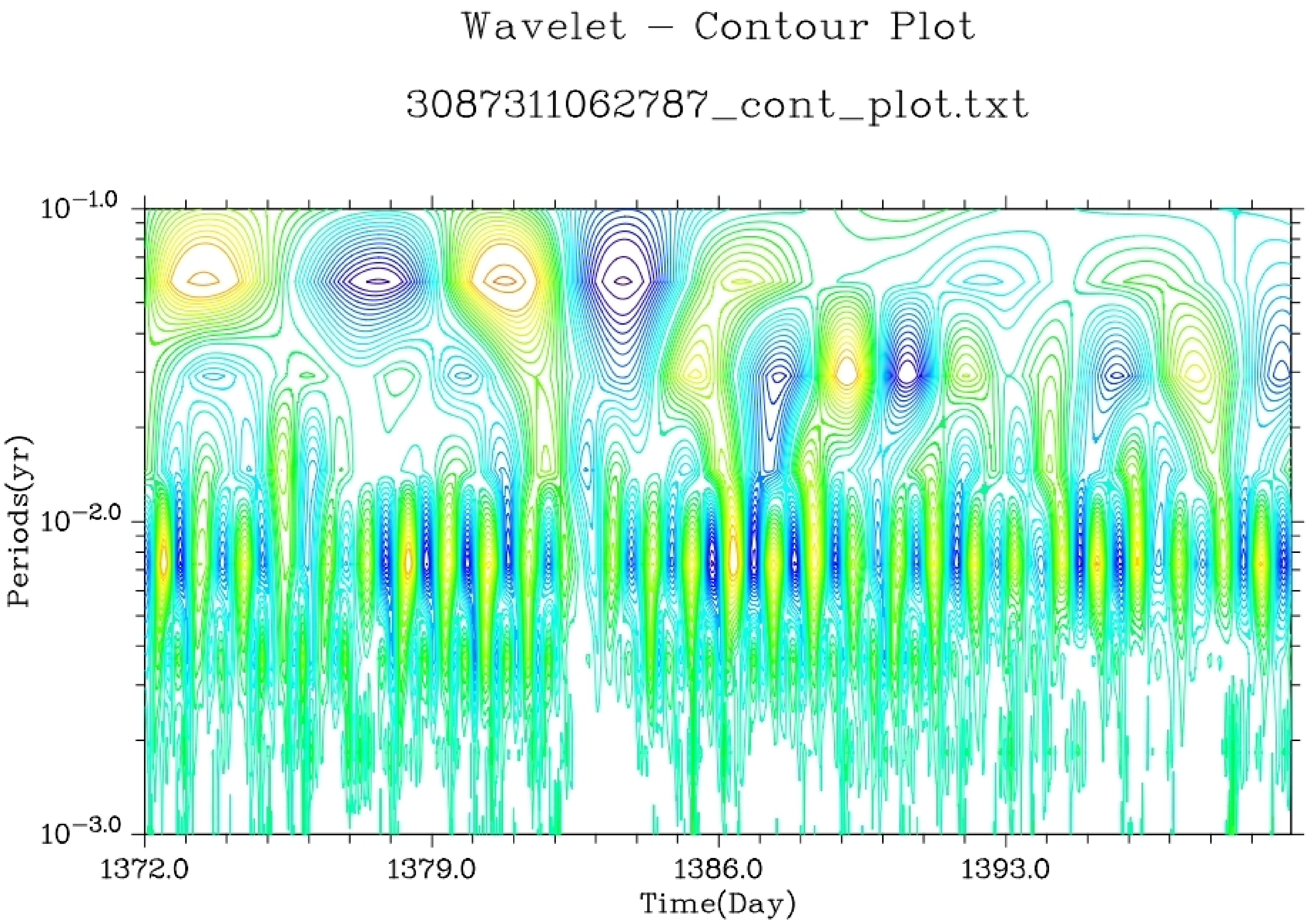


Wavelet Analysis for Temperature Data

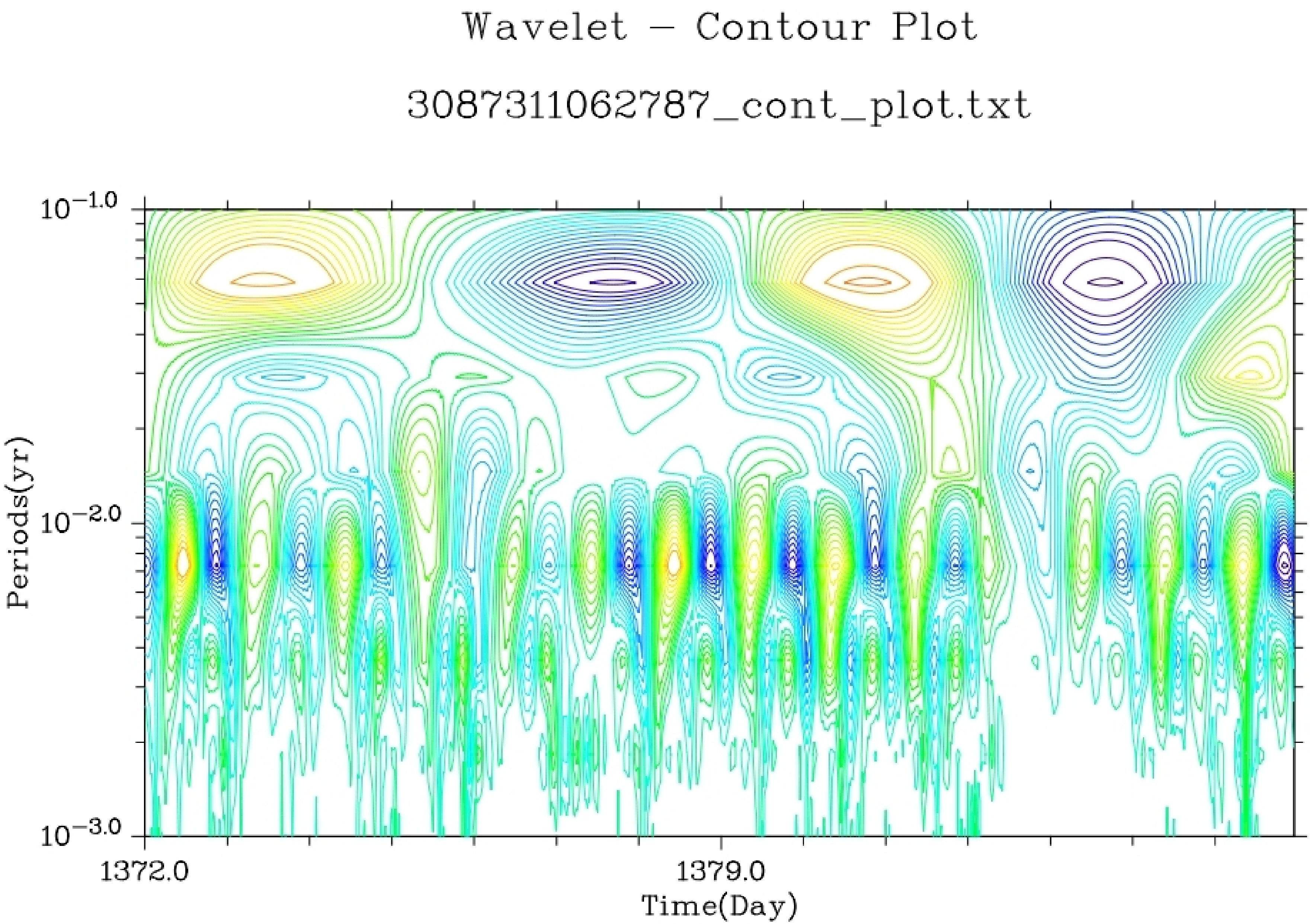


Original Signal

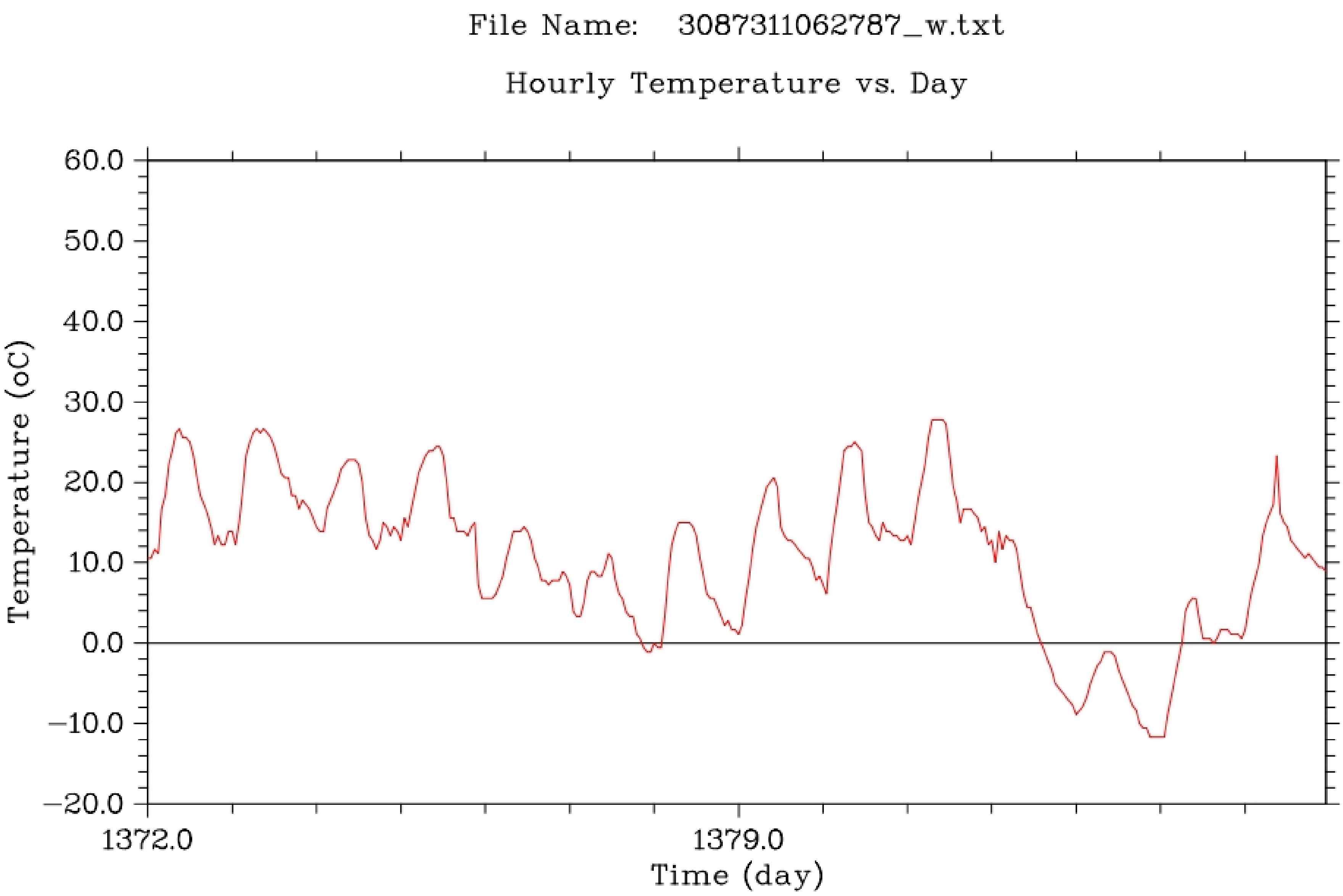
Wavelet Analysis for Temperature Data



Wavelet Analysis for Temperature Data



Wavelet Analysis for Temperature Data



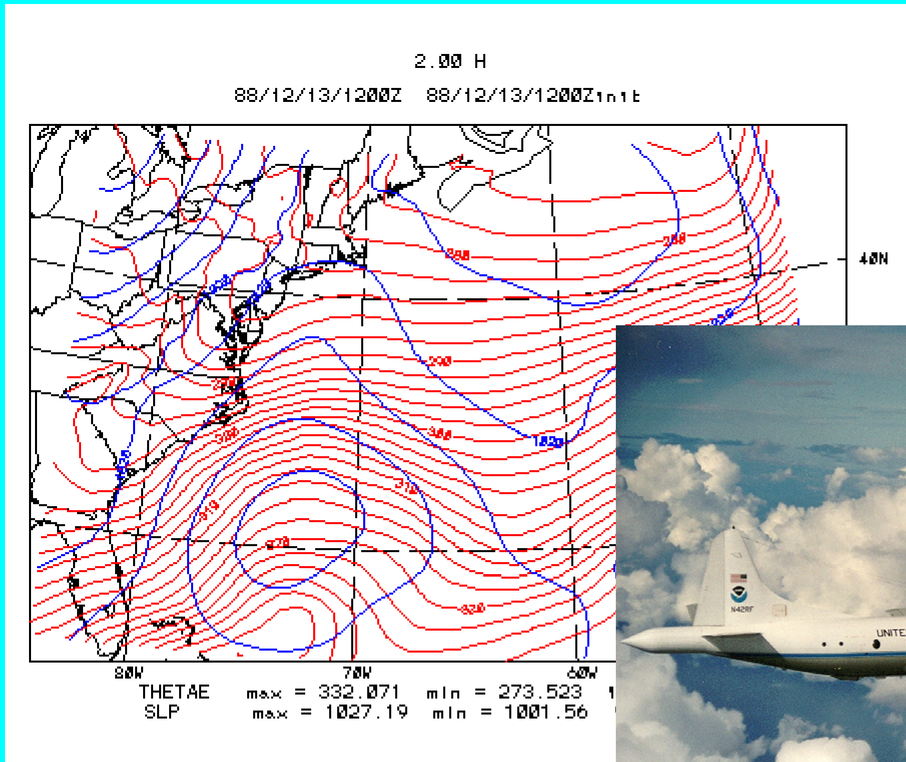
Original Signal

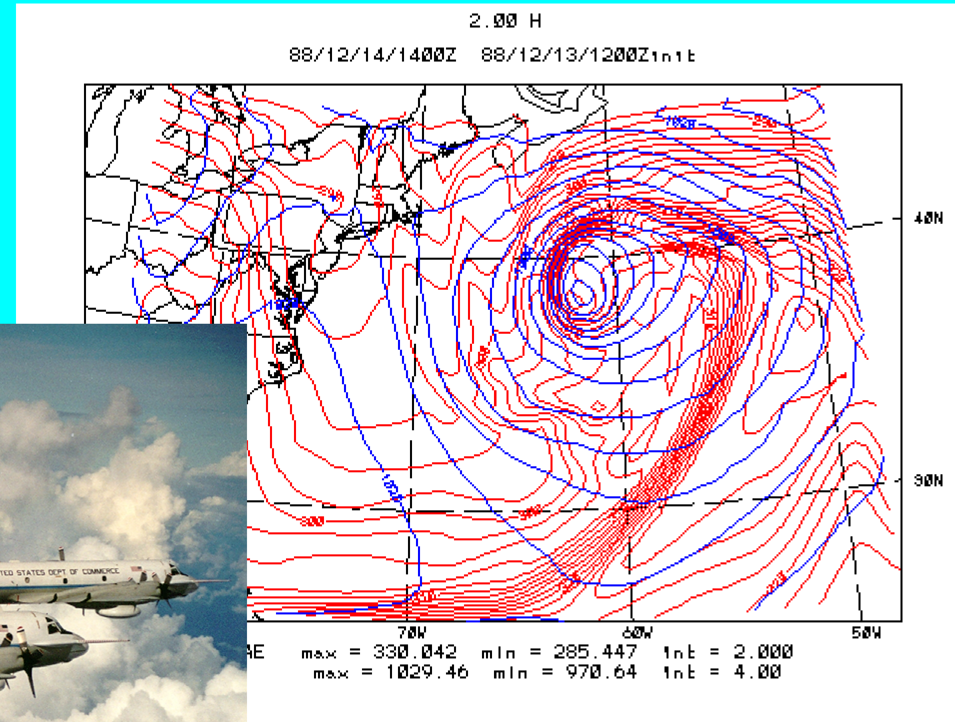
Conclusions:

1.) The structure of the fronts was found from the data collected.
2.) The model reproduced the cyclone well, even though the resolution was too coarse to reproduce the fronts.
3.) Wavelet Analysis can be useful to analyze the data.

What to do Next:

1.) Continue to improve the model, increasing resolution.
2.) Analyze the data collected and the model data to see how well the model reproduces the atmosphere.





Thank You.
Question?

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610-348-6495