Electrical Energy and Capacitance
Coulomb's Law

\[ |F| = \frac{k |q_1||q_2|}{r^2} \]

\[ k = 9 \times 10^9 \frac{N \, m^2}{C^2} \]
Find net force on charge \( q_0 \):

\[
q_0 = +20 \text{nC} \\
q_1 = +30 \text{nC} \\
q_2 = -50 \text{nC}
\]
Find net force on charge $q_0$:

$$q_0 = +20 \text{nC}$$
$$q_1 = +50 \text{nC}$$
$$q_2 = -20 \text{nC}$$
Find net force on charge $q_0$:

\[
\vec{F}_{net_0} = \vec{F}_{10} - \vec{F}_{20}
\]

Given charges:

- $q_0 = +20 \text{ nC}$
- $q_1 = +50 \text{ nC}$
- $q_2 = -20 \text{ nC}$

Using Coulomb's law:

\[
|F_{10}| = k \frac{|q_1||q_0|}{r_{10}^2} = 9 \times 10^9 \text{ Nm}^2/C^2 \left( \frac{50 \times 10^{-9} \text{ C}}{20 \times 10^{-9} \text{ C}} \right) \left( \frac{0.003 \text{ m}}{0.004 \text{ m}} \right)^2 = 1.0 \text{ N}
\]
Find net force on charge $q_0$:

$q_0 = +20 \text{nC}$
$q_1 = +50 \text{nC}$
$q_2 = -20 \text{nC}$

\[ |\vec{F}_{10}| = k \frac{|q_1| |q_0|}{r_{10}^2} = 9 \times 10^9 \frac{Nm^2}{C^2} \left( \frac{50 \times 10^{-9} C}{0.003m} \right) \left( 20 \times 10^{-9} C \right) = 1.0 \text{ N} \]

$\vec{F}_{10} = 1.0 \text{ N} \hat{i} + 0.0 \text{ N} \hat{j}$
Find net force on charge $q_0$:

$q_0 = +20 \text{ nC}$
$q_1 = +50 \text{ nC}$
$q_2 = -20 \text{ nC}$

$F_{10} = 1.0 \text{ N} \hat{i} + 0.0 \text{ N} \hat{j}$

$|F_{20}| = k \frac{|q_2||q_0|}{r_{20}^2} = 9 \times 10^9 \frac{Nm^2}{C^2} \frac{20 \times 10^{-9} C}{(0.004m)^2} \frac{20 \times 10^{-9} C}{0.003m} = 0.225 \text{ N}$
Find net force on charge $q_0$:

$q_0 = +20 \text{nC}$
$q_1 = +50 \text{nC}$
$q_2 = -20 \text{nC}$

\[ \vec{F}_{10} = 1.0 \text{ N} \hat{i} + 0.0 \text{ N} \hat{j} \]

\[ \vec{F}_{20} = 0.0 \text{ N} \hat{i} - 0.225 \text{ N} \hat{j} \]

\[ |F_{20}| = k \frac{|q_2||q_0|}{r_{20}^2} = 9 \times 10^9 \frac{Nm^2}{C^2} \left( \frac{20 \times 10^{-9} C}{0.004m} \right) \left( \frac{20 \times 10^{-9} C}{0.004m} \right) = 0.225 \text{ N} \]
Find net force on charge $q_0$:

$\mathbf{r}_1$

\[ q_0 = +20 \text{ nC} \]
\[ q_1 = +50 \text{ nC} \]
\[ q_2 = -20 \text{ nC} \]

\[ \mathbf{F}_{10} = 1.0 \text{ N} \hat{i} + 0.0 \text{ N} \hat{j} \]
\[ \mathbf{F}_{20} = 0.0 \text{ N} \hat{i} - 0.225 \text{ N} \hat{j} \]

\[ \mathbf{F}_{\text{net}_0} \]

\[ |\mathbf{F}_{\text{net}_0}| = \sqrt{(1.0 \text{ N})^2 + (-0.225 \text{ N})^2} = 1.025 \text{ N} \]

\[ \theta_o = \tan^{-1}\left(\frac{-0.225 \text{ N}}{1.0 \text{ N}}\right) = -12.68^\circ \]
Electric Field:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E}_{\text{point charge}} = \frac{\vec{F}}{q_0} = k \frac{q}{r^2}$$

Electric Field points in the direction of the force felt on a positive “test” charge.
\[ E_{net_x} = E_{1x} + E_{2x} \]
\[ E_{net_x} = E_1 \cos \theta + E_2 \cos \theta \]
\[ E_{net_x} = 2E_1 \cos \theta \]
\[ E_{net_x} = 2k \frac{q}{r^2} \cos \theta \]
\[ E_{net_x} = E_{1x} + E_{2x} \]
\[ E_{net_x} = E_1 \cos \theta + E_2 \cos \theta \]
\[ E_{net_x} = 2 E_1 \cos \theta \]
\[ E_{net_x} = 2 k \frac{q}{r^2} \cos \theta \]
\[ E_{net_x} = 2 k \frac{q}{r^2} \left( \frac{a}{r} \right) = 2 k a \frac{q}{r^3} \]
\[ E_{net_x} = \frac{2 k a q}{\left( \sqrt{a^2 + y^2} \right)^3} \]
\[ E_{net_x} = \frac{2 k a q}{\left( a^2 + y^2 \right)^{3/2}} \]
\[ E_{net_x} = \frac{2kaq}{(a^2 + y^2)^{3/2}} \]

\[ E_{net_x} = \frac{2(9 \times 10^9 \text{Nm}^2/C^2)(0.1 m)2 \times 10^{-3} C}{((0.1 m)^2 + (0.3 m)^2)^{3/2}} = 1.14 \times 10^8 N/C \]
\[ q_0 = 3.0 \text{ nC} \]

\[ E_{\text{net}} = \frac{2kaq}{(a^2 + y^2)^{3/2}} \]

\[ F_0 = q_0 E = 4 \times 10^{-9} C \left(1.14 \times 10^8 N/C\right) = +0.456 N \]

\[ q_1 = 2.0 \text{ mC} \]

\[ q_2 = -2.0 \text{ mC} \]

\[ E_{\text{net}} = \frac{2 \left(9 \times 10^9 \frac{Nm^2}{C^2}\right)(0.1 m)2 \times 10^{-3} C}{\left((0.1 m)^2 + (0.3m)^2\right)^{3/2}} = 1.14 \times 10^8 N/C \]
\[ q_0 = -5.0 \text{ nC} \]

\[ \vec{F}_0 = q_0 \vec{E} = -5 \times 10^{-9} \text{ C} \left( 1.14 \times 10^8 \text{ N/C} \right) = -0.57 \text{ N} \]

\[ q_1 = 2.0 \text{ mC} \]

\[ q_2 = -2.0 \text{ mC} \]

\[ E_{\text{net}} = \frac{2kaq}{\left( a^2 + y^2 \right)^{3/2}} \]

\[ = \frac{2 \left( 9 \times 10^9 \frac{Nm^2}{C^2} \right) (0.1 \text{ m}) 2 \times 10^{-3} \text{ C}}{\left( (0.1 \text{ m})^2 + (0.3 \text{ m})^2 \right)^{3/2}} \]

\[ = 1.14 \times 10^8 \text{ N/C} \]
Find electric field at point p.
Find electric field at point p.

\[ E_{net} = E_1 + E_2 = k \frac{q_1}{r_1^2} + k \frac{q_2}{r_2^2} \]

\[ E_{net} = 9 \times 10^9 \frac{Nm^2}{C^2} \left( \frac{2 \times 10^{-3} \, C}{(3.0 \, m)^2} + \frac{6 \times 10^{-3} \, C}{(2.0 \, m)^2} \right) = 1.5 \times 10^7 \, N/C \]
36°C

-24°C
1 line for each 3C
1 line for each 3C
Other Charge Configurations

Two Negatively Charged Objects

A Positively and a Negatively Charged Object
Electric Field Line Patterns for Objects with Unequal Amounts of Charge
Gauss's Law

\[ \Phi_E = \vec{E} \cdot \vec{A} = |E||A| \cos \theta = \frac{q_{\text{enc}}}{\epsilon_o} \]

\[ k = \frac{1}{4 \pi \epsilon_o} \]

\[ \epsilon_o = 8.85 \times 10^{-12} \frac{C^2}{N \, m^2} \]

\[ \Phi_E = \vec{E} \cdot \vec{A} = |E||A| \cos \theta \]
Point Charge
Point Charge

Draw Gaussian Surface. (Sphere)
Point Charge

\[ EA = \frac{q_{enc}}{\varepsilon_0} \]

\[ E(4\pi r^2) = \frac{q}{\varepsilon_0} \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} = k \frac{q}{r^2} \]

Draw Gaussian Surface. (Sphere)
Gauss's Law

\[ \Phi_E = \vec{E} \cdot \hat{A} = |E||A| \cos \theta = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ k = \frac{1}{4\pi \varepsilon_0} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \]

\[ \lambda = \frac{\Delta q}{\Delta l} \quad \text{Linear Charge Density} \]

\[ \sigma = \frac{\Delta q}{\Delta A} \quad \text{Area Charge Density} \]

\[ \rho = \frac{\Delta q}{\Delta V} \quad \text{Volume Charge Density} \]
Line of Charge
Line of Charge

\[ A = 2\pi r L \]
Line of Charge

\[ A = 2\pi r L \]

\[ E\text{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \]

\[ E(2\pi r L) = \frac{\lambda L}{\varepsilon_0} \]

\[ E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r} = 2k \frac{\lambda}{r} \]
\[
\phi_E = \frac{q_{\text{enc}}}{\varepsilon_0}
\]

\[
EA + EA = \frac{\sigma A}{\varepsilon_0}
\]

\[
2EA = \frac{\sigma A}{\varepsilon_0}
\]

\[
E = \frac{\sigma}{2\varepsilon}
\]
Sheet of Charge

\[ E = \frac{\sigma}{2\epsilon} \]
Sheet of Charge

\[ E = \frac{\sigma}{2\varepsilon} \]
Sheet of Charge

\[ E = \frac{\sigma}{2\epsilon} + \frac{\sigma}{2\epsilon} = \frac{\sigma}{\epsilon} \]
Sheet of Charge

\[ E = \frac{\sigma}{2\epsilon} + \frac{\sigma}{2\epsilon} = \frac{\sigma}{\epsilon} \]
Two plates each with an area of 2.0 square meters, have an equal and opposite charge of 80 mC and are separated by 3.0 cm. A proton starts from rest on the positive plate. What is the velocity of the proton as it reaches the negative plate?

\[ F_{net} = F_E \]
\[ ma = qE \]
\[ a = \frac{qE}{m} \]

\[ v^2 = v_0^2 + 2a \Delta x \]
Two plates each with an area of 2.0 square meters, have an equal and opposite charge of 80 mC and are separated by 3.0 cm. A proton starts from rest on the positive plate. What is the velocity of the proton as it reaches the negative plate?

\[
F_{\text{net}} = F_E \\
ma = qE \\
a = \frac{qE}{m}
\]

\[
v^2 = v_o^2 + 2a \Delta x \\
v = \sqrt{v_o^2 + 2a \Delta x} \\
v = \sqrt{2 \left( \frac{qE}{m} \right) \Delta x}
\]
Two plates each with an area of 2.0 square meters, have an equal and opposite charge of 8 mC and are separated by 0.03 cm. A proton starts from rest on the positive plate. What is the velocity of the proton as it reaches the negative plate?

\[ F_{net} = F_E \]
\[ ma = qE \]
\[ a = \frac{qE}{m} \]

\[ v^2 = v_o^2 + 2a \Delta x \]
\[ v = \sqrt{v_o^2 + 2a \Delta x} \]
\[ v = \sqrt{2 \left( \frac{qE}{m} \right) \Delta x} \]

\[ v = \sqrt{2 \left( \frac{qE}{m} \right) \Delta x} = \sqrt{2 \left( \frac{qE}{m} \right) \Delta x} = 5.1 \times 10^6 \text{ m/s} \]
Two plates have an electric field between them of 10000 N/C. Particle has a mass of 0.002 grams and a charge of +2e.

\[ V_o = 2 \times 10^5 \text{ m/s} \hat{i} \]

\[ F_y = F_E \]
\[ ma_y = qE \]
\[ a_y = \frac{qE}{m} \]

\[ F_x = 0.0 \text{ N} \]
\[ a_x = 0.0 \text{ m/s}^2 \]
Two plates have an electric field between them of 10000 N/C. Particle has a mass of 0.002 grams and a charge of +2e.

\[ E = 10000 \, N/C \]

\[ V_0 = 2 \times 10^5 \, m/s \hat{i} \]

\[ \Delta y \]

\[ F_y = F_E \]

\[ ma_y = qE \]

\[ a_y = \frac{qE}{m} \]

\[ F_x = 0.0 \, N \]

\[ a_x = 0.0 \, m/s^2 \]
Work done on particle in an electric field.
Change in potential energy in a constant $E$ field.

\[
W_{AB} = F_x \Delta x = q E_x (x_f - x_i)
\]

\[
W = qE \Delta x = \Delta KE = -\Delta PE
\]

\[
\Delta PE = -W = -qE \Delta x
\]
Proton starts from rest.

\[ PE_i + KE_i = PE_f + KE_f \]
\[ KE_f = PE_i - PE_f = \Delta PE \]
\[ \frac{1}{2} m v_f^2 = qE \Delta x \]
\[ v_f = \sqrt{\frac{2qE \Delta x}{m}} \]
Proton starts from rest.

\[ PE_i + KE_i = PE_f + KE_f \]

\[ KE_f = PE_i - PE_f = \Delta PE \]

\[ \frac{1}{2} m v_f^2 = qE \Delta x \]

\[ v_f = \sqrt{\frac{2qE \Delta x}{m}} \]

\[ v_f = \sqrt{\frac{2(1.6 \times 10^{-19} \text{C}) \times 1500 \text{ N/m} \times 0.03 \text{m}}{1.67 \times 10^{-27} \text{kg}}} \]

\[ v_f = 9.2 \times 10^4 \text{ m/s} \]
Electric Potential:

\[ \Delta V = V_B - V_A = \frac{\Delta PE}{q} \]

1 Volt (V) = 1 \( \frac{J}{C} \)

\[ \Delta V = -E_x \Delta x \]

\[ E_x = \frac{-\Delta V}{\Delta x} \]
Electric Potential:

\[ \Delta V = V_B - V_A = \frac{\Delta \text{PE}}{q} \]

1 Volt (V) = 1 \( \frac{J}{C} \)

\[ \Delta V = -E_x \Delta x \]

\[ E_x = \frac{-\Delta V}{\Delta x} \]

\[ E_x = \frac{-(0.0 V - 10000V)}{0.05m} = 2 \times 10^5 \text{ V/m} \]

\[ V_A = 10000 \text{V} \quad V_B = 0 \text{V} \]

\[ \Delta x = 5.0 \text{ cm} \]
$V_{\text{point charge}} = k \frac{q}{r}$

Electron Volt (eV) is defined as the kinetic energy that an electron gains when accelerated through a potential difference of 1 Volt.

$1 \, eV = 1.6 \times 10^{-19} \, CV = 1.60 \times 10^{-19} \, J$
1. Smoke particles pick up a negative charge.

2. Smoke particles are attracted to the collecting plates.

3. Collecting plates are knocked to remove the smoke particles.

- Waste gases without smoke particles
- Positively charged collecting plate
- Negatively charged metal grid
- Waste gases containing smoke particles
Electrostatic precipitator

- high-voltage transformer/rectifier
  access panel
- rapper for discharge electrodes
- insulator
  clean air
- high-voltage wire support
- high-voltage discharge electrode
- grounded collecting surface (collection electrode)
- inspection door
- wire weight
- collection hopper

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Capacitance and Capacitors:

\[ C \equiv \frac{Q}{\Delta V} \]

1 Farad \((F) = 1 \, C/V\)
Parallel Plate Capacitors:

\[ C = \frac{q}{\Delta V} \]
\[ C = \frac{\sigma A}{Ed} \]
\[ C = \frac{\sigma A}{\left( \frac{\sigma}{\varepsilon_0} \right) d} \]
\[ C = \varepsilon_0 \frac{A}{d} \]
Parallel Plate Capacitors:

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ C = 8.85 \times 10^{-12} \frac{c^2}{N \ m^2} \left( \frac{2 \times 10^{-4} \ m^2}{1 \times 10^{-3} \ m} \right) = 1.77 \times 10^{-12} \text{ F} = 1.77 \text{ pF} \]

\[ A = 2 \times 10^{-4} \ m^2 \]
\[ d = 1 \times 10^{-3} \ m \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \ m^2} \]
Parallel Connection

$C_{eqv} = C_1 + C_2 + C_3$
Series Connection

\[ \frac{1}{C_{eqv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \]
<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant</th>
<th>Dielectric Strength (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacuum</td>
<td>1.000000</td>
<td>-</td>
</tr>
<tr>
<td>air</td>
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<td>$3 \times 10^6$</td>
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<tr>
<td>Bakelite</td>
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</tr>
<tr>
<td>Silicone Oil</td>
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<td>$15 \times 10^6$</td>
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</tbody>
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