Physics 280
Fundamentals of Physics III

Week One:

Oscillations

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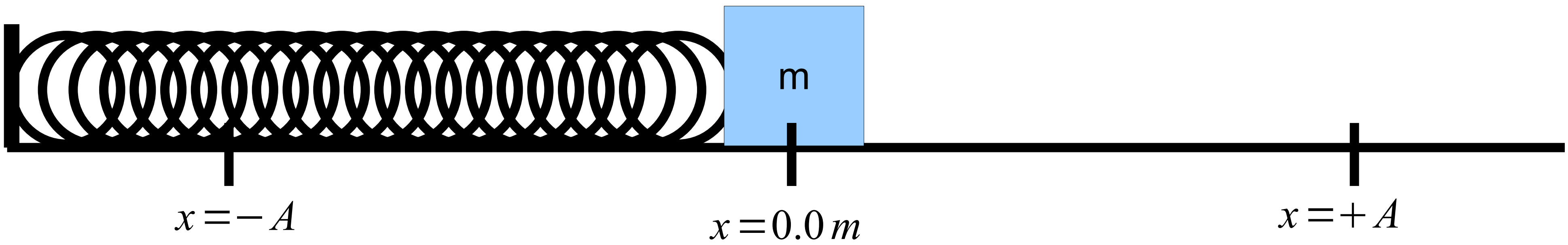
SHM

Simple Harmonic Motion

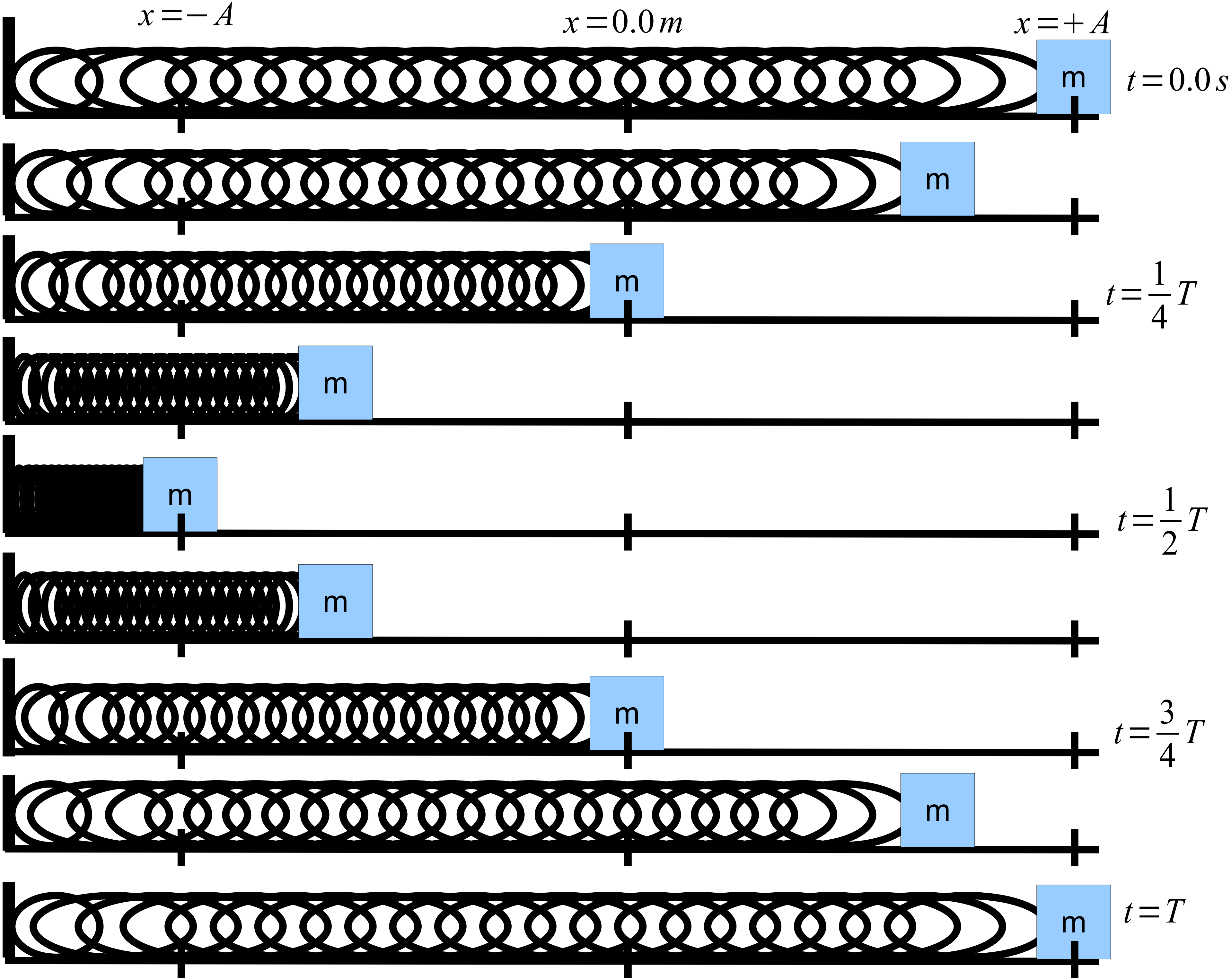
Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



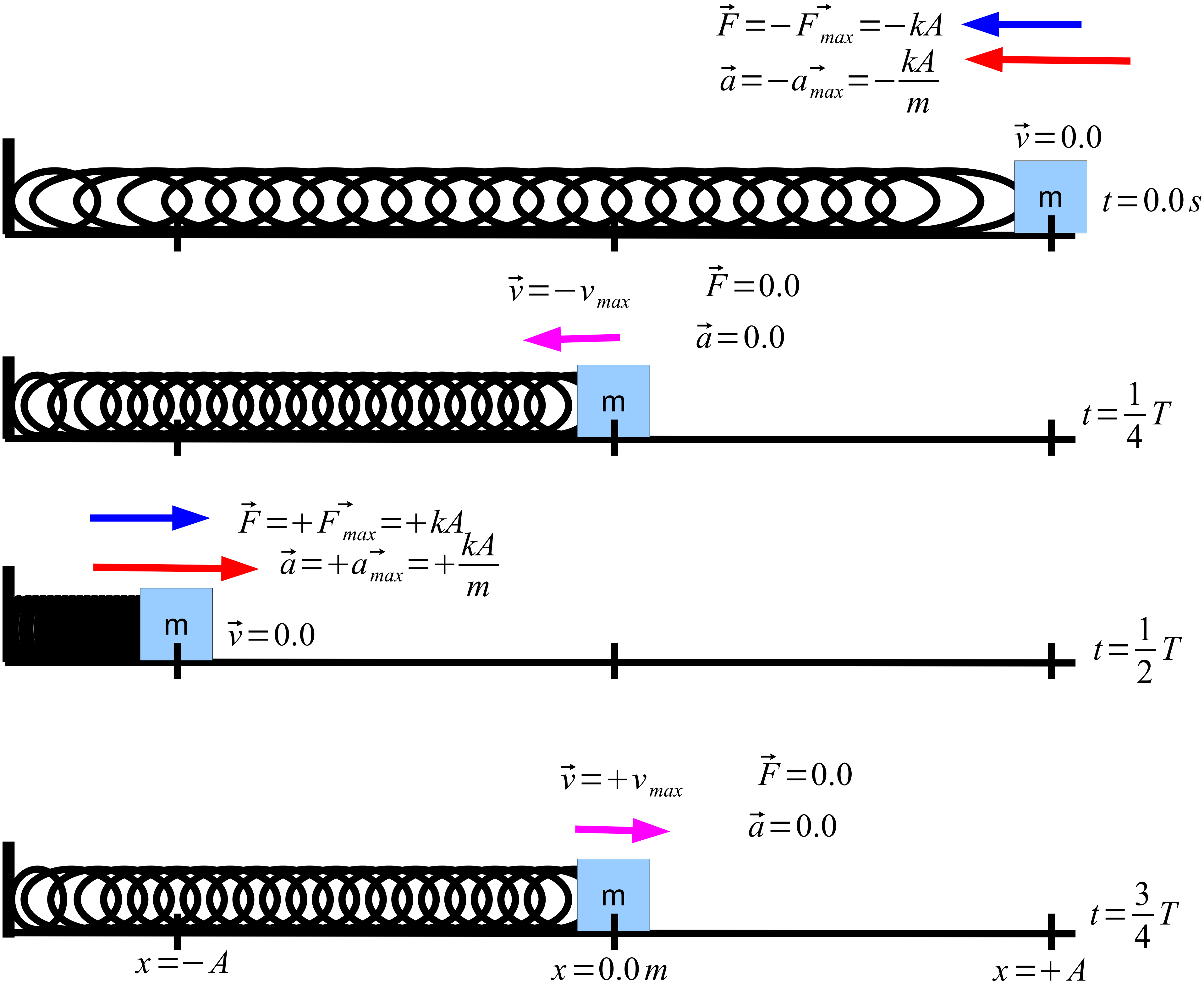
PERIOD (T) – Time to complete one oscillation. Measured in seconds.

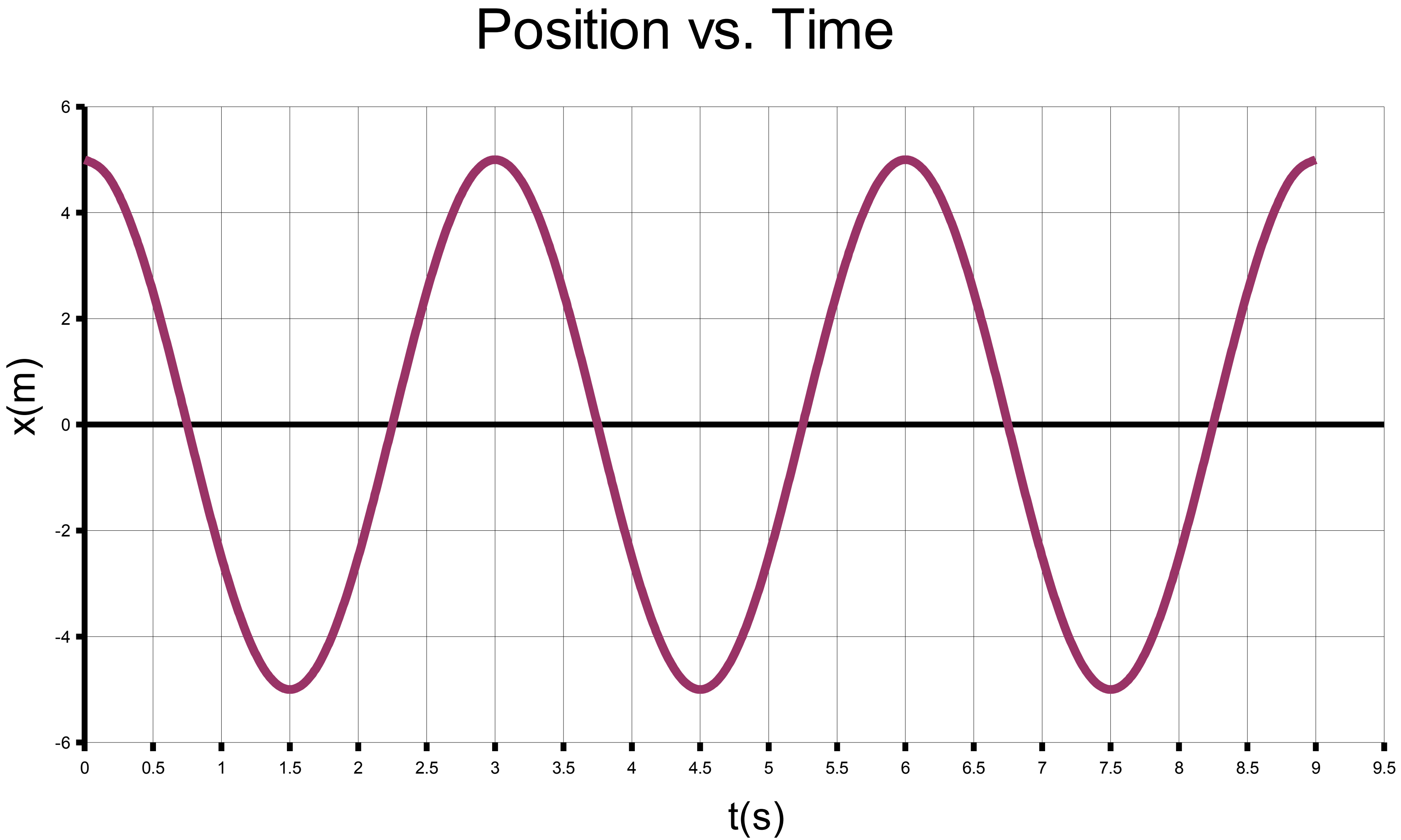
FREQUENCY (f) – Number of oscillations per seconds. Measured in Hertz (1 Hz = 1 1/s).

$$f = \frac{1}{T}$$



$x=-A$
$x=0.0\,m$
$x=+A$
m
$t=0.0\,s$
$t=\frac{1}{4}T$
$t=\frac{1}{2}T$
$t=\frac{3}{4}T$
$t=T$


$\vec{F} = -\vec{F}_{max} = -kA$
$\vec{a} = -\vec{a}_{max} = -\frac{kA}{m}$
$\vec{v} = 0.0$
m
$t = 0.0\,s$
$\vec{v} = -v_{max}$
$\vec{F} = 0.0$
$\vec{a} = 0.0$
m
$t = \frac{1}{4}T$
$\vec{F} = +\vec{F}_{max} = +kA$
$\vec{a} = +\vec{a}_{max} = +\frac{kA}{m}$
m
$\vec{v} = 0.0$
$t = \frac{1}{2}T$
$\vec{v} = +v_{max}$
$\vec{F} = 0.0$
$\vec{a} = 0.0$
m
$t = \frac{3}{4}T$
$x = -A$
$x = 0.0\,m$
$x = +A$

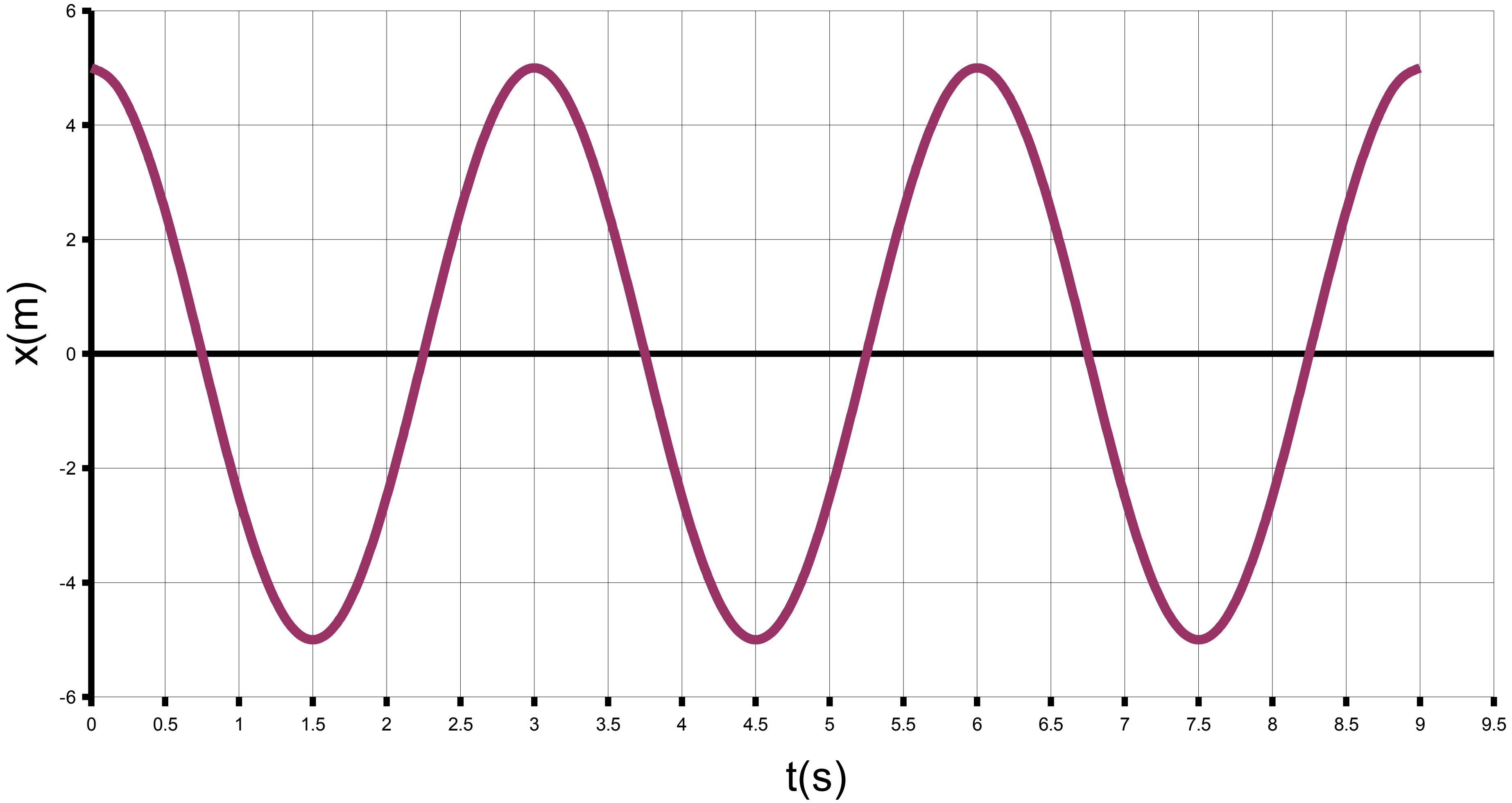


$$x(t) = A\cos(\theta)$$

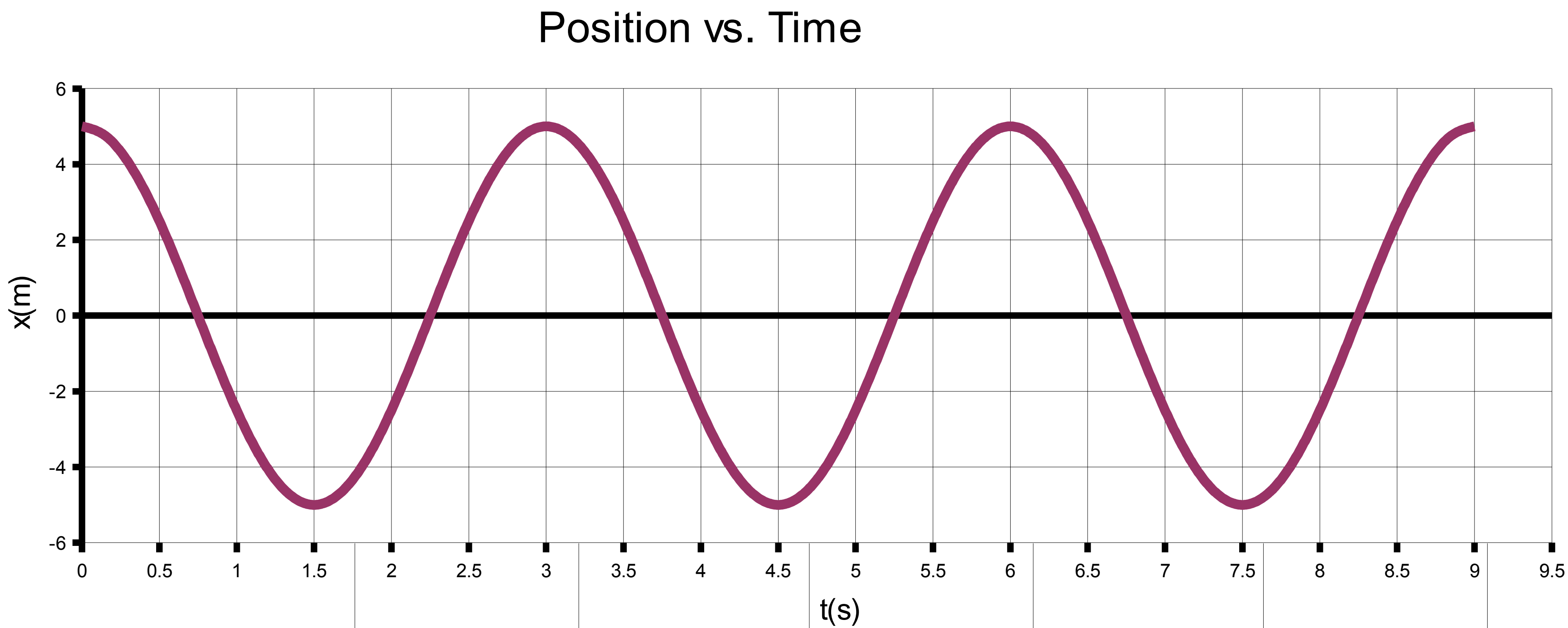
Angular Frequency

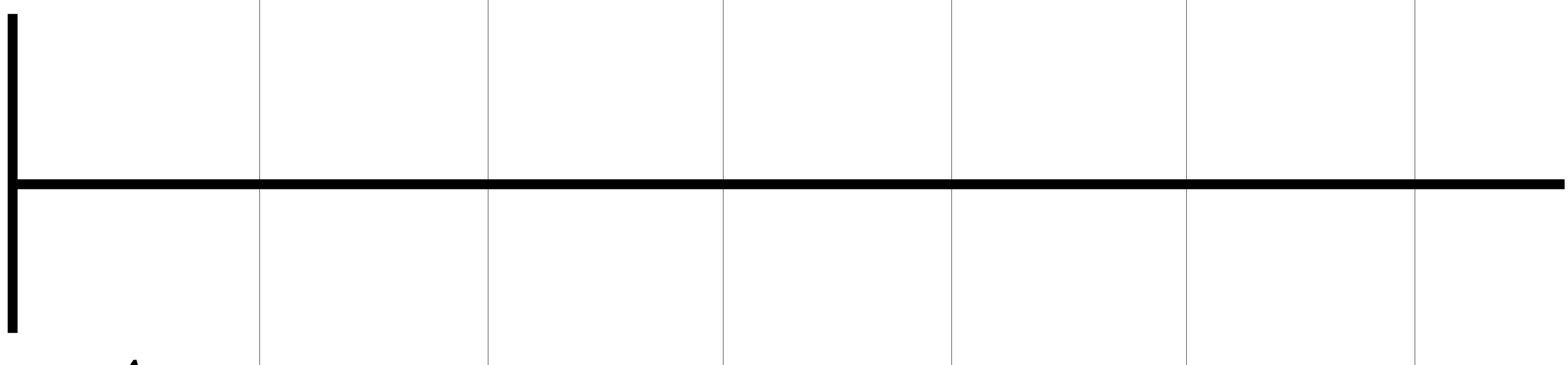
$$\omega = \frac{2\pi}{T}$$

Position vs. Time

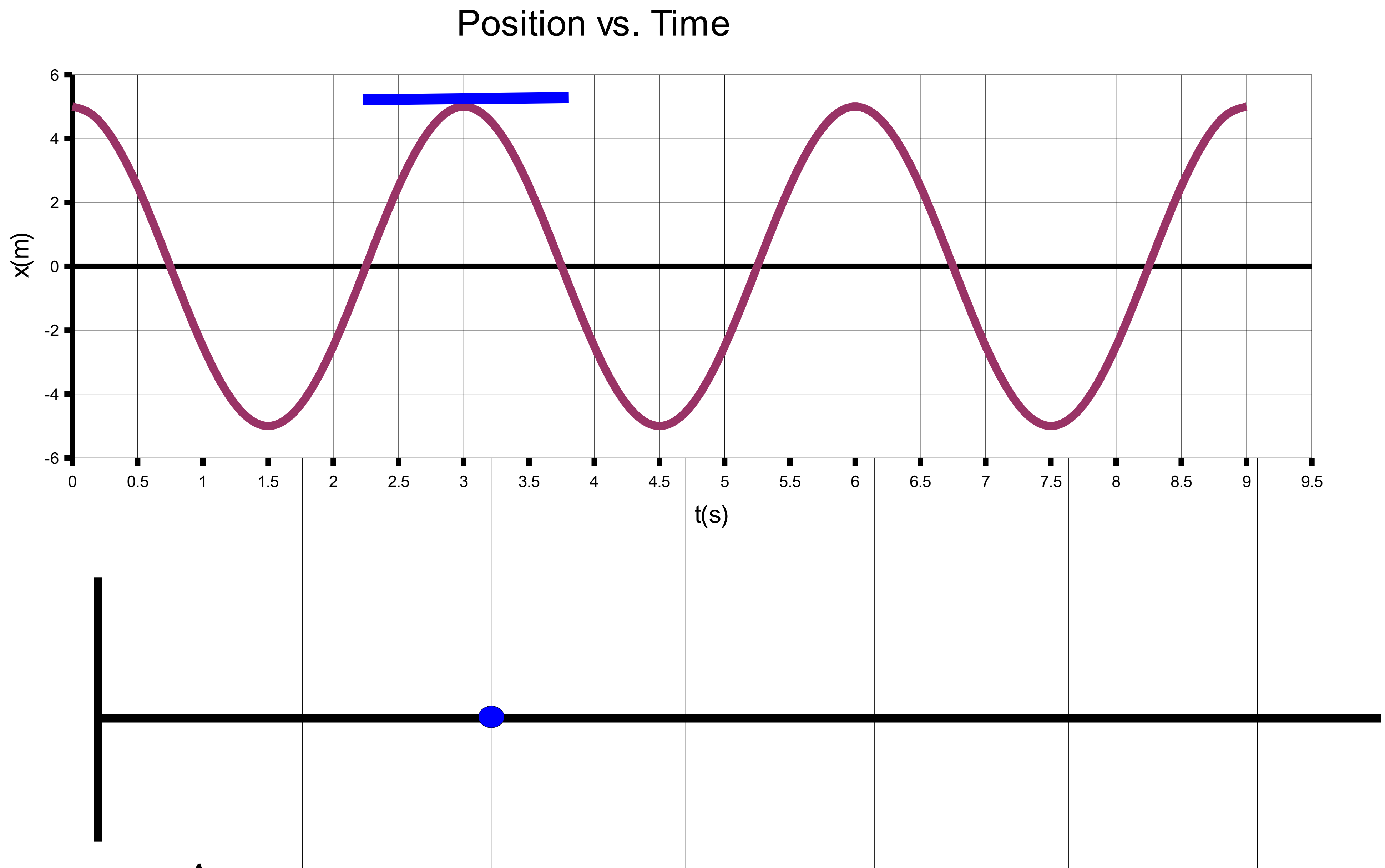


$$x(t) = A\cos(\omega t)$$

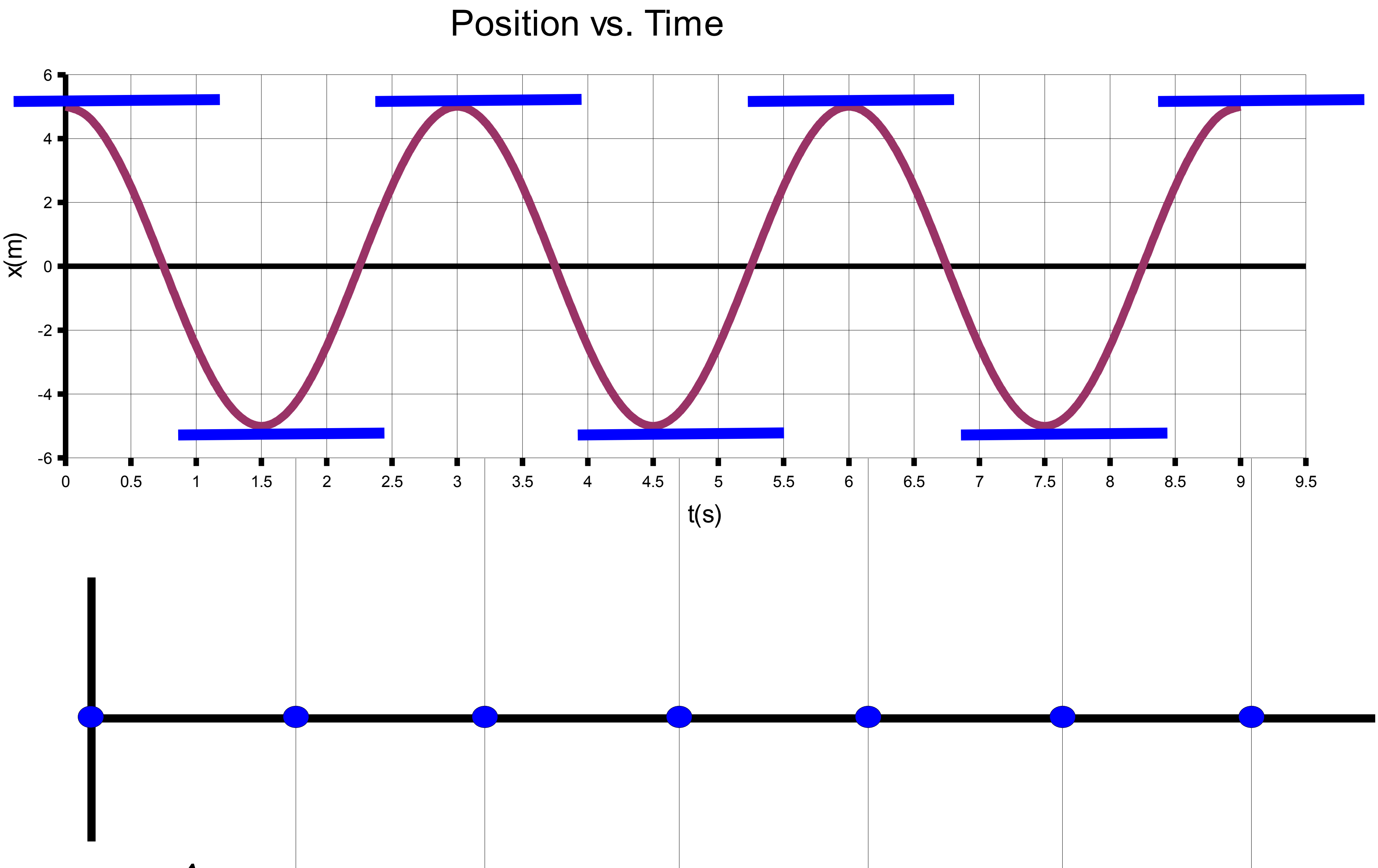




$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

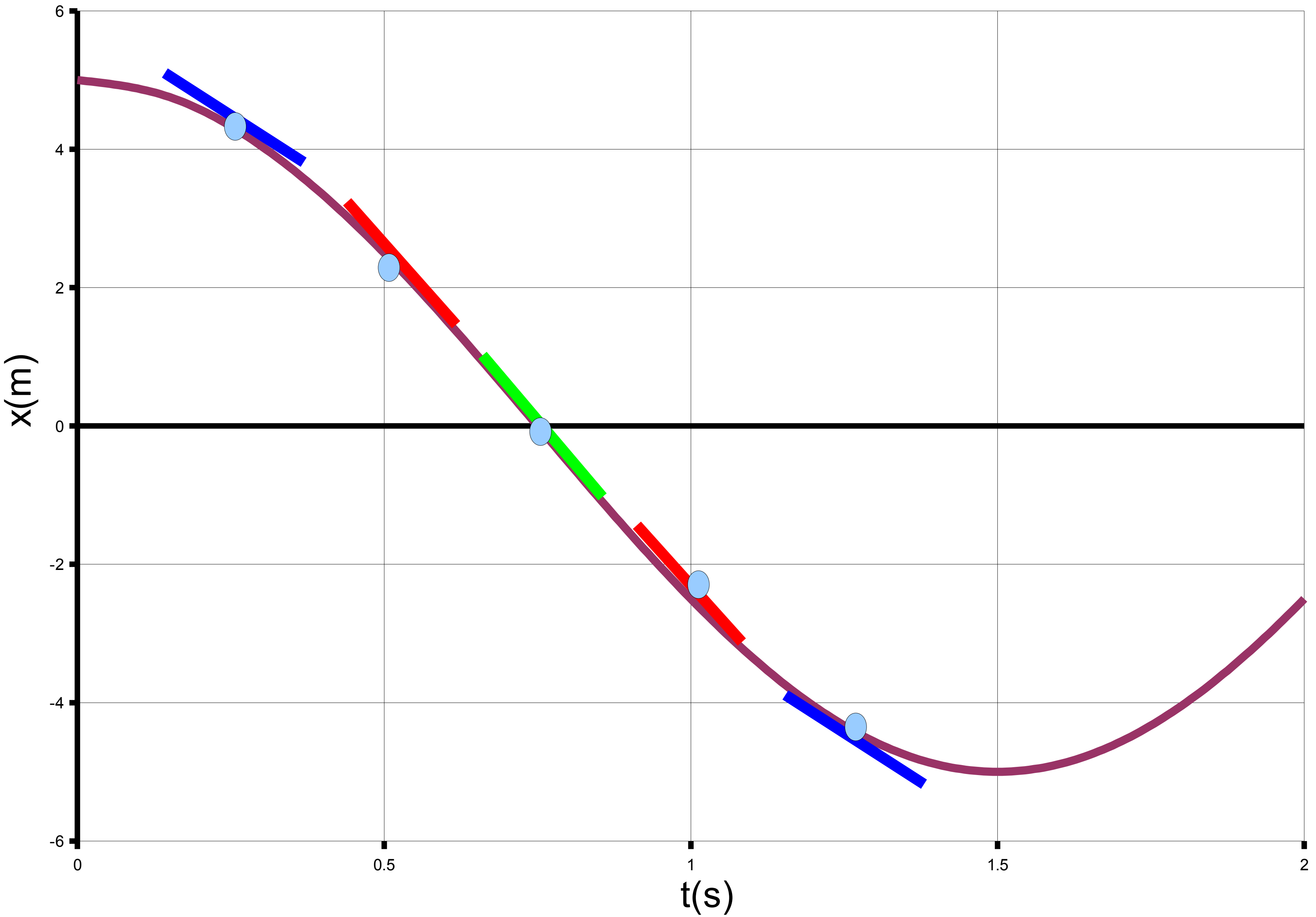


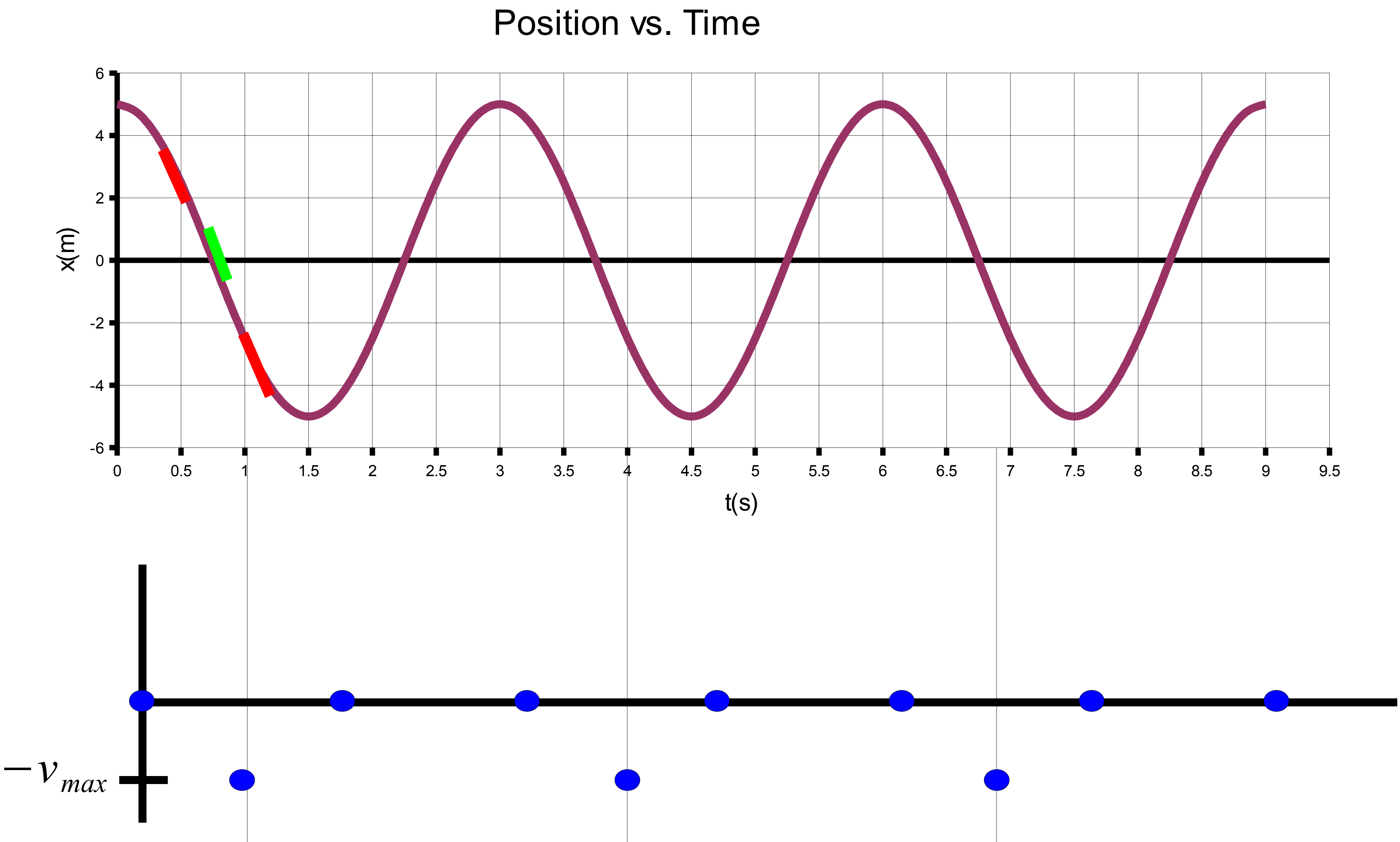
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$



$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

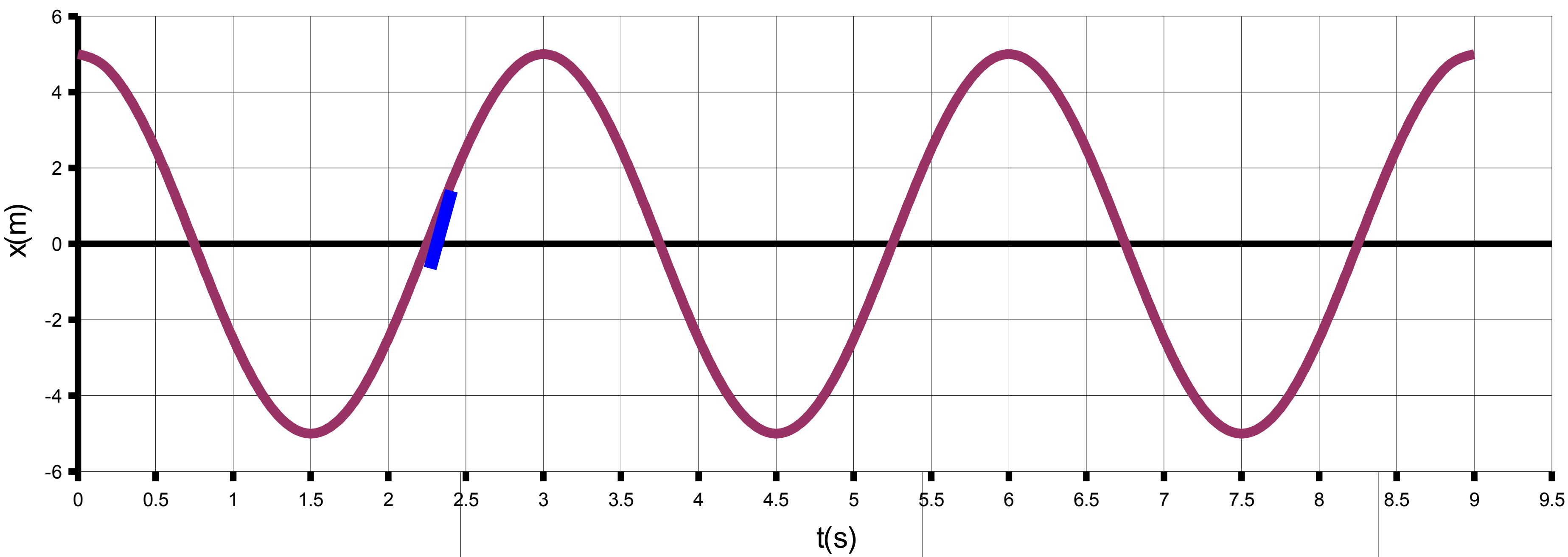
Position vs. Time

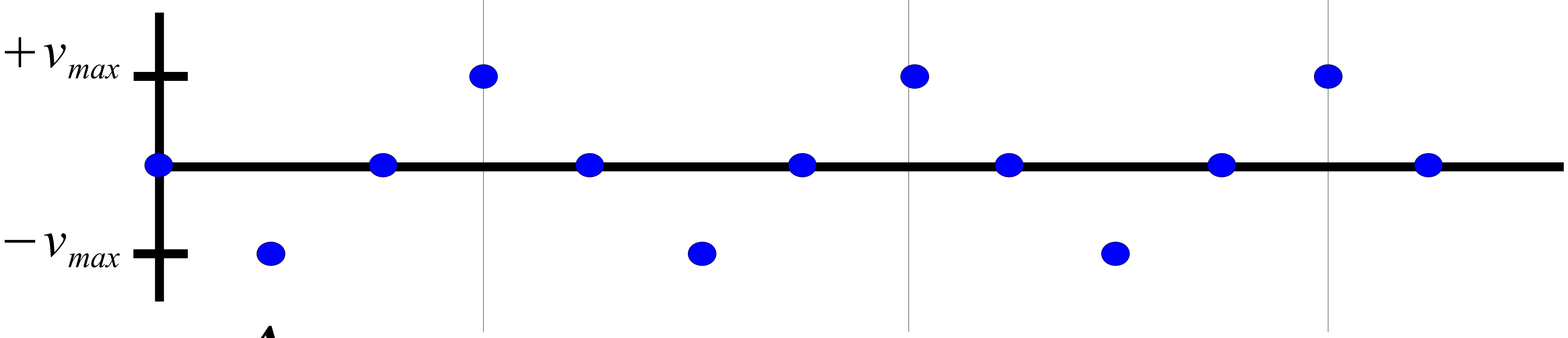




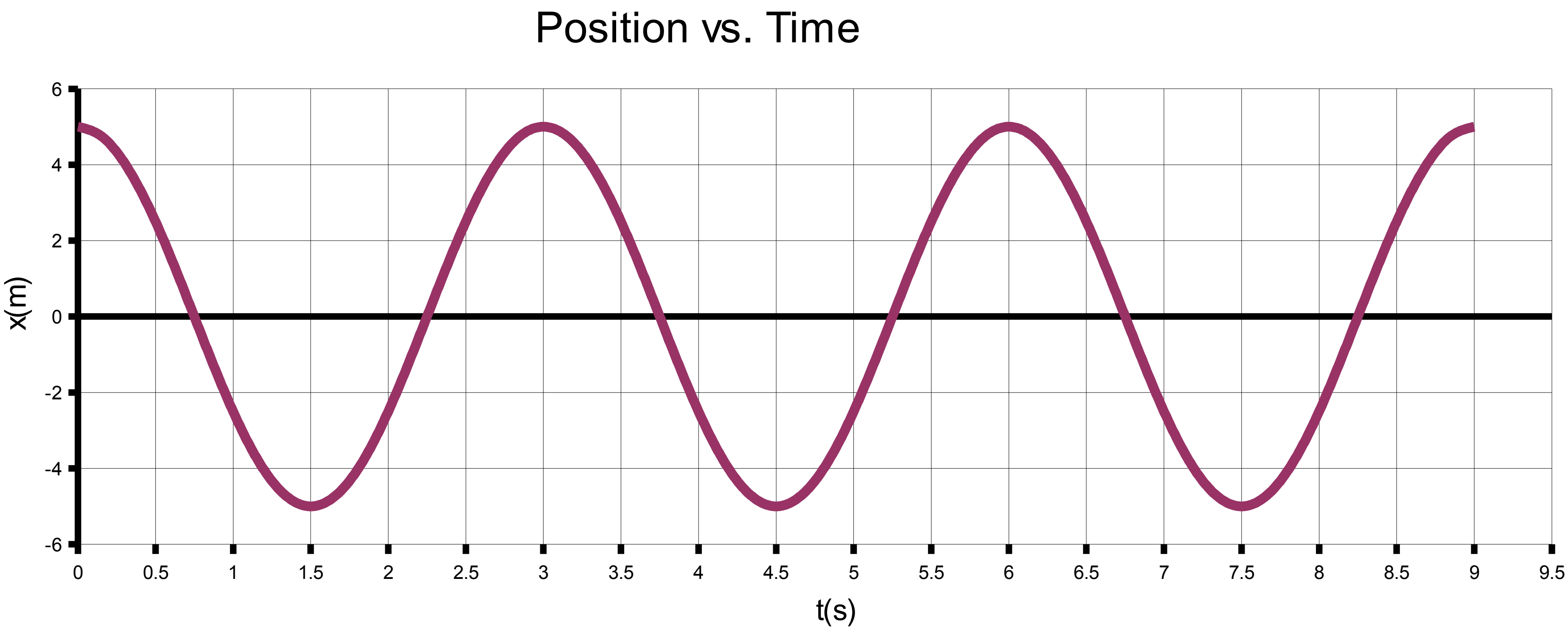
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

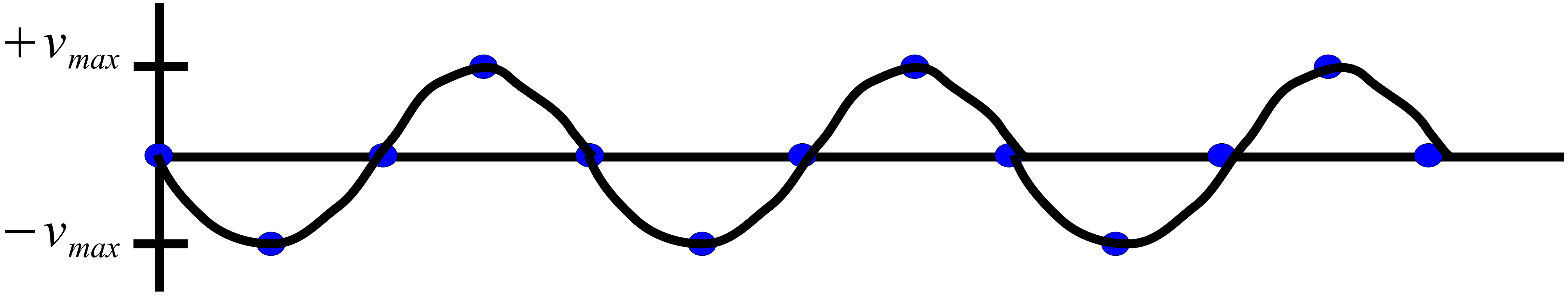
Position vs. Time





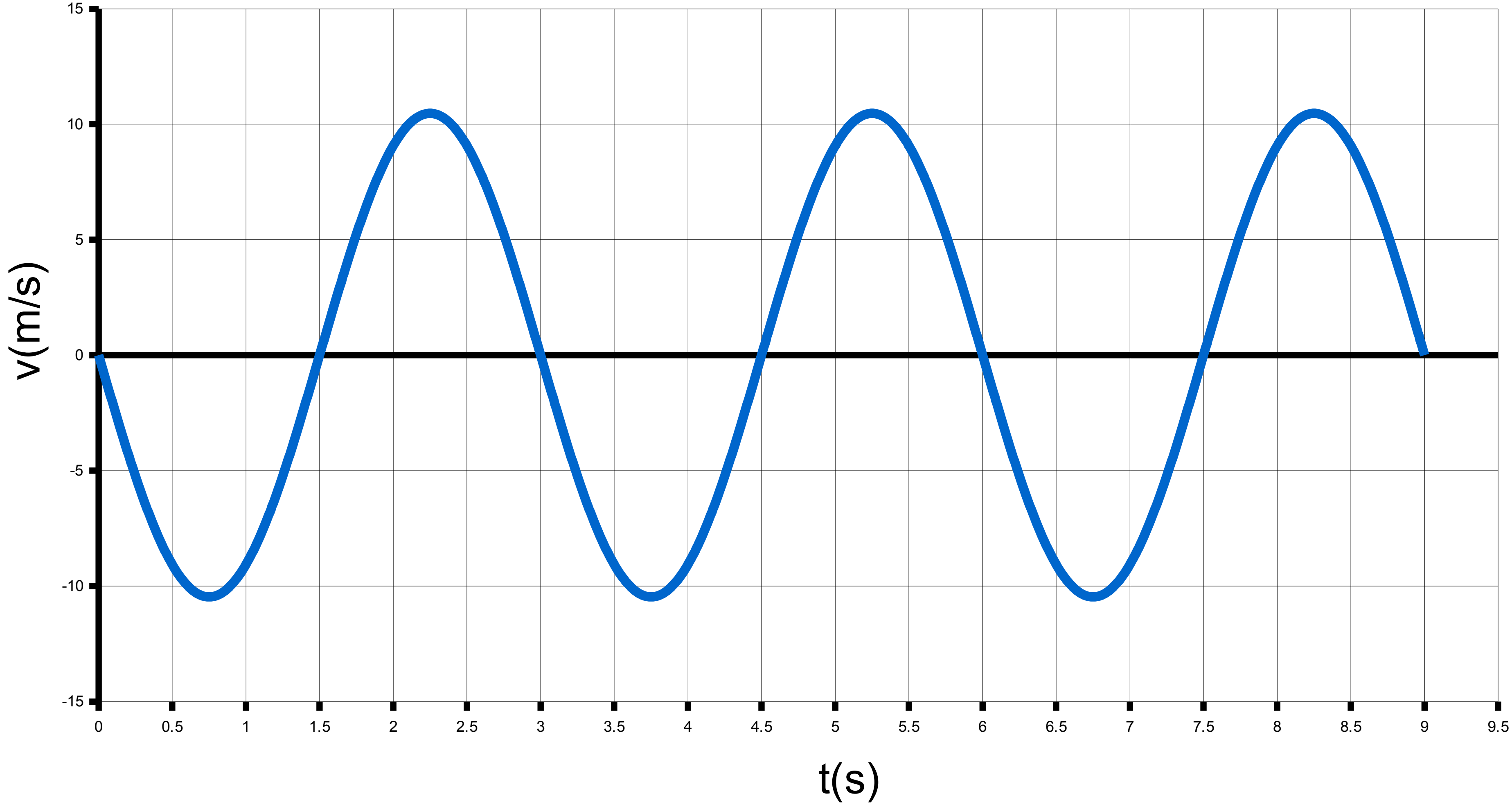
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$





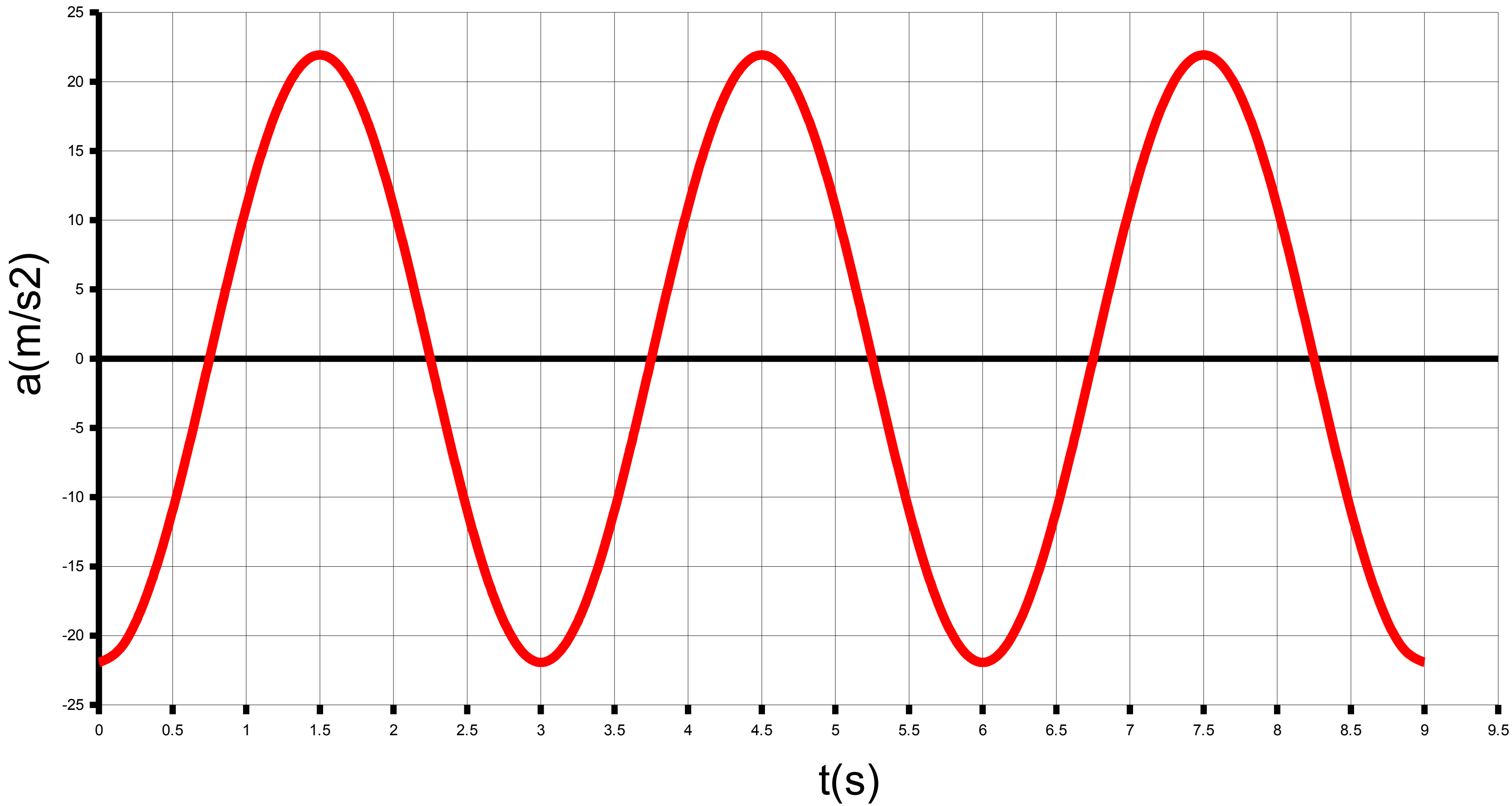
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

Velocity vs. Time



$$v(t) = -v_{max} \sin(\omega t)$$

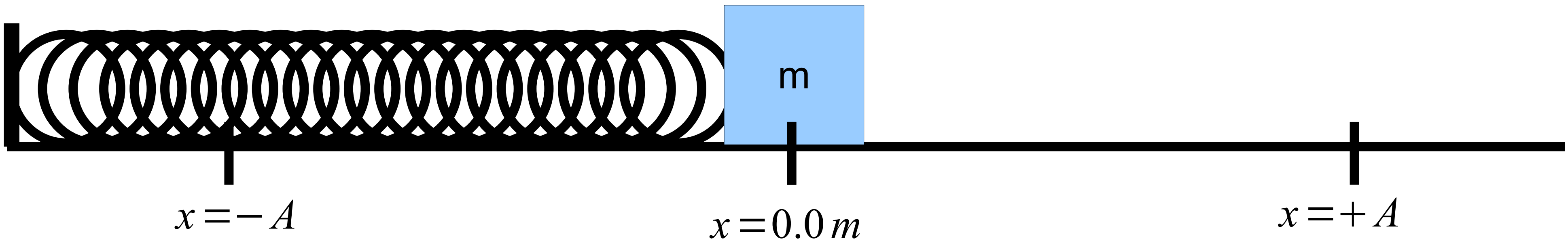
Acceleration vs. Time



$$a(t) = -a_{max} \cos(\omega t)$$

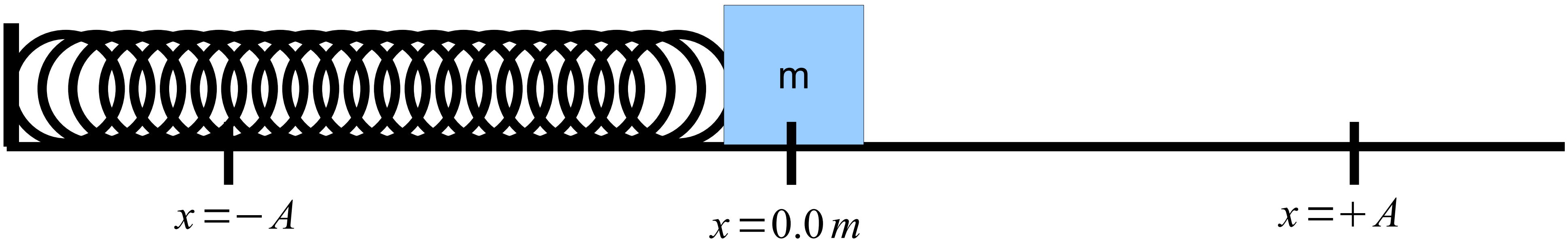
A Different Approach….

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.

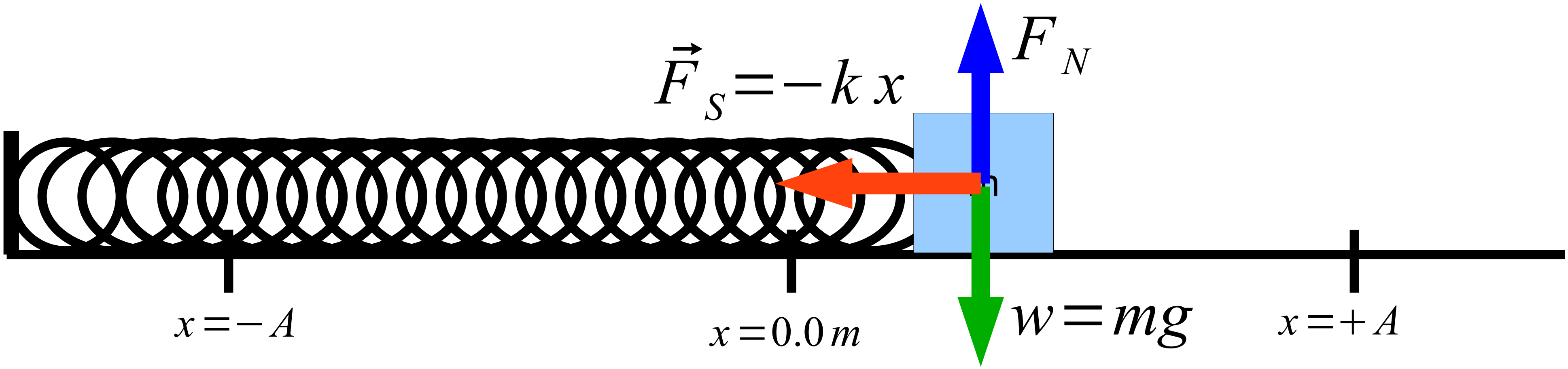


$$x(t)=A\cos(\omega t+\phi)$$

$$v(t)=\frac{dx}{dt}=-A\omega\sin(\omega t+\phi)$$

$$a(t)=\frac{dv}{dt}=-A\omega^2\cos(\omega t+\phi)$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

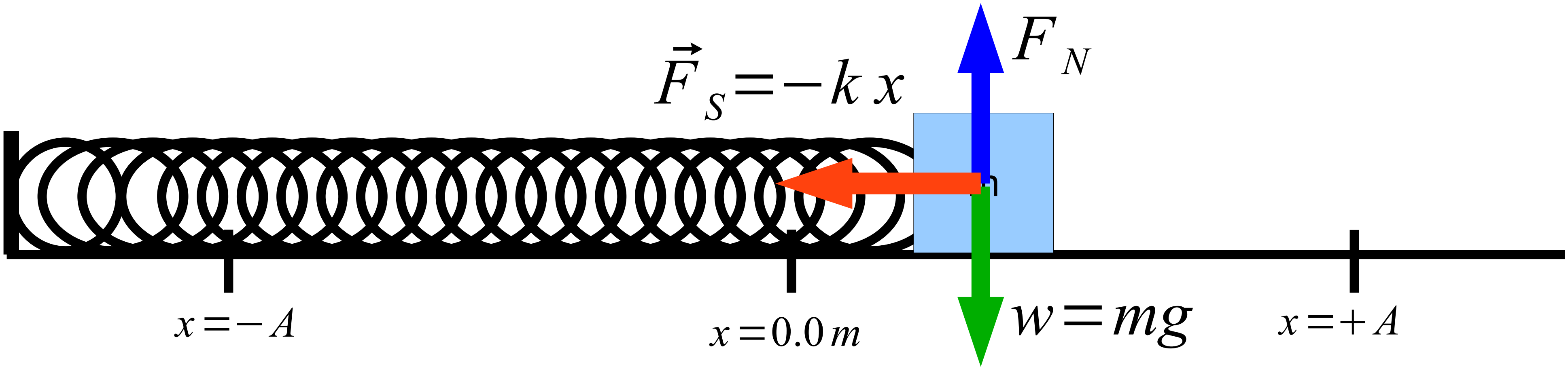
$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

$$\sum F_y = m a_y$$

$$F_N - mg = 0$$

$$F_N = mg$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

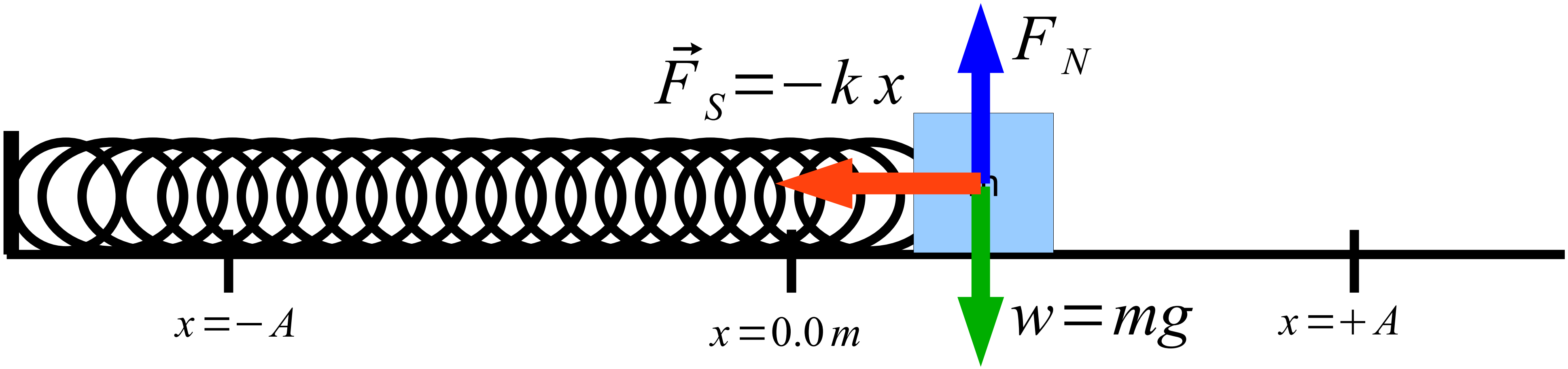
$$\sum F_x = m\,a_x$$

$$-k\,x = m\frac{d^2x}{dt^2}$$

$$-k\left[A\cos(\omega t)\right] = m\left[-A\omega^2\cos(\omega t)\right]$$

$$k = m\,\omega^2$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

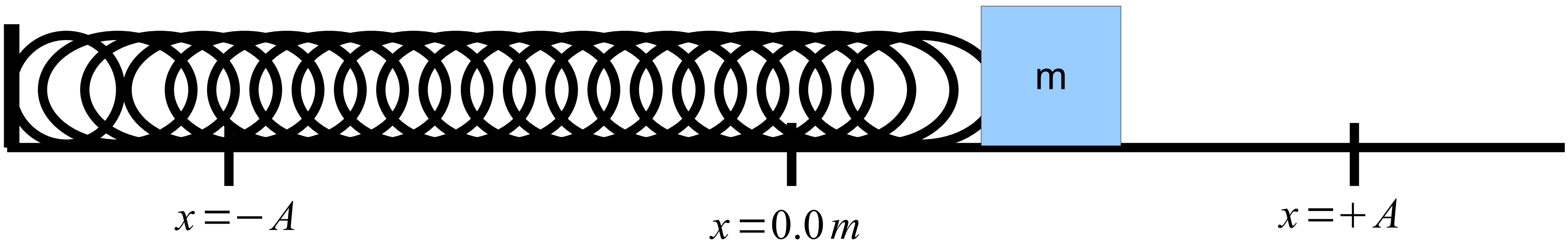
$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



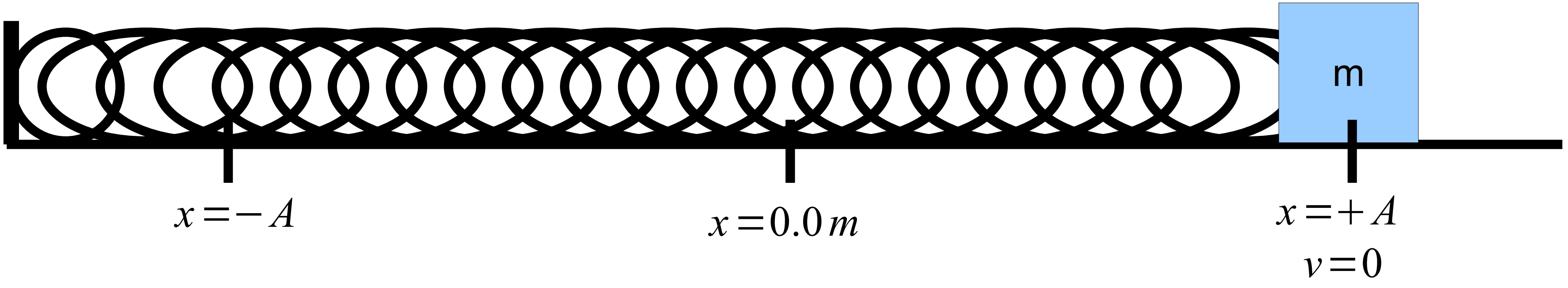
$$E_{tot}=U_s+K$$

$$E_{tot}=\frac{1}{2}kx^2+\frac{1}{2}mv^2$$

When the mass is at x = A, the velocity is zero.

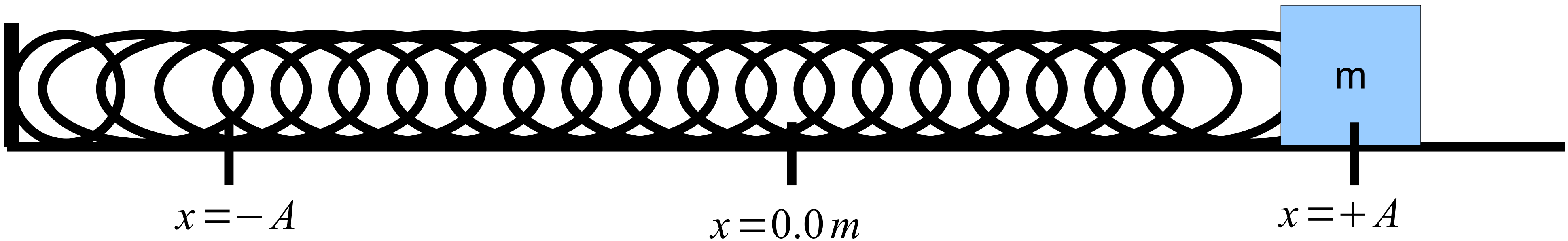
$$E_{tot} = U_s + K$$

$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$



$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \frac{1}{2} k A^2 + 0 = \frac{1}{2} k A^2$$

When the mass is at x = A, the velocity is zero.



$$E_{tot}=\frac{1}{2}k\,x^2+\frac{1}{2}m\,v^2$$

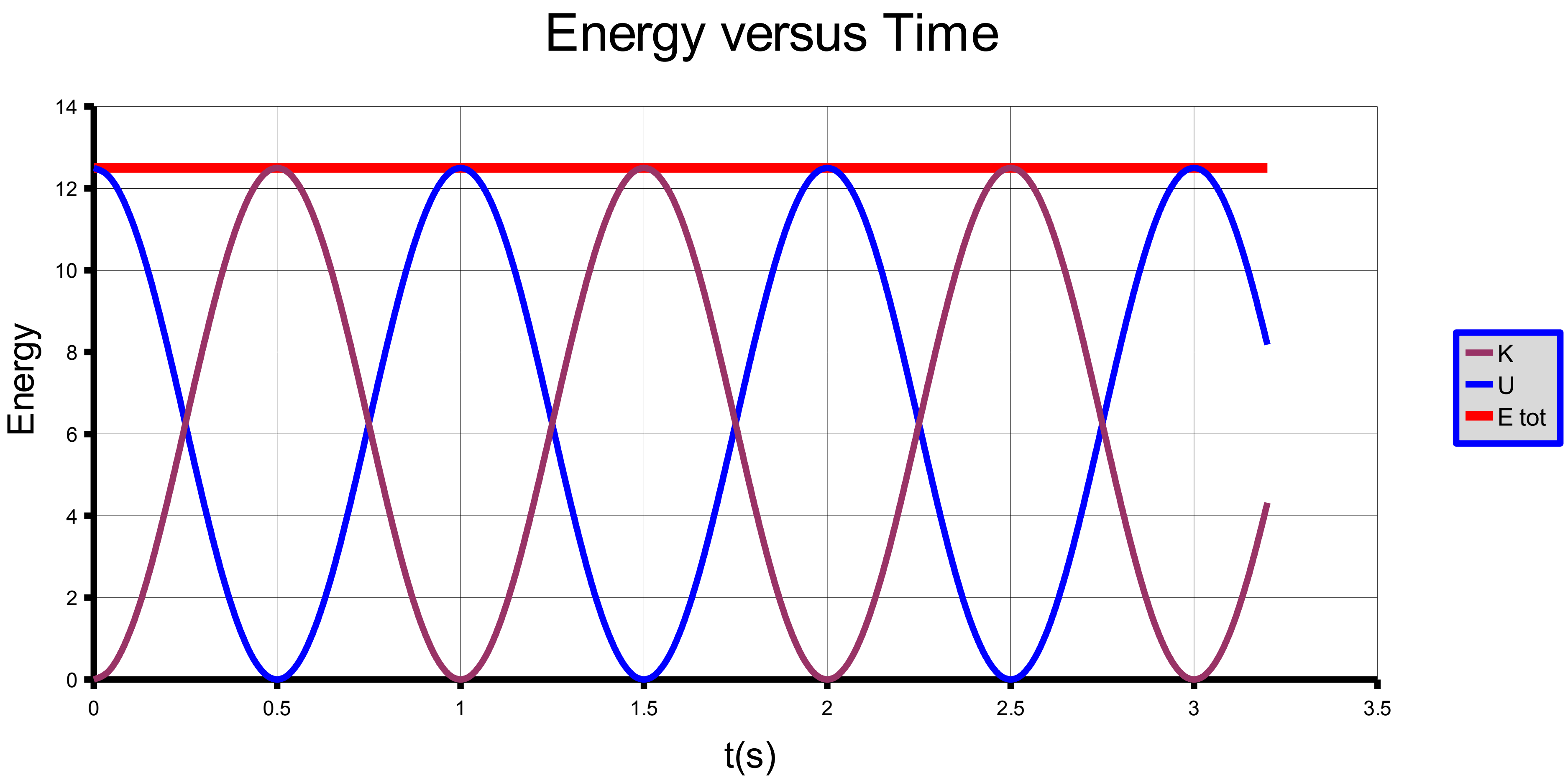
$$E_{tot}=\frac{1}{2}k\left(A\cos(\omega t)\right)^2+\frac{1}{2}m\left(-A\,\omega\sin(\omega t)\right)^2$$

$$E_{tot}=\frac{1}{2}k\,A^2\cos^2(\omega t)+\frac{1}{2}m\,A^2\omega^2\sin^2(\omega t) \qquad \omega=\sqrt{\frac{k}{m}}$$

$$E_{tot}=\frac{1}{2}k\,A^2\cos^2(\omega t)+\frac{1}{2}m\,A^2\left(\frac{k}{m}\right)\sin^2(\omega t)$$

$$E_{tot}=\frac{1}{2}k\,A^2\left(\cos^2(\omega t)+\sin^2(\omega t)\right) \qquad \sin^2\theta+\cos^2\theta=1$$

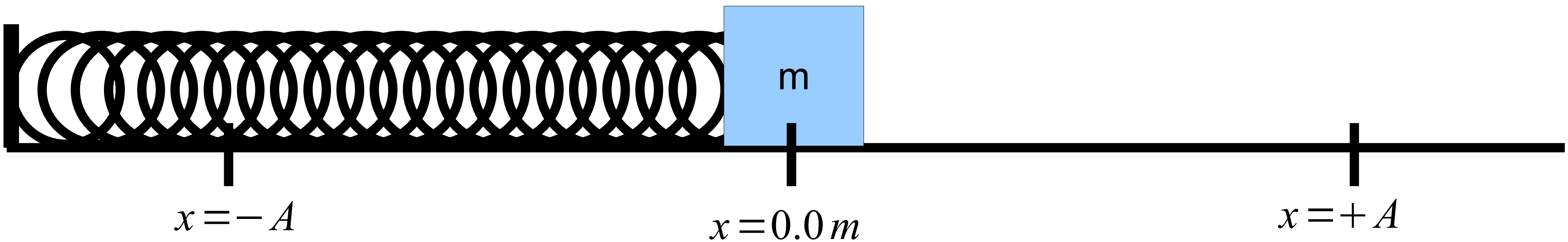
$$E_{tot}=\frac{1}{2}k\,A^2$$



Energy versus Time
Energy
t(s)
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
K
U
E tot

When the mass is at x = 0.0 m, there is no energy stored in the spring, and the velocity is equal to the maximum velocity.

$$\frac{1}{2}k x^2+\frac{1}{2}m v^2=\frac{1}{2}k A^2$$

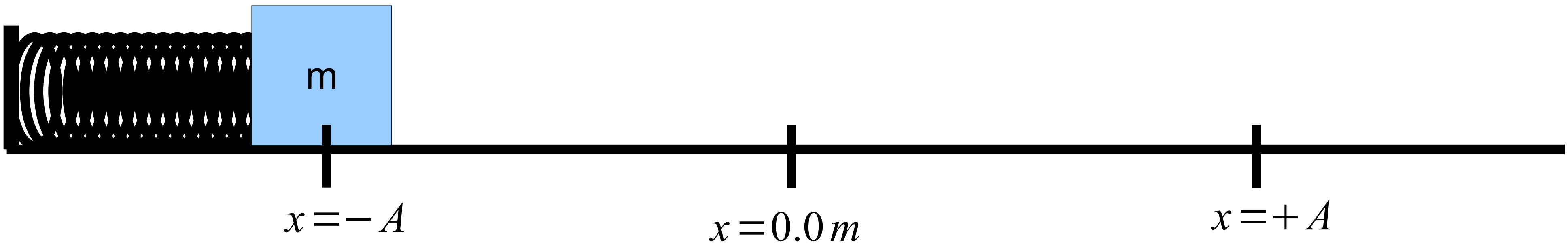


$$0+\frac{1}{2}m v_{max}^2=\frac{1}{2}k A^2$$

$$v_{max}=A\sqrt{\frac{k}{m}}$$

When the mass is at x = 0.0 m, there is no energy stored in the spring, and the velocity is equal to the maximum velocity.

$$F_s = -k\,x$$

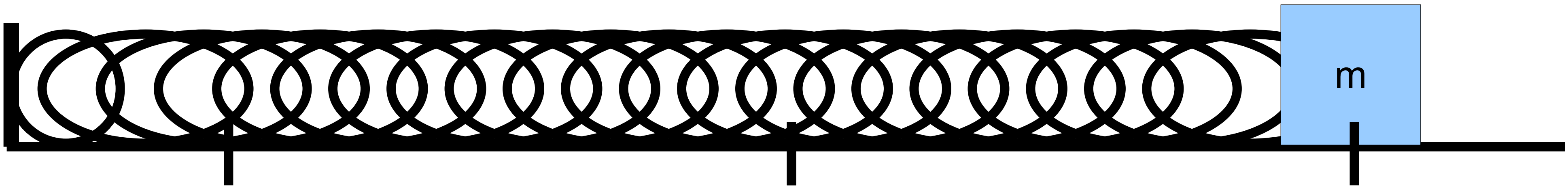


$$F_{net} = F_s$$

$$ma = -kx$$

$$a_{max} = -\left(\frac{k}{m}\right)(-A) = \left(\frac{k}{m}\right)A$$

Energy can be used to find a function for the magnitude of the velocity at any position (x).



$x=-A$ $\quad$ $x=0.0\,m$ $\quad$ $x=+A$

$$\frac{1}{2}k\,x^2+\frac{1}{2}m\,v^2=\frac{1}{2}k\,A^2$$

$$\frac{1}{2}m\,v^2=\frac{1}{2}k\,A^2-\frac{1}{2}k\,x^2$$

$$m\,v^2=k\,A^2-k\,x^2$$

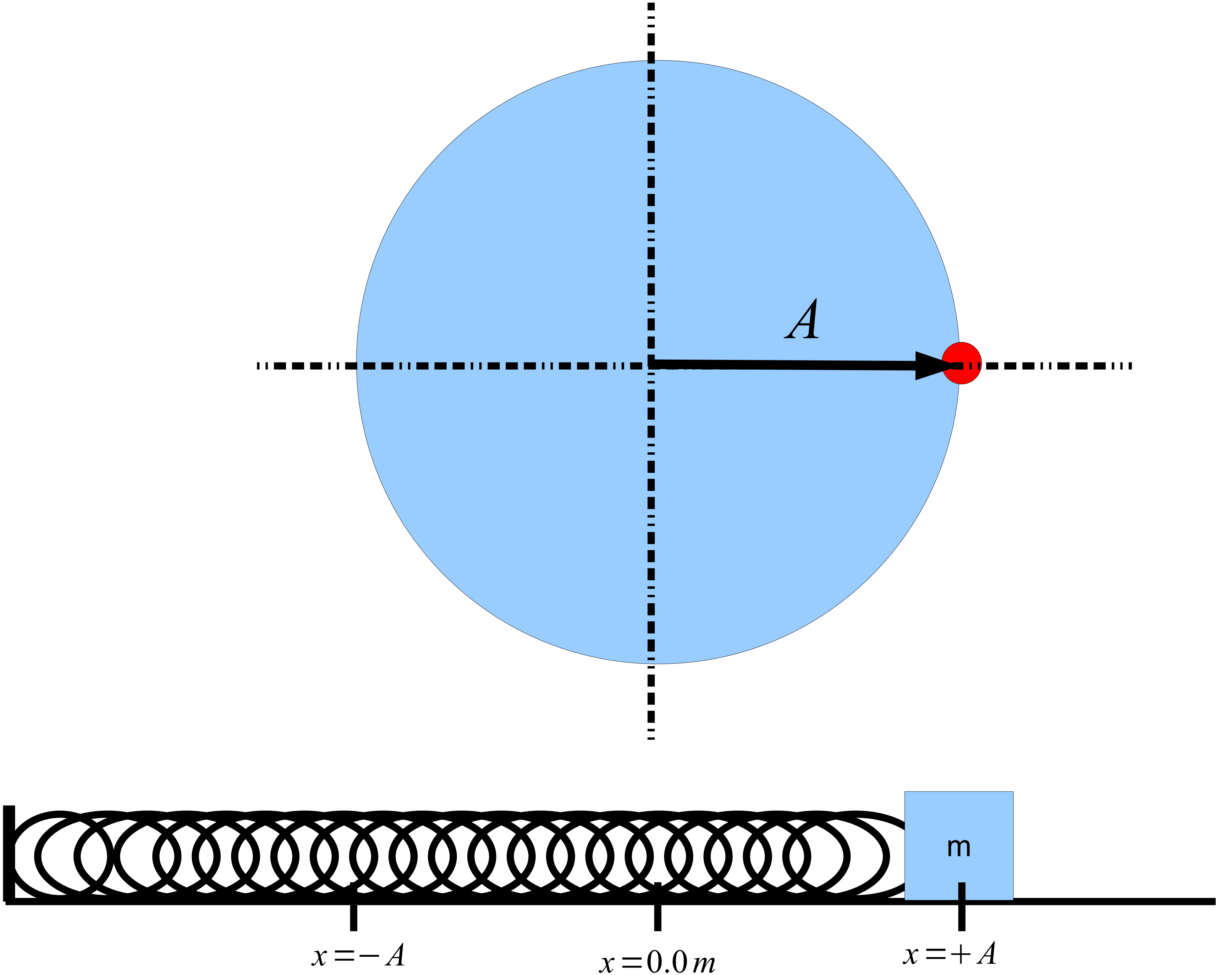
$$m\,v^2=k\left(A^2-x^2\right)$$

$$v^2=\frac{k}{m}\left(A^2-x^2\right)$$

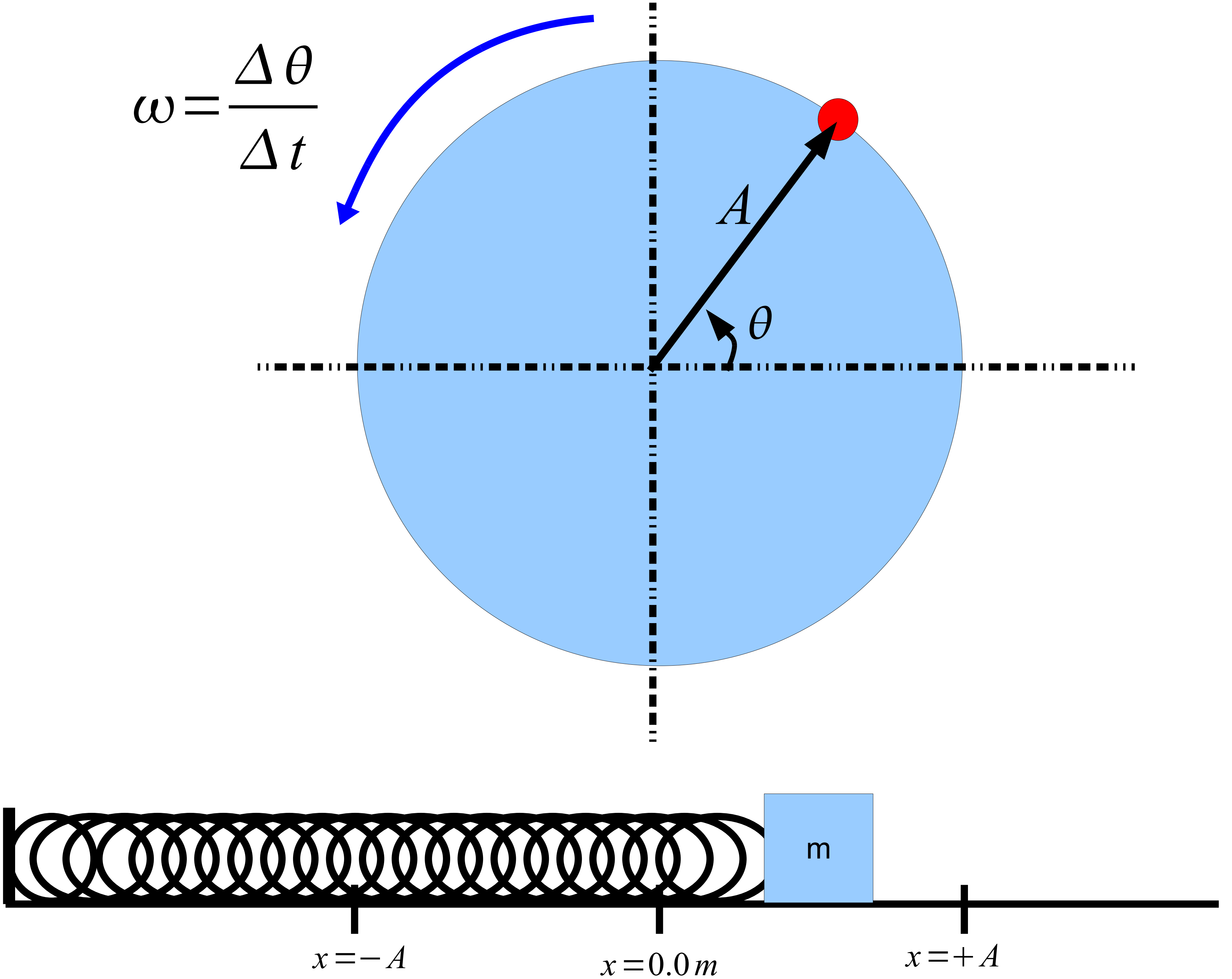
$$|v|=\sqrt{\frac{k}{m}\left(A^2-x^2\right)}$$

$$|v(x)|=\sqrt{\frac{k}{m}\left(A^2-x^2\right)}$$

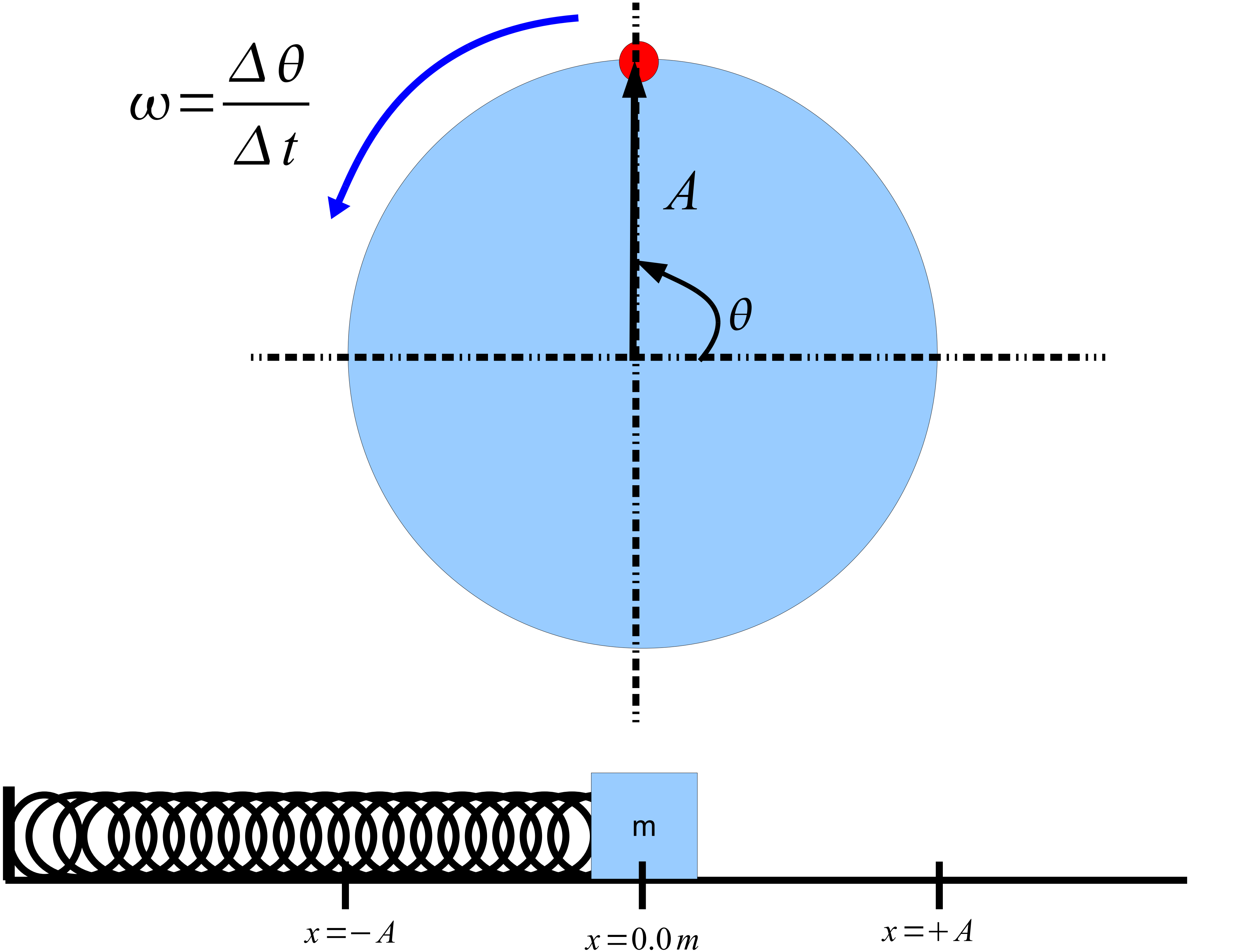
Simple Harmonic Motion and Circular Motion



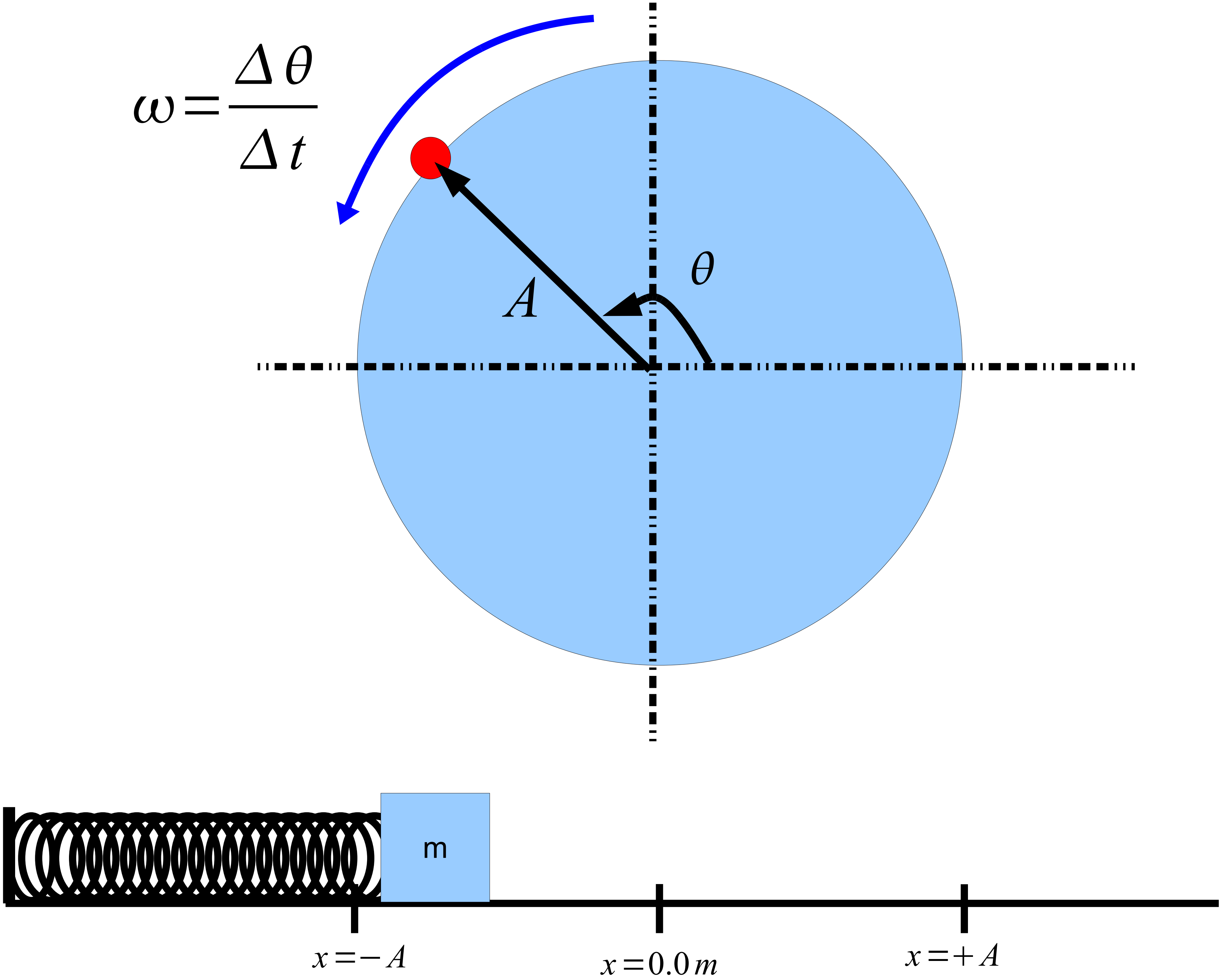
A
m
x = − A
x = 0.0 m
x = + A

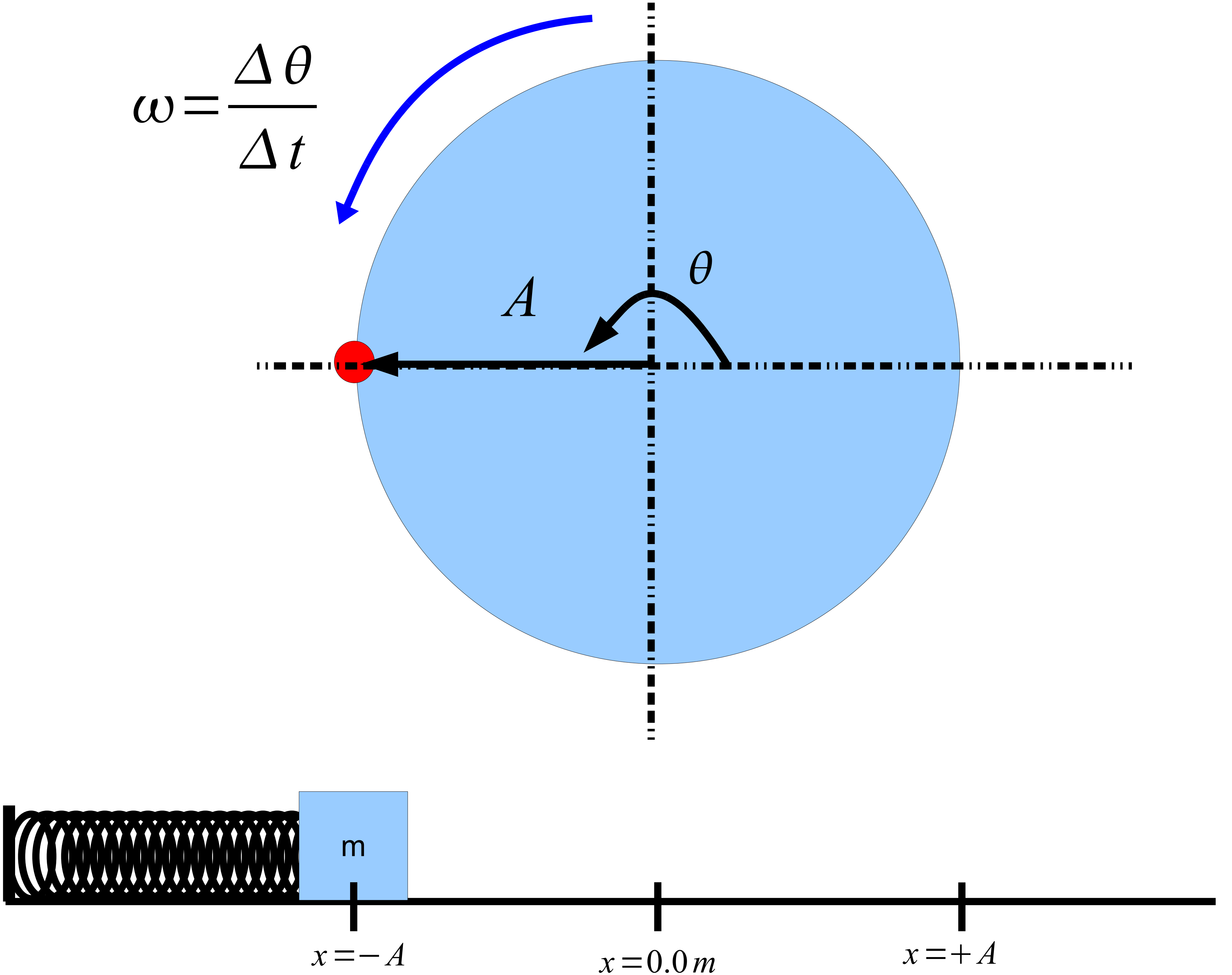


$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$

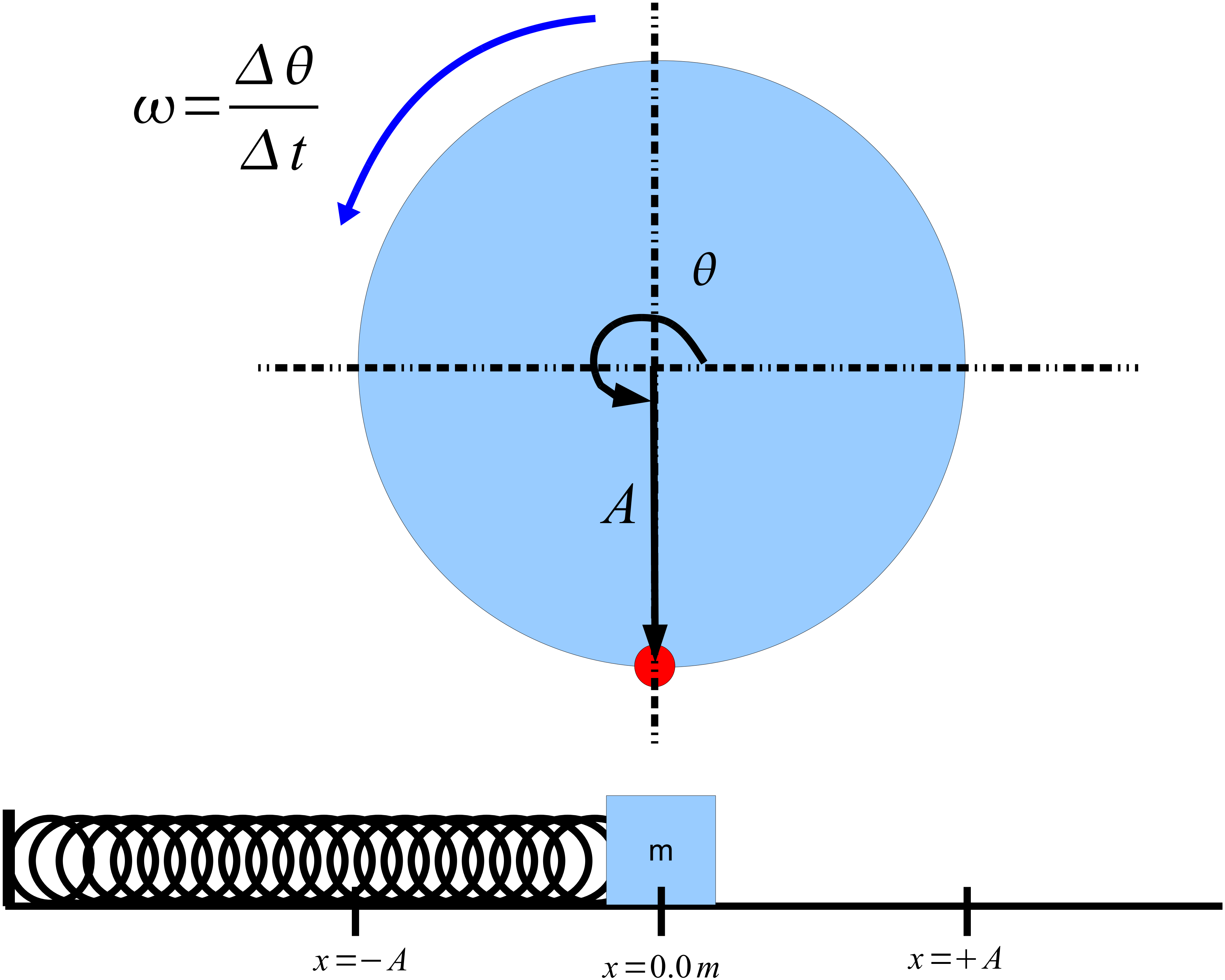


$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$

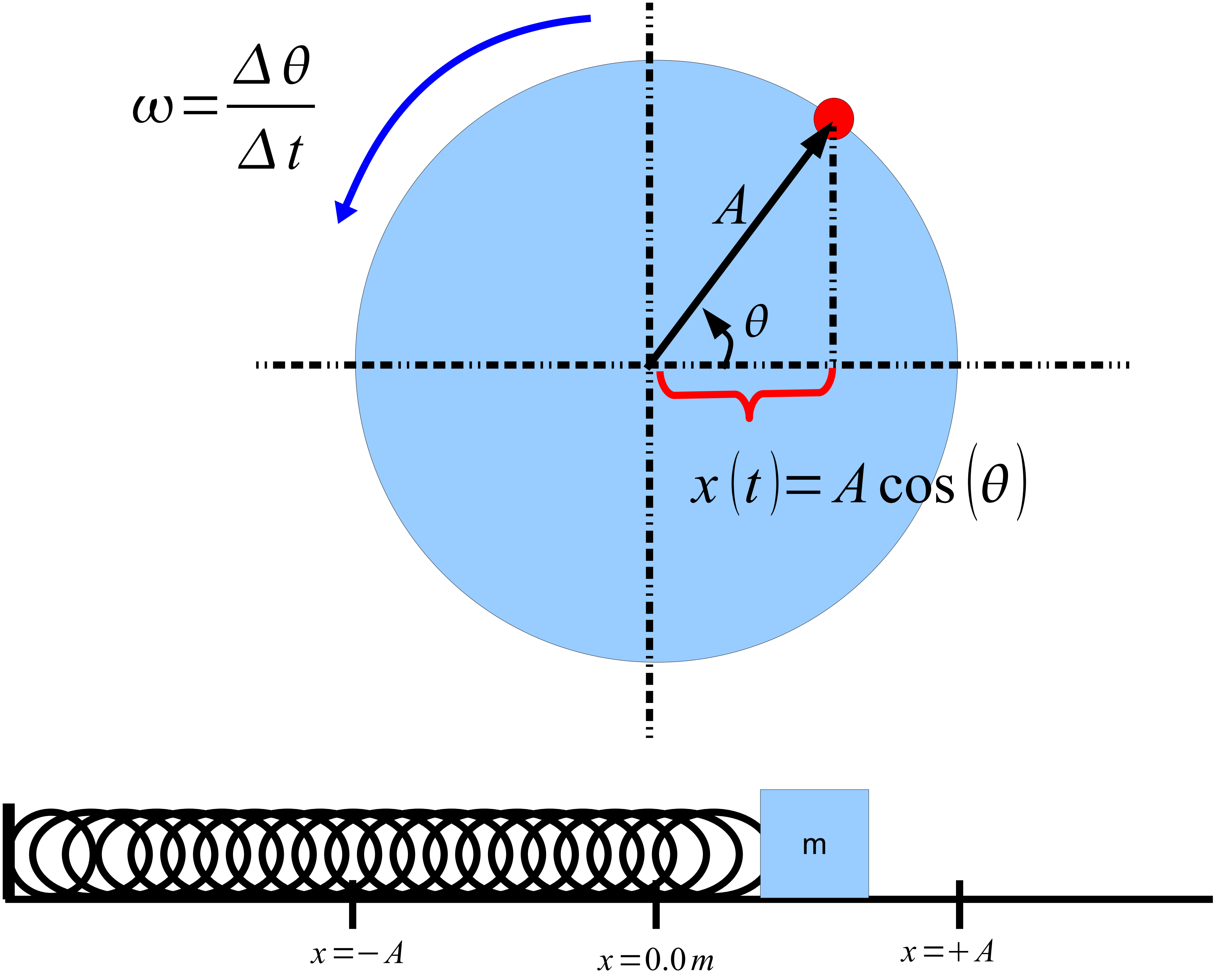

$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$



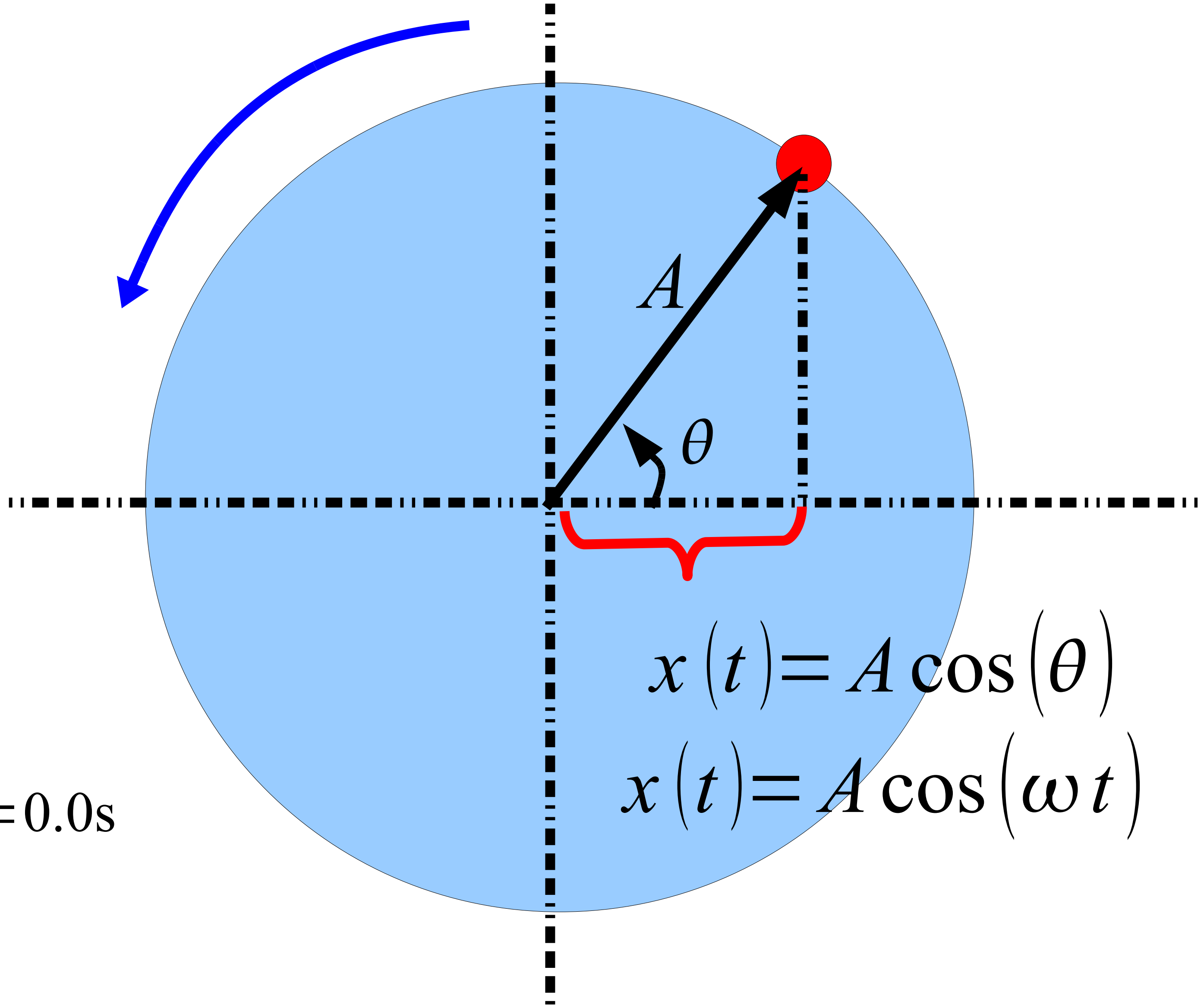
$\omega = \frac{\Delta\theta}{\Delta t}$
θ
A
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$\omega = \frac{\Delta\theta}{\Delta t}$
θ
A
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
$x(t) = A\cos(\theta)$
m
$x = -A$
$x = 0.0\,m$
$x = +A$



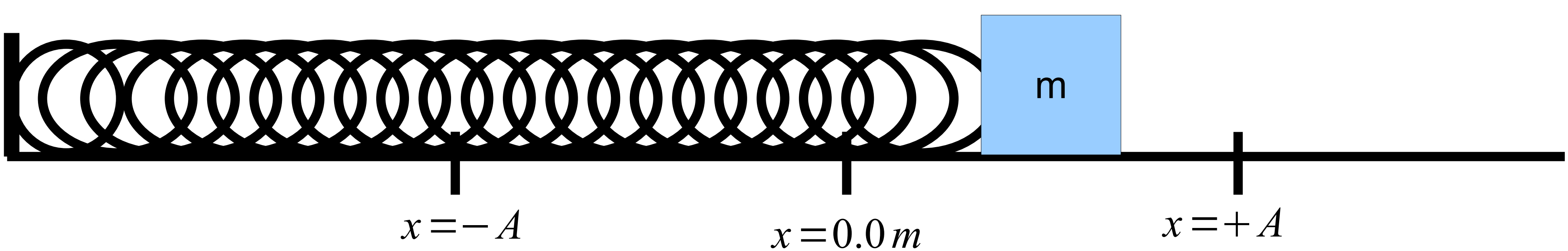
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{\theta - \theta_o}{t - t_o}$$

$$\theta_o = 0^o \text{ when } t = 0.0\text{s}$$

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega t$$



$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

Solutions must be cos, sin, or exponential.

$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$|v_{max}| = A\omega$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

$$|a_{max}| = A\omega^2$$

$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)\left(A\cos\left(\omega t + \phi\right)\right) = -A\omega^2\cos\left(\omega t + \phi\right)$$

$$\frac{k}{m} = \omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos\left(\omega t + \phi\right)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \phi\right)$$

$$\left|v_{max}\right| = A\omega$$

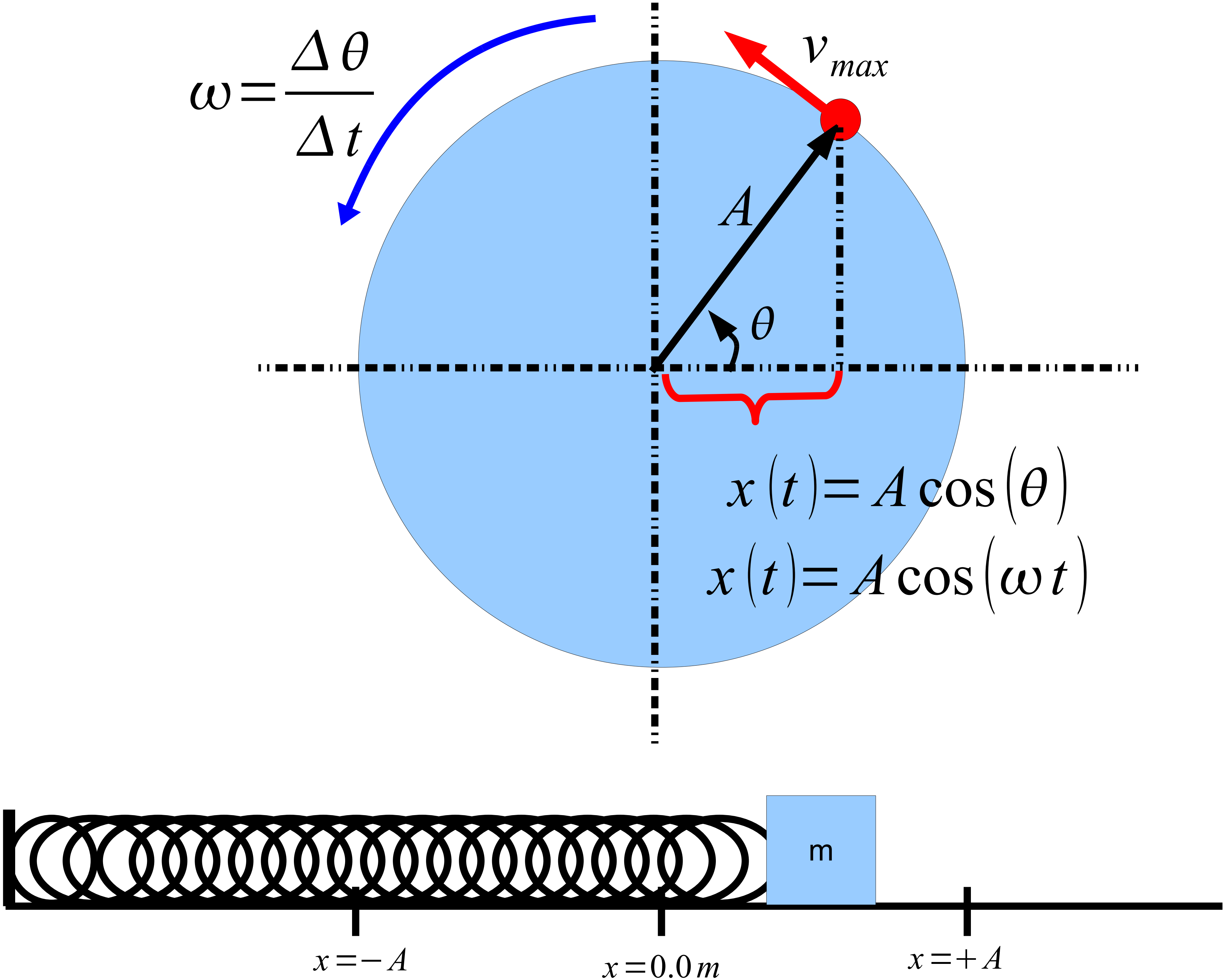
$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin\left(\omega t + \phi\right)$$

$$\left|a_{max}\right| = A\omega^2$$

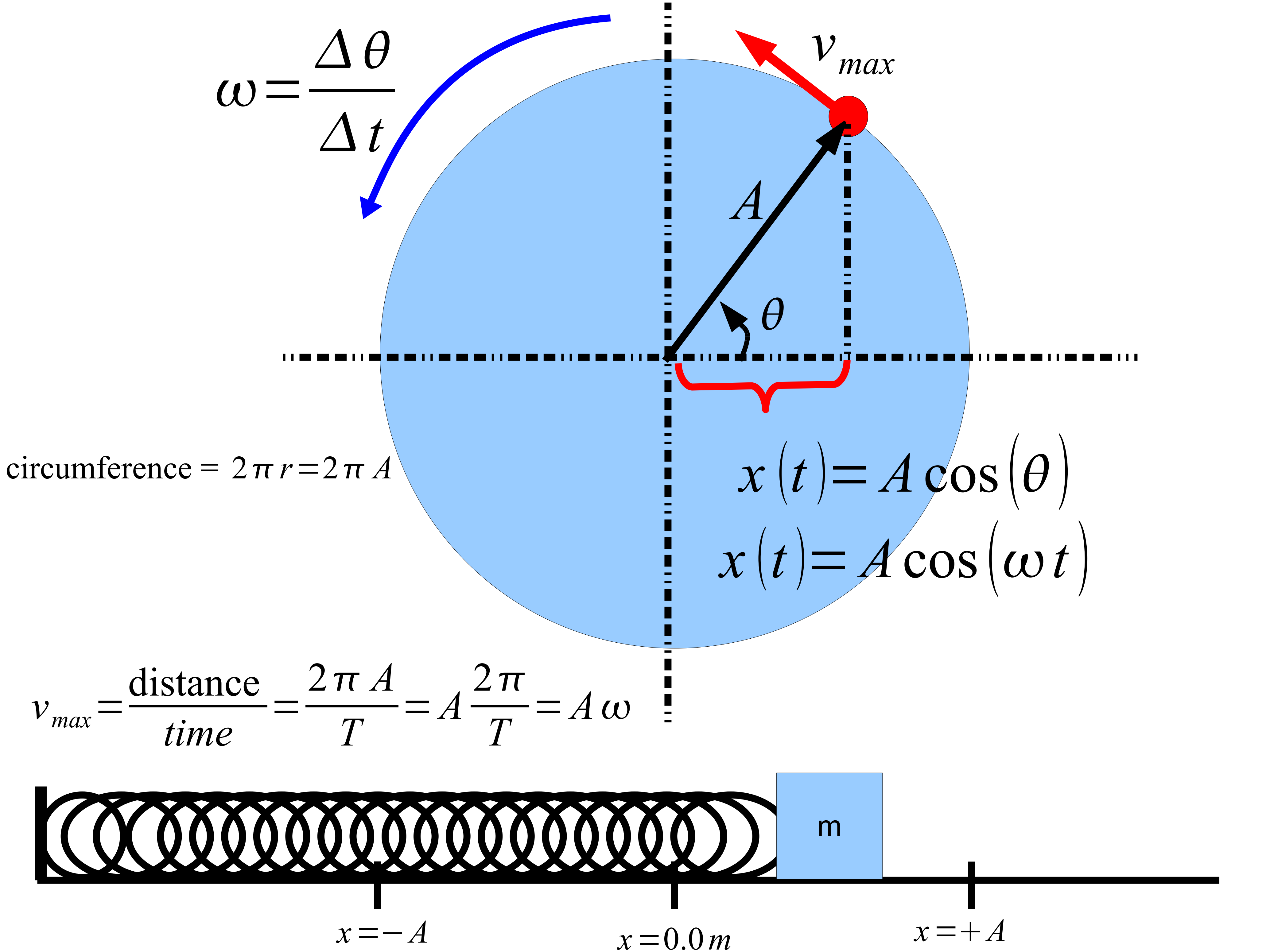
$$x(t) = A\cos(\omega t)$$

$$v(t) = -v_{max}\sin(\omega t) = -A\omega\sin(\omega t)$$

$$a(t) = -a_{max}\cos(\omega t) = -A\omega^2\cos(\omega t)$$



$\omega = \frac{\Delta \theta}{\Delta t}$
v_{max}
A
θ
$x(t) = A\cos(\theta)$
$x(t) = A\cos(\omega t)$
m
$x = -A$
$x = 0.0\,m$
$x = +A$


$\omega = \frac{\Delta\theta}{\Delta t}$
v_{max}
A
θ
circumference = $2\pi r = 2\pi A$
$x(t) = A\cos(\theta)$
$x(t) = A\cos(\omega t)$
$v_{max} = \frac{\text{distance}}{time} = \frac{2\pi A}{T} = A\frac{2\pi}{T} = A\omega$
m
$x = -A$
$x = 0.0\,m$
$x = +A$

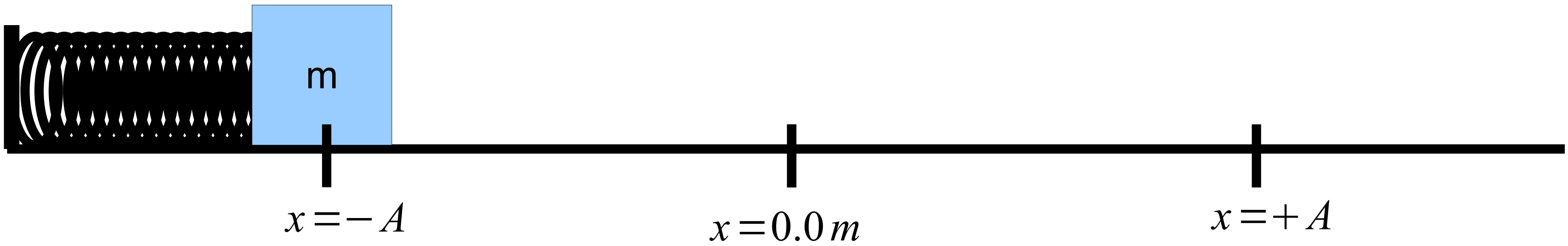
Recall:

$$0+\frac{1}{2}m\,v_{max}^{2}=\frac{1}{2}k\,A^{2}$$

$$v_{max}=A\sqrt{\frac{k}{m}}$$

$$v_{max}=A\sqrt{\frac{k}{m}}=A\,\omega$$

$$\omega=\sqrt{\frac{k}{m}}$$



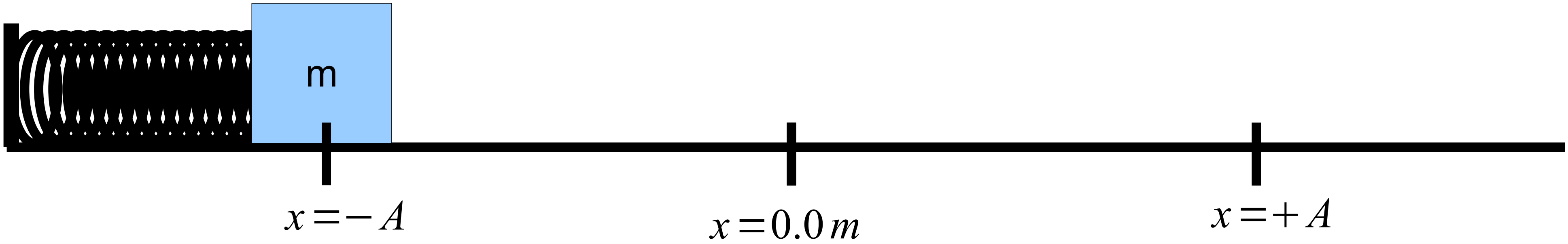
$$F_{net} = F_s$$

$$ma = -k\,x$$

$$m\left(-A\,\omega^2 \cos\left(\omega\,t\right)\right) = -k\left(A\,\omega \cos\left(\omega\,t\right)\right)$$

$$m = k\,\omega$$

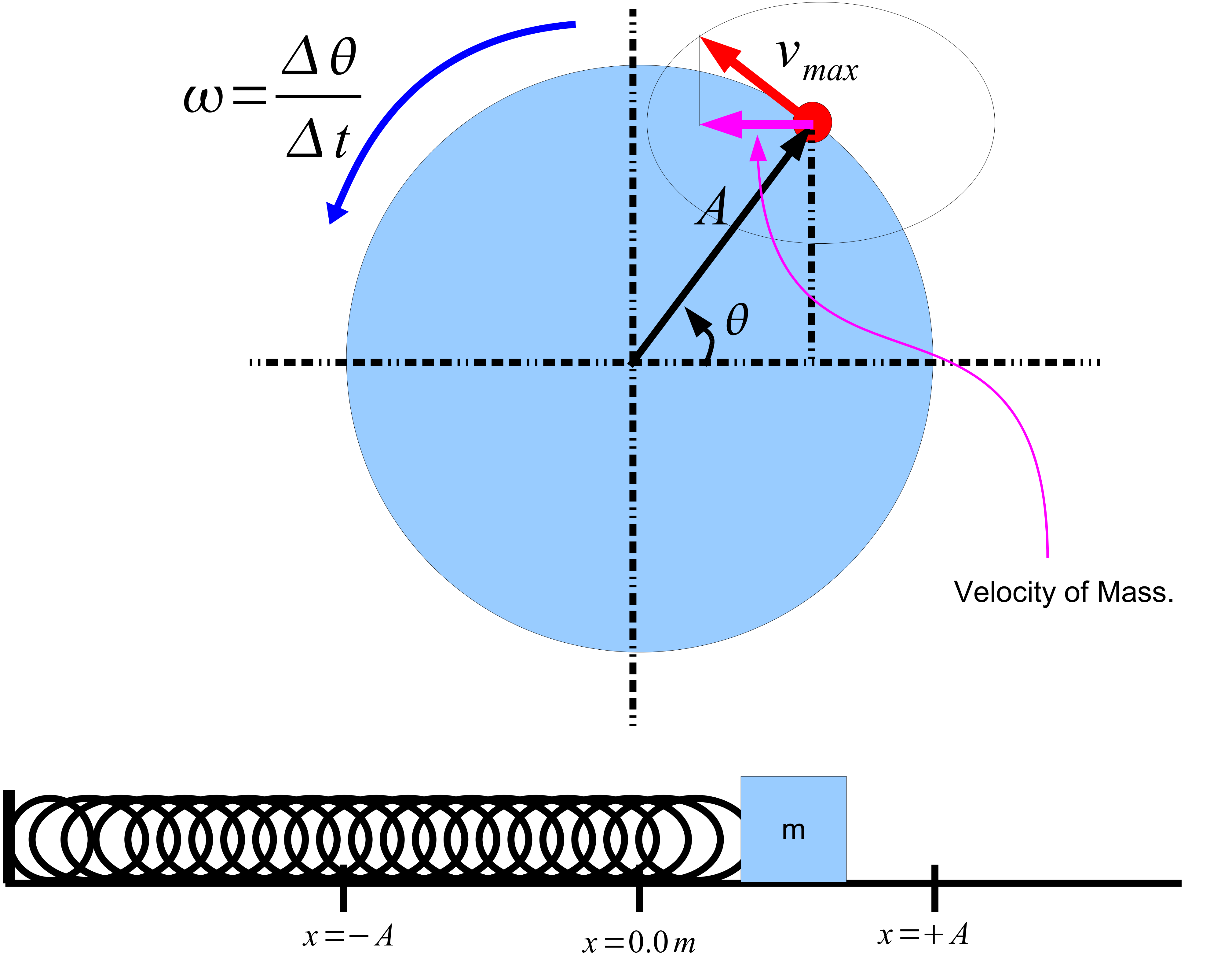
$$\omega = \sqrt{\frac{k}{m}}$$



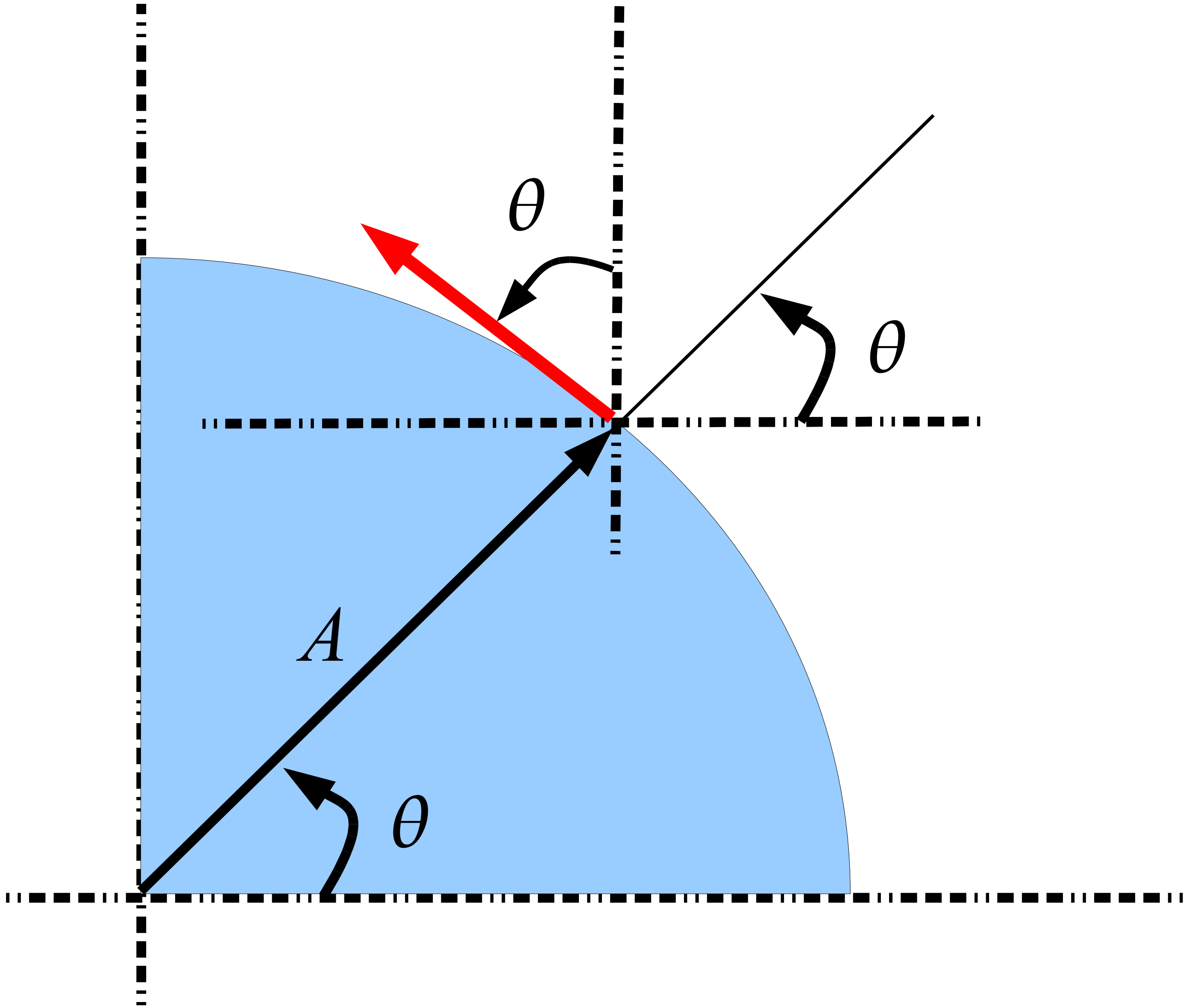
$$\omega = \sqrt{\frac{k}{m}}$$

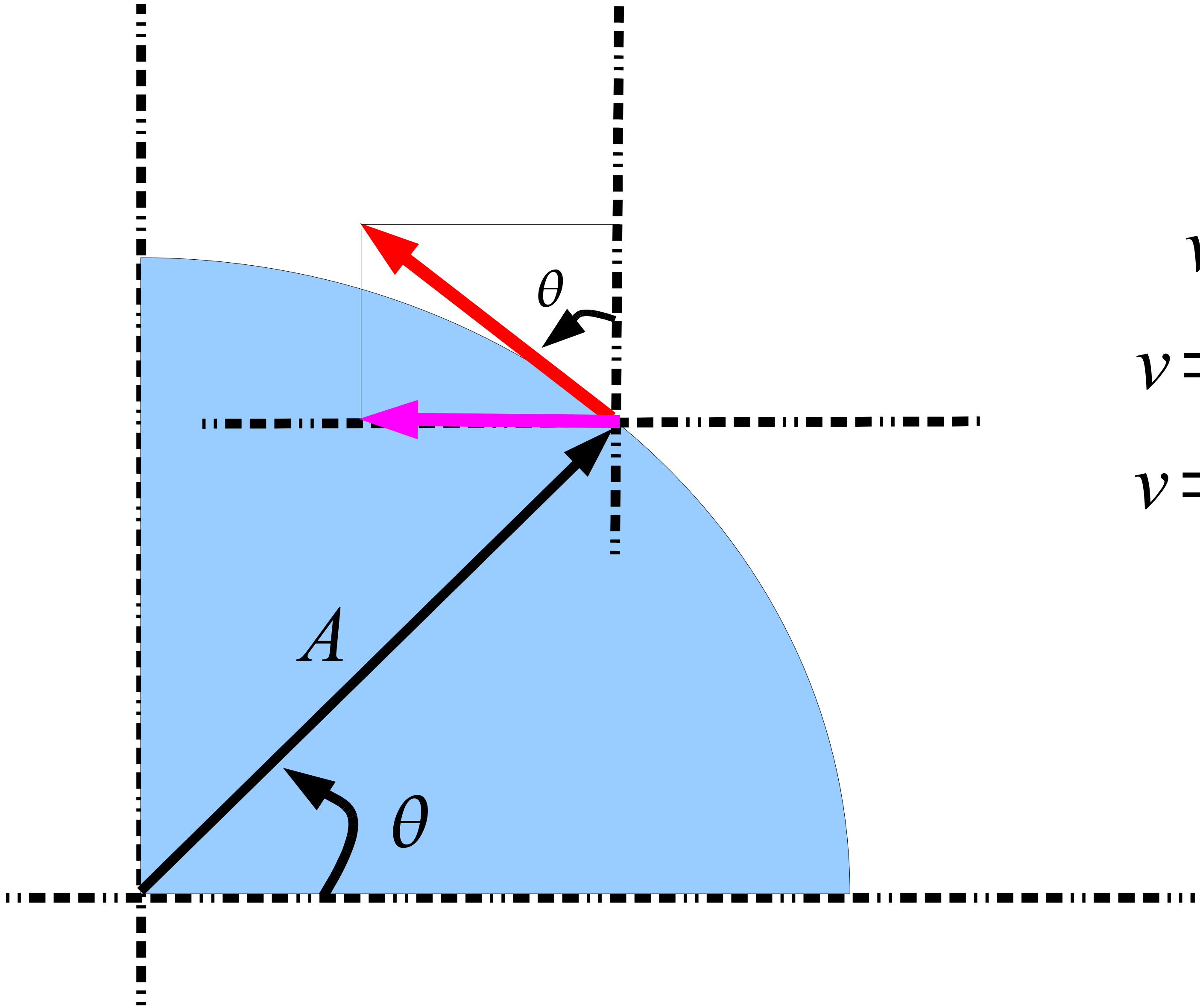
$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$



$\omega = \frac{\Delta\theta}{\Delta t}$
v_{max}
A
θ
Velocity of Mass.
m
$x = -A$
$x = 0.0\,m$
$x = +A$


θ
θ
A
θ

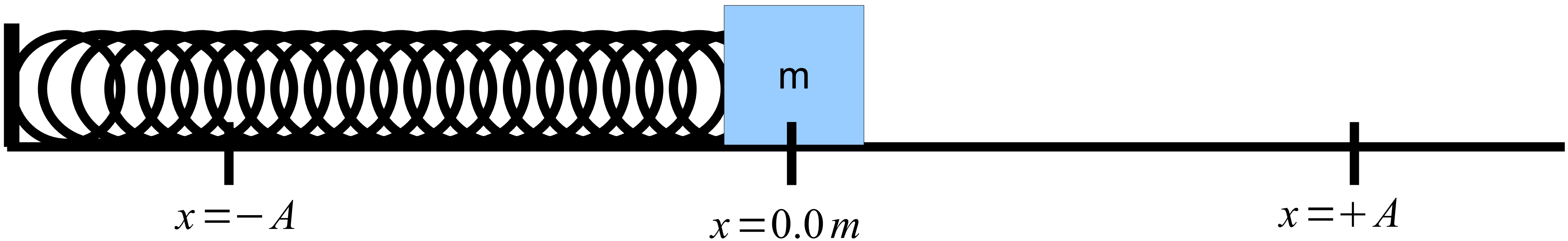


$$v = -v_{max} \sin \theta$$

$$v = -v_{max} \sin(\omega t)$$

$$v = -A \omega \sin(\omega t)$$

So what we know so far ....



$$x(t) = A\cos(\omega t)$$

$$v(t) = -v_{max}\sin(\omega t) = -A\omega\sin(\omega t)$$

$$a(t) = -a_{max}\cos(\omega t) = -A\omega^2\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$|v_{max}| = A\omega = A\sqrt{\frac{k}{m}}$$

$$|a_{max}| = A\omega^2 = A\left(\frac{k}{m}\right)$$

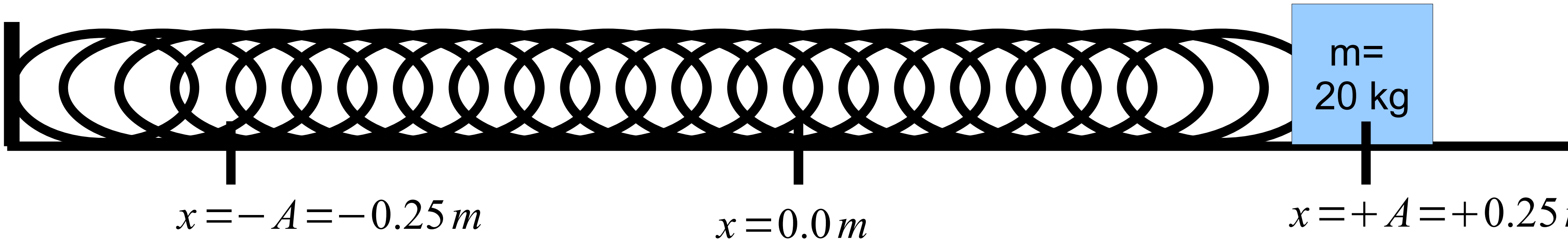
$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v(x) = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$k = 100\ N/m$ **Mass is released from rest at x = +A = 0.25 m**



$$A = 0.25\, m$$

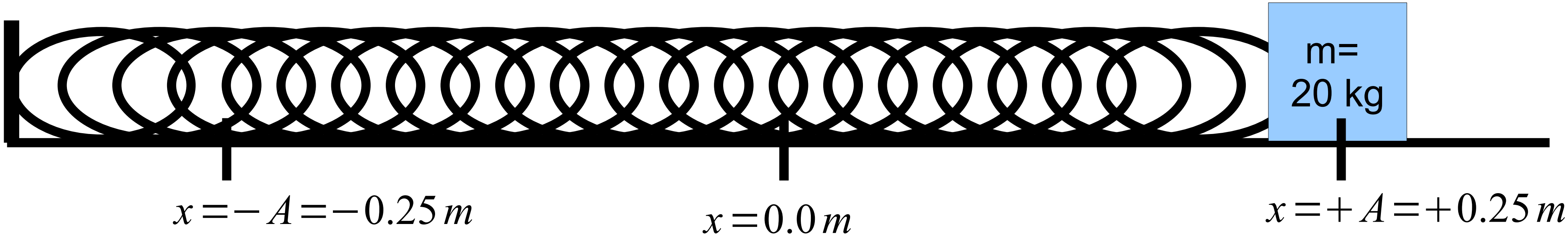
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100\ N/m}{20\, kg}} = 2.24\, s^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.24\, s^{-1}} = 2.81\, s$$

$$v_{max} = A\omega = (0.25\, m)(2.24\, s^{-1}) = 0.56\, m/s$$

$$a_{max} = A\omega^2 = (0.25\, m)(2.24\, s^{-1})^2 = 1.25\, m/s^2$$

$k = 100\ N/m$ **Mass is released from rest at x = +A = 0.25 m**



$$A = 0.25\ m$$

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$$v_{max} = A\omega = (0.25\ m)(2.24\ s^{-1}) = 0.56\ m/s$$

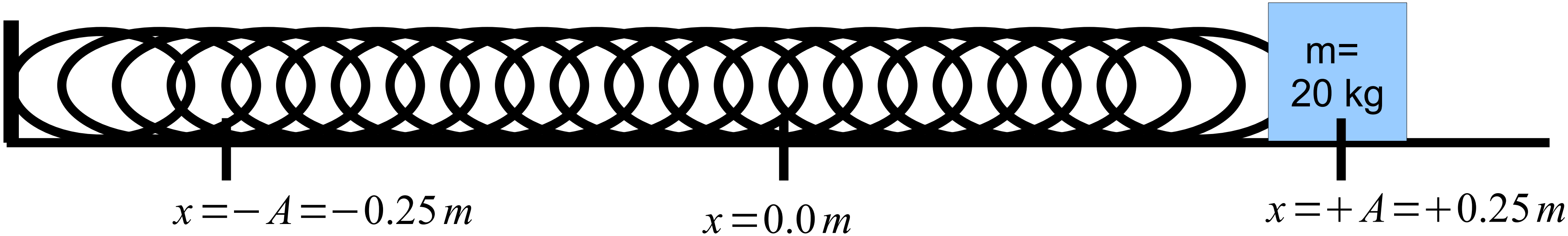
$$a_{max} = A\omega^2 = (0.25\ m)(2.24\ s^{-1})^2 = 1.25\ m/s^2$$

Check:

$$v_{max} = A\sqrt{\frac{k}{m}} = 0.25\ m\sqrt{\frac{100\ N/m}{20\ kg}} = 0.56\ m/s$$

$$a_{max} = A\frac{k}{m} = (0.25\ m)\frac{100\ N/m}{20\ kg} = 1.25\ m/s^2$$

$k = 100\,N/m$ **Mass is released from rest at x = +A = 0.25 m**



$$A = 0.25\,m$$

$$\omega = \sqrt{\frac{k}{m}} = 2.24\,s^{-1}$$

$$T = \frac{2\pi}{\omega} = 2.81\,s$$

$$v_{max} = A\omega = 0.56\,m/s$$

$$a_{max} = A\omega^2 = 1.25\,m/s^2$$

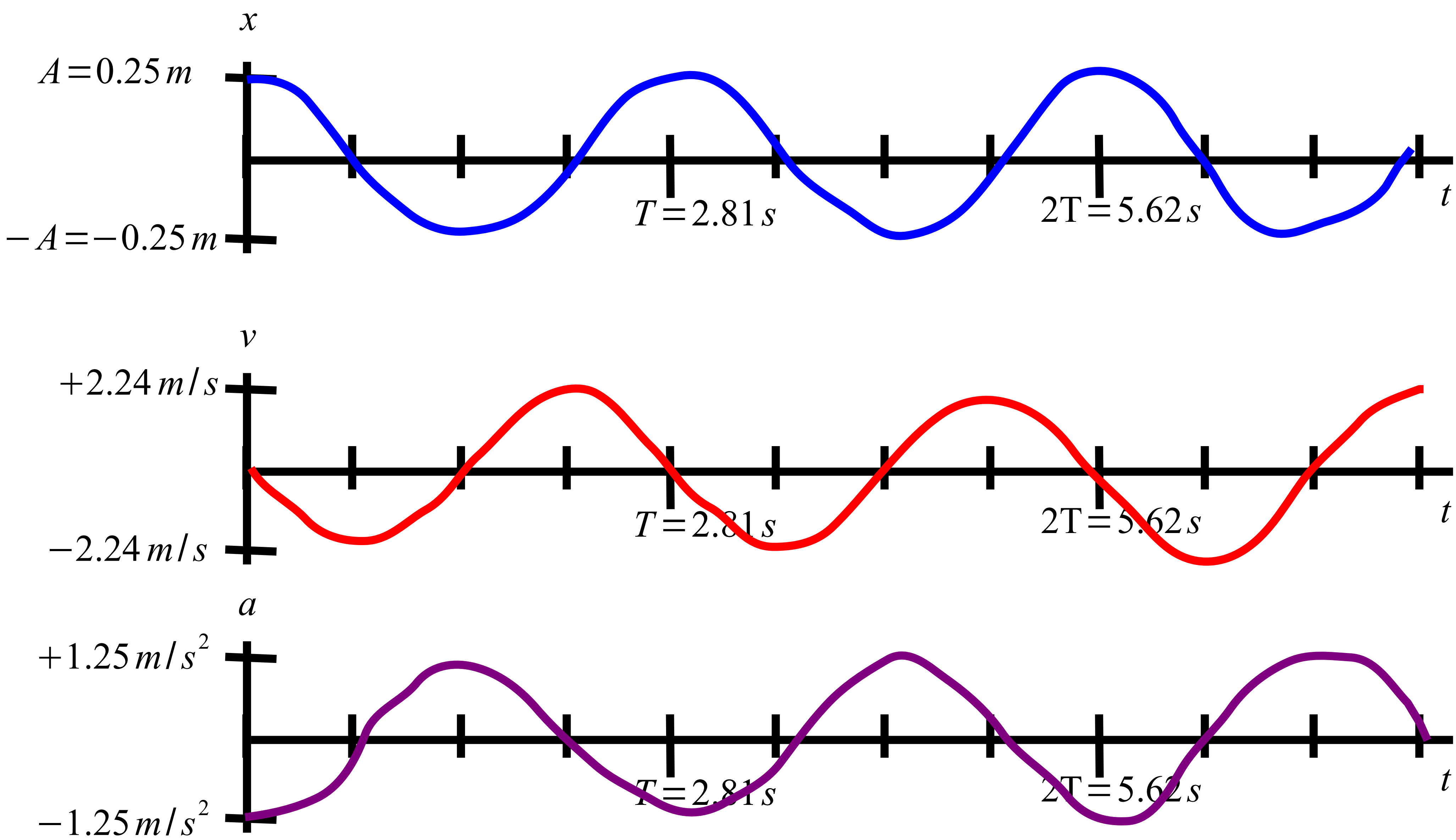
$$x(t) = A\cos(\omega t) = 0.25\,m\cos(2.24\,s^{-1}\,t)$$

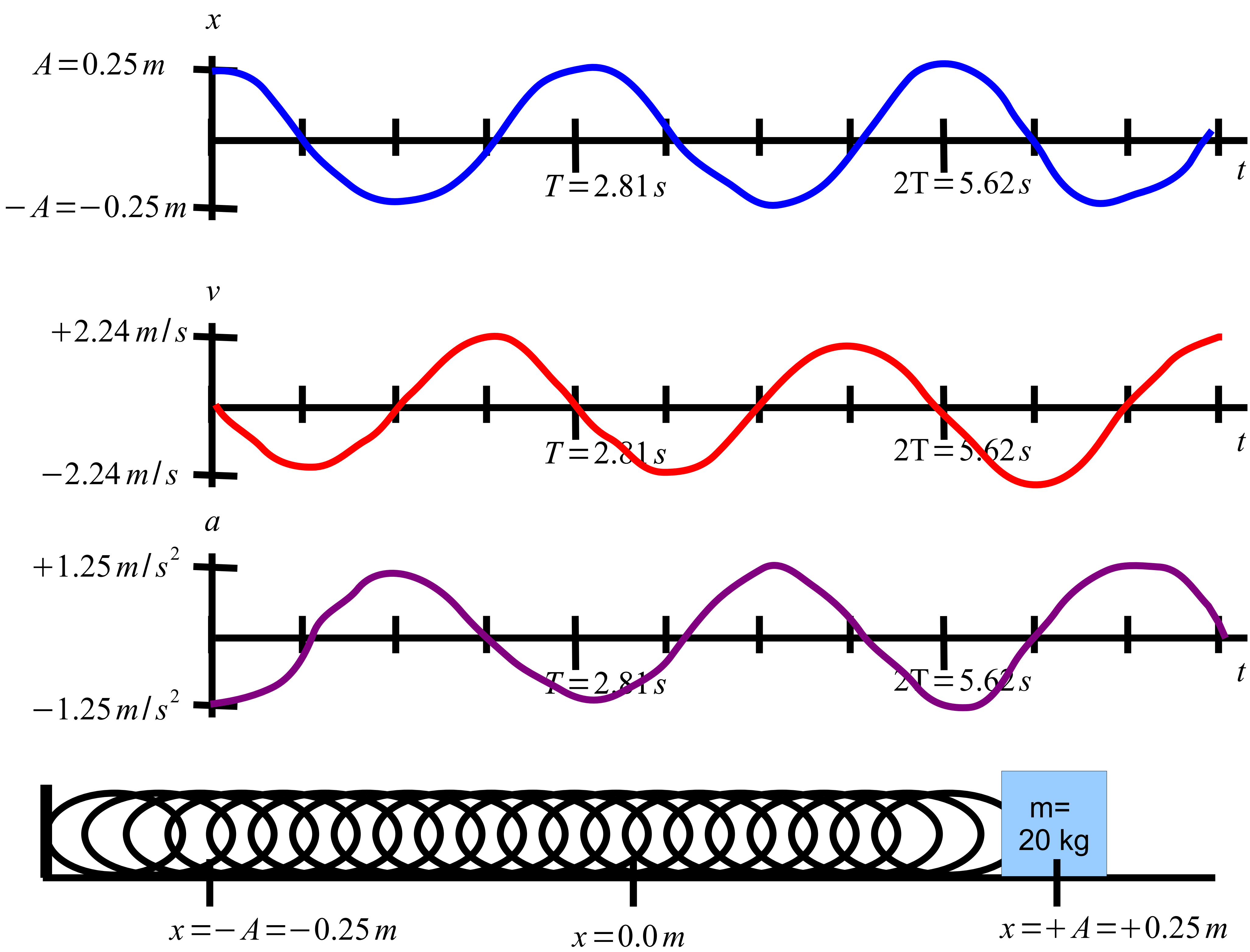
$$v(t) = -A\omega\sin(\omega t) = -0.56\,m/s\sin(2.24\,s^{-1}\,t)$$

$$a(t) = -A\omega^2\cos(\omega t) = -1.25\,m/s^2\cos(2.24\,s^{-1}\,t)$$

$$x(t) = A\cos(\omega t) = 0.25\,m\cos(2.24\,s^{-1}\,t)$$
$$v(t) = -A\omega\sin(\omega t) = -0.56\,m/s\,\sin(2.24\,s^{-1}\,t)$$
$$a(t) = -A\omega^2\cos(\omega t) = -1.25\,m/s^2\cos(2.24\,s^{-1}\,t)$$

You should be able to roughly draw out plots.



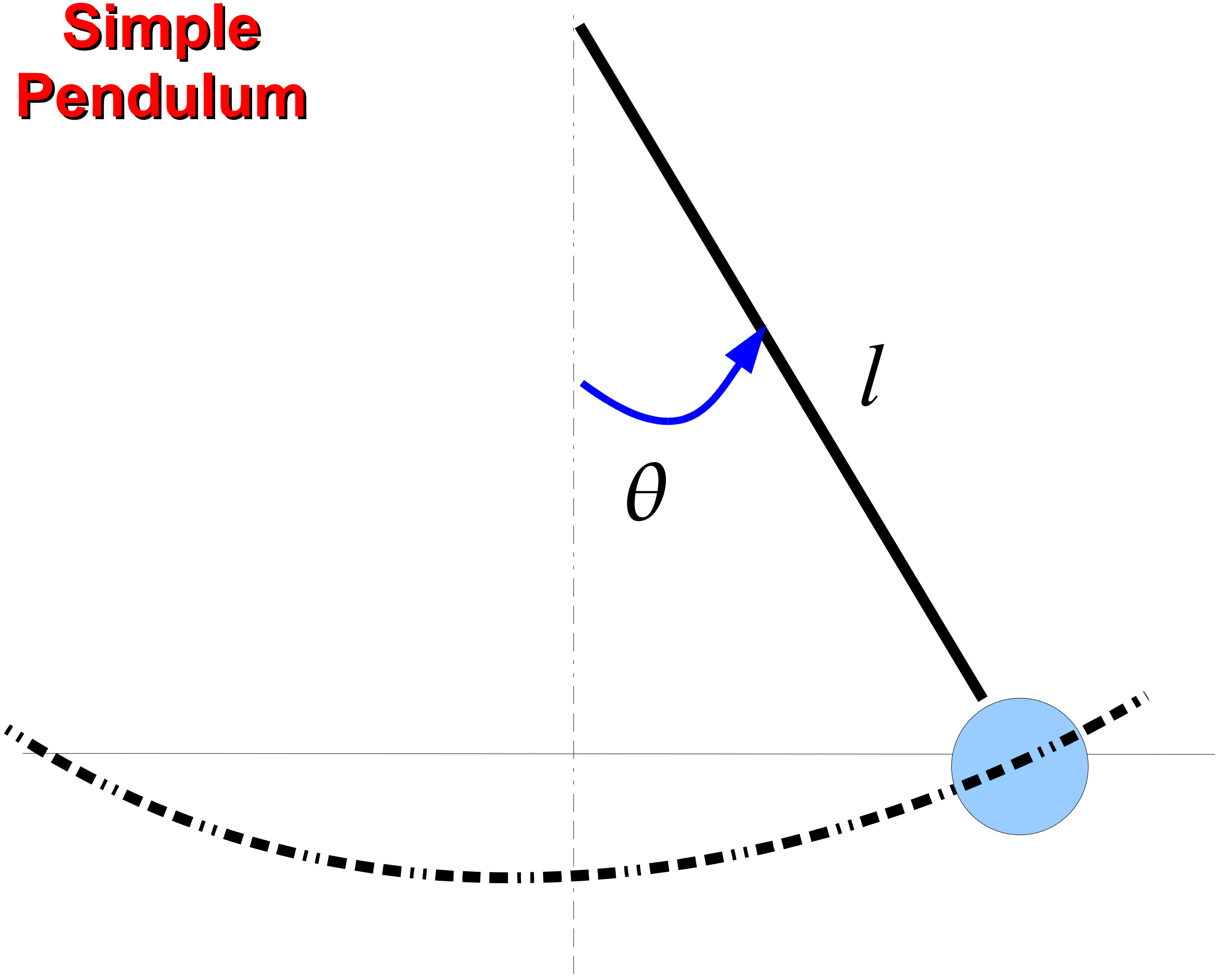


x
A=0.25 m
−A=−0.25 m
T=2.81 s
2T=5.62 s
t
v
+2.24 m/s
−2.24 m/s
T=2.81 s
2T=5.62 s
t
a
+1.25 m/s2
−1.25 m/s2
T=2.81 s
2T=5.62 s
t
m=
20 kg
x=−A=−0.25 m
x=0.0 m
x=+A=+0.25 m

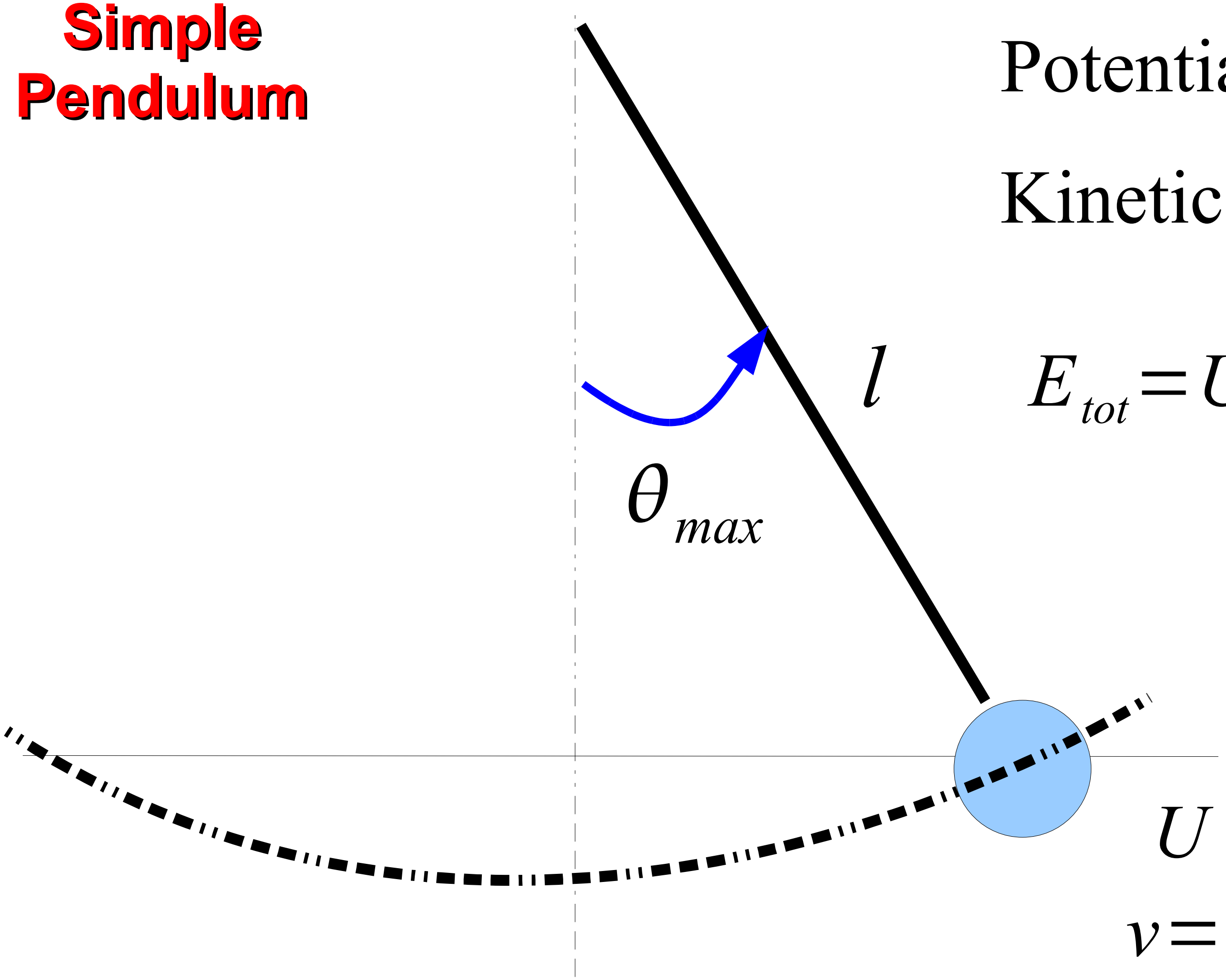
Potential Energy

Simple Pendulum

Simple Pendulum



Simple Pendulum



Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$

$U = U_{max}$

$v = 0.0\, m/s$

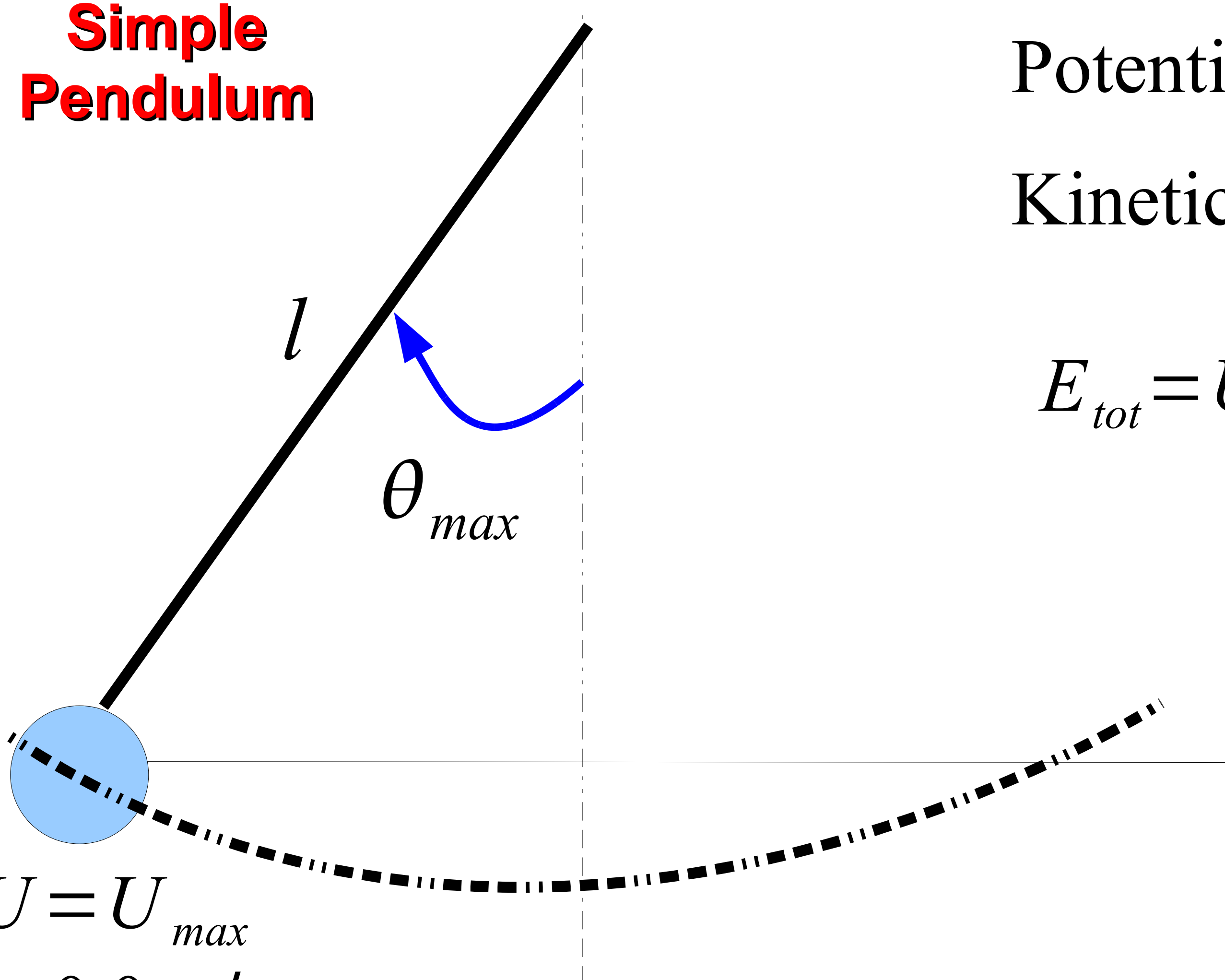
$K = 0.0\, J$

Simple Pendulum

Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$



$U = U_{max}$

$v = 0.0\, m/s$

$K = 0.0\, J$

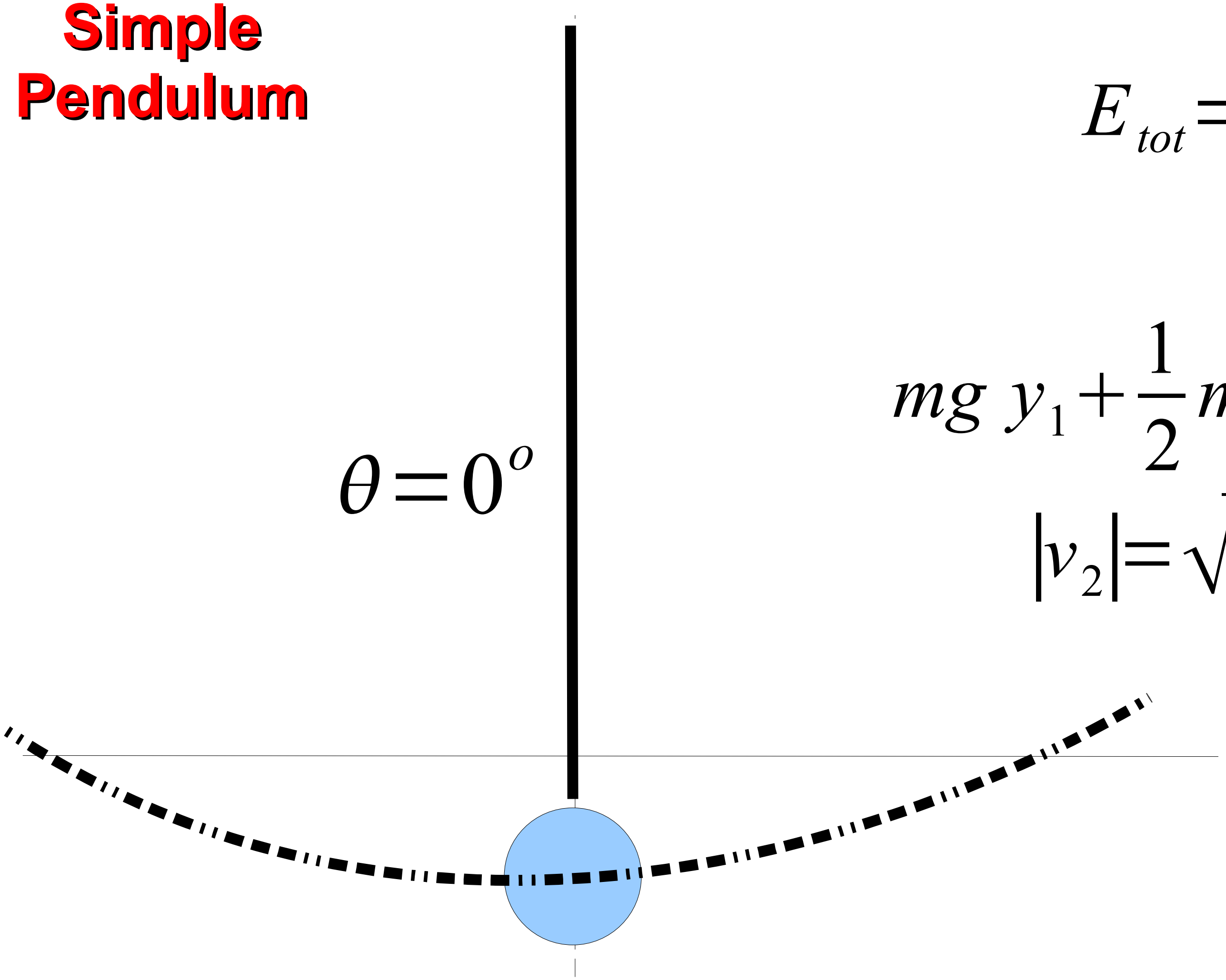
Simple Pendulum

$$E_{tot} = mgy + \frac{1}{2} m v^2$$

$$E_1 = E_2$$

$$mg\, y_1 + \frac{1}{2} m v_1^2 = mg\, y_2 + \frac{1}{2} m v_2^2$$

$$|v_2| = \sqrt{v_1^2 + 2g\left(y_1 - y_2\right)}$$



$$\theta = 0^o$$

$$U = U_{min}$$

$$v = v_{max}$$

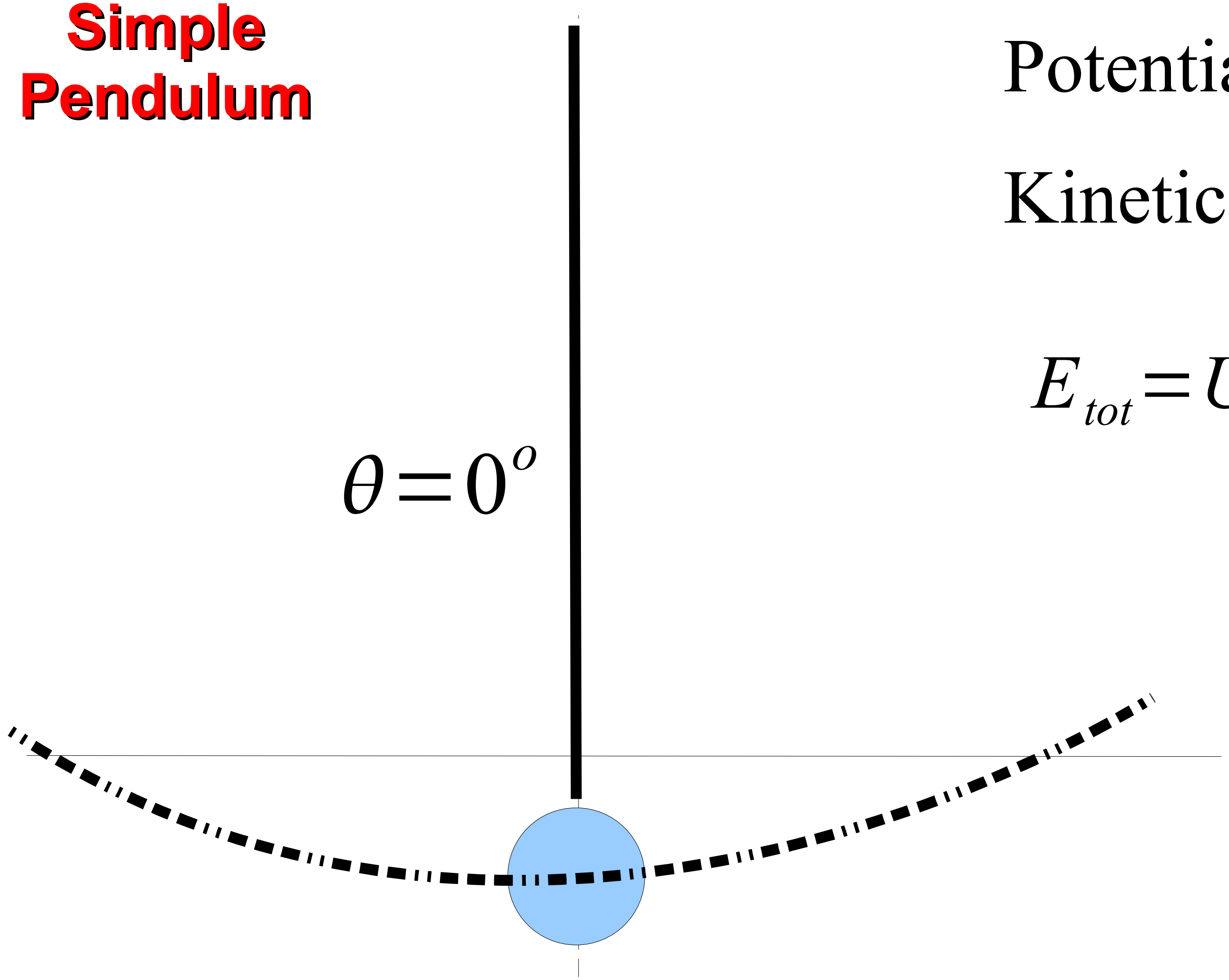
$$K = \frac{1}{2} m v_{max}^2$$

Simple Pendulum

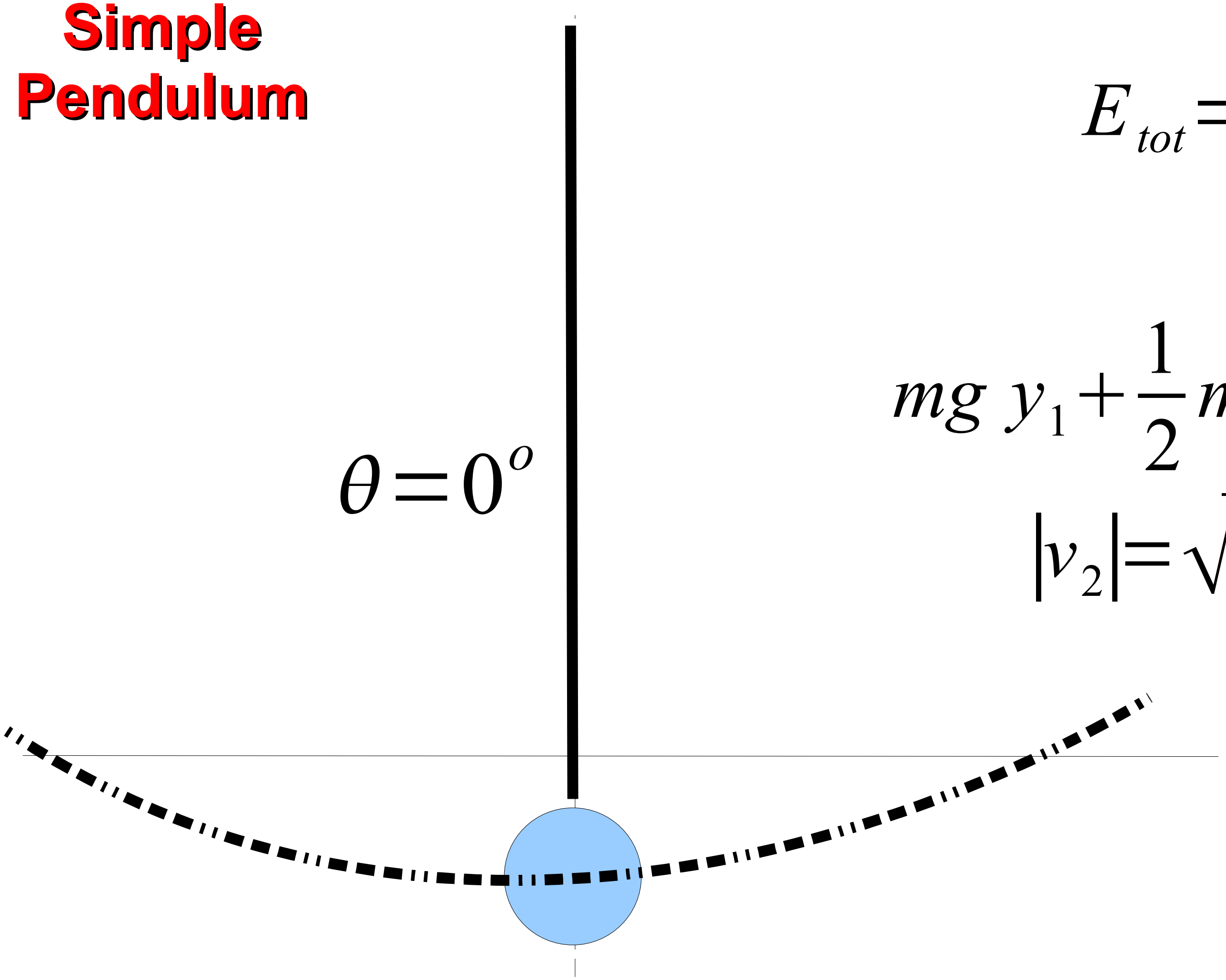
Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$



Simple Pendulum



$$E_{tot} = mgy + \frac{1}{2} m v^2$$

$$E_1 = E_2$$

$$mg\, y_1 + \frac{1}{2} m v_1^2 = mg\, y_2 + \frac{1}{2} m v_2^2$$

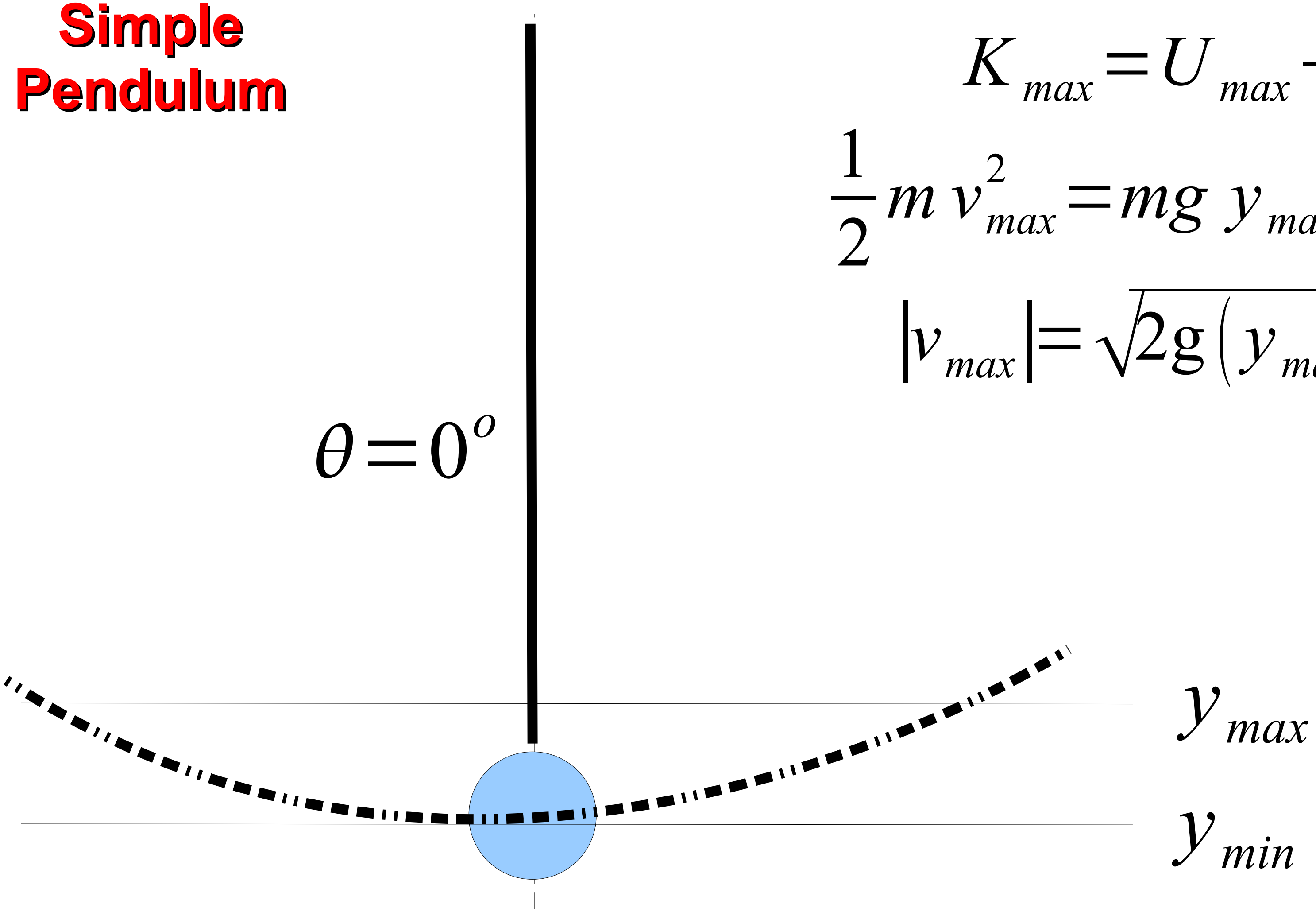
$$|v_2| = \sqrt{v_1^2 + 2g\left(y_1 - y_2\right)}$$

Simple Pendulum

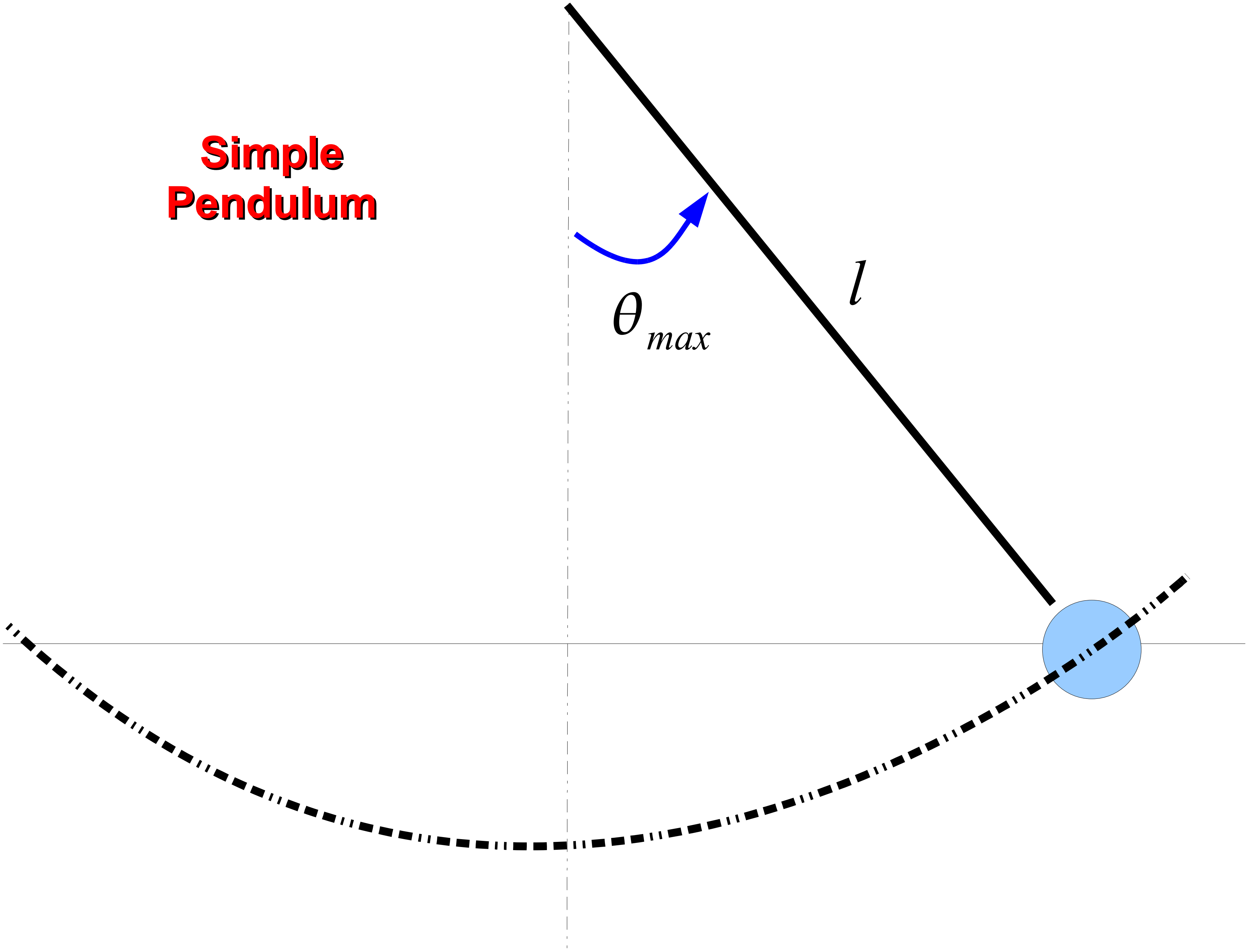
$$K_{max} = U_{max} - U_{min}$$

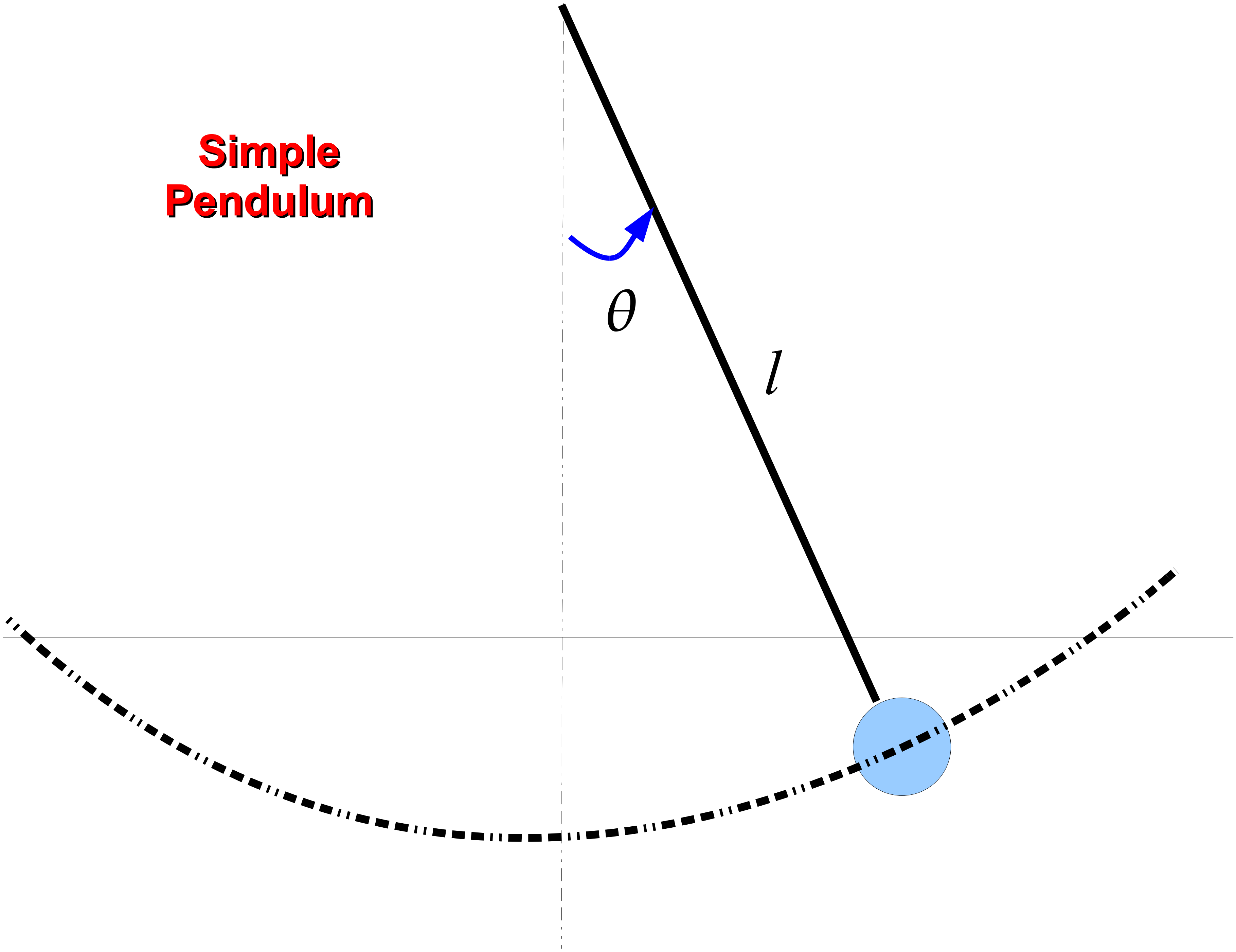
$$\frac{1}{2} m v_{max}^2 = mg\, y_{max} - mg\, y_{min}$$

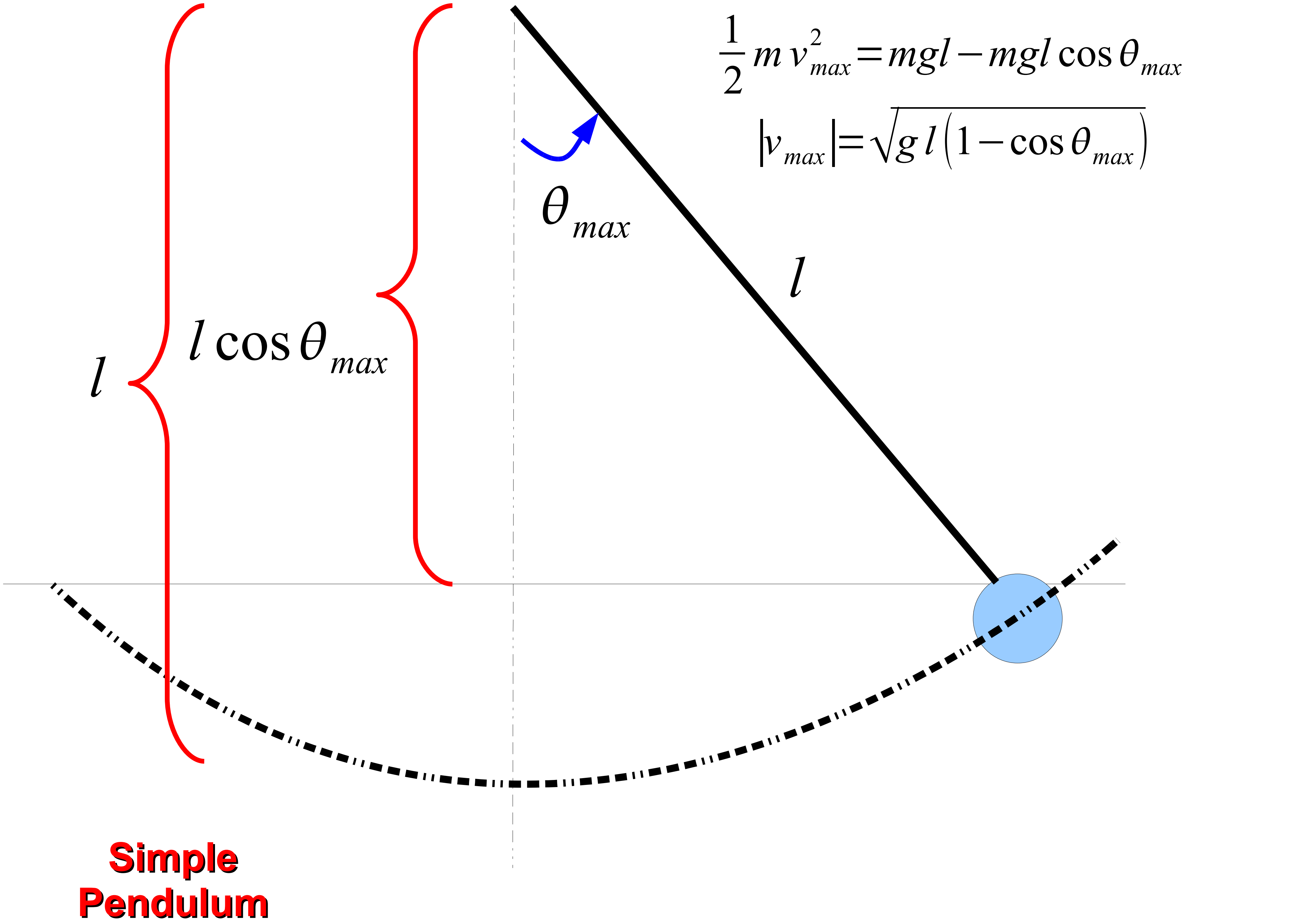
$$|v_{max}| = \sqrt{2g\left(y_{max} - y_{min}\right)}$$



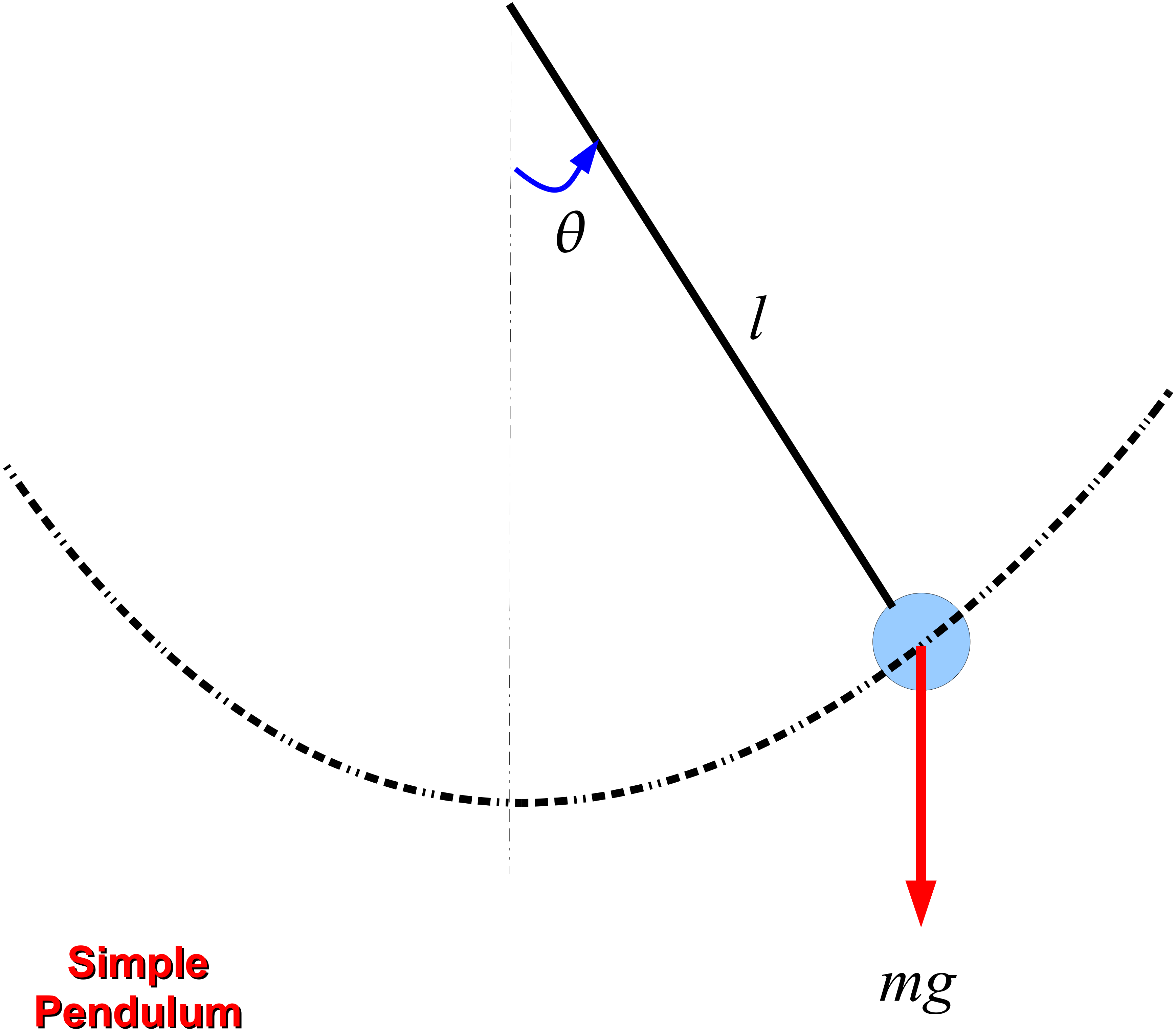
Simple Pendulum



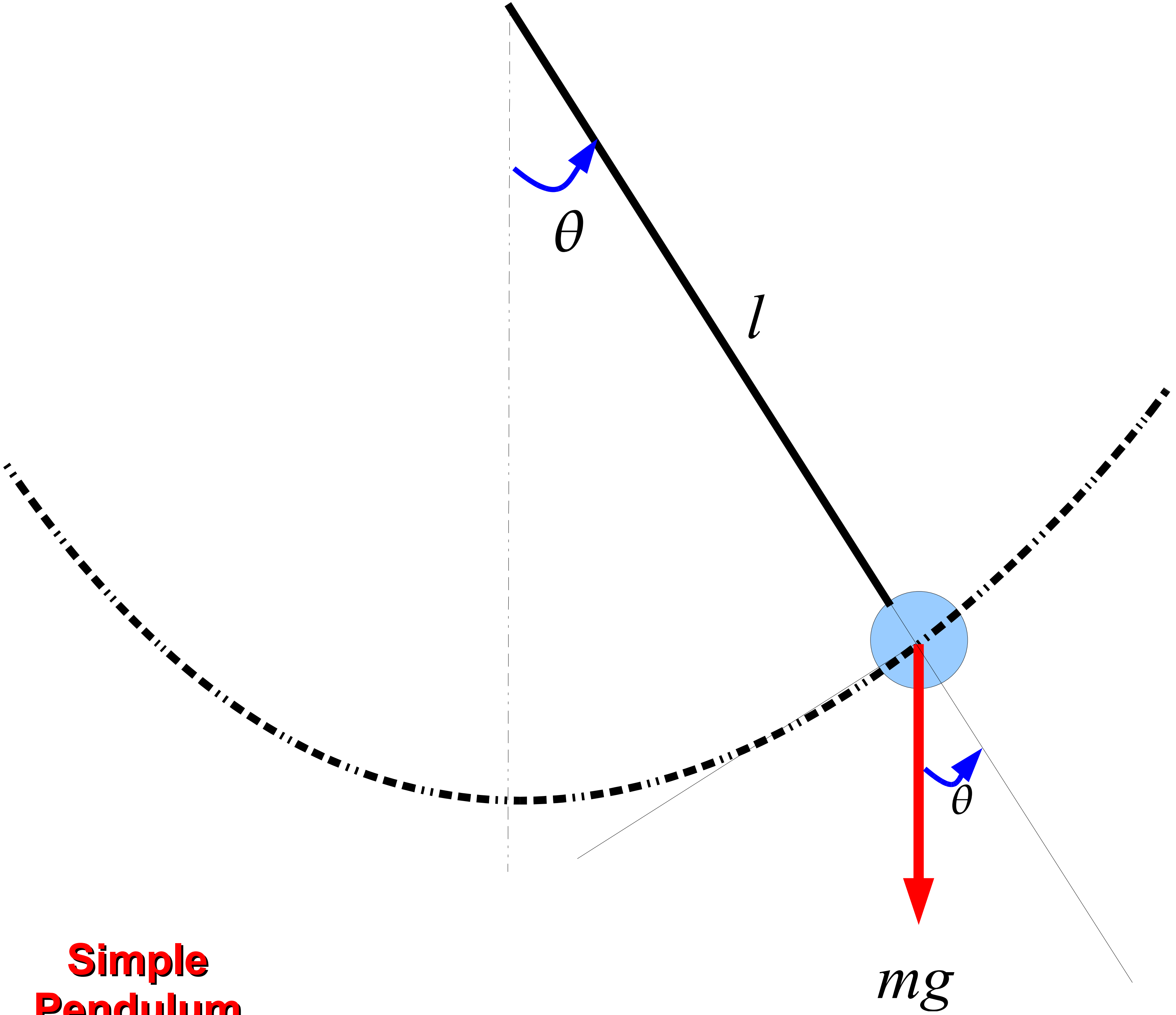

Simple
Pendulum
θ
l



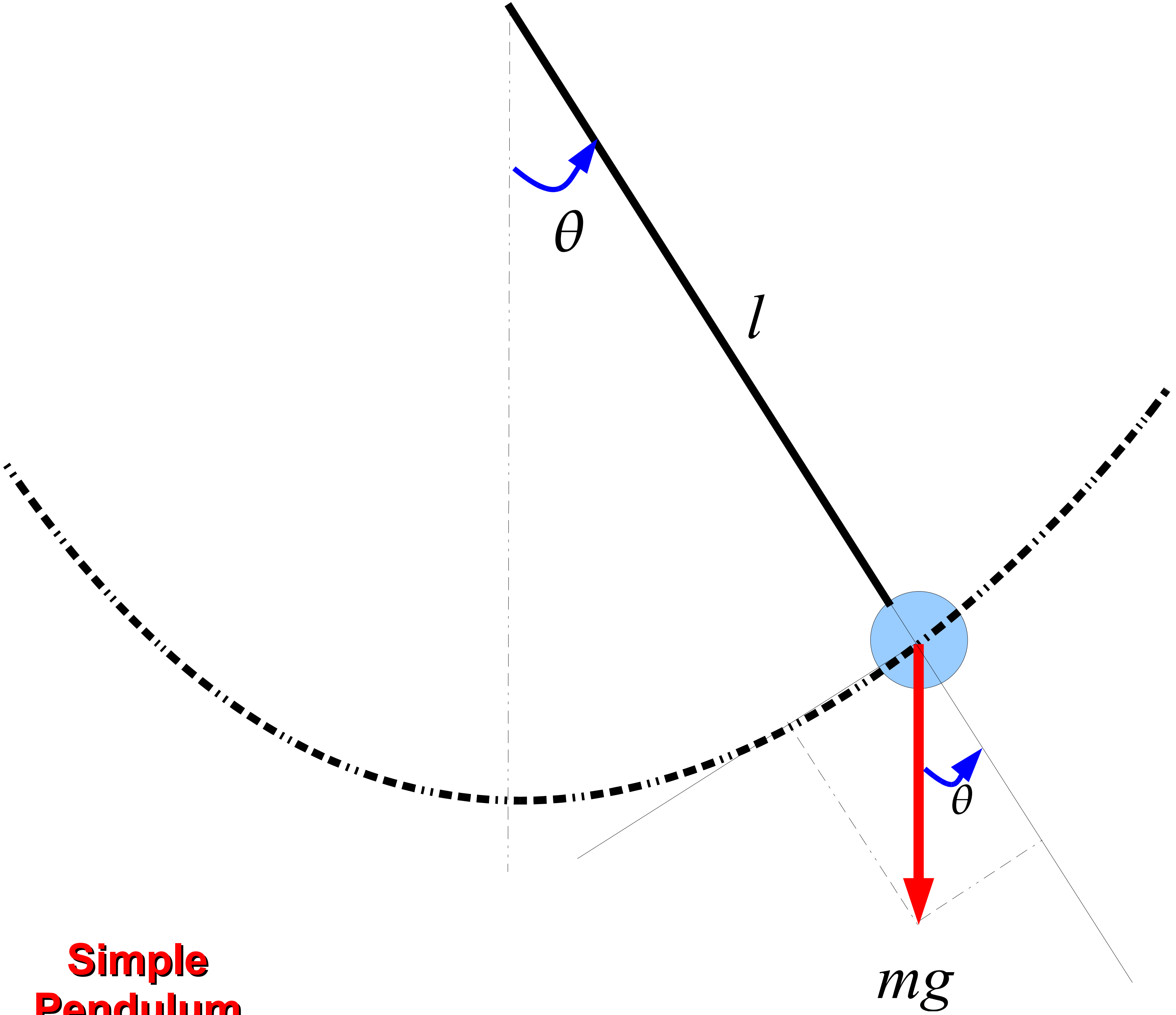
$\frac{1}{2}mv_{max}^2 = mgl - mgl\cos\theta_{max}$
$|v_{max}| = \sqrt{gl(1-\cos\theta_{max})}$
θ_{max}
l
$l\cos\theta_{max}$
l
Simple
Pendulum



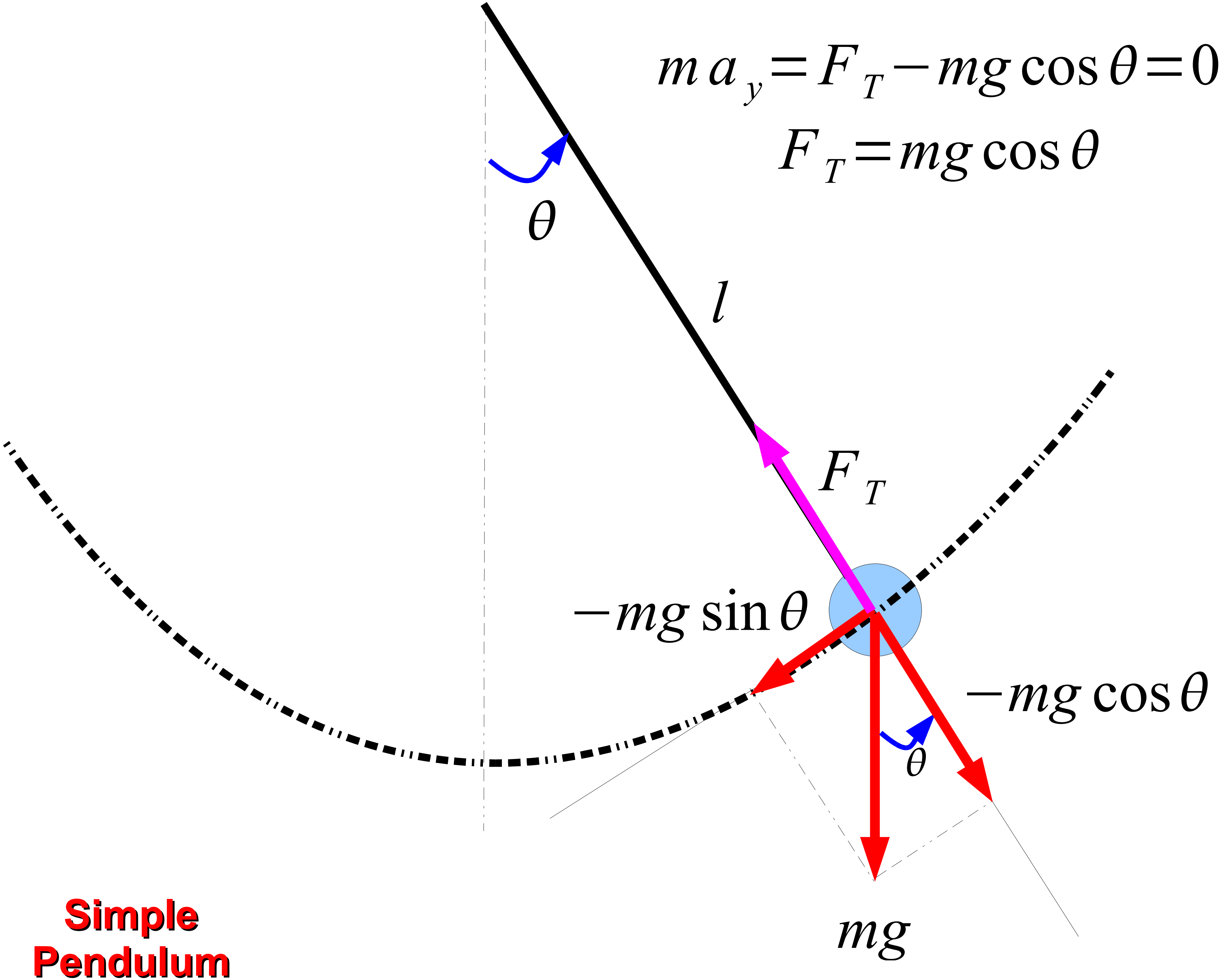
Simple Pendulum



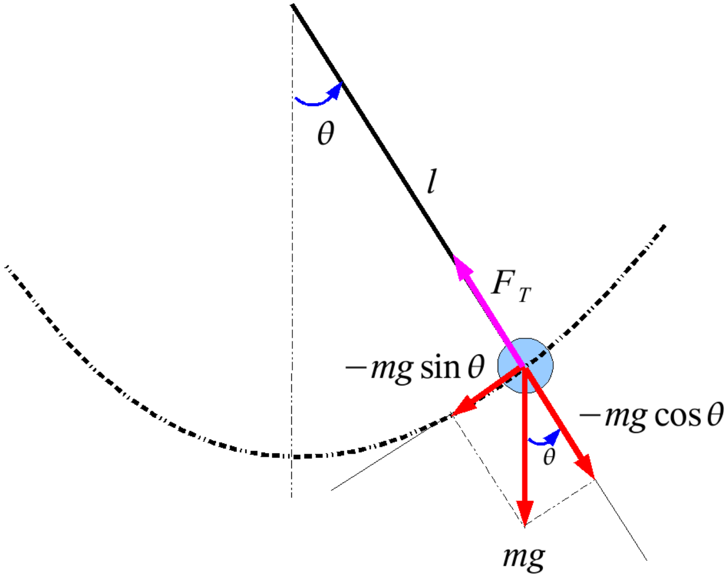
Simple Pendulum



Simple Pendulum



$m a_y = F_T - mg\cos\theta = 0$
$F_T = mg\cos\theta$
θ
l
F_T
$-mg\sin\theta$
$-mg\cos\theta$
θ
mg
Simple Pendulum



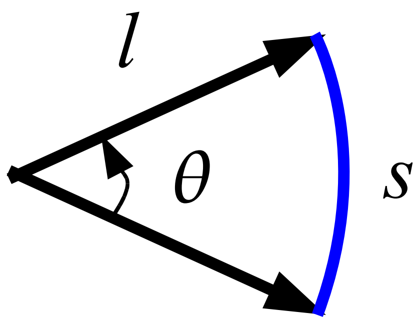
$$F_{s_{net}} = m a_s$$

$$-mg \sin\theta = m \frac{d^2 s}{dt^2}$$

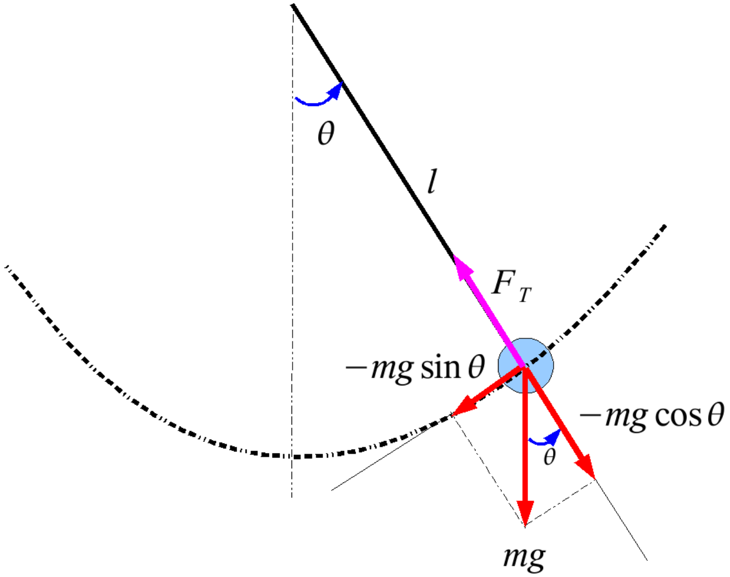
$$\sin\theta \approx \theta$$

$$-g\theta = \frac{d^2 s}{dt^2}$$

$$s = l\theta \qquad \theta = \frac{s}{l}$$



Simple Pendulum



$$F_{net} = m\,a$$

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

$$\sin\theta \approx \theta$$

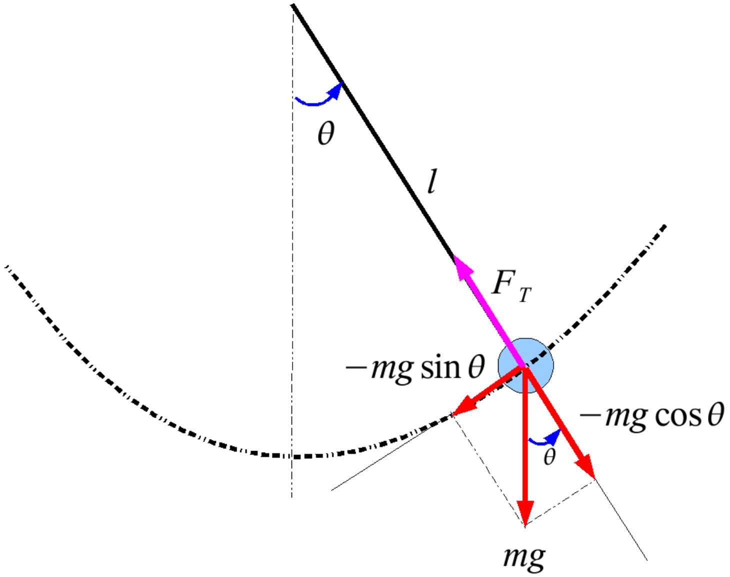
$$-g\,\theta = \frac{d^2 s}{dt^2}$$

$$s = l\,\theta$$

$$\frac{ds}{dt} = l\frac{d\,\theta}{dt}$$

$$\frac{d^2 s}{dt^2} = l\frac{d^2\theta}{dt^2}$$

Simple Pendulum



$$F_{net} = m\,a$$

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

$$\sin\theta \approx \theta$$

$$-g\,\theta = \frac{d^2 s}{dt^2}$$

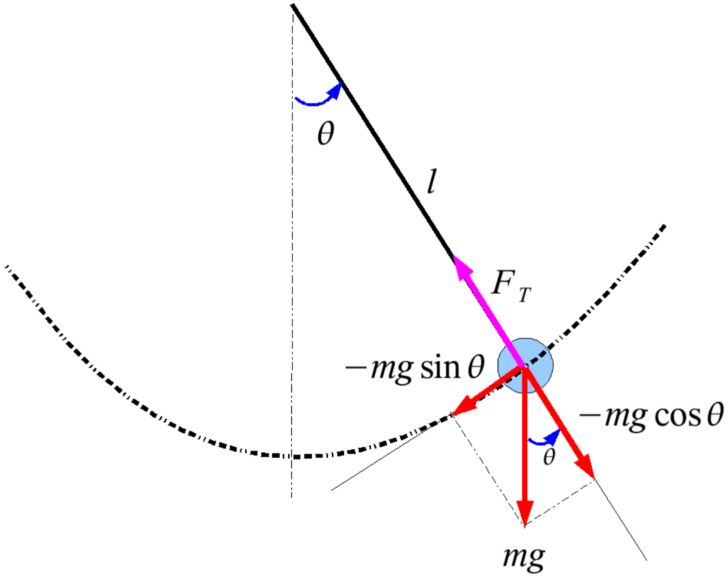
$$-g\,\theta = l\frac{d^2 \theta}{dt^2}$$

$$\theta = \theta_{max}\cos(\omega t)$$

$$\frac{d\theta}{dt} = -\theta_{max}\,\omega\sin(\omega t)$$

$$\frac{d^2\theta}{dt^2} = -\theta_{max}\,\omega^2\cos(\omega t)$$

Simple Pendulum



$$-g\theta = l\frac{d^2\theta}{dt^2}$$

$$-g\theta_{max}\cos(\omega t) = l\left(\theta_{max}\omega^2\cos(-\omega t)\right)$$

$$g = l\omega^2$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\theta = \theta_{max}\cos(\omega t)$$

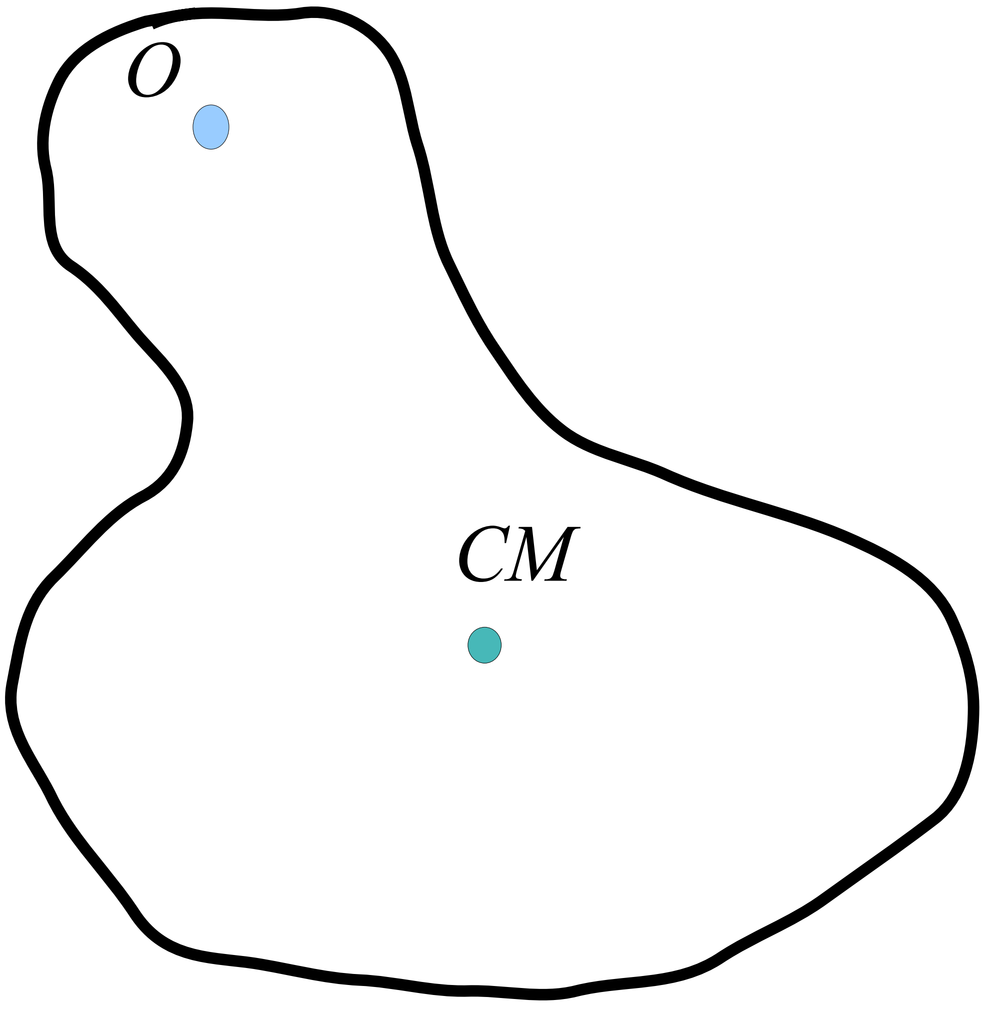
$$\frac{d\theta}{dt} = -\theta_{max}\omega\sin(\omega t)$$

$$\frac{d^2\theta}{dt^2} = -\theta_{max}\omega^2\cos(\omega t)$$

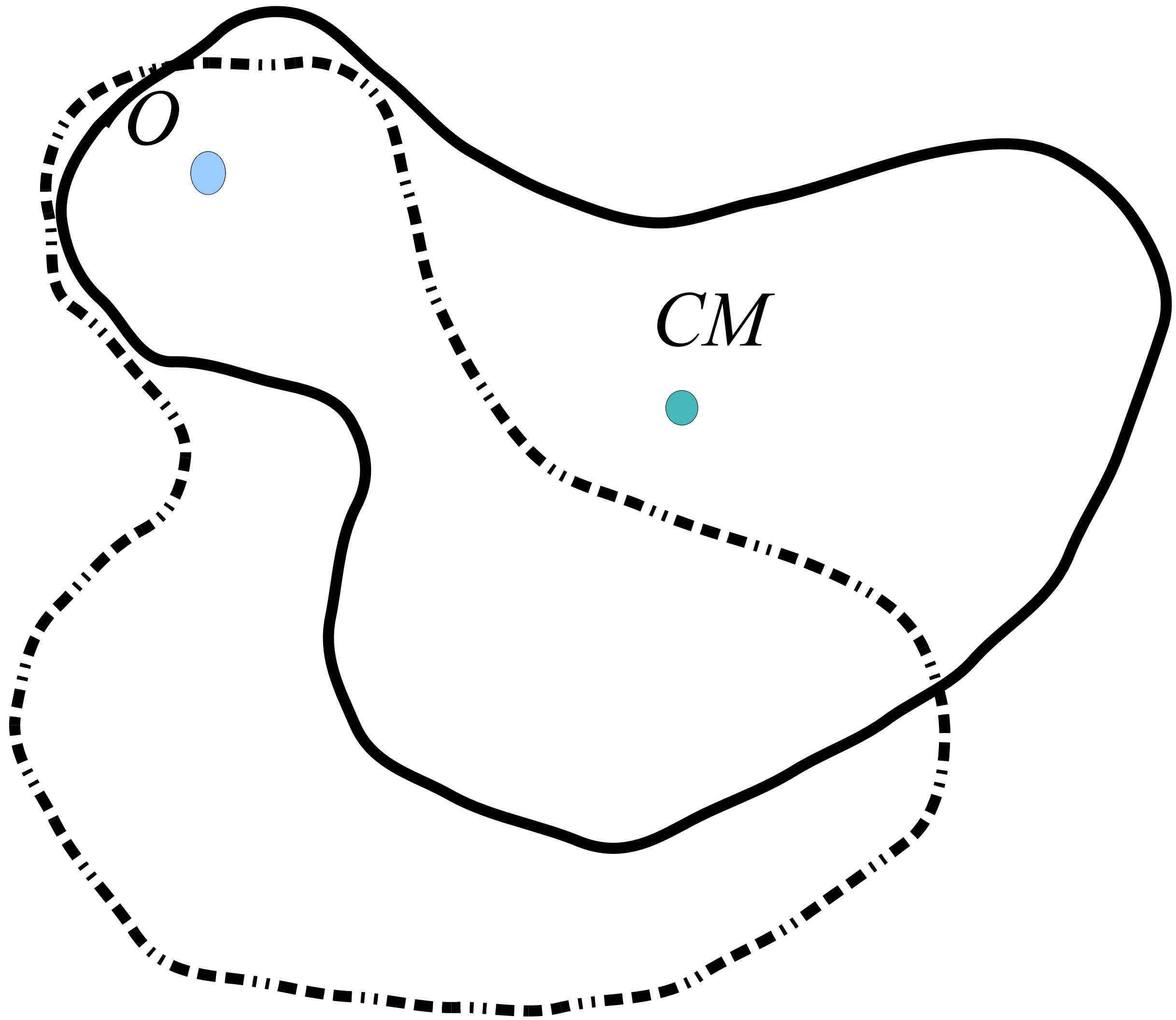
Simple Pendulum

Physical Pendulum

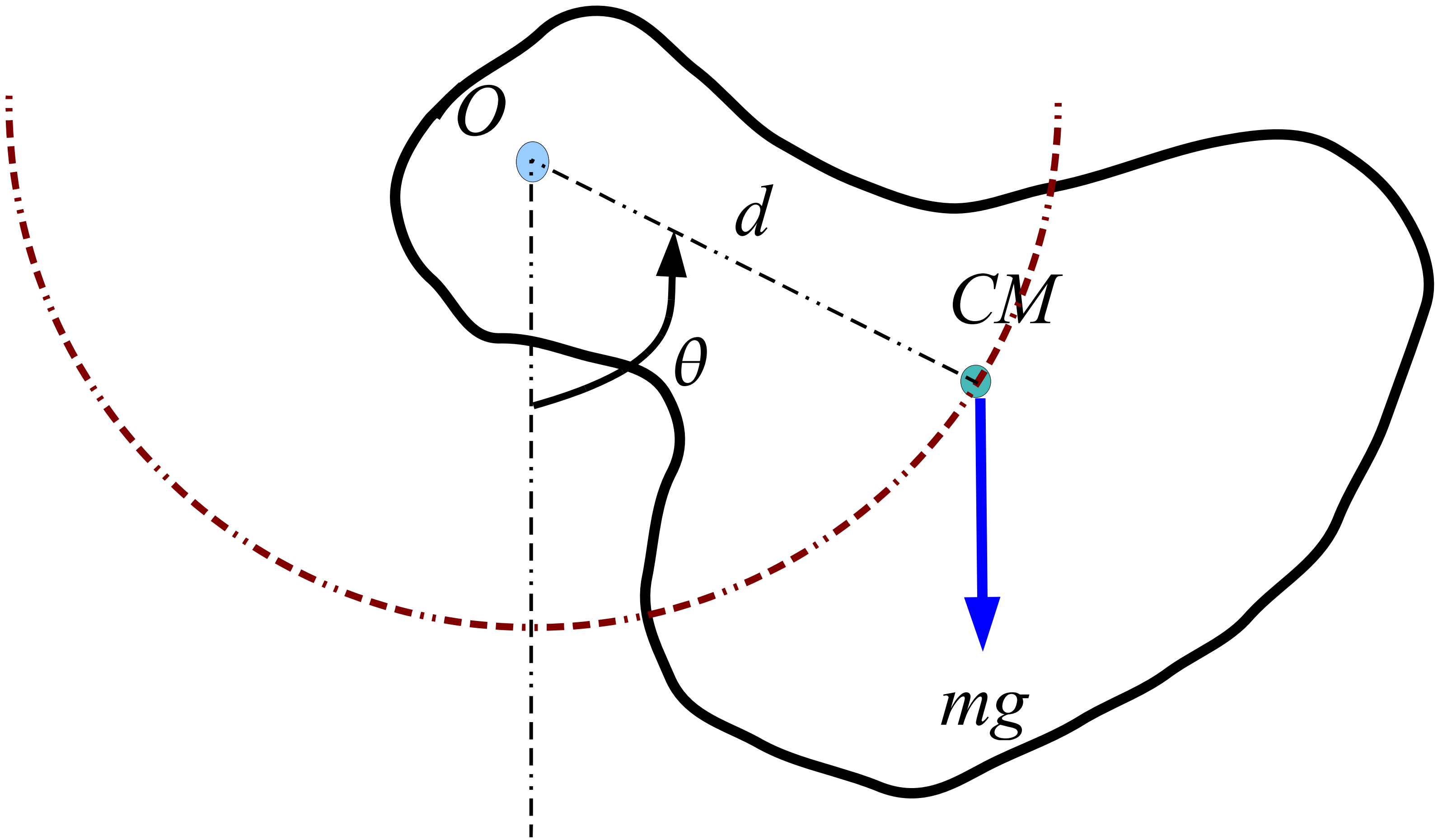
Physical Pendulum



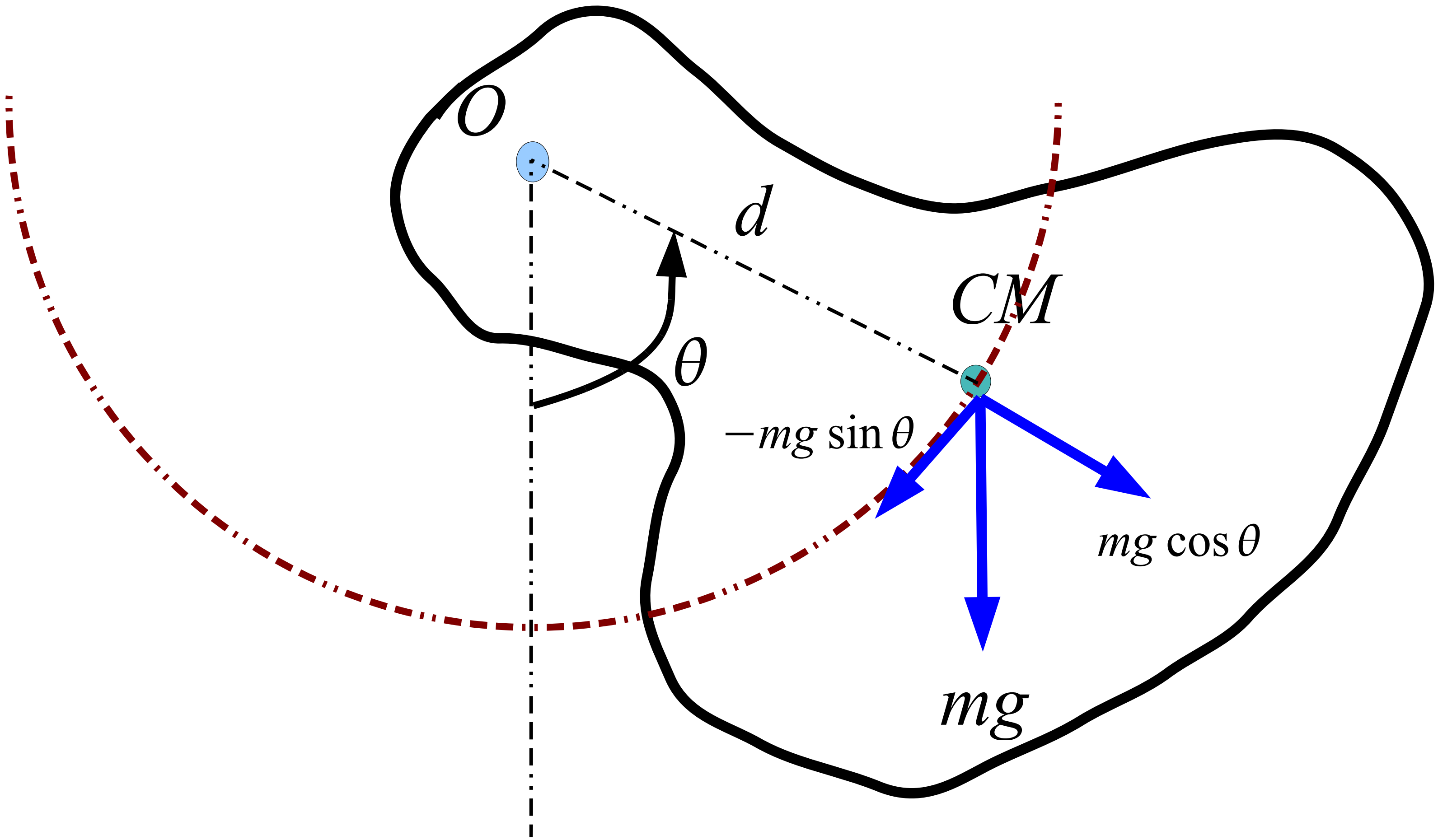
Physical Pendulum



Physical Pendulum



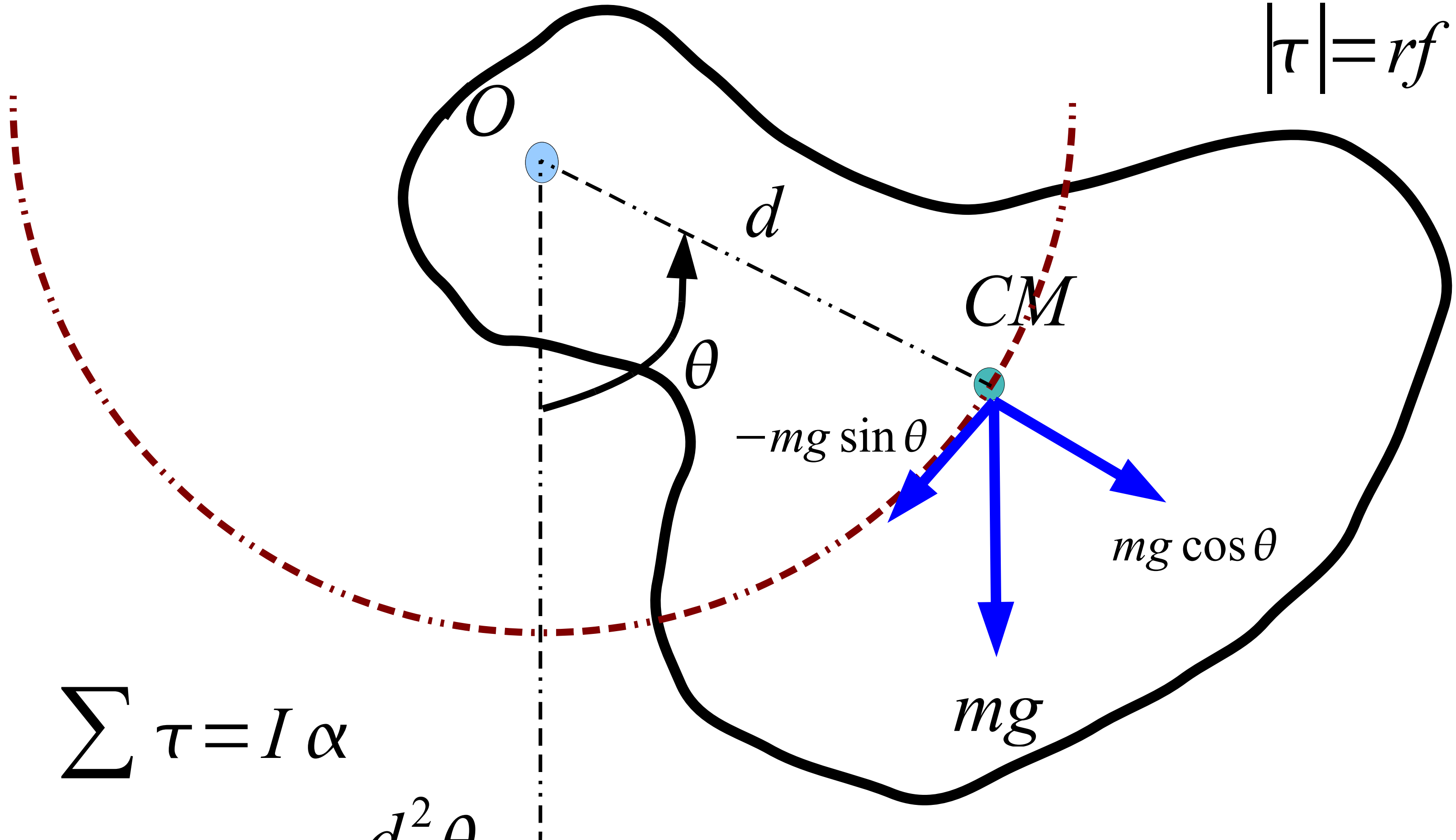
Physical Pendulum



Physical Pendulum

$$\vec{\tau} = \vec{r} \; X \; \vec{F}$$

$$|\tau| = rf \sin\theta$$



$$\sum \tau = I\,\alpha$$

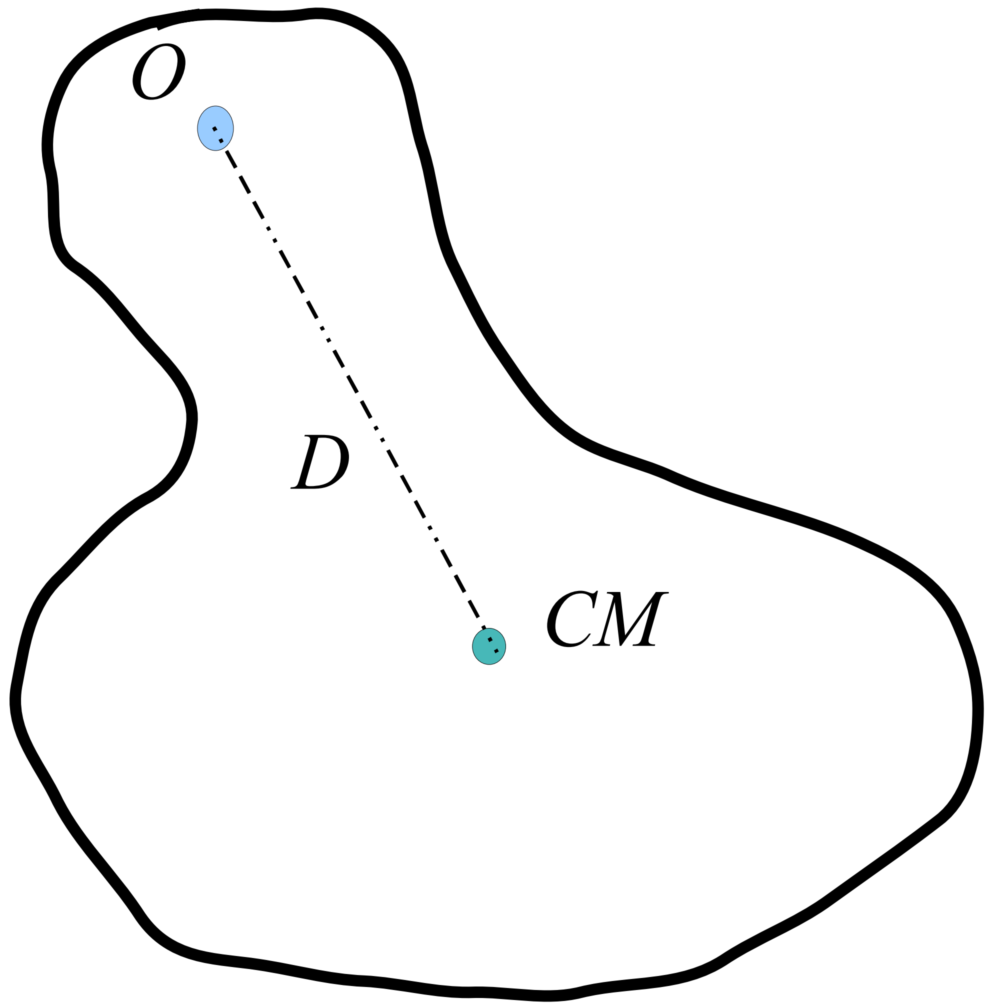
$$-mgd \sin\theta = I \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} \approx -\left(\frac{mgd}{I}\right)\theta$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

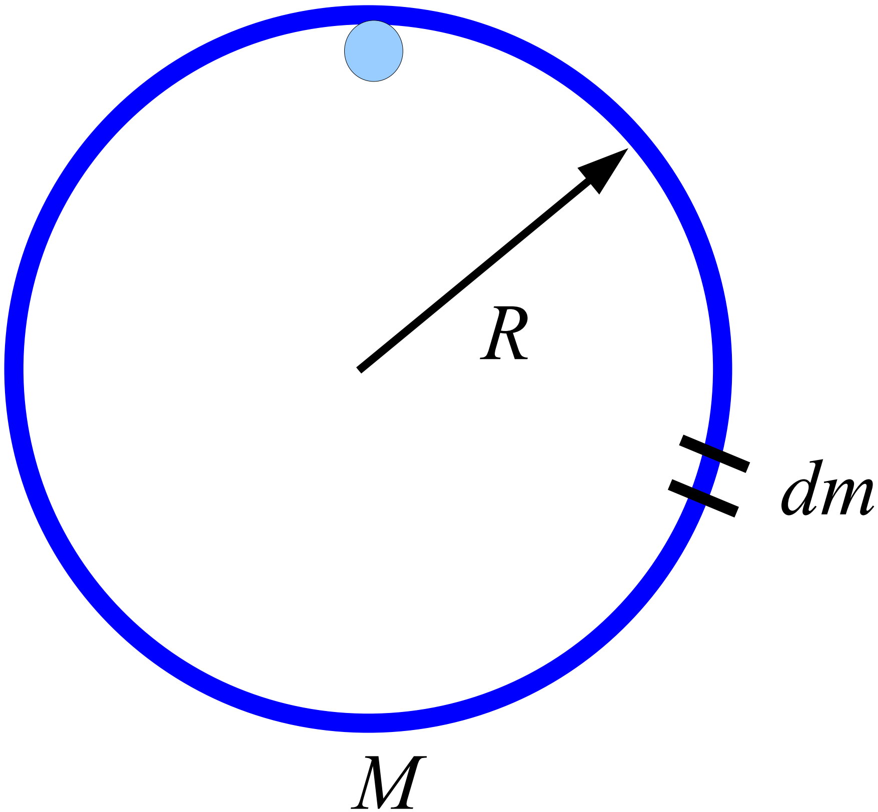
RECALL:



$$I_{CM} = \int r^2 \, dm$$

$$I = I_{CM} + MD^2$$

Hula Hoop on a Peg



$$I_{CM} = \int r^2 dm$$

$$I_{CM} = R^2 \int_0^M dm$$

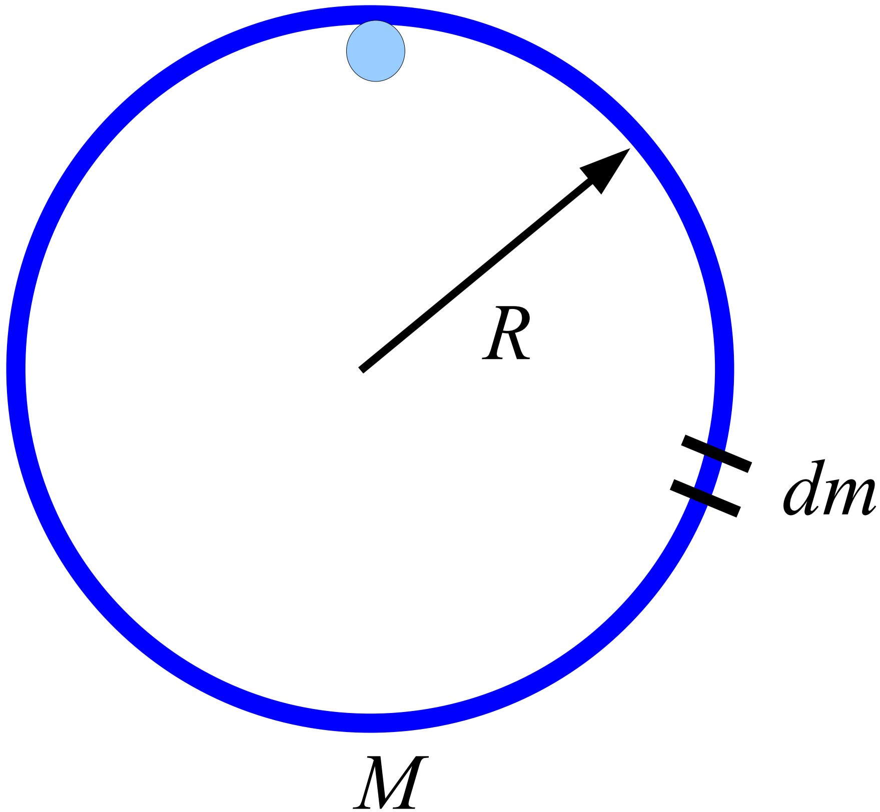
$$I_{CM} = MR^2$$

$$I = I_{CM} + MD^2$$

$$I = M\,R^2 + M\,R^2$$

$$I = 2\mathrm{MR}^2$$

Hula Hoop on a Peg



$$I_{CM} = \int r^2\, dm$$

$$I_{CM} = R^2 \int_0^M dm$$

$$I_{CM} = MR^2$$

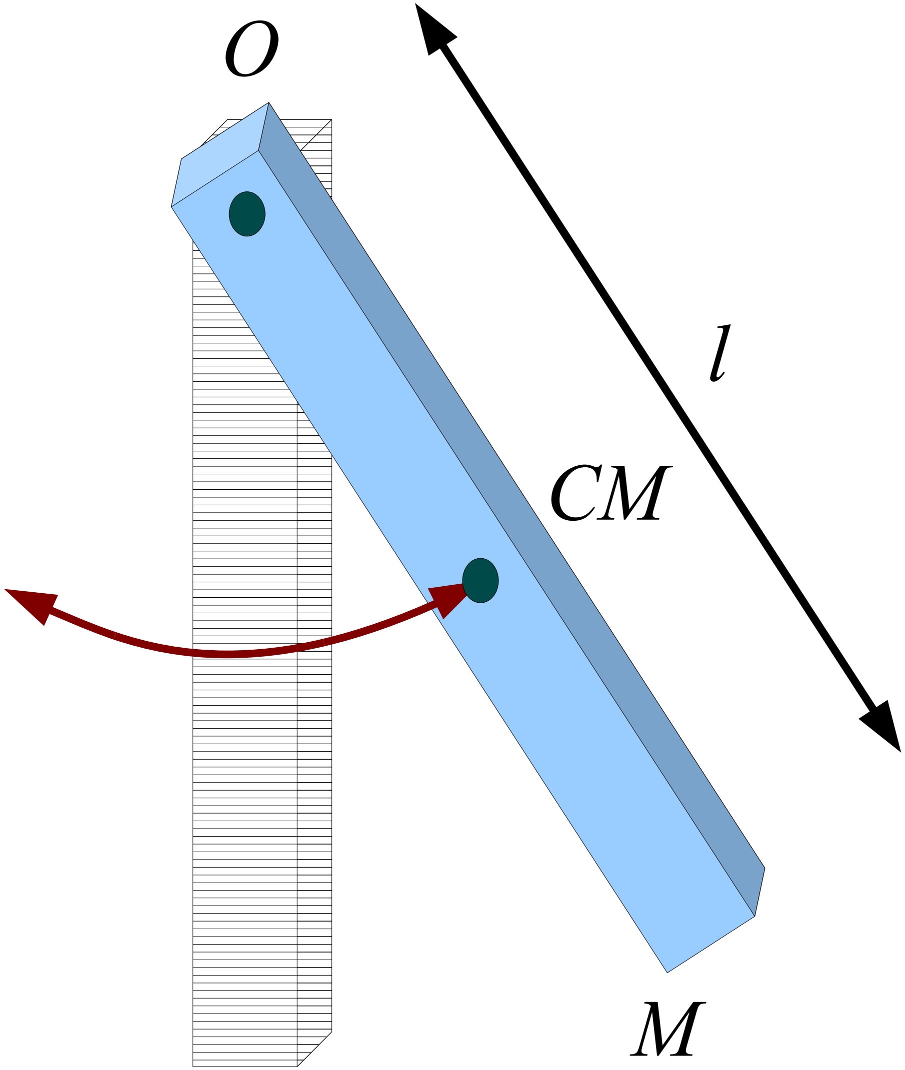
$$I = I_{CM} + MD^2$$

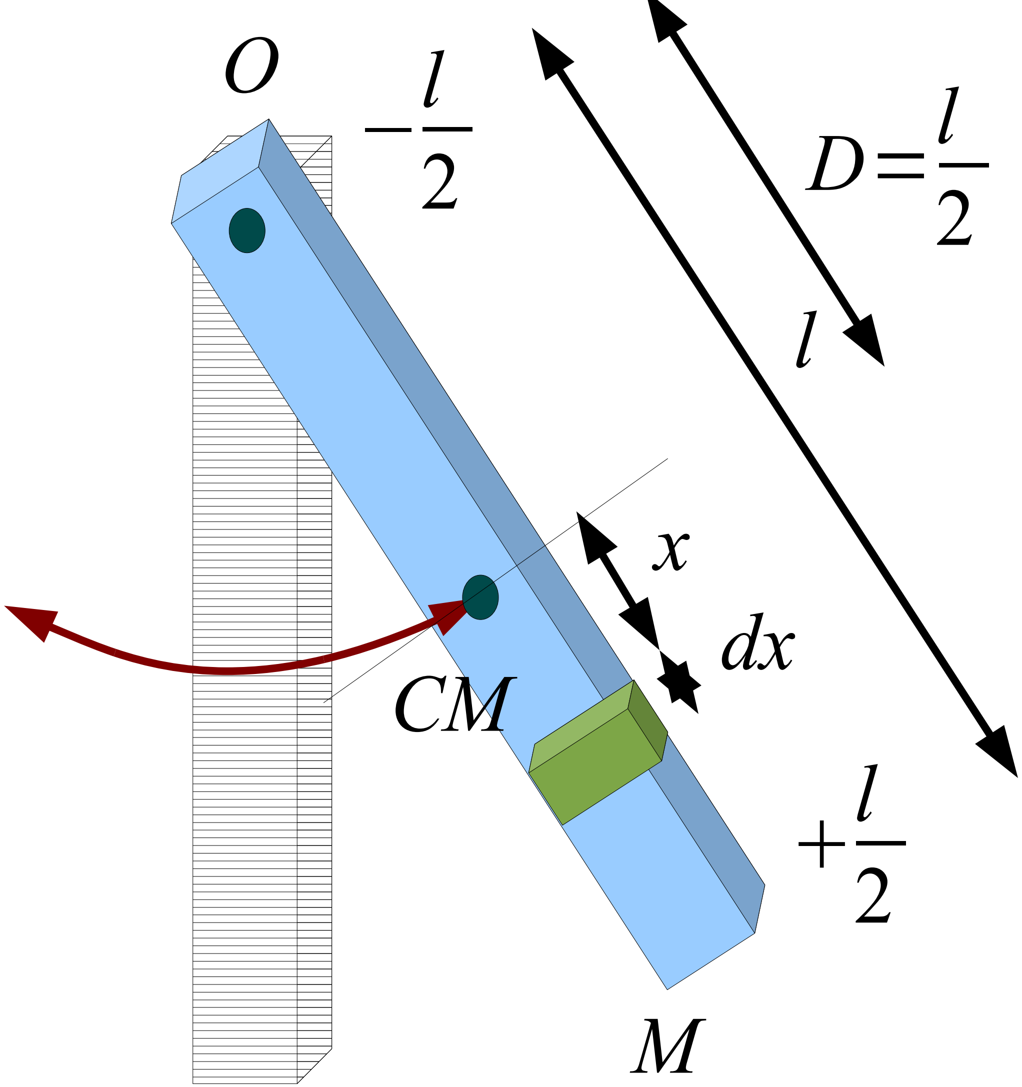
$$I = M\,R^2 + M\,R^2$$

$$I = 2\mathrm{MR}^2$$

$$T = 2\pi\sqrt{\frac{I}{mgD}}$$

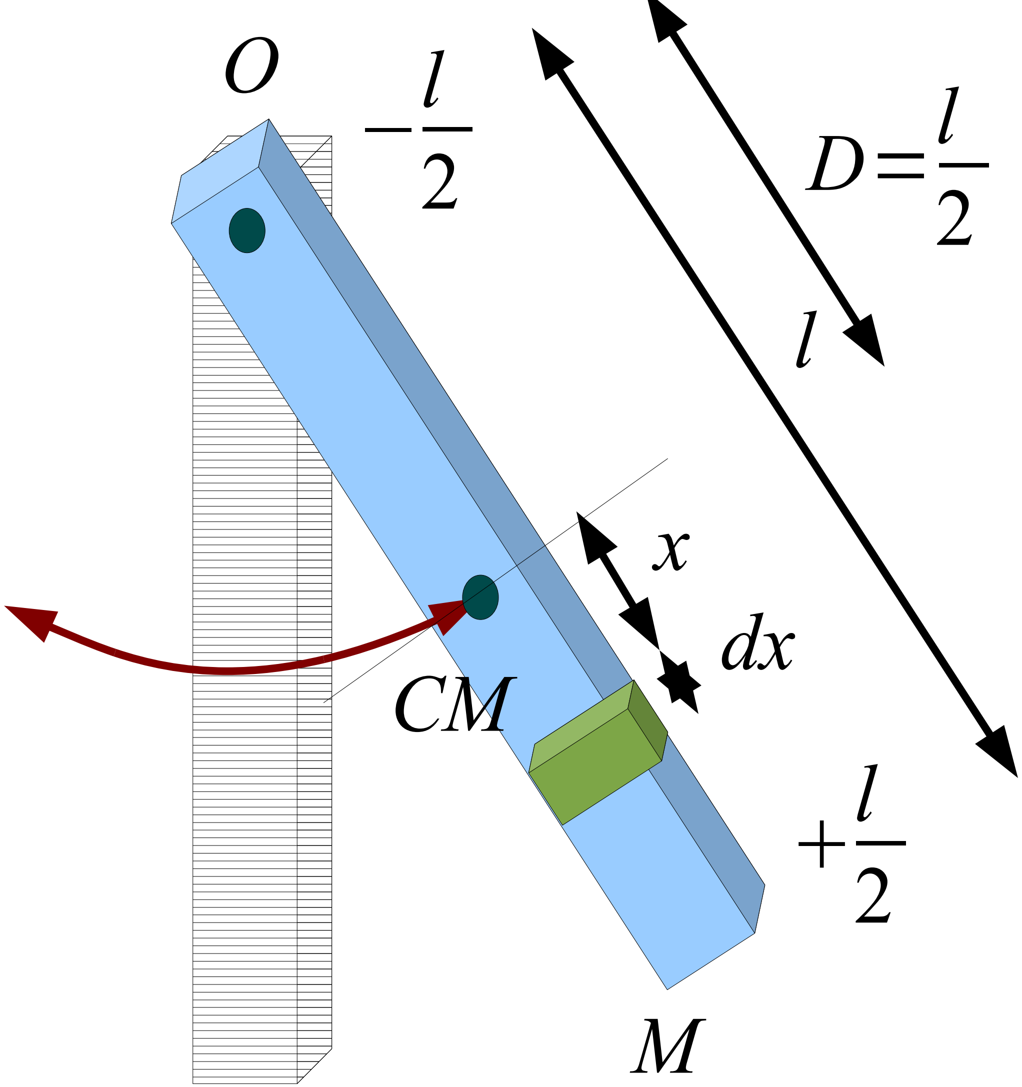
$$T = 2\pi\sqrt{\frac{2\mathrm{MR}^2}{MgR}} = 2\pi\sqrt{\frac{2\mathrm{R}}{g}}$$


O
l
CM
M



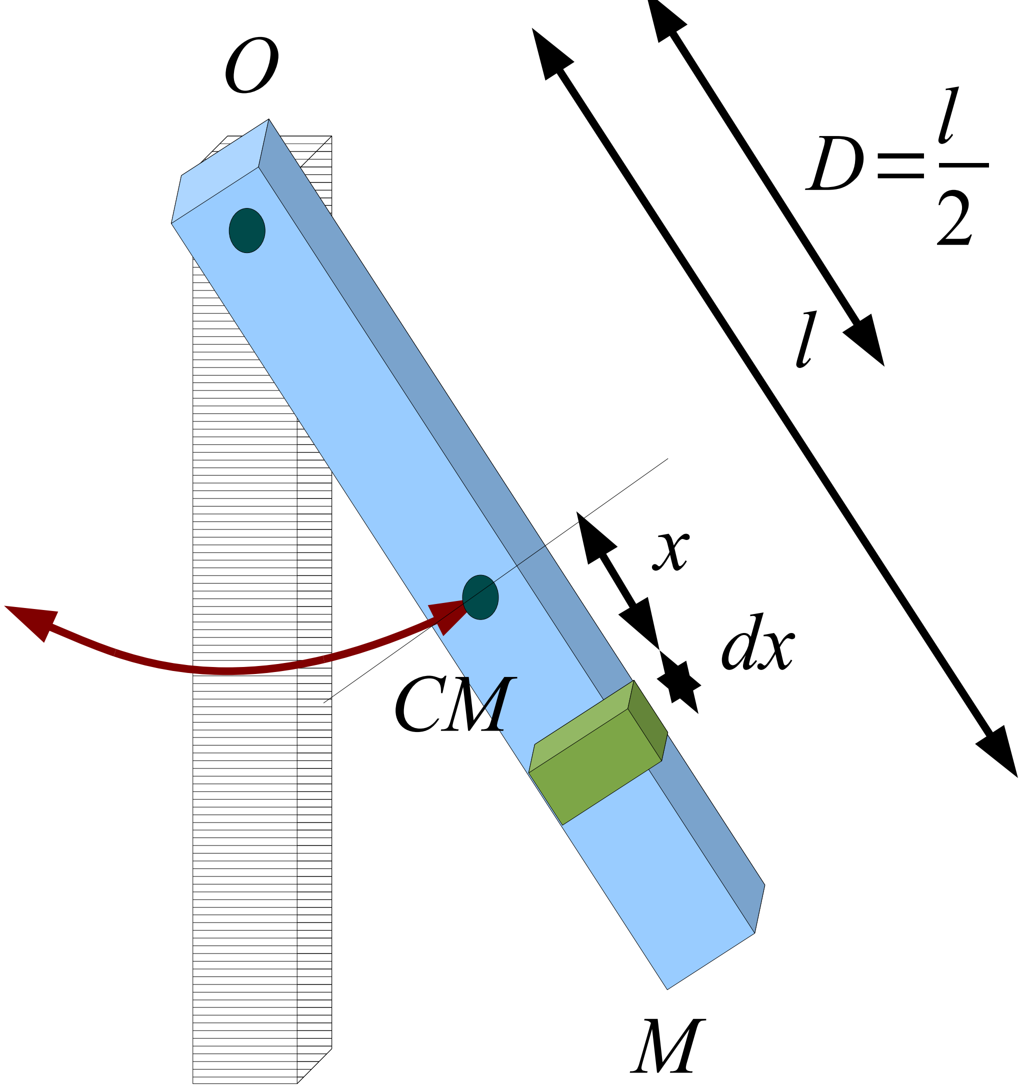
$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$

$$I_{CM} = \int r^2\, dm$$

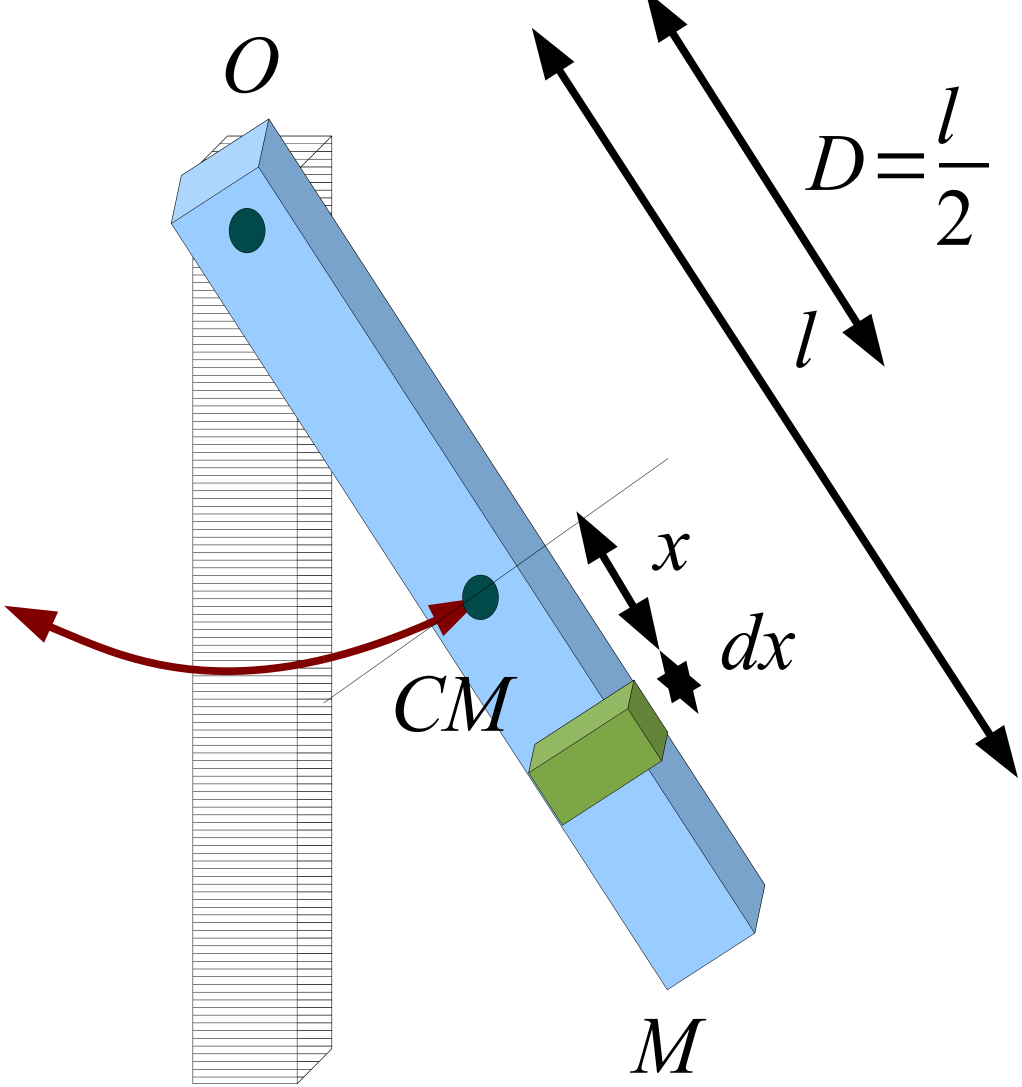
$$I_{CM} = \int_{-l/2}^{+l/2} x^2 (\lambda\, dx)$$

$$I_{CM} = \lambda \int_{-l/2}^{+l/2} x^2\, dx$$

$$I_{CM} = \lambda \left[\frac{1}{3} x^3 \right]_{-l/2}^{+l/2}$$

$$I_{CM} = \lambda \left(\frac{1}{3} \frac{l^3}{8} - \left(-\frac{1}{3} \frac{l^3}{8} \right) \right)$$

$$I_{CM} = \lambda \left(\frac{l^3}{12} \right) = \frac{M}{l} \left(\frac{l^3}{12} \right) = \frac{1}{12} M\, l^2$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$

$$I_{CM} = \int r^2\, dm$$

$$I_{CM} = \int_{-l/2}^{+l/2} x^2 (\lambda\, dx)$$

$$I_{CM} = \lambda \int_{-l/2}^{+l/2} x^2\, dx$$

$$I_{CM} = \lambda \left[\frac{1}{3} x^3 \right]_{-l/2}^{+l/2}$$

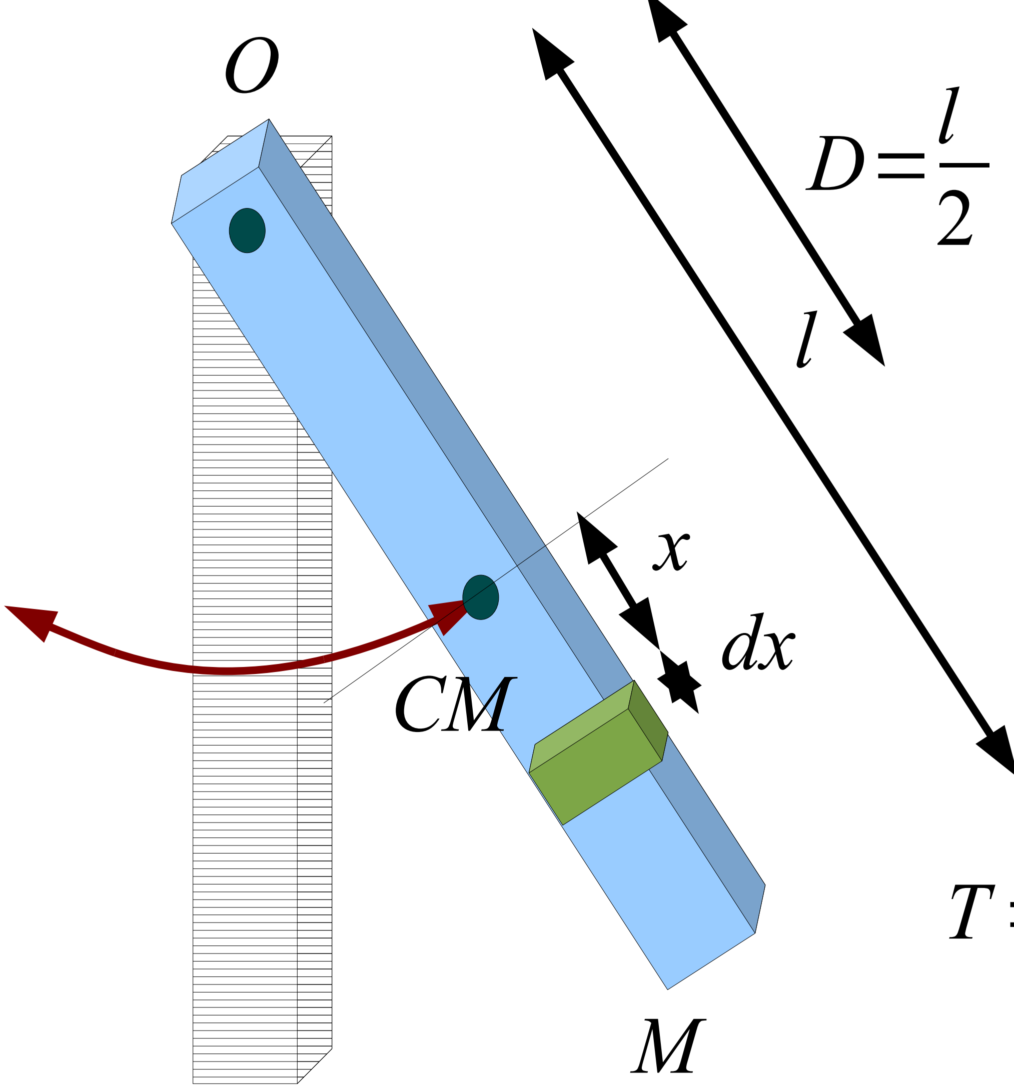
$$I_{CM} = \lambda \left(\frac{1}{3} \frac{l^3}{8} - \left(-\frac{1}{3} \frac{l^3}{8} \right) \right)$$

$$I_{CM} = \lambda \left(\frac{l^3}{12} \right) = \frac{M}{l} \left(\frac{l^3}{12} \right) = \frac{1}{12} M l^2$$

$$I = I_{CM} + MD^2$$

$$I = \frac{1}{12} M l^2 + M \left(\frac{l^2}{4} \right)$$

$$I = \frac{1}{12} M l^2 + \frac{3}{12} M (l^2) = \frac{4}{12} M l^2 = \frac{1}{3} M l^2$$



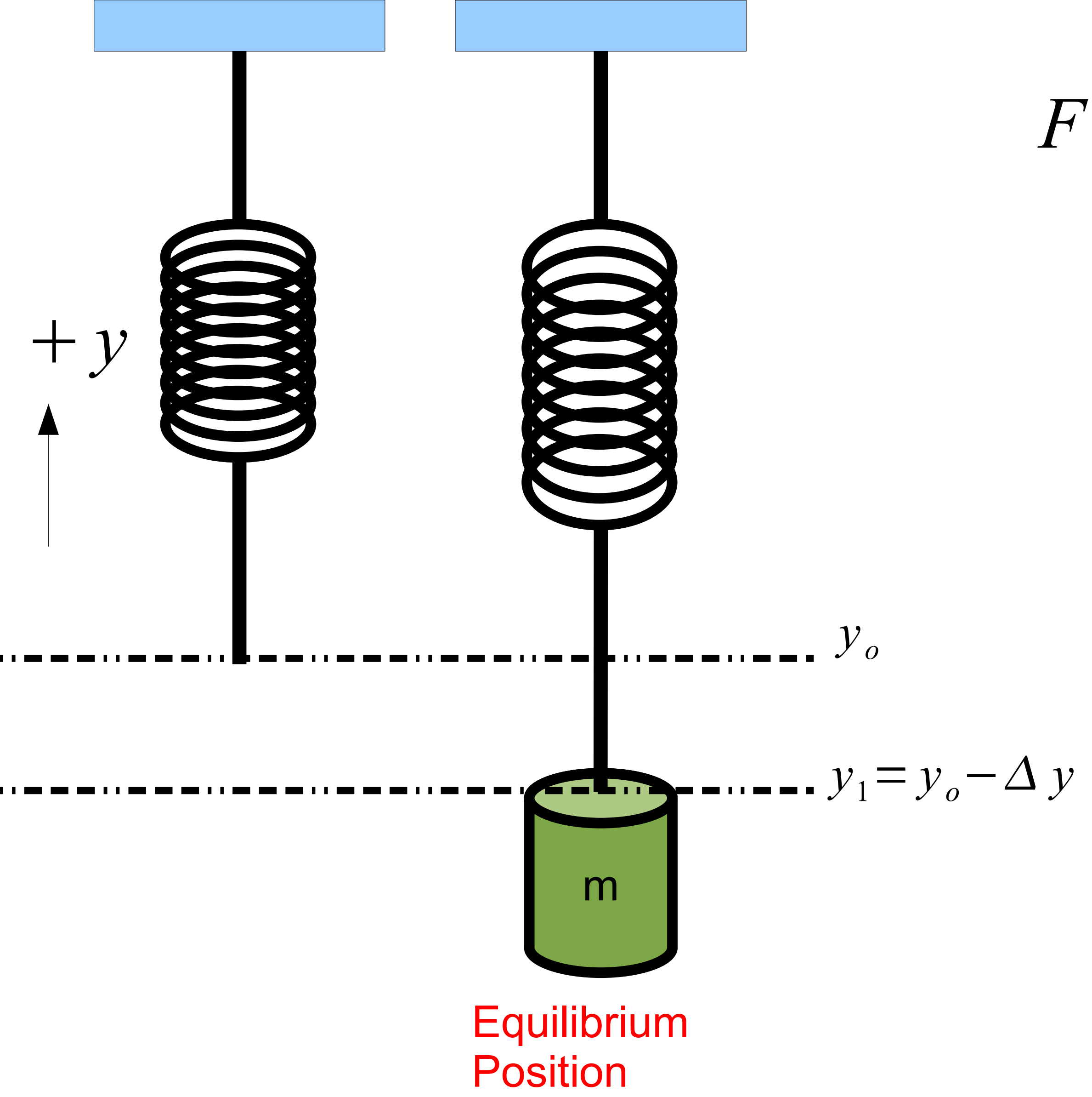
$$T = 2\pi \sqrt{\frac{I}{mgD}}$$

$$T = 2\pi \sqrt{\frac{1/3\, M\, l^2}{Mg\left(l/2\right)}} = 2\pi \sqrt{\frac{2}{3}\,\frac{l}{g}}$$

$$I = I_{CM} + MD^2$$

$$I = \frac{1}{12} M\, l^2 + M\left(\frac{l^2}{4}\right)$$

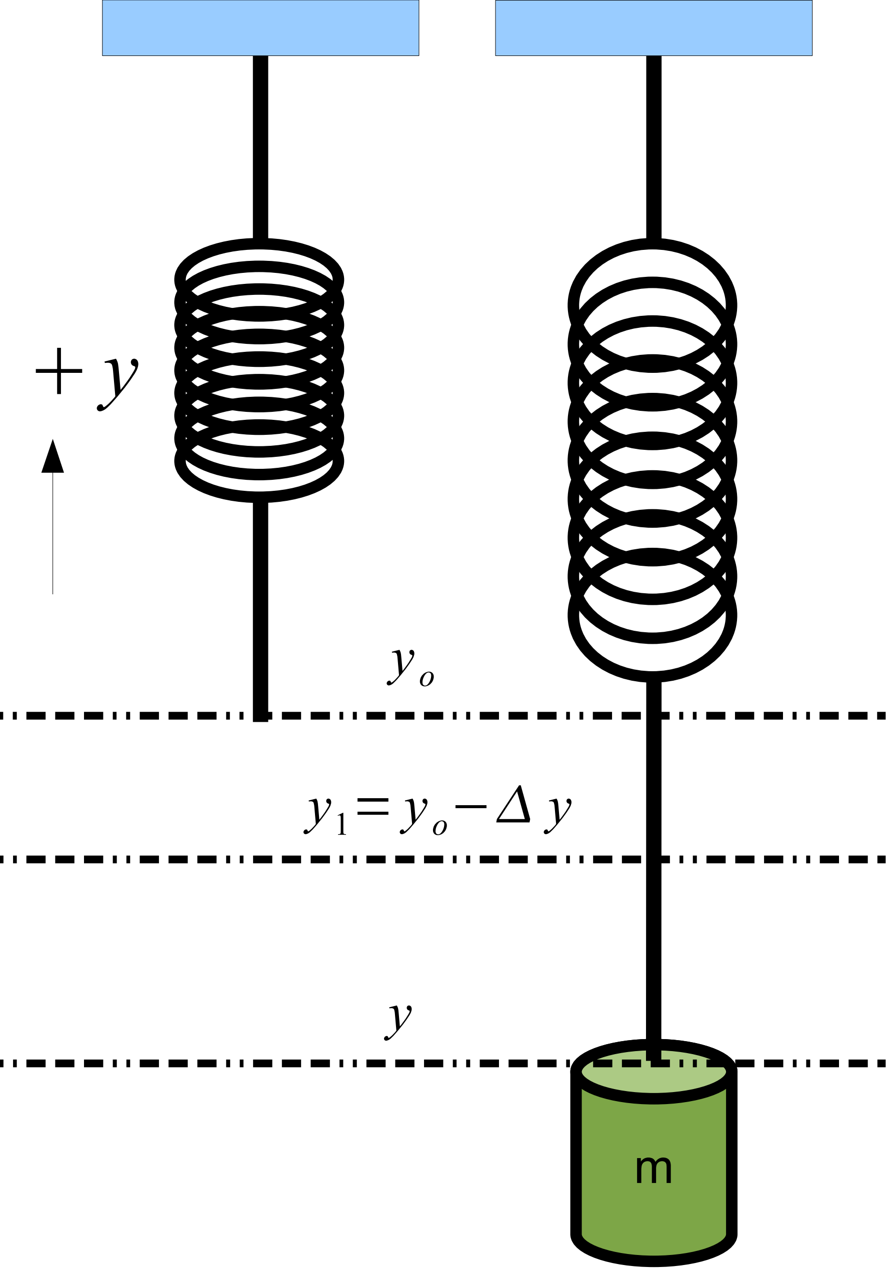
$$I = \frac{1}{12} M\, l^2 + \frac{3}{12} M\left(l^2\right) = \frac{4}{12} M\, l^2 = \frac{1}{3} M\, l^2$$



$$F_{net} = F_s - w = k\,\Delta y - mg = 0$$

$$k\,\Delta y = mg$$

$$k\left(y_o - y_1\right) = mg$$



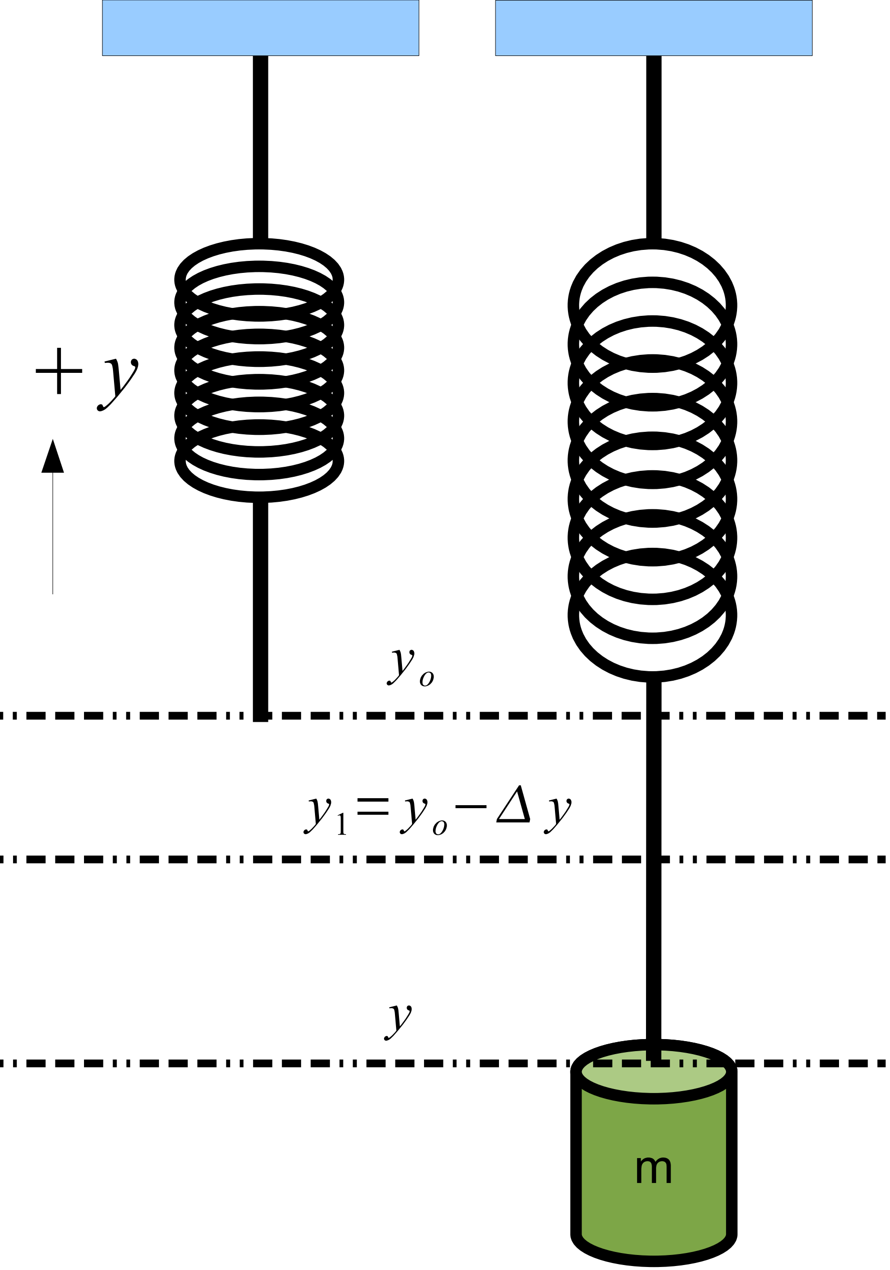
$$F_{net} = F_s - w = k\,\Delta y - mg = 0$$

$$k\,\Delta y = mg$$

$$k\left(y_o - y_1\right) = mg$$

$$F_{net} = -k\left(y - y_o\right) - mg$$

$$ma = -k\left(y - y_o\right) - mg$$



$$F_{net} = F_s - w = k\Delta y - mg = 0$$

$$k\Delta y = mg$$

$$k(y_o - y_1) = mg$$

$$F_{net} = -k(y - y_o) - mg$$

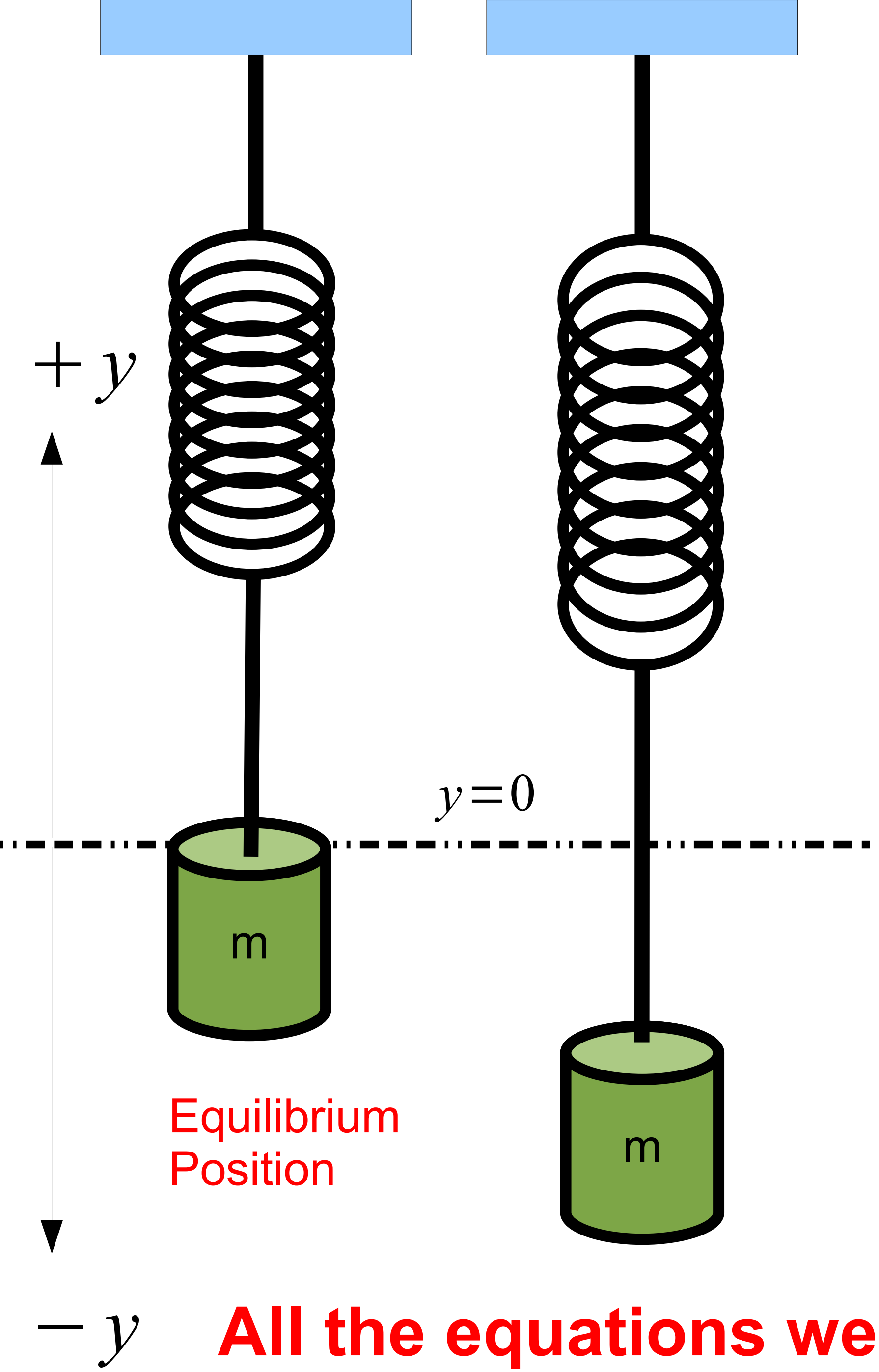
$$ma = -k(y - y_o) - mg$$

$$ma = -k(y - y_o) - k(y_o - y_1)$$

$$ma = -(ky - ky_o) - (ky_o - ky_1)$$

$$ma = -ky + ky_o - ky_o + ky_1$$

$$ma = -k(y - y_1)$$



$$F_{net}=F_s-w=k\,\Delta\,y-mg=0$$

$$k\,\Delta\,y=mg$$

$$k\left(y_o-y_1\right)=mg$$

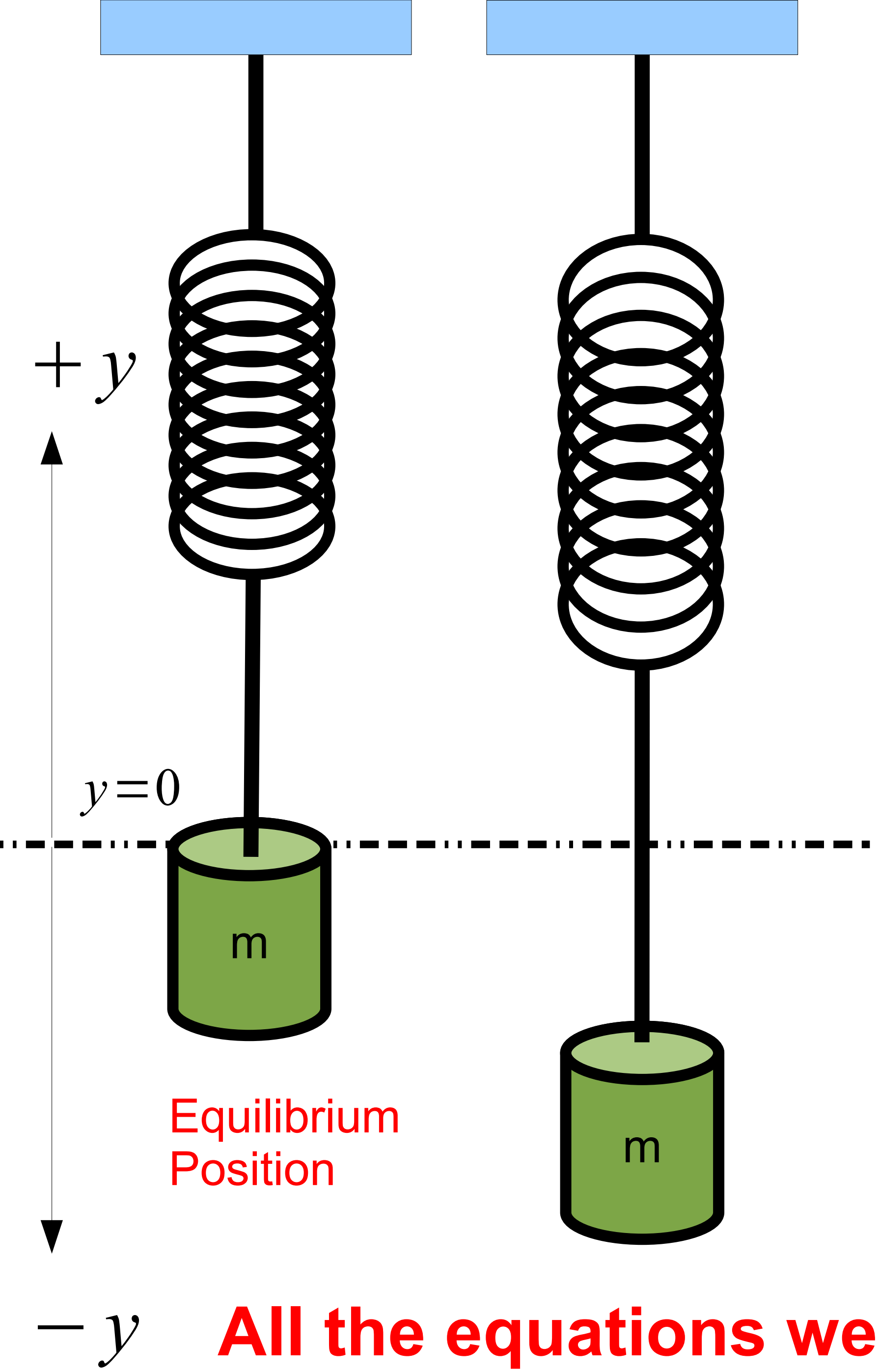
$$F_{net}=-k\left(y-y_o\right)-mg$$

$$ma=-k\left(y-y_1\right)$$

Set y_1 as new equilibrium position $y_1=0$

$$ma=-ky$$

All the equations we used for the horizontal spring on a frictionless table can be used here, as long as we take the equilibrium position to be y = 0.



$$F_{net} = F_s - w = k\,\Delta y - mg = 0$$

$$k\,\Delta y = mg$$

$$k\left(y_o - y_1\right) = mg$$

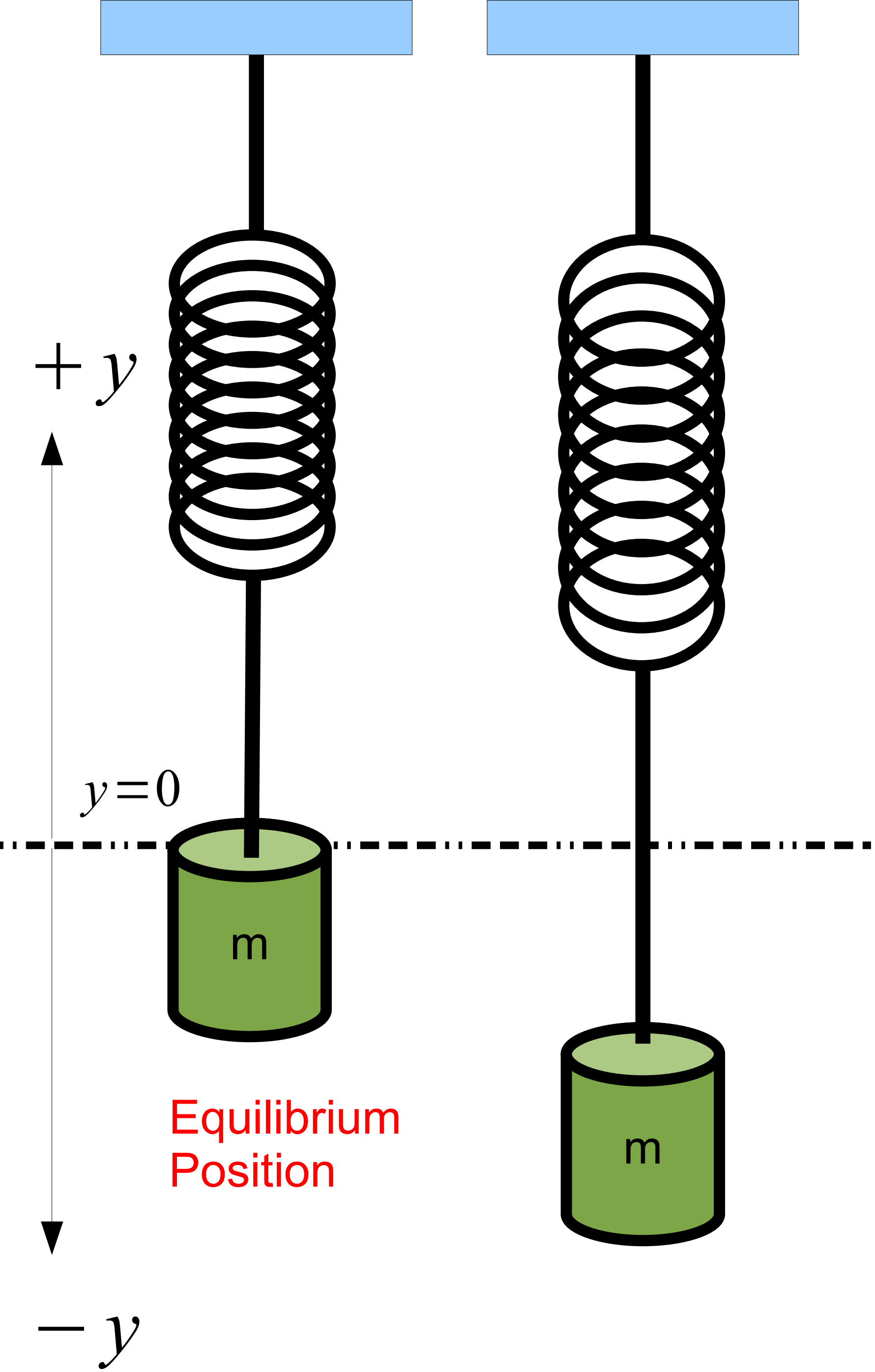
$$F_{net} = -k\left(y - y_o\right) - mg$$

$$ma = -k\left(y - y_1\right)$$

Set y_1 as new equilibrium position $y_1 = 0$

$$ma = -ky$$

All the equations we used for the horizontal spring on a frictionless table can be used here, as long as we take the equilibrium position to be y = 0.



A

$$\omega=\sqrt{\frac{k}{m}}$$

$$T=\frac{2\pi}{\omega}$$

$$v_{max}=A\omega$$

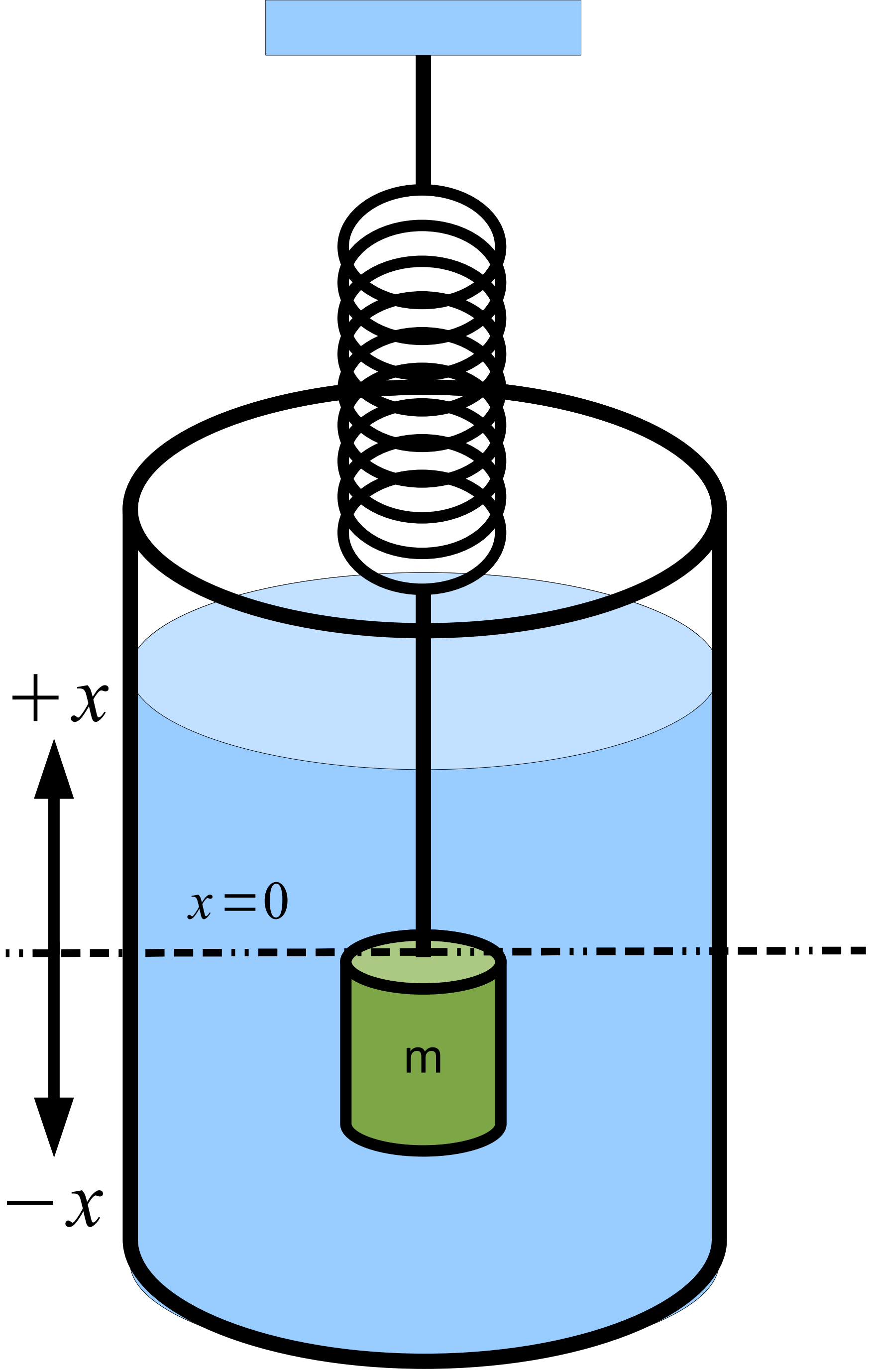
$$a_{max}=A\omega^2$$

$$y(t)=A\cos(\omega t)$$

$$v_y(t)=-A\omega\sin(\omega t)$$

$$a_y(t)=-A\omega^2\cos(\omega t)$$

Damped Harmonic Motion



$$-kx-bv=ma$$

$$x=e^{-\alpha t}\sin\left(\omega t\right)$$

$$\alpha=\frac{b}{2\text{m}}$$

$$\omega=\sqrt{\frac{k}{m}-\left(\frac{b}{2\text{m}}\right)^2}$$

$$\omega=\sqrt{\omega_o^2-\left(\frac{b}{2\text{m}}\right)^2}$$

Figure 13.20 Plots of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$x = e^{-\alpha t}\sin(\omega t)$$

$$\alpha = \frac{b}{2\mathrm{m}}$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2\mathrm{m}}\right)^2}$$

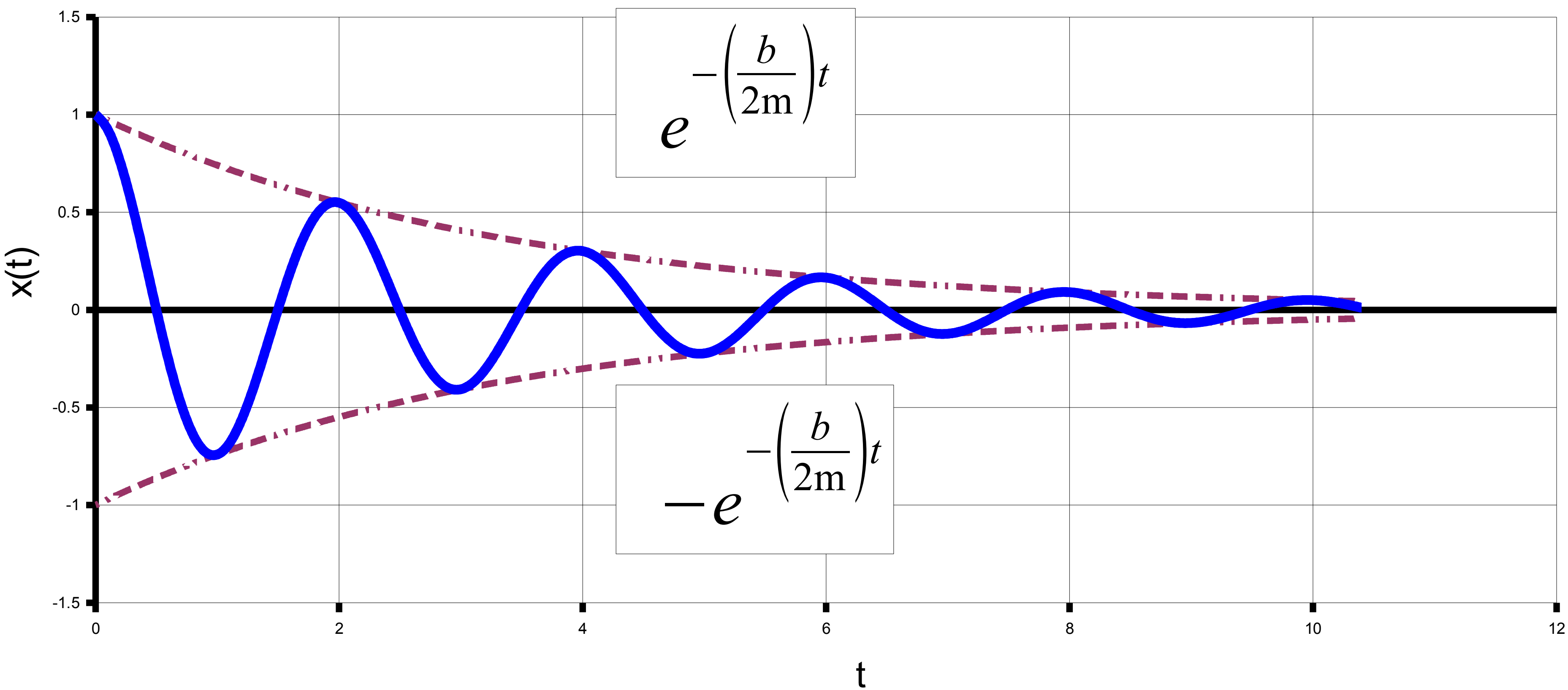
$$b_c = \sqrt{4mk}$$

a.) Underdamped $b < b_c$

b.) Critically Damped $b = b_c$

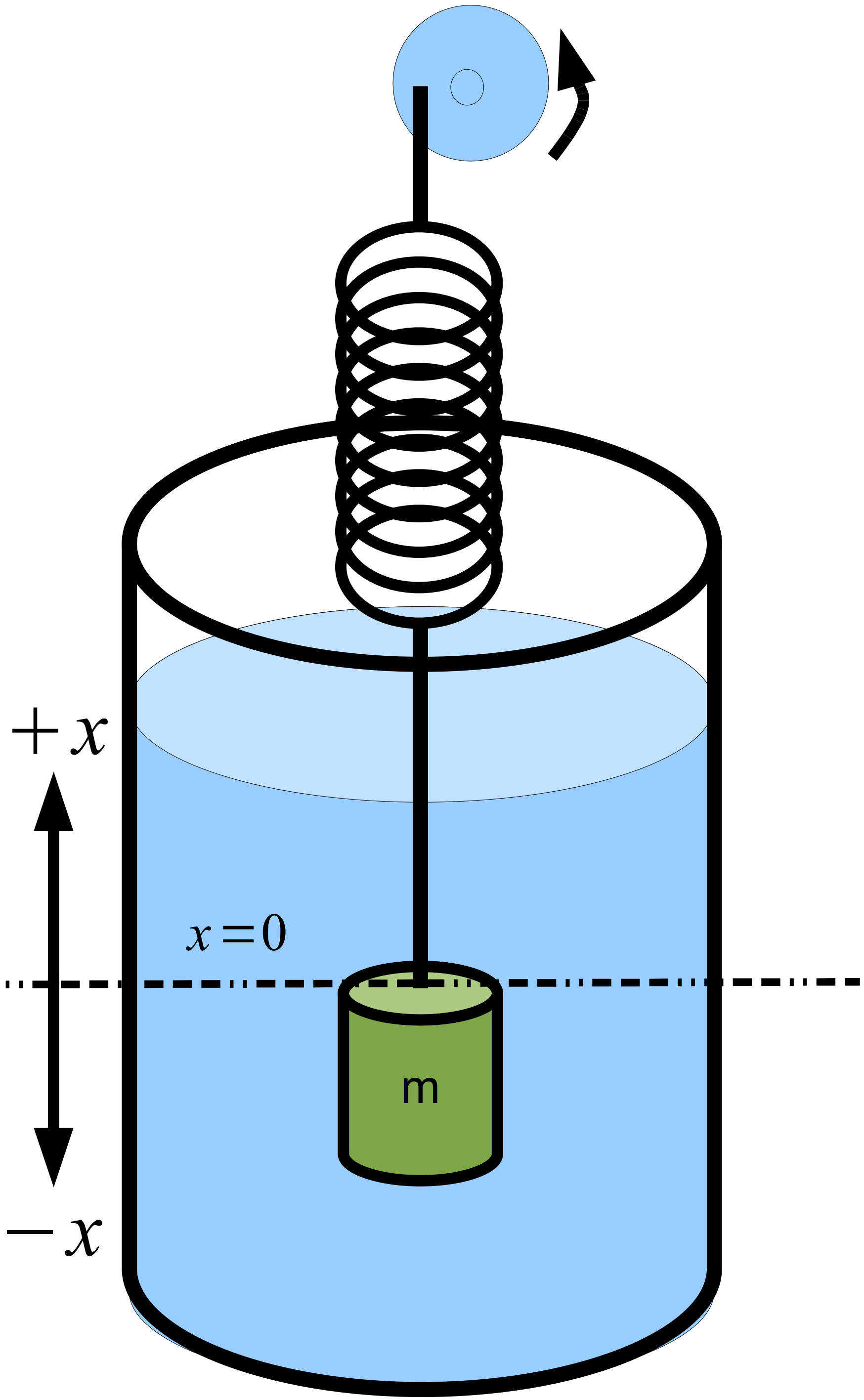
c.) Overdamped $b > b_c$

Under Damped Harmonic Motion



m	**10**
k	**100**
b	**6**
wo	**3.16**
w	**3.15**

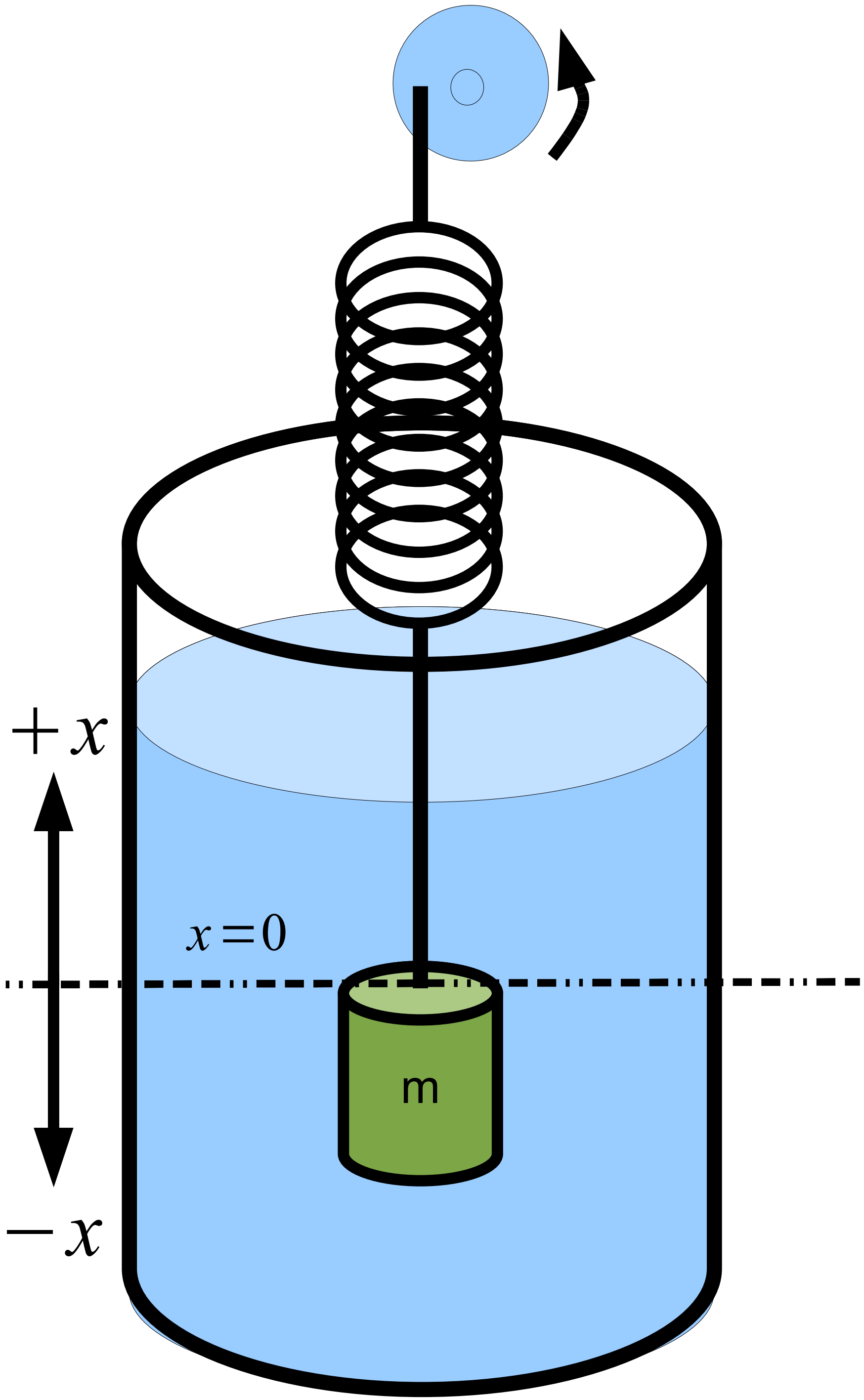
Forced Damped Harmonic Motion



$$-kx - b\frac{dx}{dt} + F_o \sin(\omega t) = m\frac{d^2 x}{dt^2}$$

$$x(t) = A\sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$



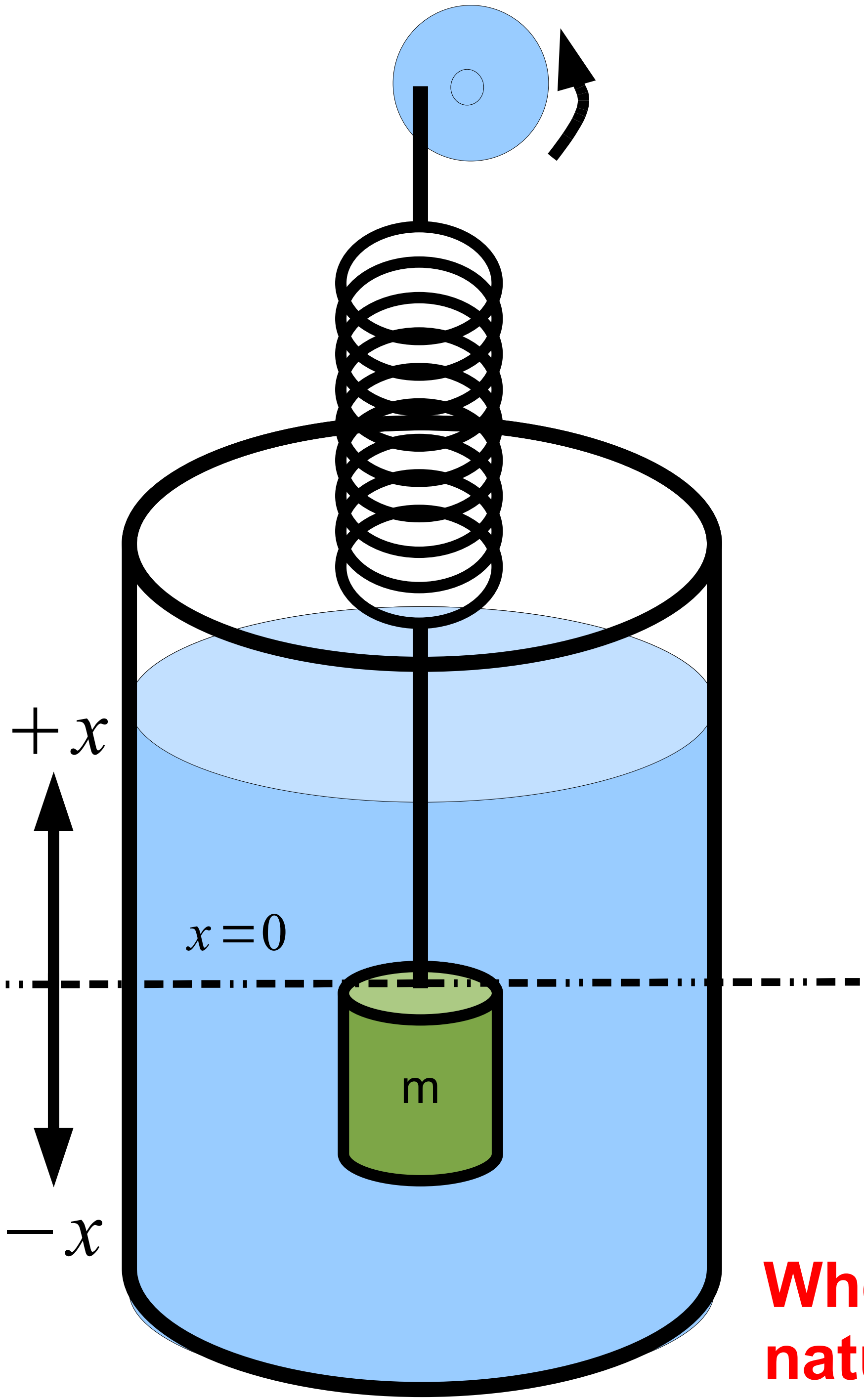
$$-kx - bv + F_o \sin(\omega t) = ma$$

$$x(t) = A\sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$

When $\omega = \omega_o$

$$A_{max} = \frac{F_o}{\sqrt{b^2\omega^2}}$$



$$-kx - bv + F_o \sin(\omega t) = ma$$

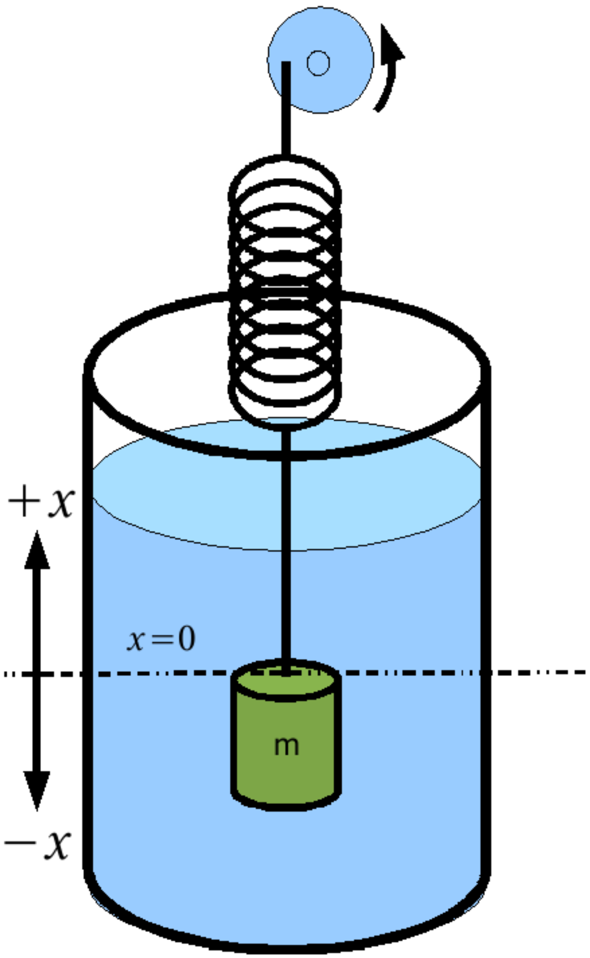
$$x(t) = A \sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$

When $\omega = \omega_o$

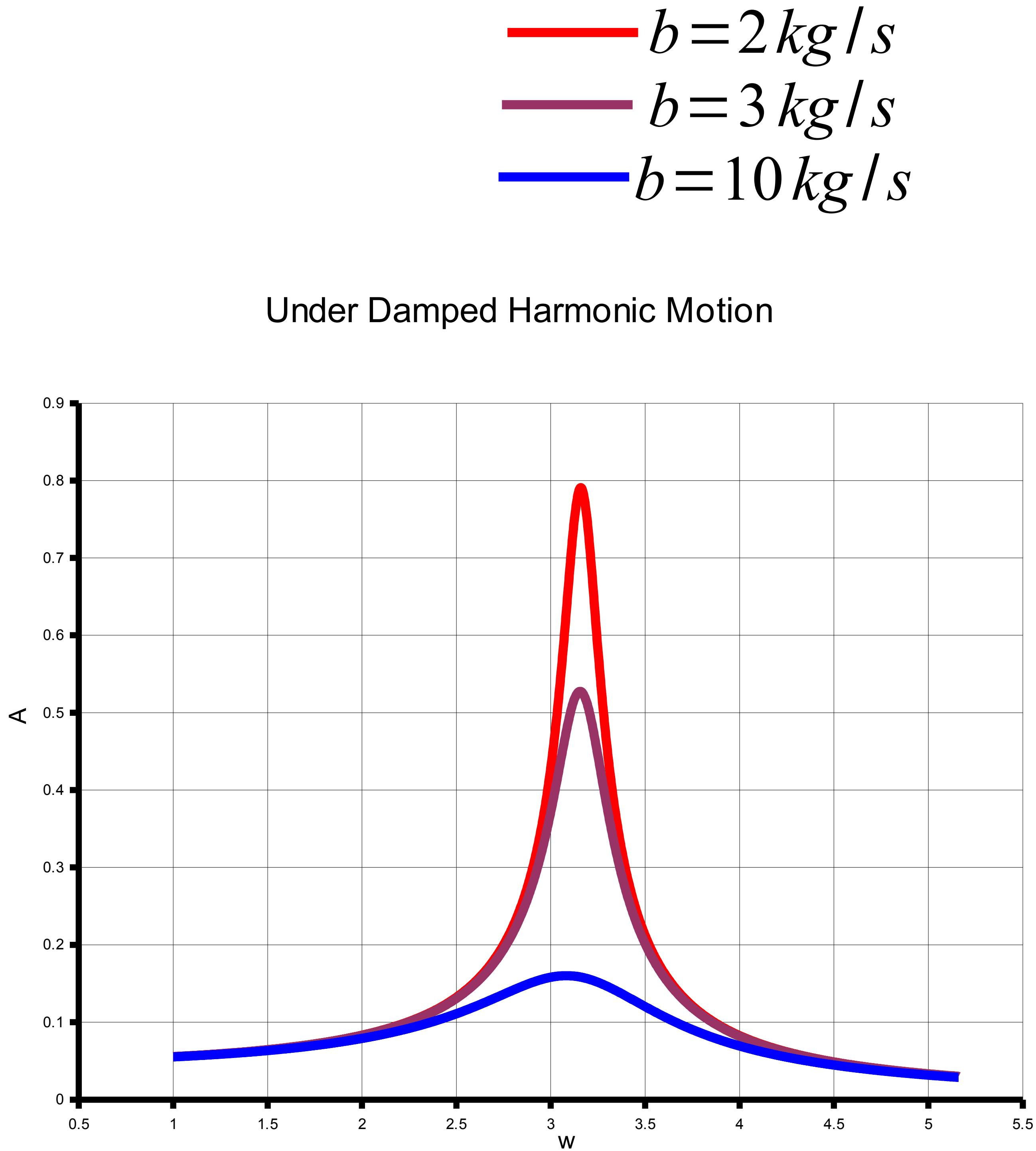
$$A_{max} = \frac{F_o}{\sqrt{b^2\omega^2}}$$

When the driving frequency equals the natural frequency, the amplitude reaches a maximum.



When $\omega = \omega_o = 3.16\, s^{-1}$

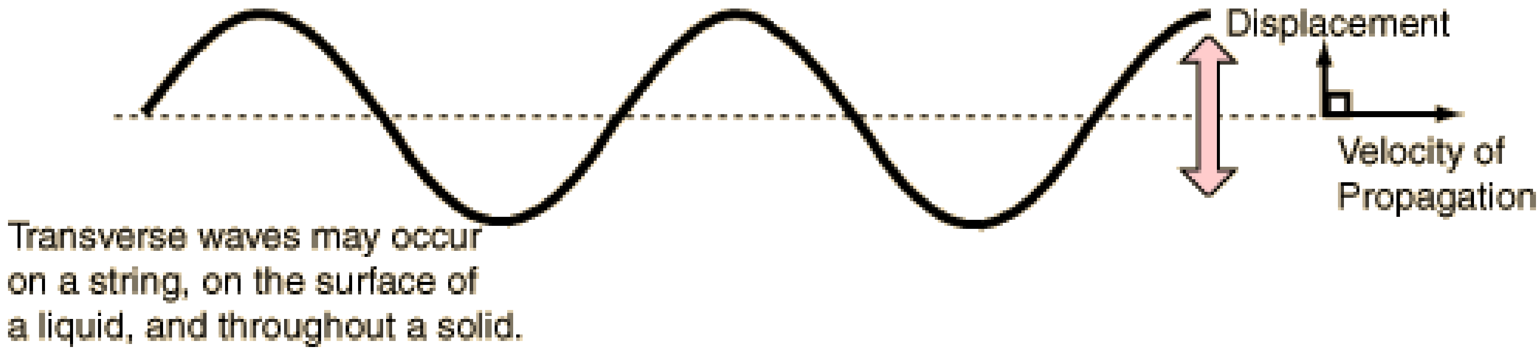
$$A_{max} = \frac{F_o}{\sqrt{b^2 \omega^2}}$$



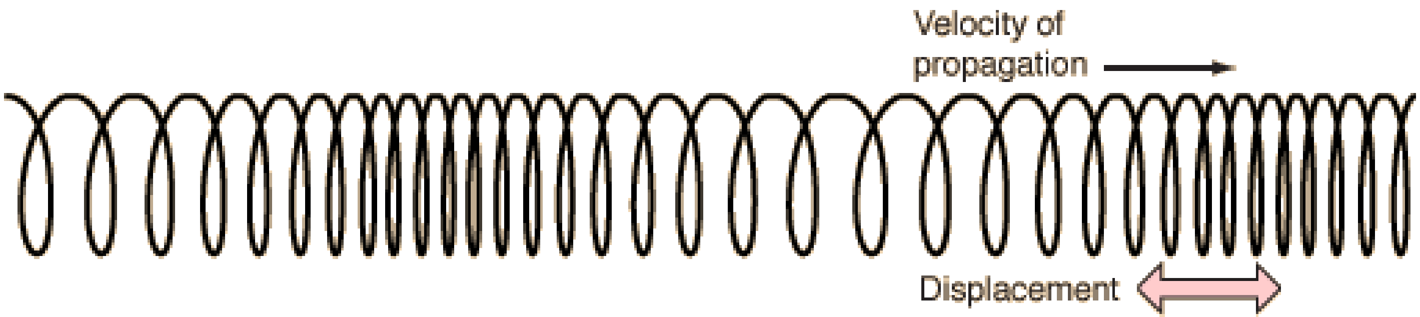
Waves

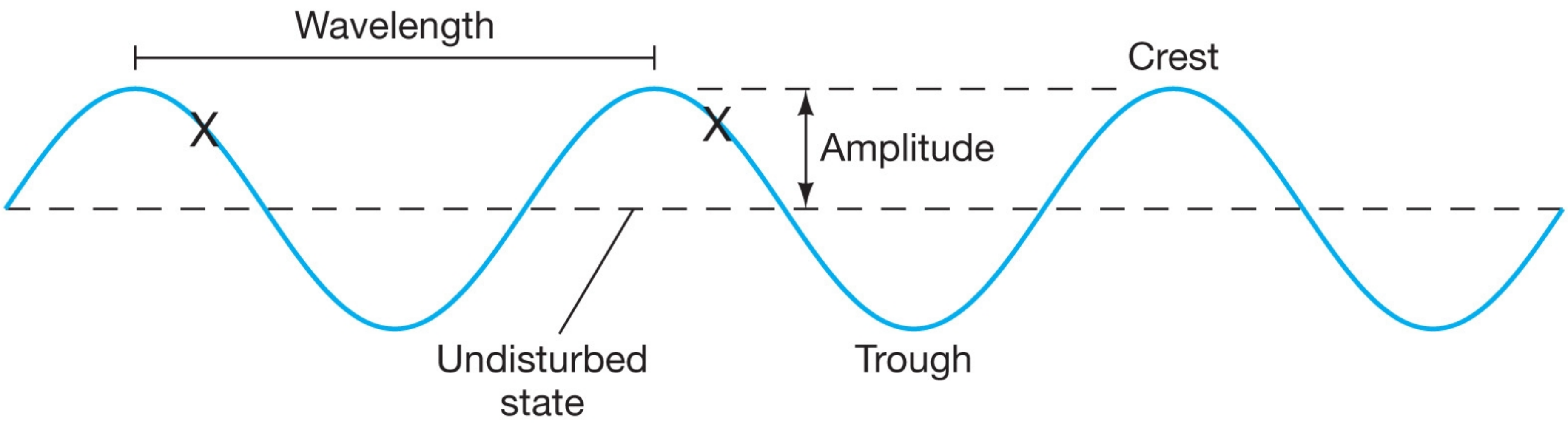
Types of Waves (or Pulses)

Transverse Wave – Velocity of wave is perpendicular to motion of the particles of the medium.



Longitudinal Wave – Velocity of wave is parallel to motion of the particles of the medium.





Amplitude – Maximum displacement of medium from the equilibrium position, i.e., the height of a crest or depth of a trough.

Wavelength – The length of a wave.

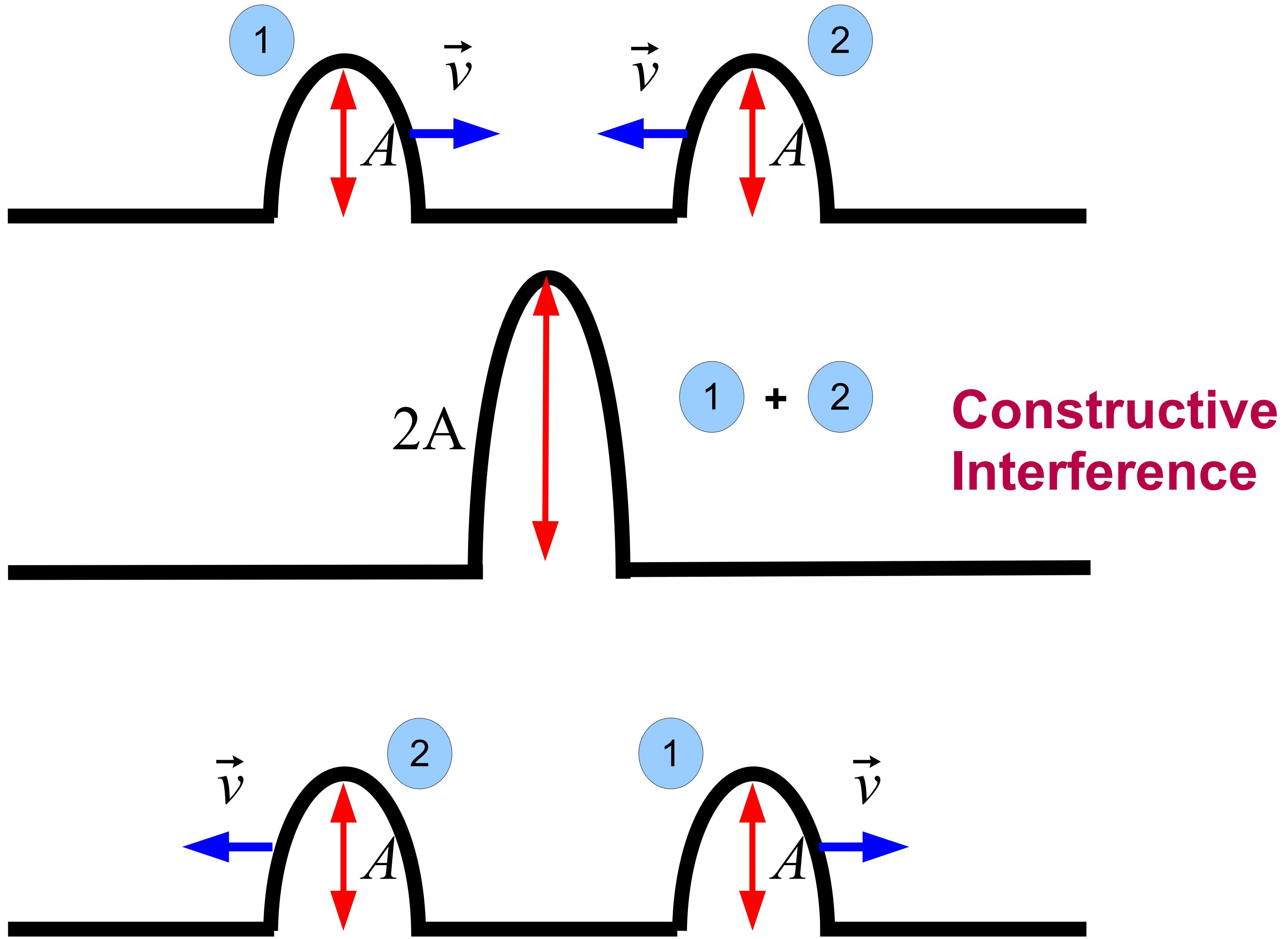
Period – The time for one wave to pass by.

Frequency – The number of waves per second.

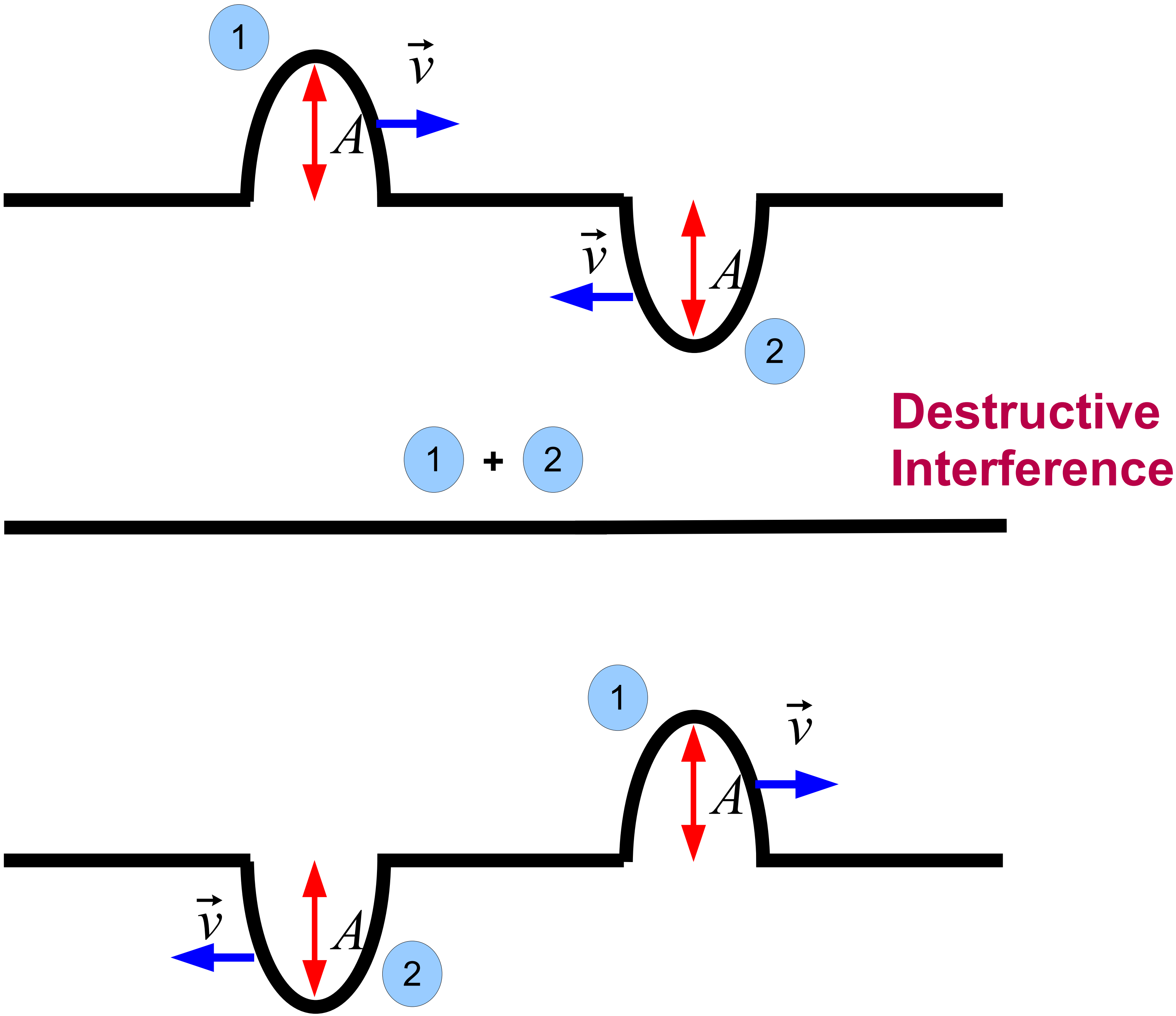
Superposition – The resultant wave of two or more waves moving through a medium is just the algebraic sum of each wave.

Interference – If two or more waves encounter each other, they seem to pass through one another totally unaffected.

Ripples on a Pond

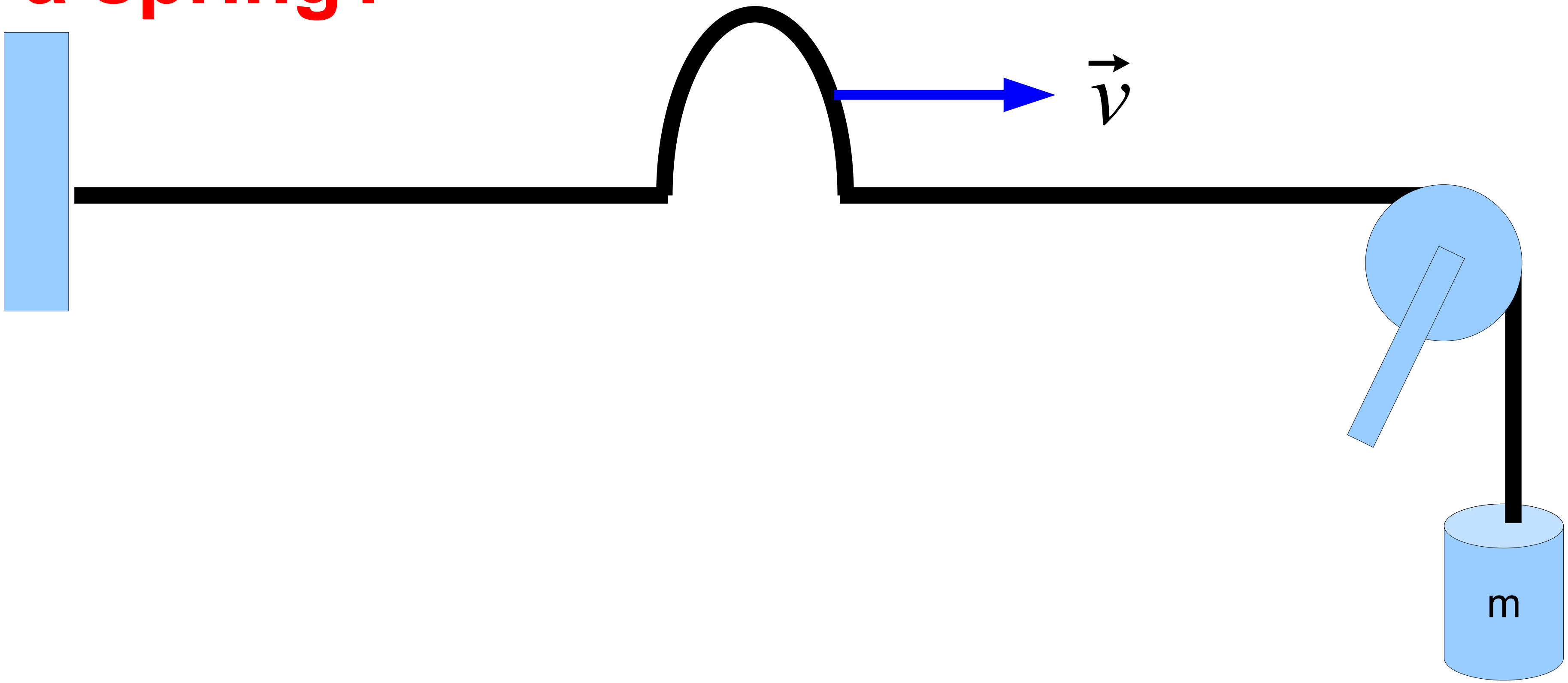


Superposition and Interference



Superposition and Interference

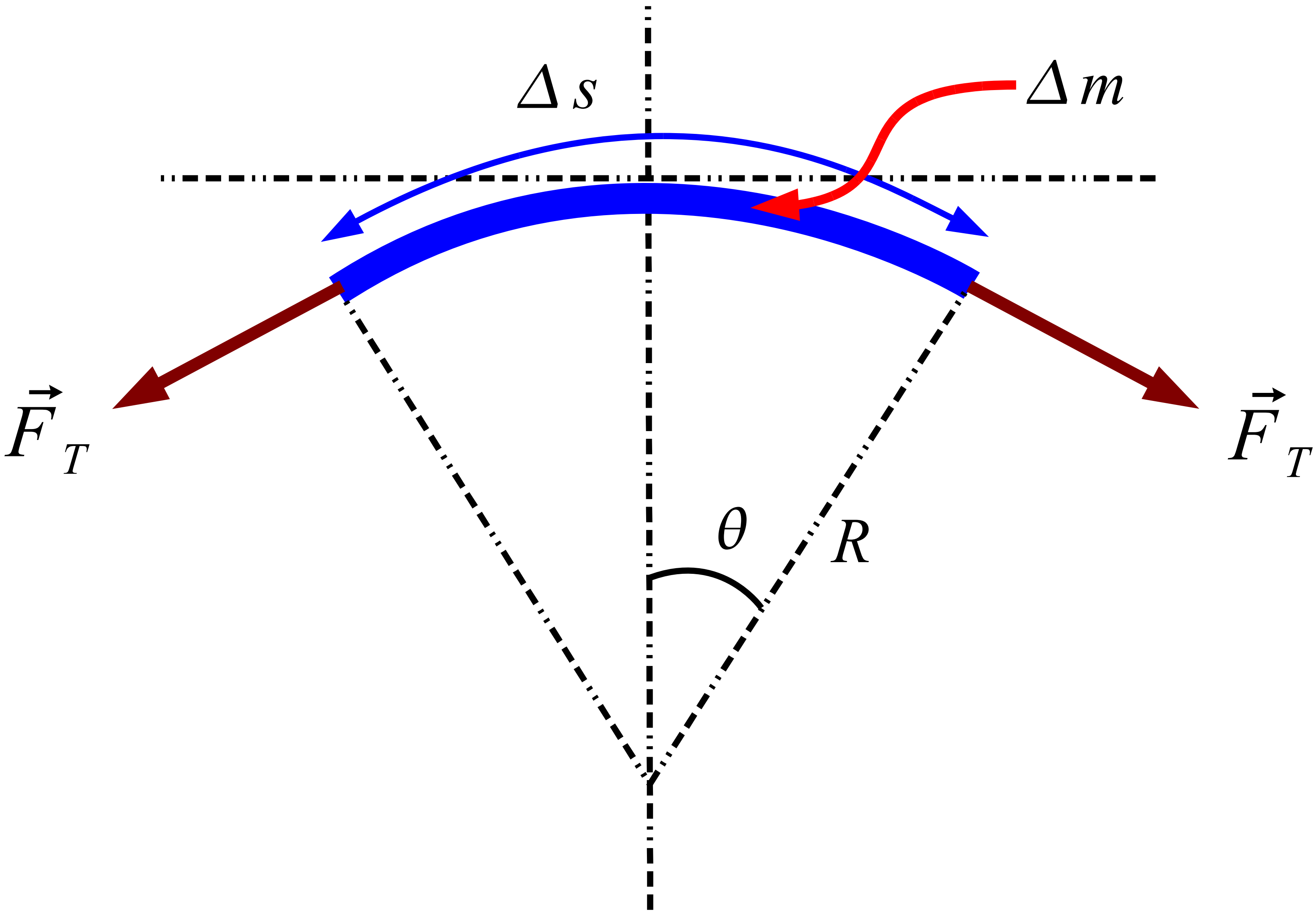
What is the speed of a pulse or wave on a spring?

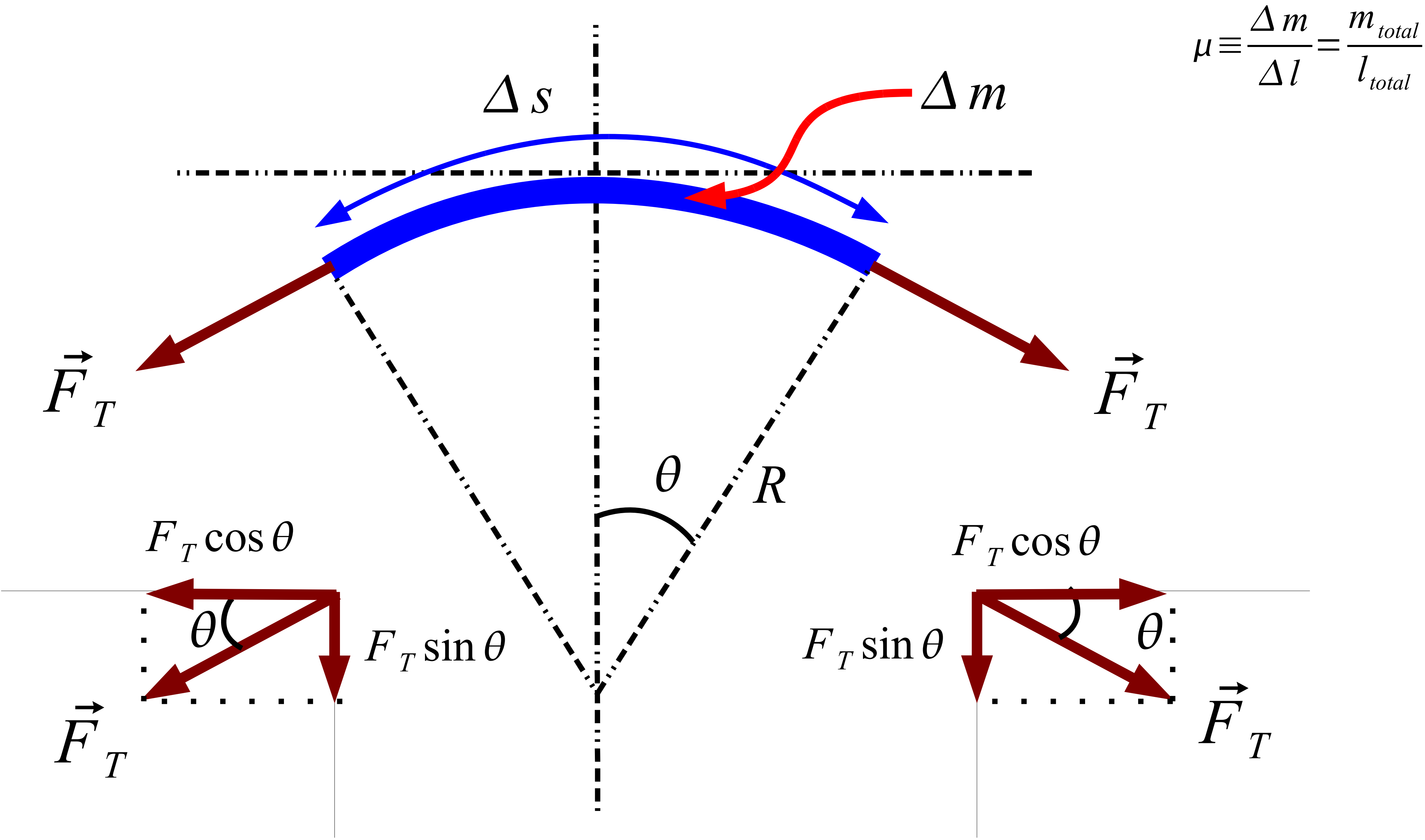


Linear Density

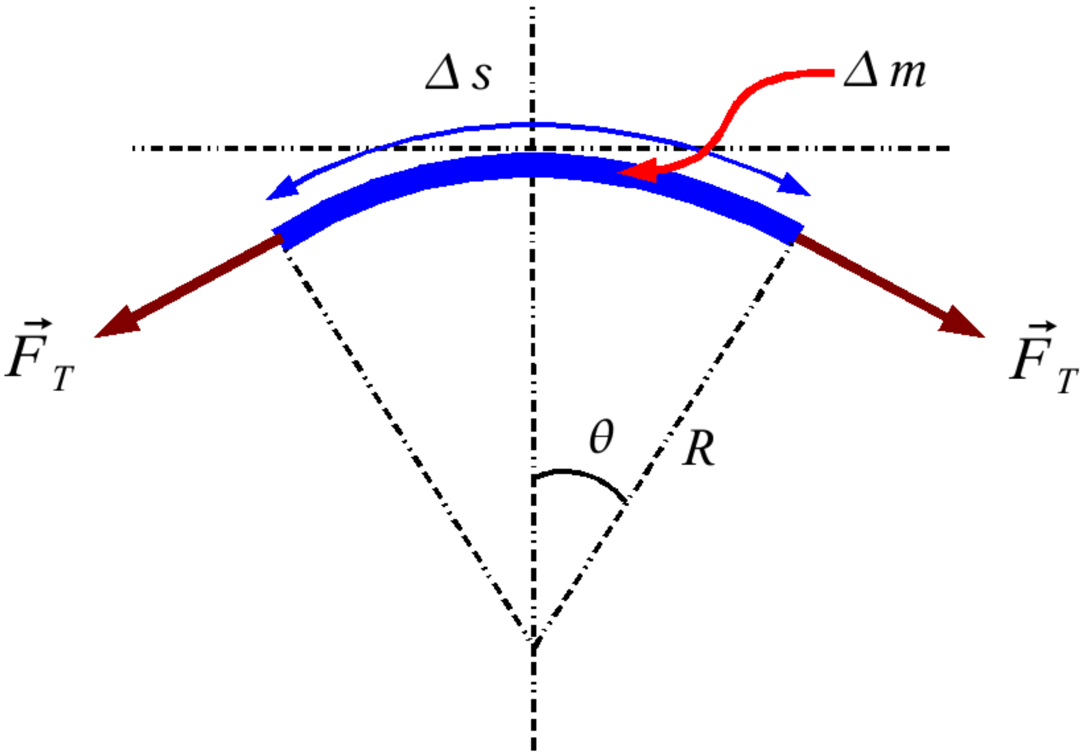
$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$





Horizontal Components Cancel. Vertical components both point toward the center.



$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

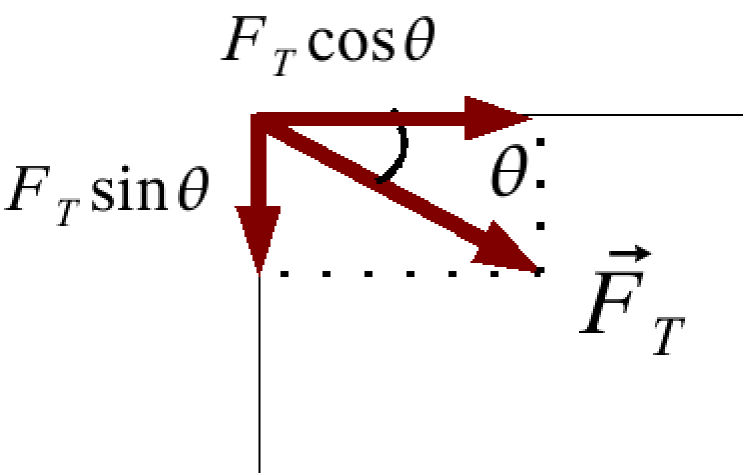
$$\sin\theta \approx \theta \text{ for } \ll \theta$$

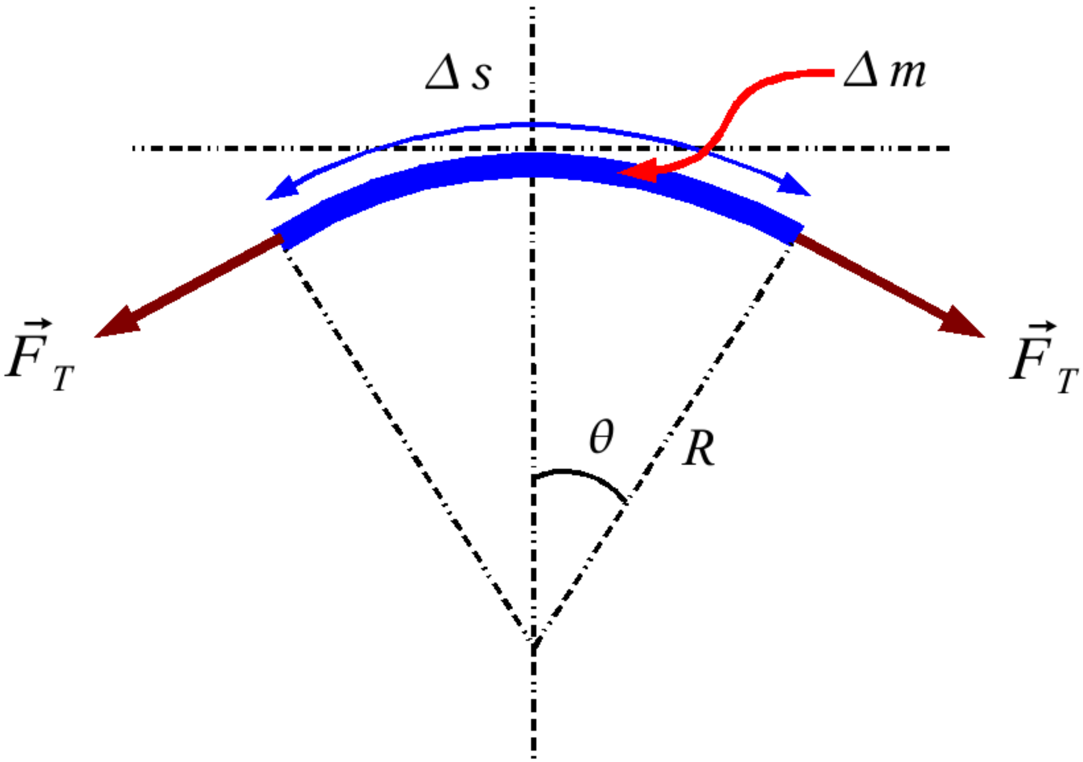
$$\Delta s = R(2\theta)$$

$$\Delta m = \mu \Delta s = \mu R(2\theta)$$

$$F_c = \Delta m\, a_c$$

$$2F_T \sin\theta = \Delta m \frac{v^2}{R}$$



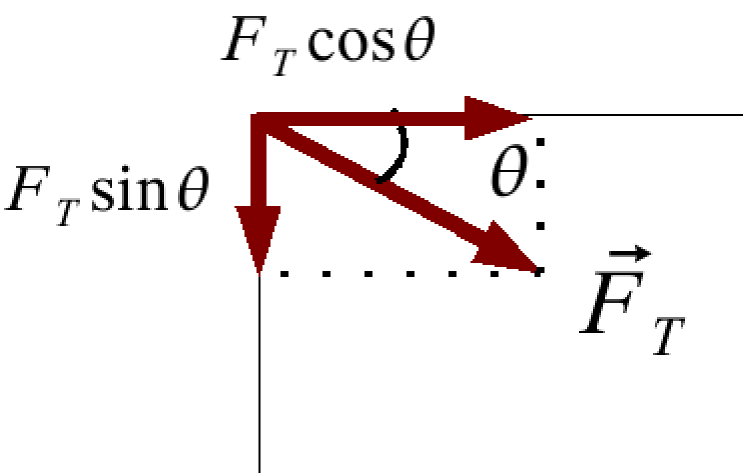


$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

$$\sin\theta \approx \theta \text{ for } \ll \theta$$

$$\Delta s = R(2\theta)$$

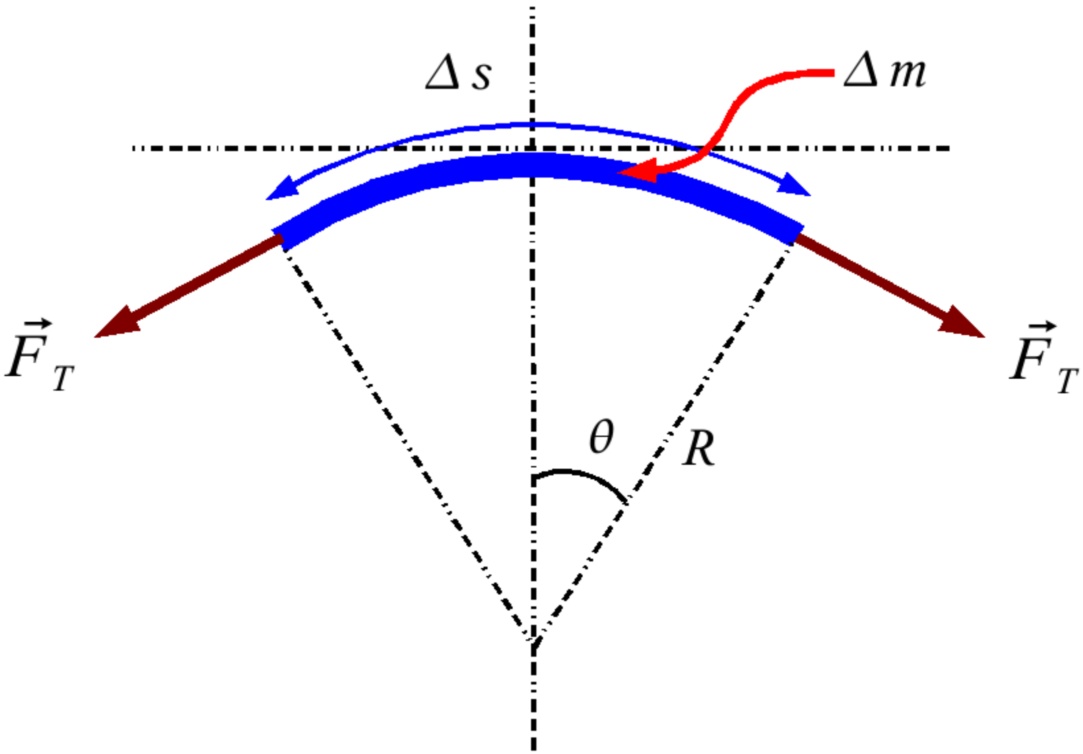
$$\Delta m = \mu \Delta s = \mu R(2\theta)$$



$$F_c = \Delta m\, a_c$$

$$2 F_T \sin\theta = \Delta m \frac{v^2}{R}$$

$$2 F_T \theta = (\mu 2 \theta R) \frac{v^2}{R}$$

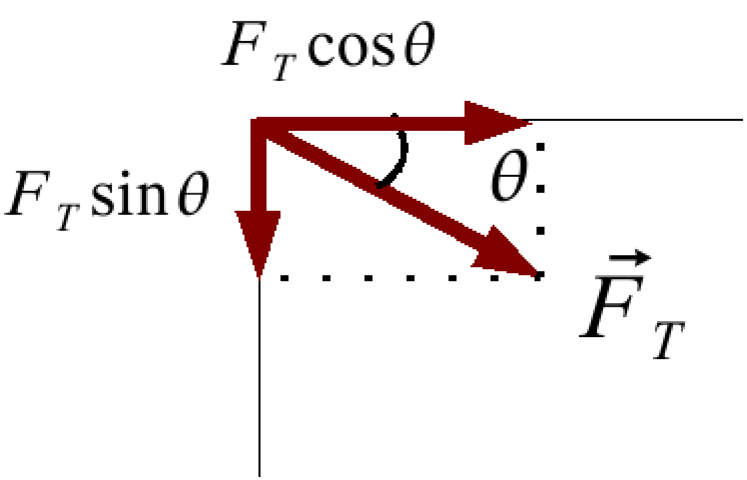


$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

$$\sin\theta \approx \theta \ \text{ for } \ll \theta$$

$$\Delta s = R(2\theta)$$

$$\Delta m = \mu \Delta s = \mu R(2\theta)$$



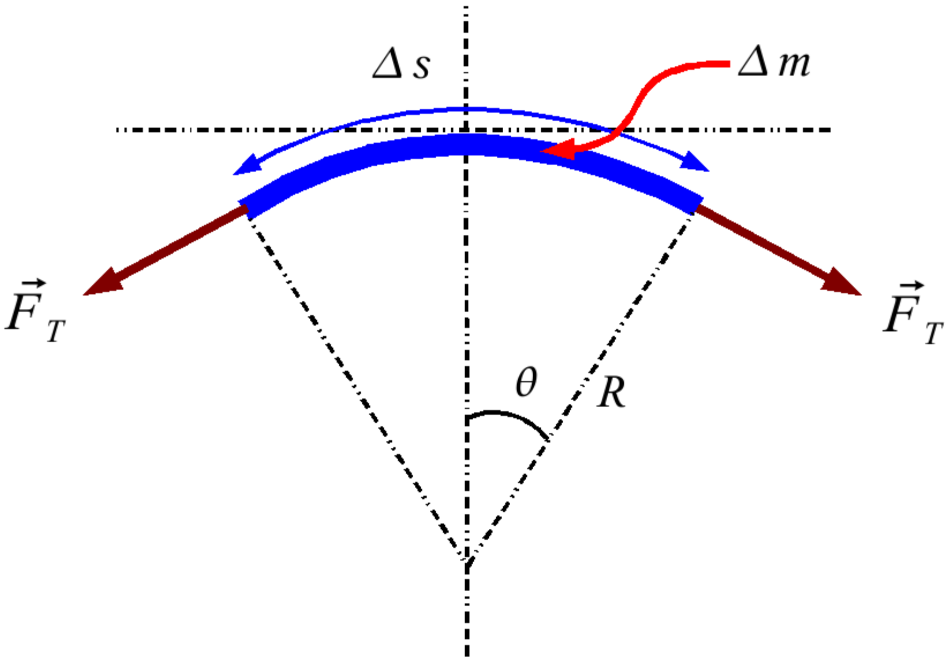
$$F_c = \Delta m\, a_c$$

$$2F_T \sin\theta = \Delta m \frac{v^2}{R}$$

$$\cancel{2} F_T \cancel{\theta} = (\mu \cancel{2} \cancel{\theta} \cancel{R}) \frac{v^2}{\cancel{R}}$$

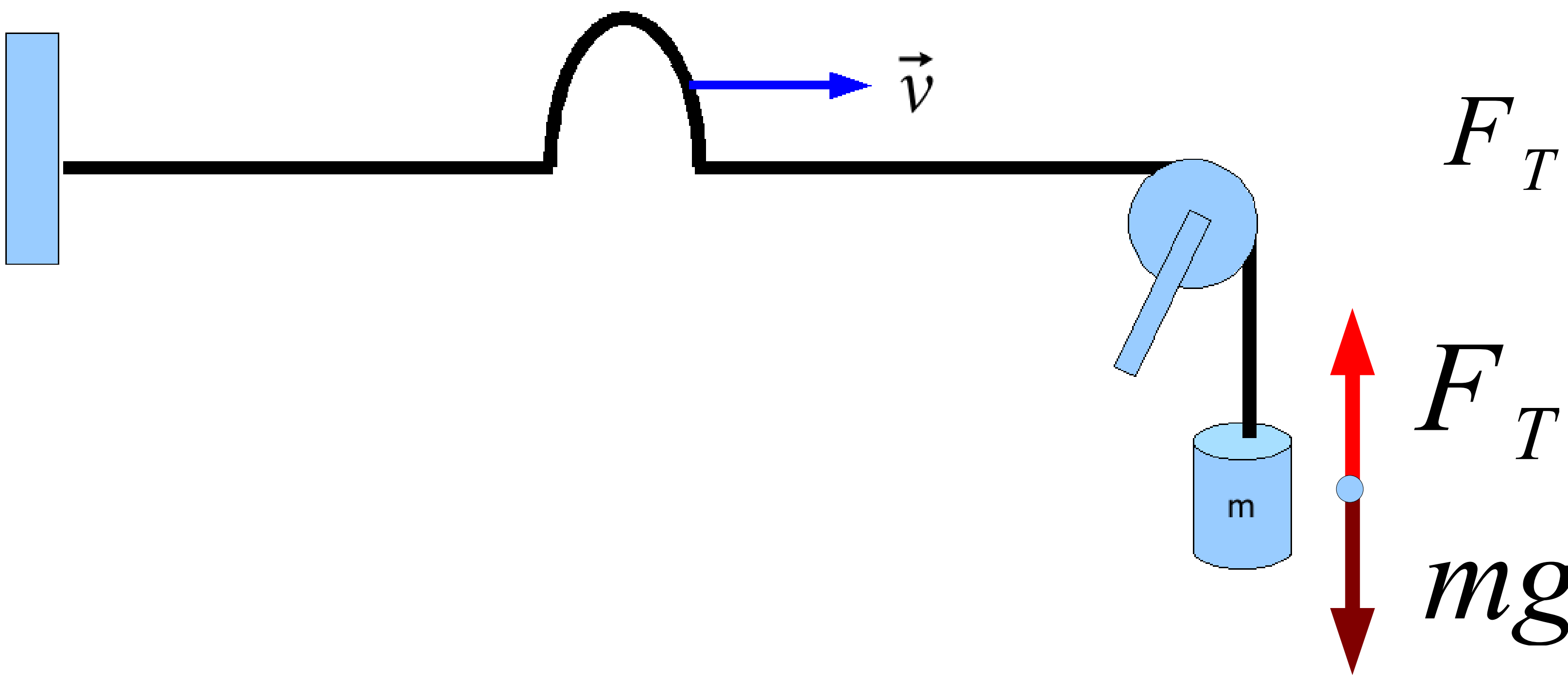
$$F_T = \mu v^2$$

$$v = \sqrt{\frac{F_T}{\mu}}$$



$$v = \sqrt{\frac{F_T}{\mu}}$$

Velocity of a wave on a string.



$$F_T = mg$$

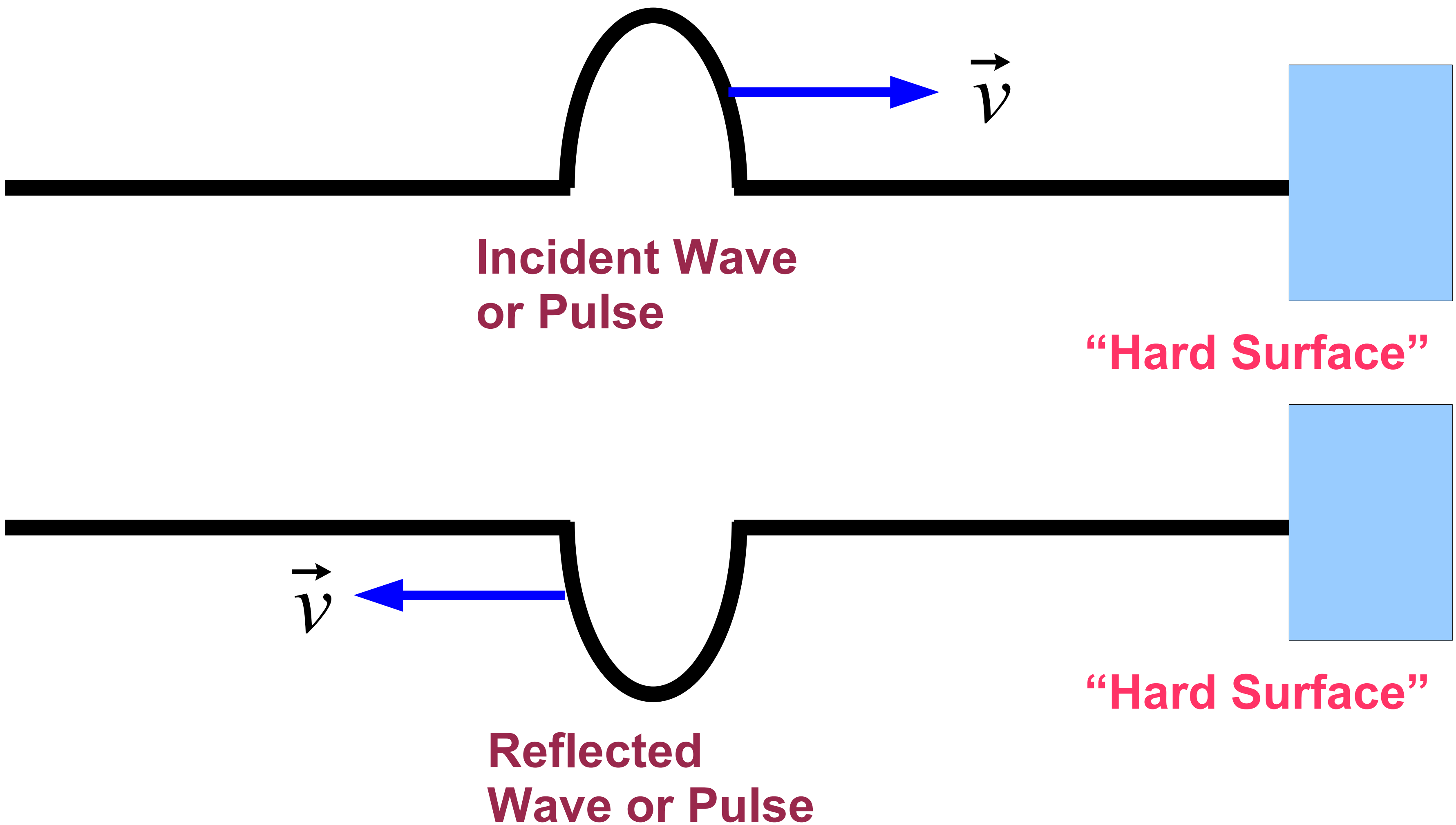
In General ....

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

$$v_{\text{string}} = \sqrt{\frac{F_T}{\mu}}$$

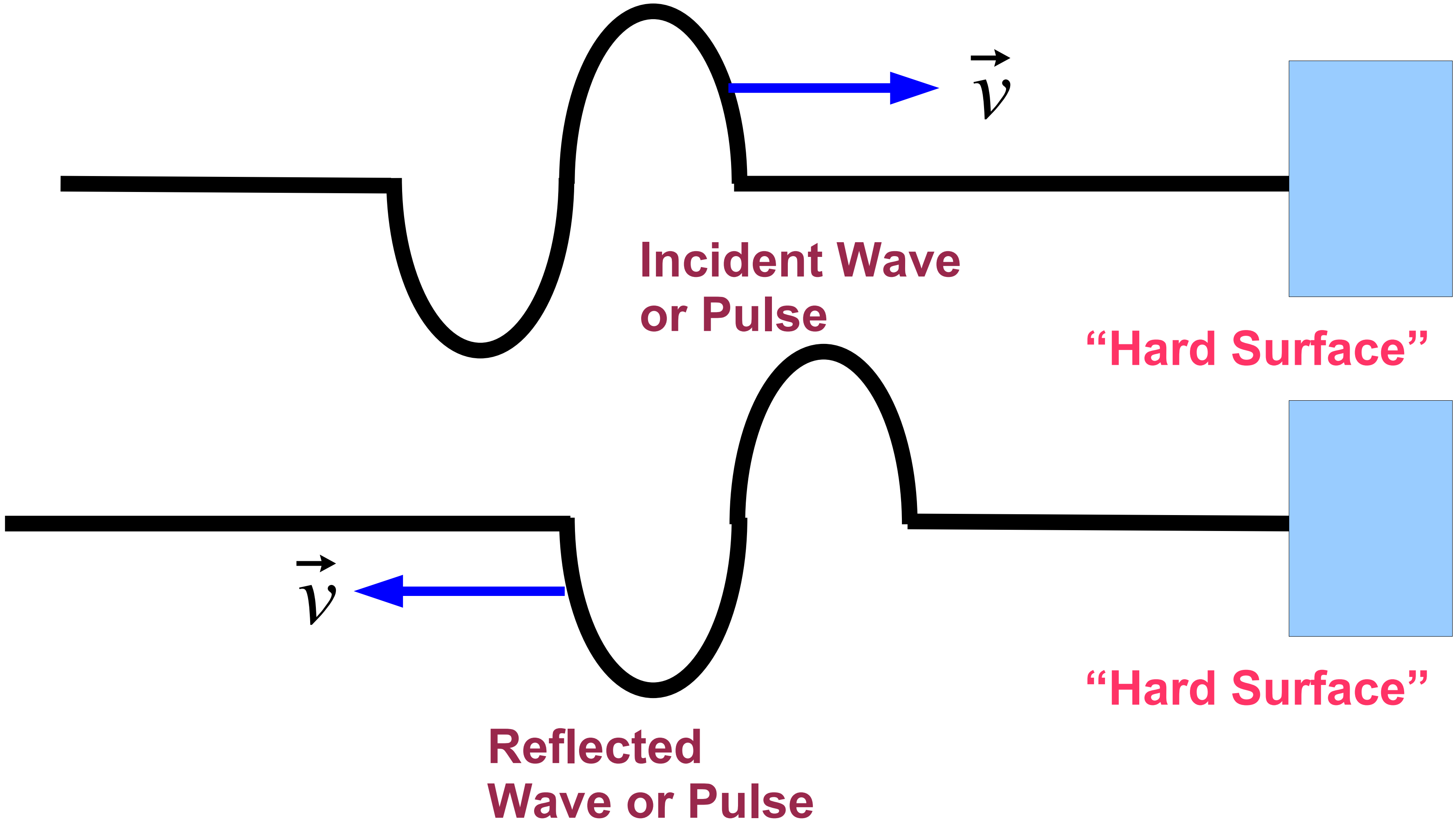
$$v_{\text{medium}} = \sqrt{\frac{B}{\rho}}$$

REFLECTIONS



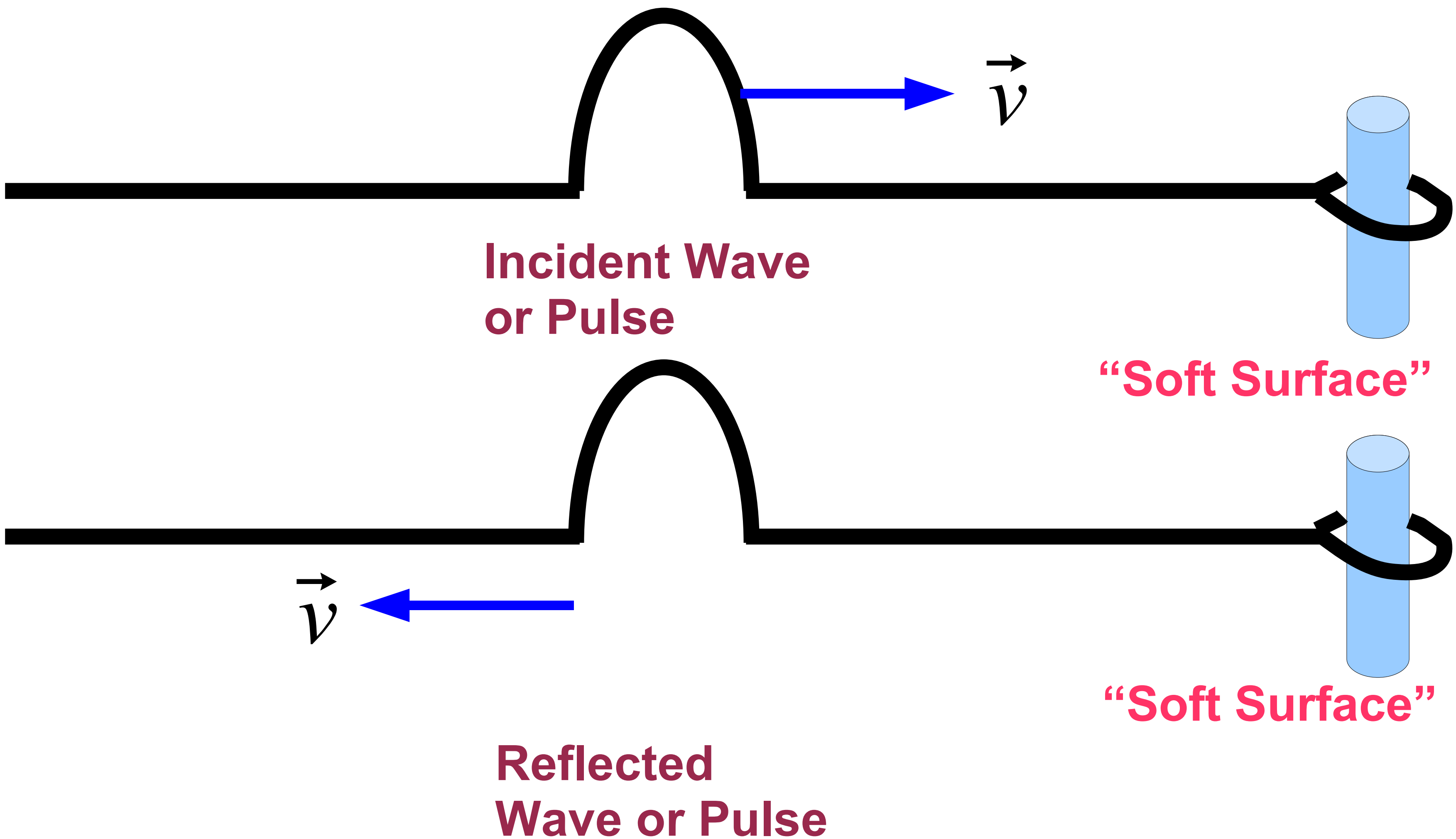
Waves reflected off a “Hard” surface are 180° out of phase with respect to the incident wave.

REFLECTIONS REFLECTIONS



Waves reflected off a "Hard" surface are 180° out of phase with respect to the incident wave.

REFLECTIONS REFLECTIONS



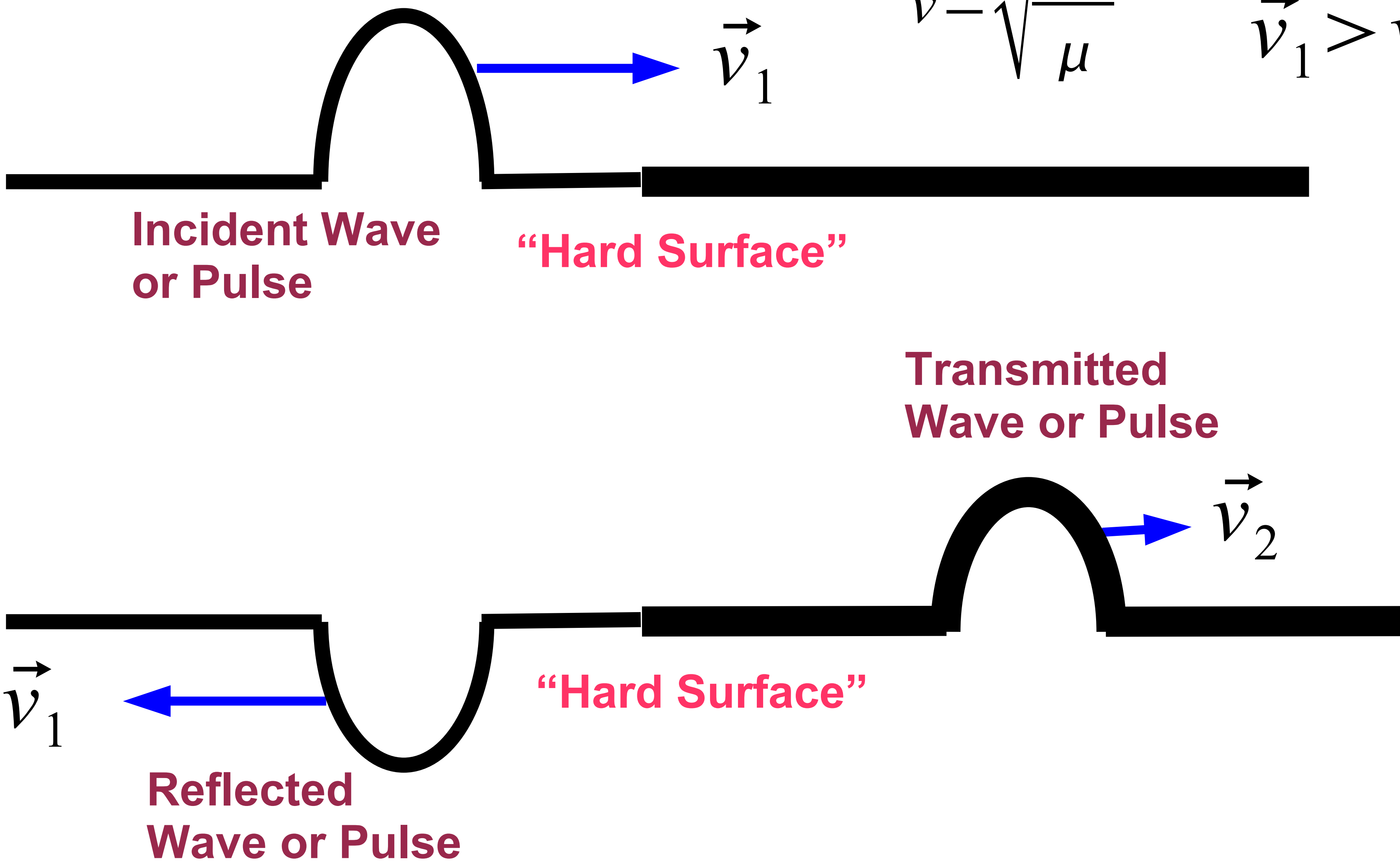
Waves reflected off a "Soft" surface are in phase (0° out of phase) with respect to the incident wave.

REFLECTIONS

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$\mu_1 < \mu_2$$

$$\vec{v}_1 > \vec{v}_2$$



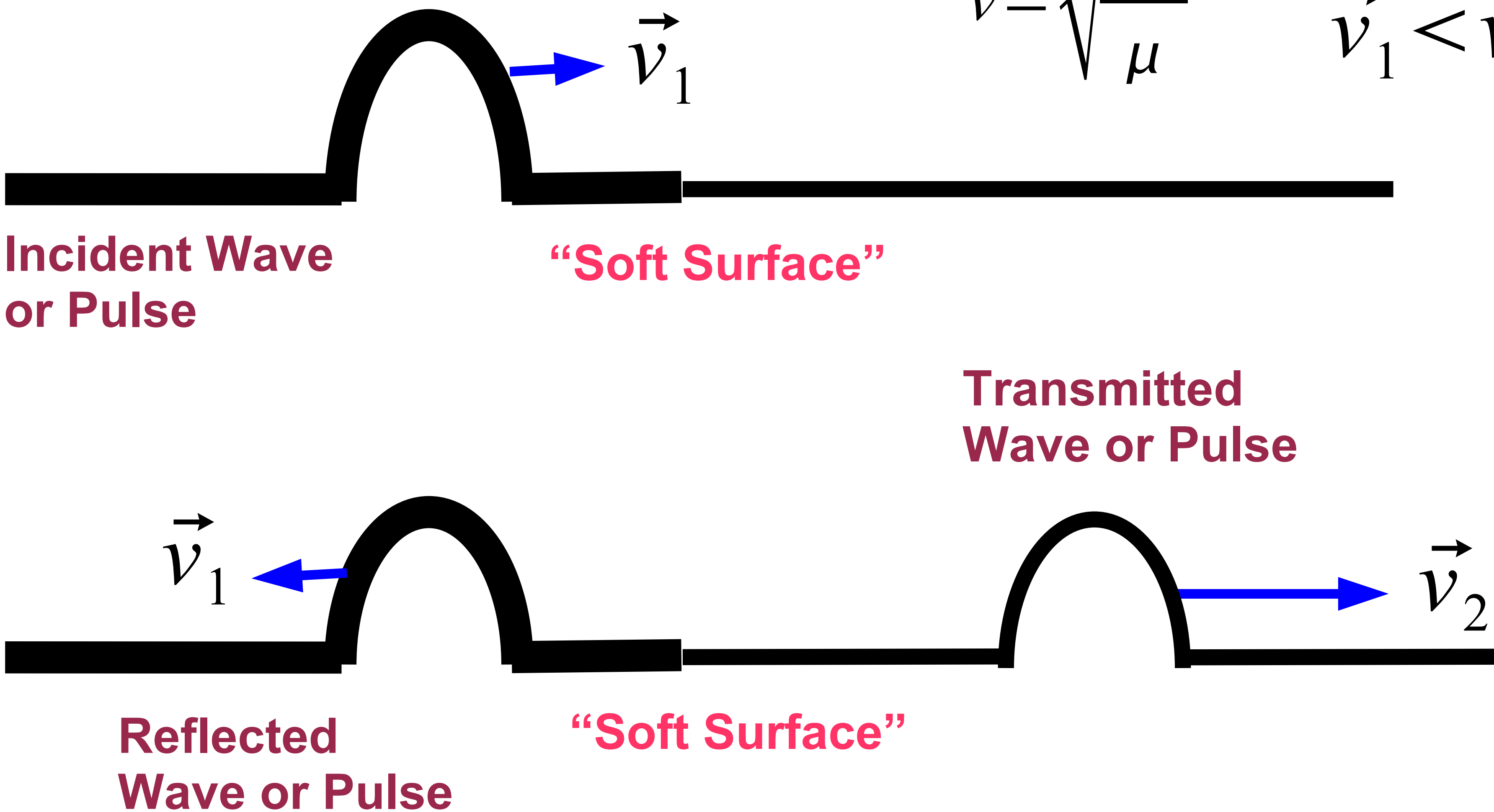
Waves reflected off a “Hard” surface are 180° out of phase with respect to the incident wave.

REFLECTIONS

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$\mu_1 > \mu_2$$

$$\vec{v}_1 < \vec{v}_2$$



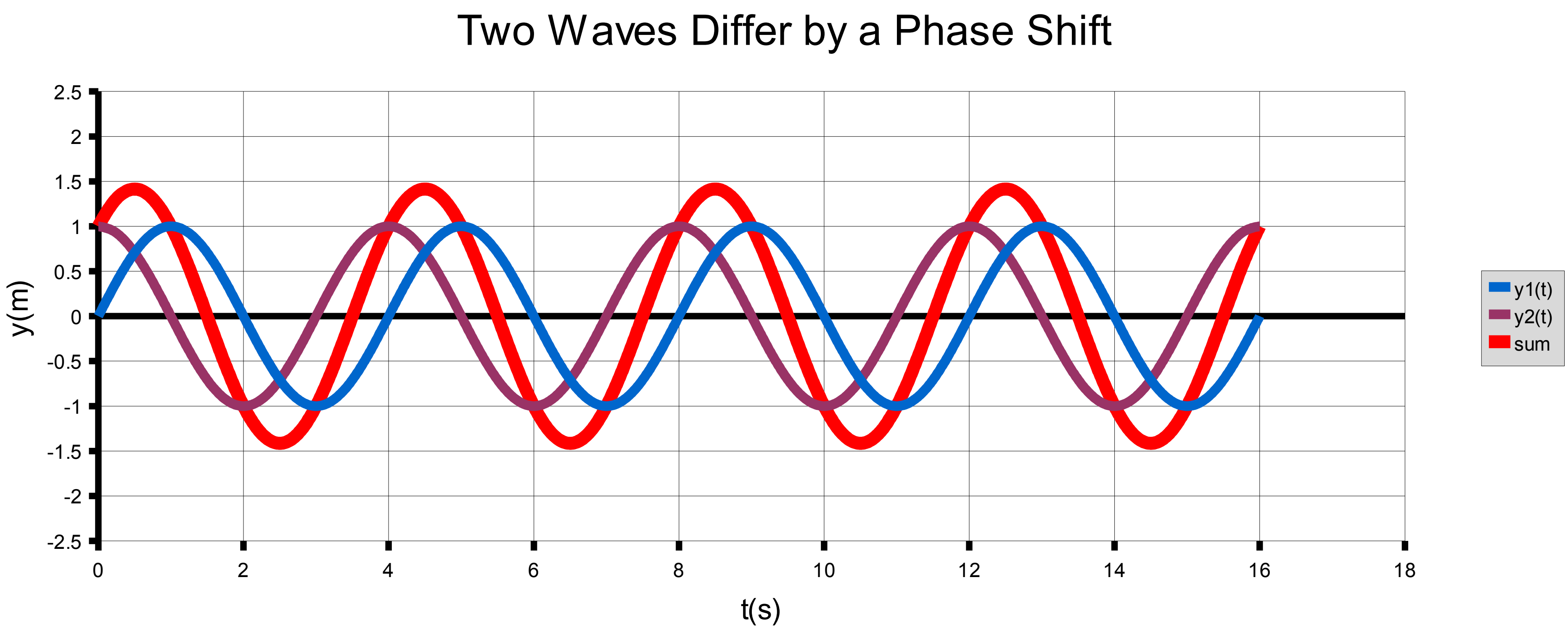
Waves reflected off a "Soft" surface are in phase with respect to the incident wave.

II. Waves Phenomena

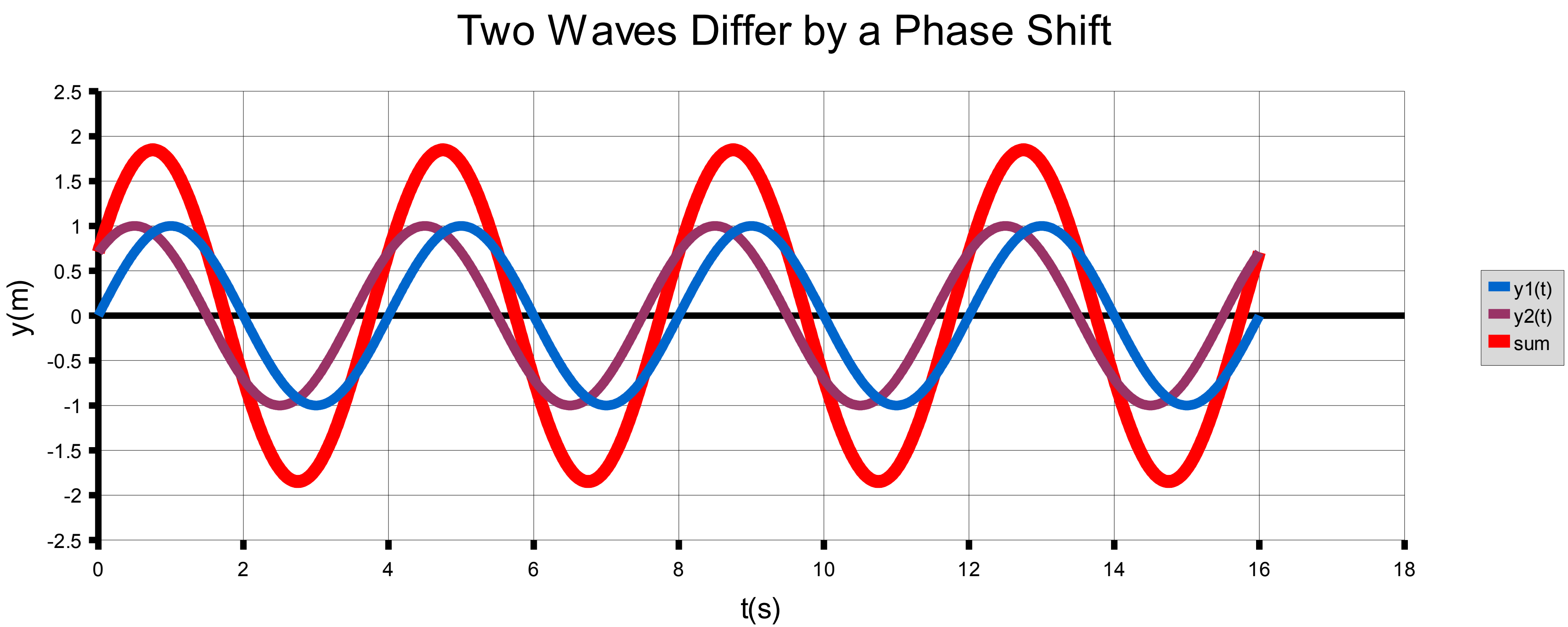
Interference and Superposition:

Two Waves that differ by a phase shift.

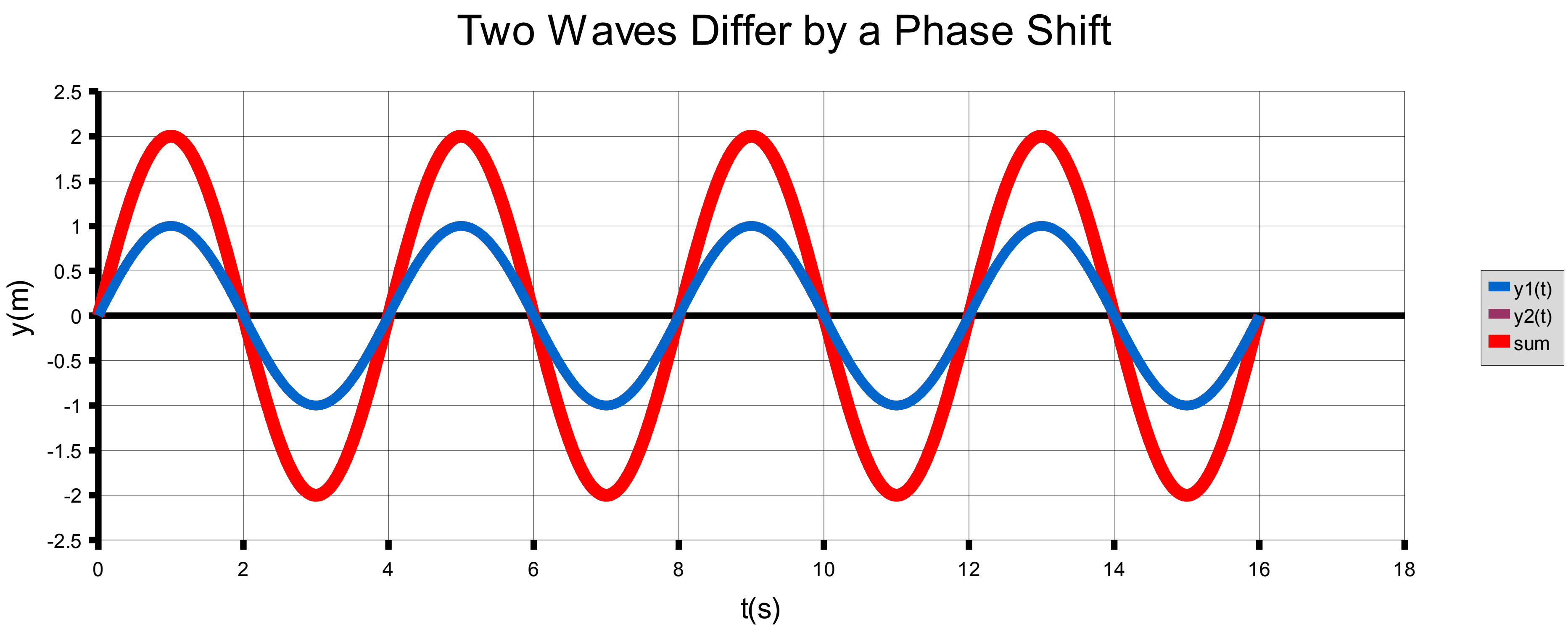
$$\text{Phase Shift } \phi = \frac{\pi}{2} \, radians = 90^{O}$$



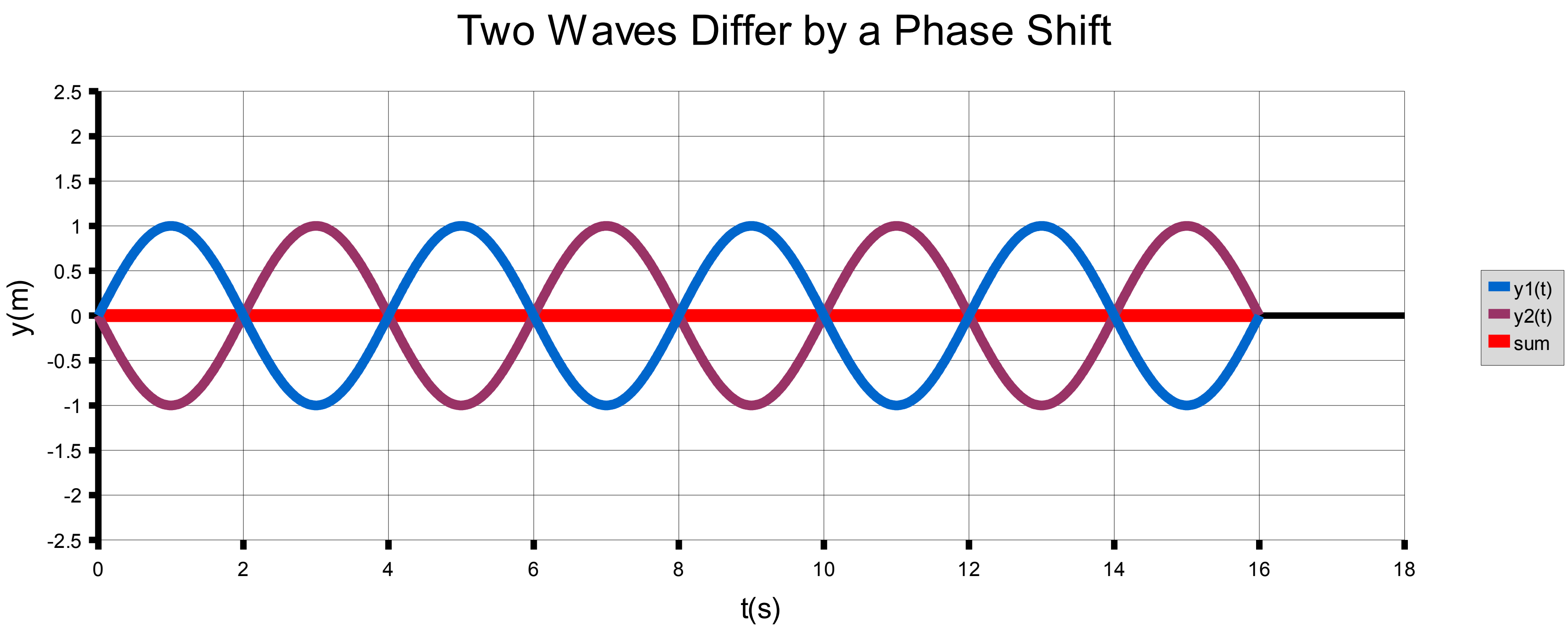
$$\text{Phase Shift } \phi = \frac{\pi}{4}\, radians = 45^{O}$$



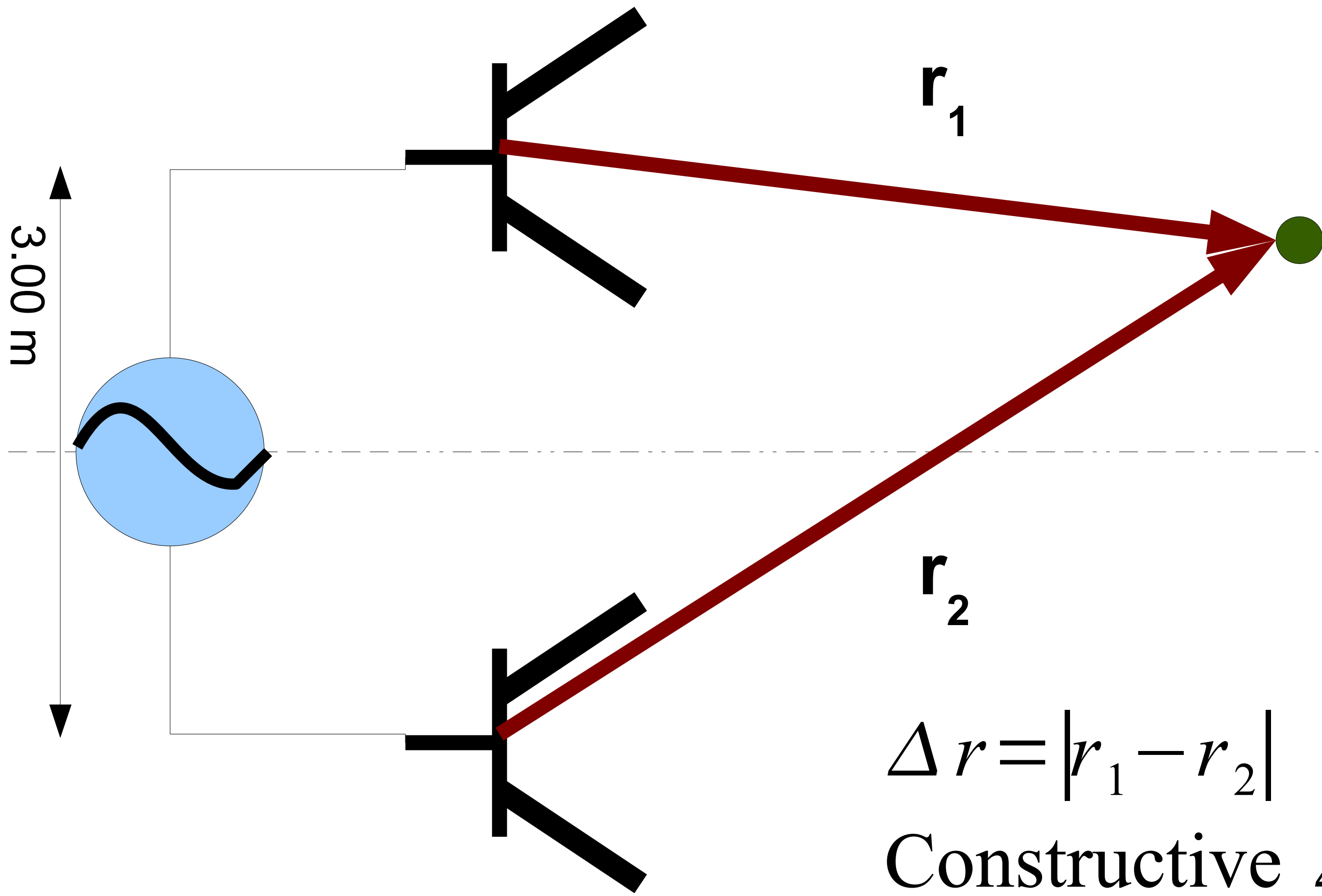
Phase Shift $\phi = 0\, radians = 0^{O}$



$$\text{Phase Shift } \phi = \pi \; radians = 180^{O}$$



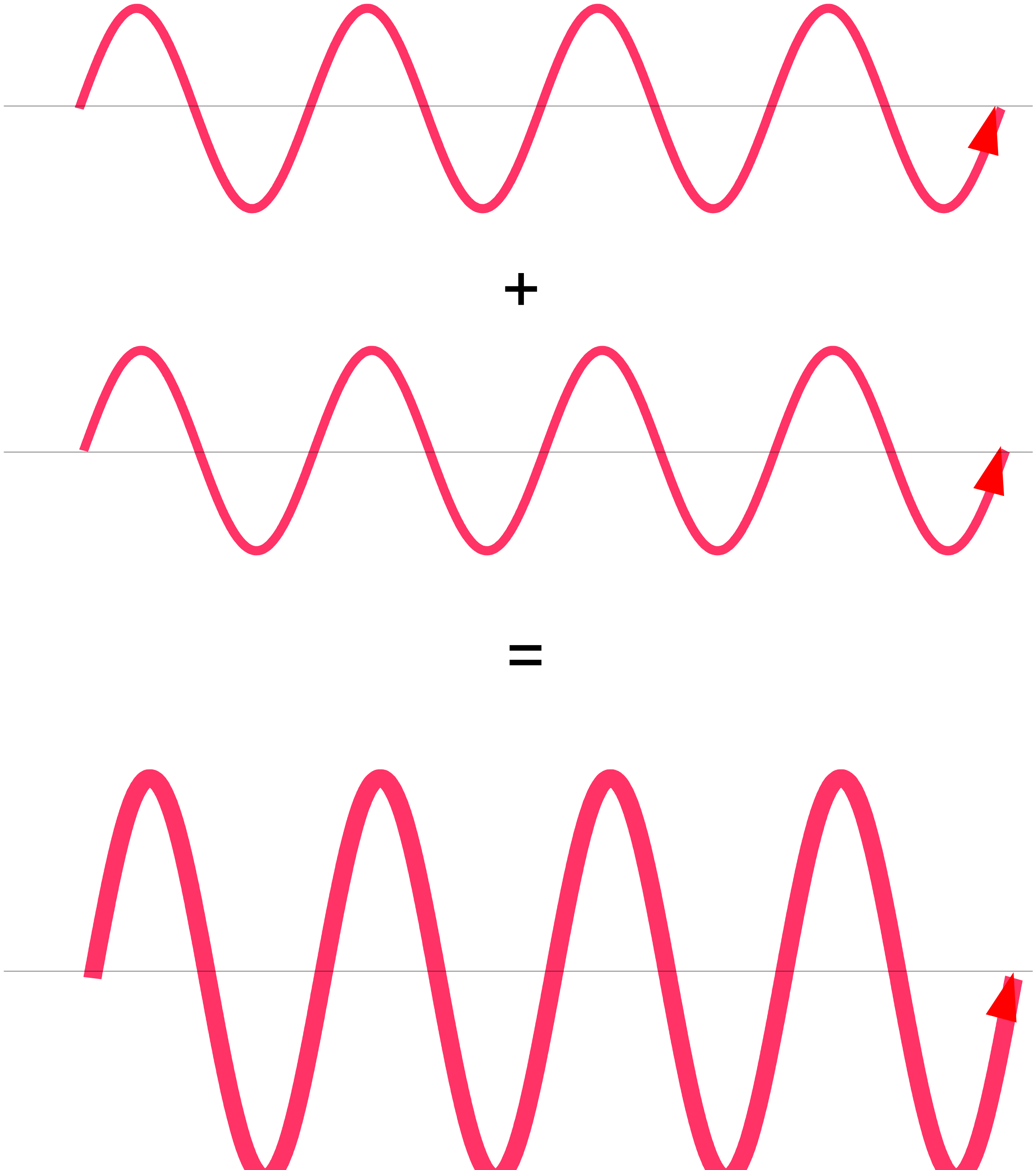
The study of waves vary by a phase shift has applications in the "acoustics" of buildings.



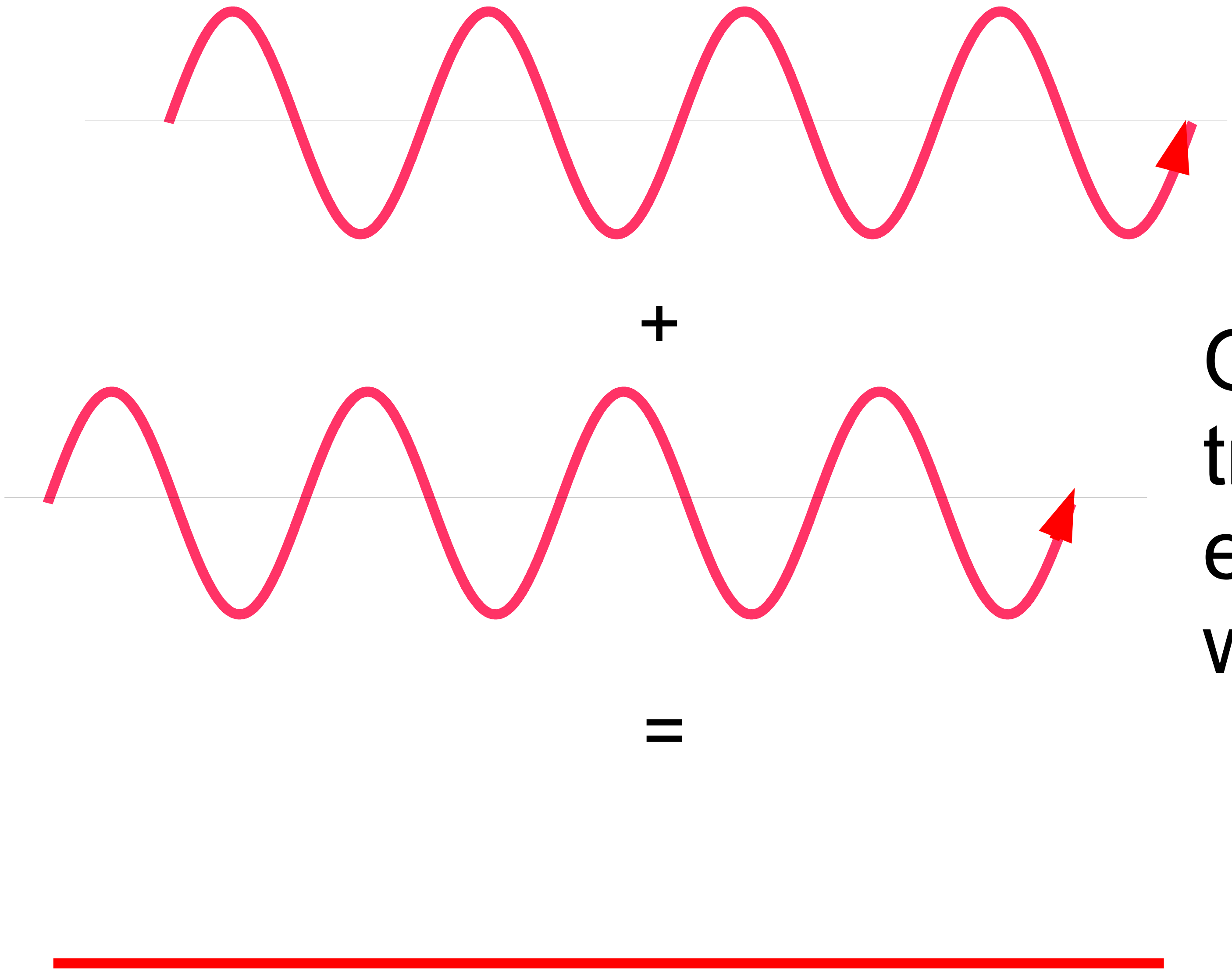
$$\Delta r = |r_1 - r_2|$$

$$\text{Constructive} \quad \Delta r = n\lambda \qquad n = 0,1,2,\ldots$$

$$\text{Destructive} \quad \Delta r = \frac{n}{2}\lambda \qquad n = 1,3,5,\ldots$$



Two Waves that traveled the same distance.



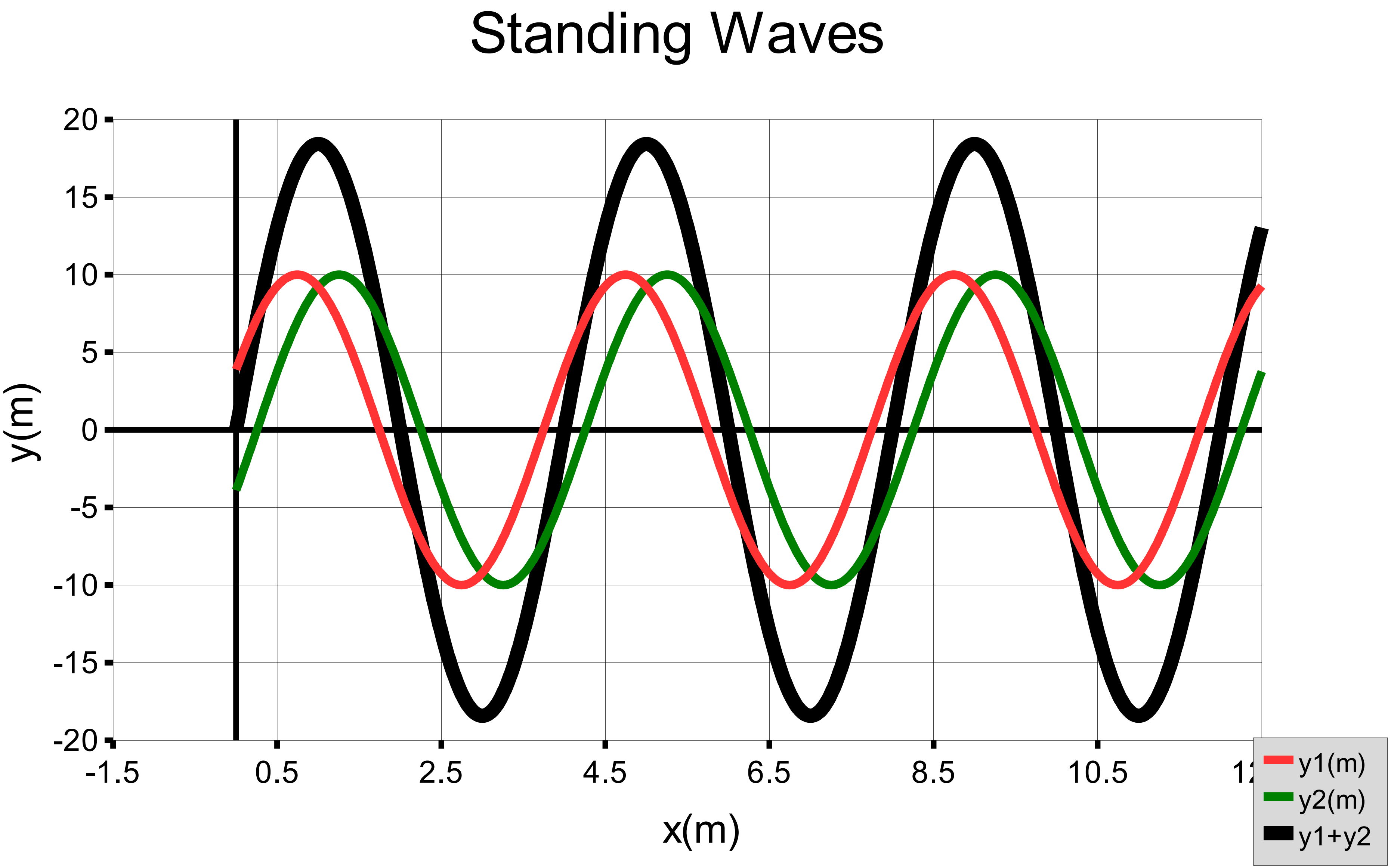
One wave travels and extra ½ of a wavelength.

Standing Waves:

Two Identical Waves moving in two different directions.

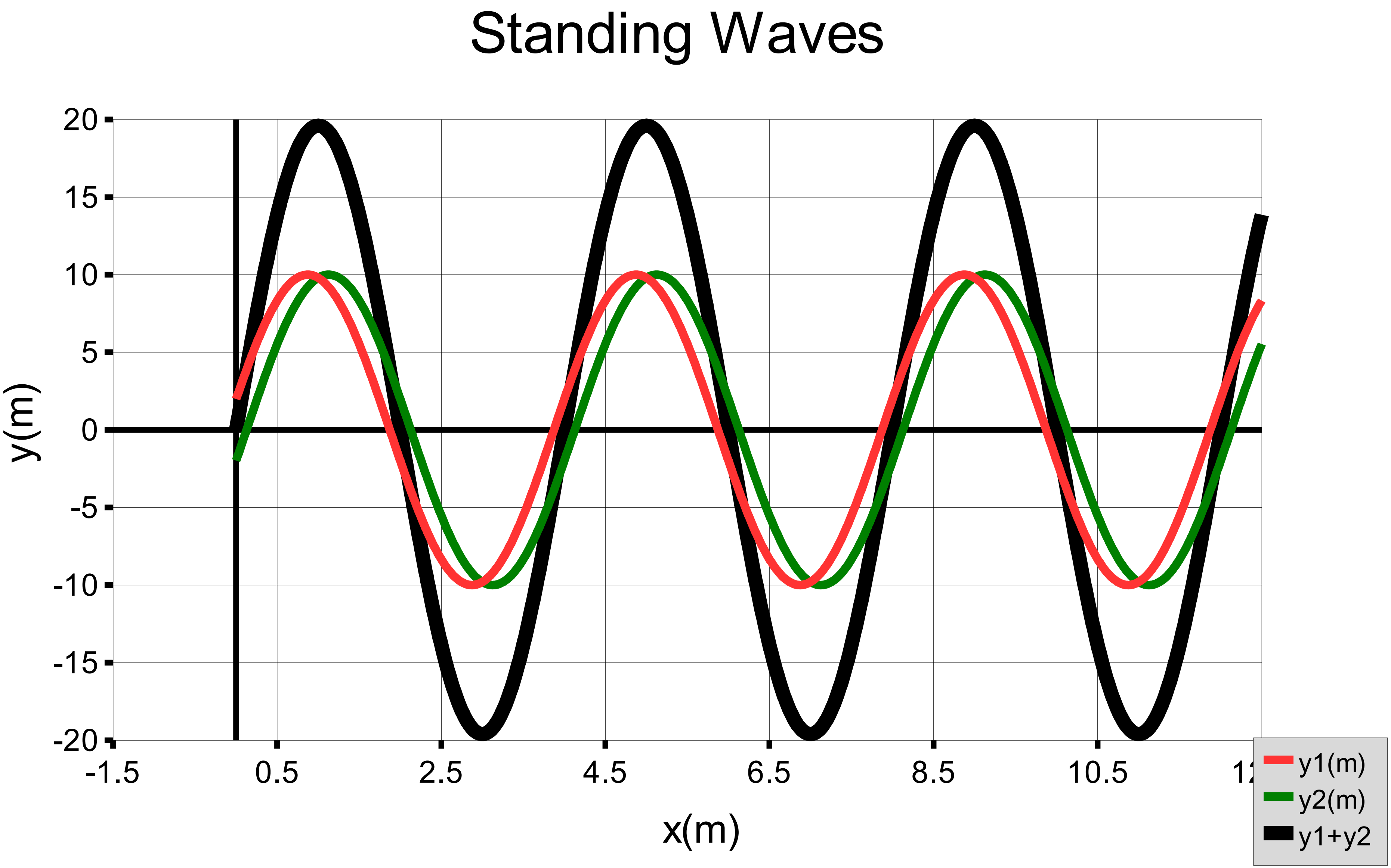
t=-0.2 s

START

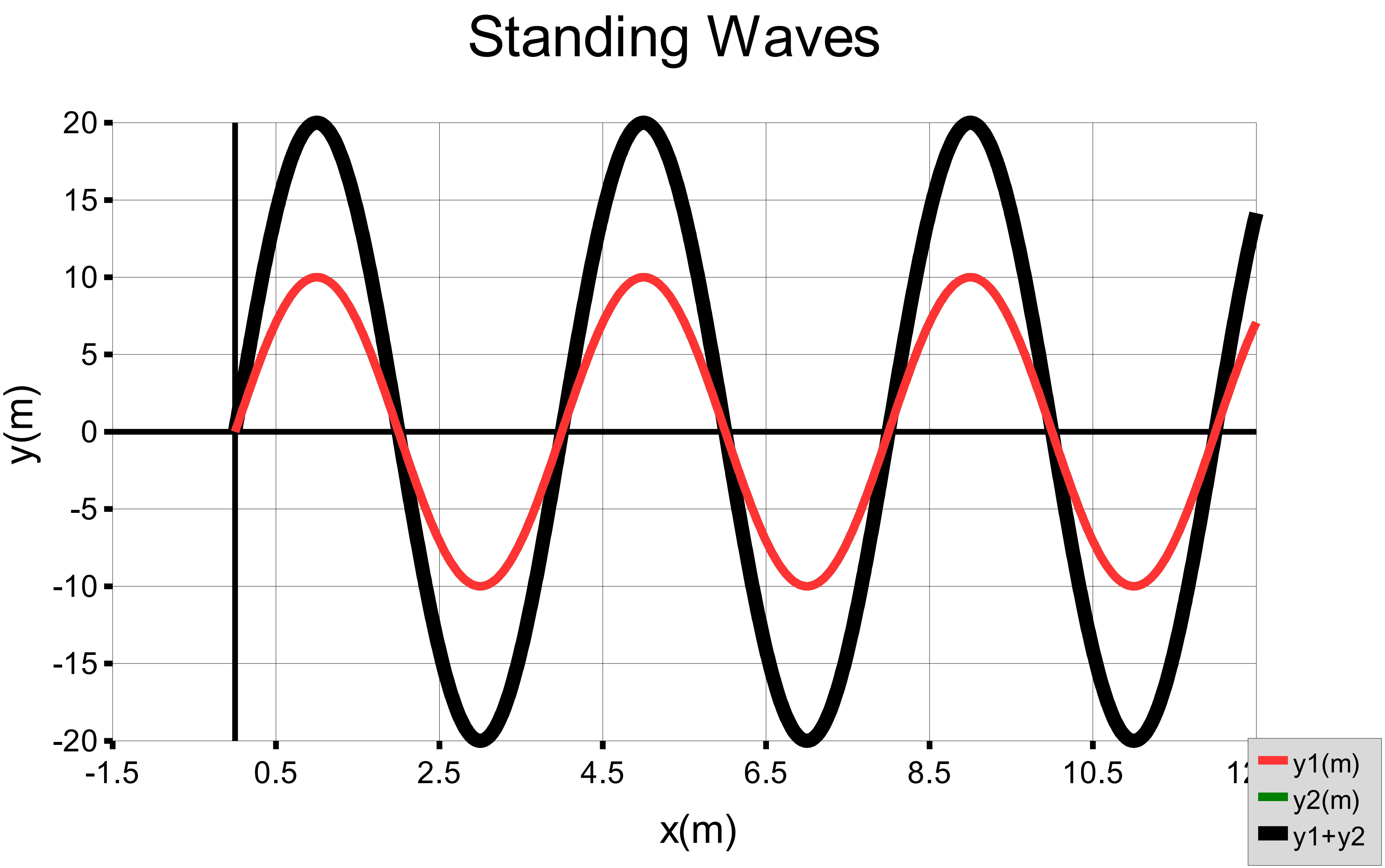


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=-0.1 s

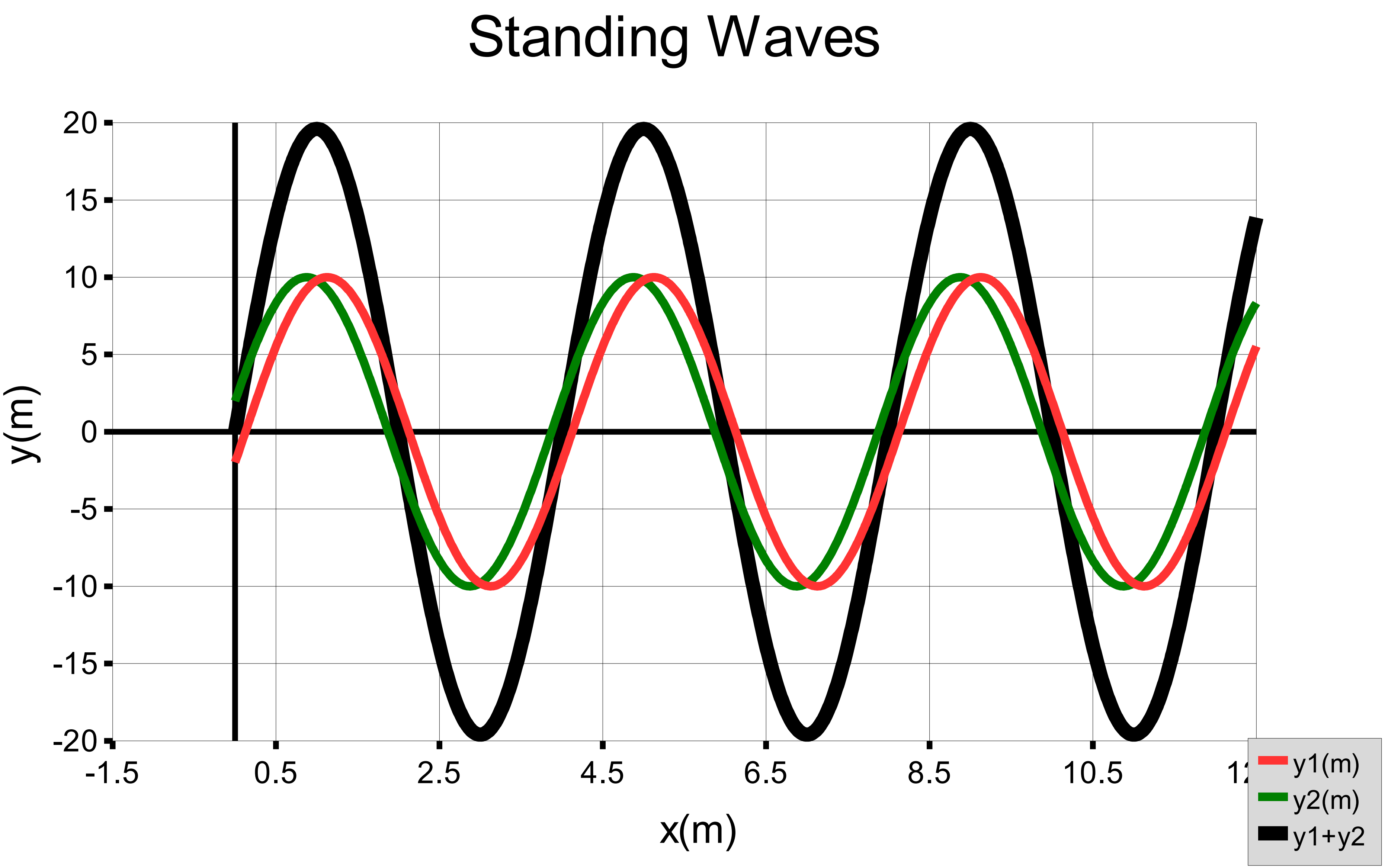

Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.0 s



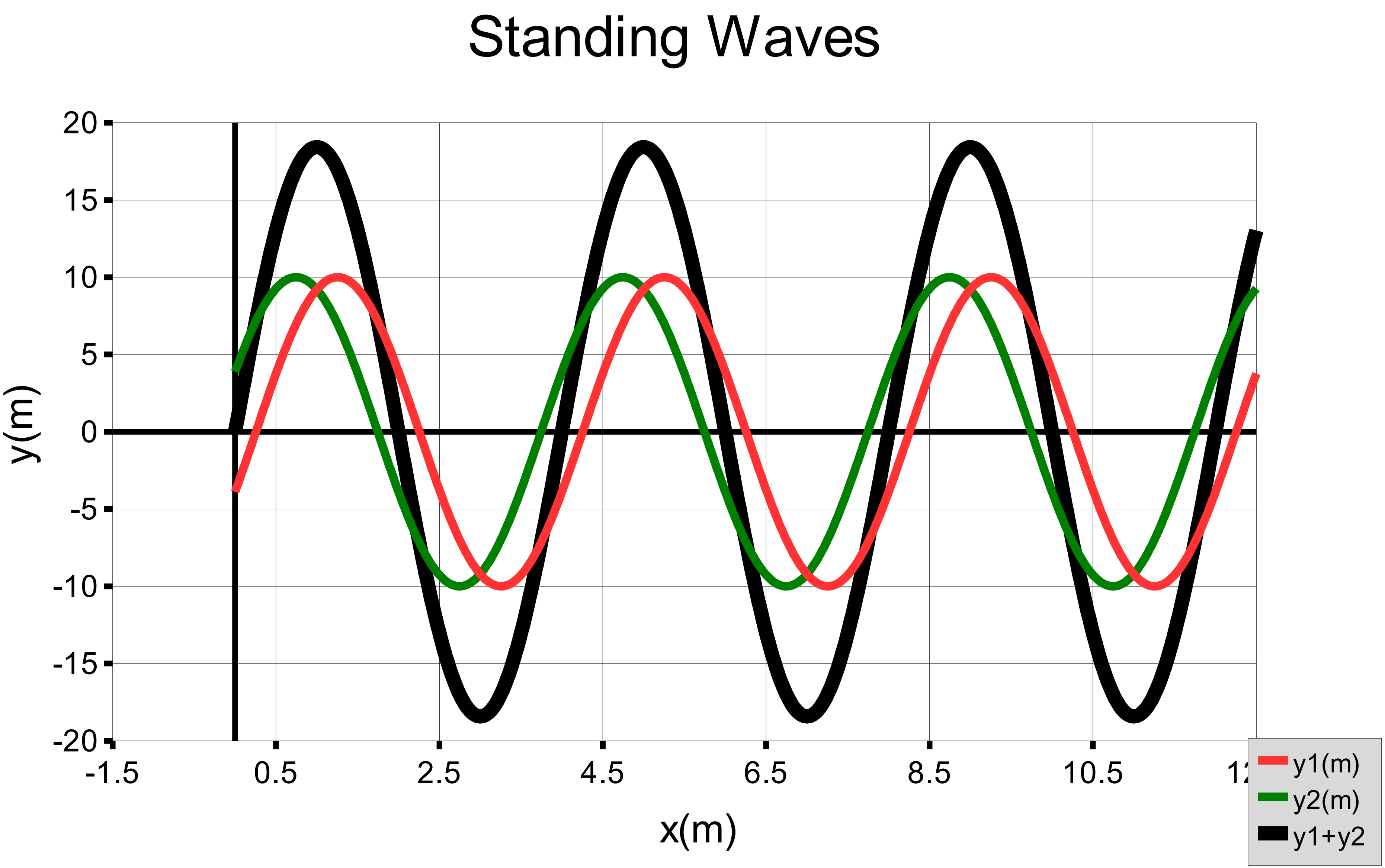
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.1 s



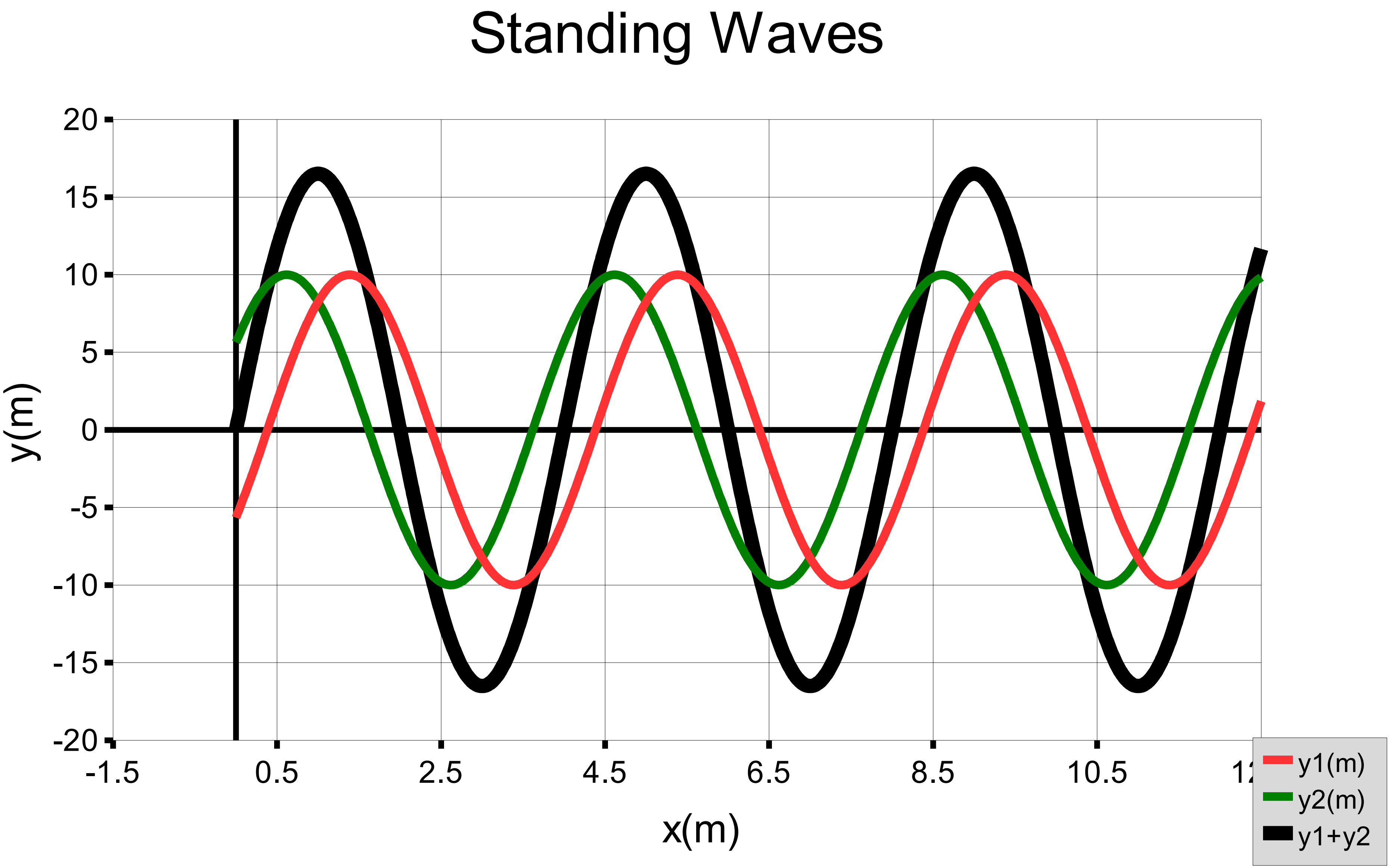
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.2 s



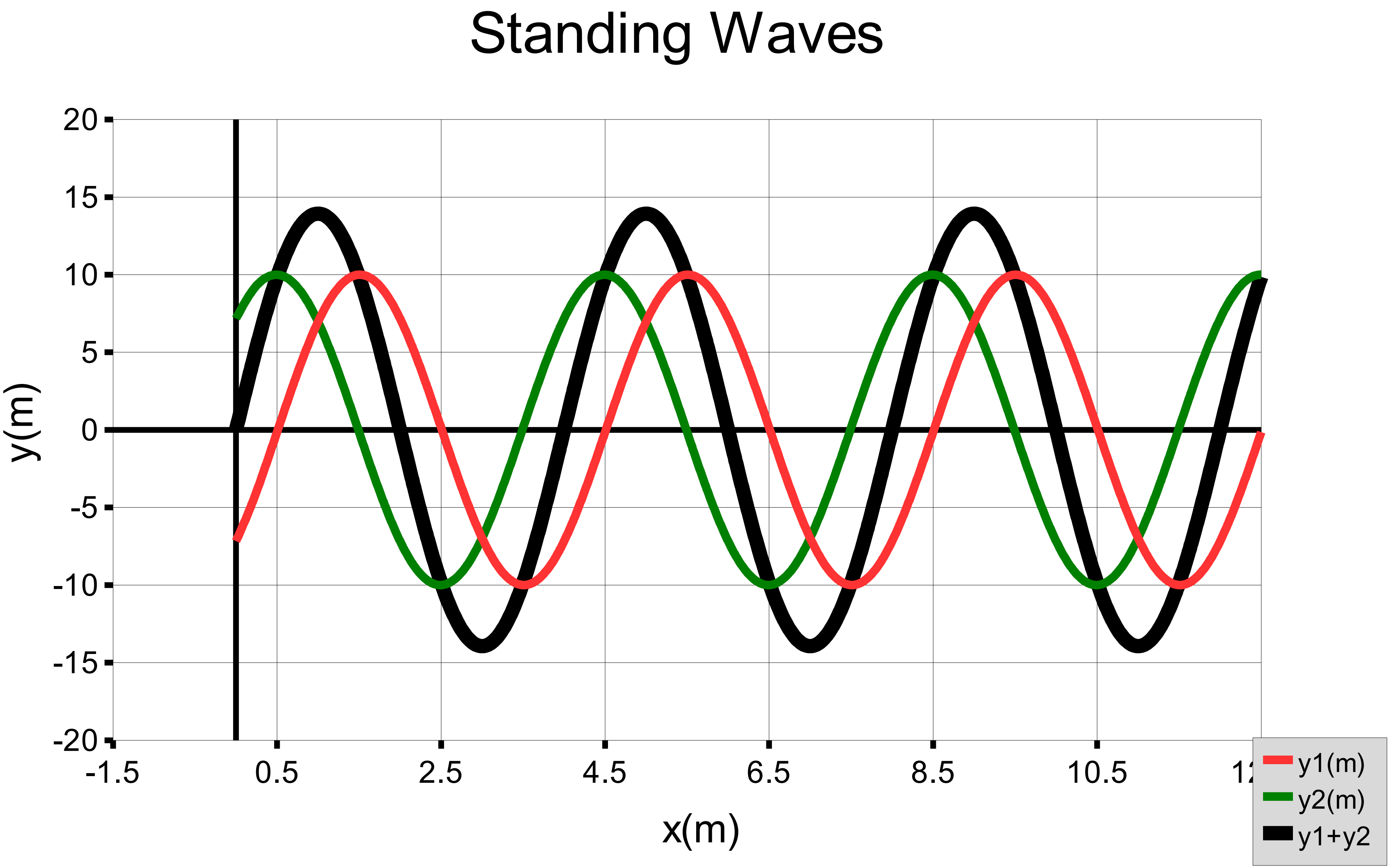
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.3 s



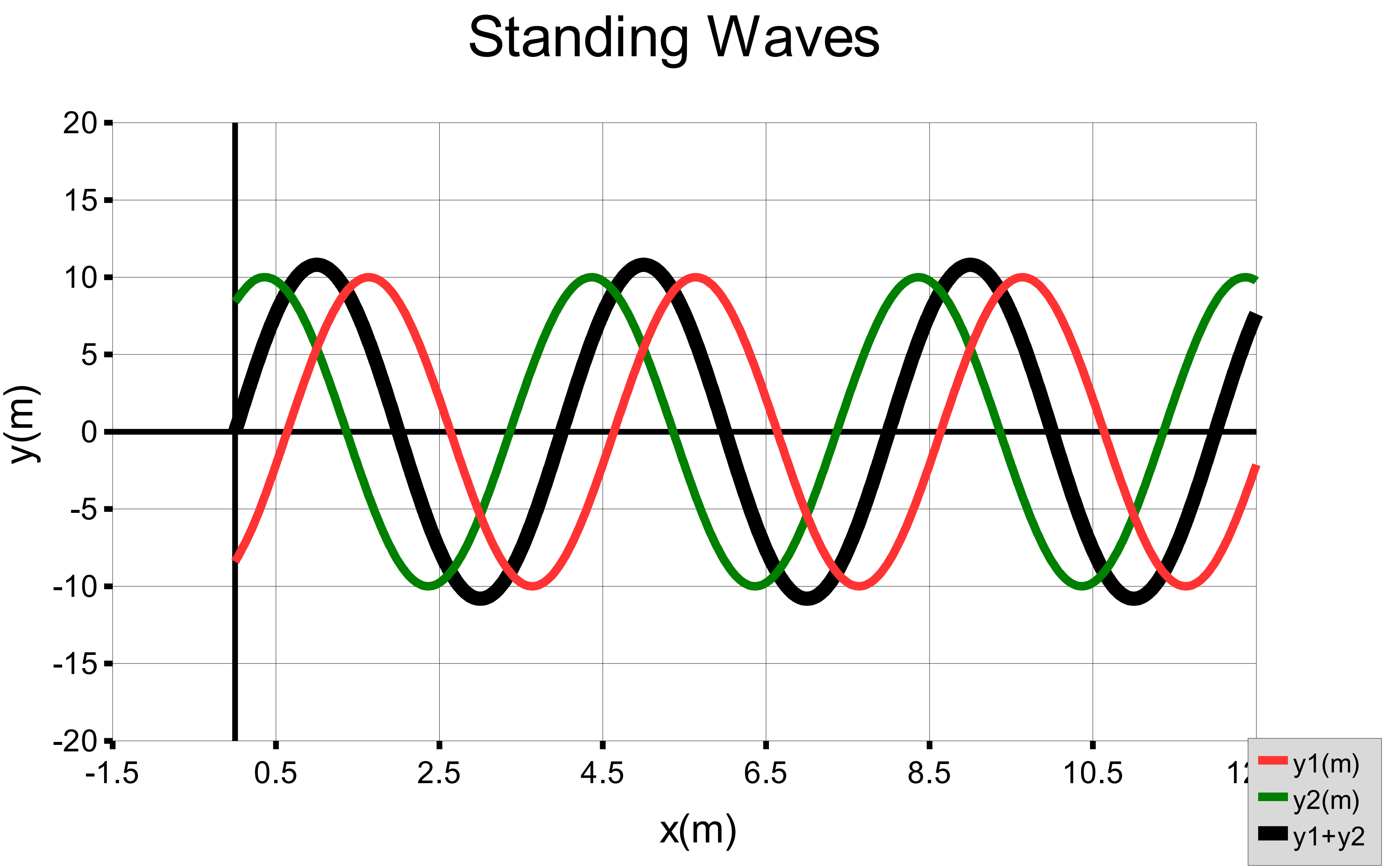
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.4 s

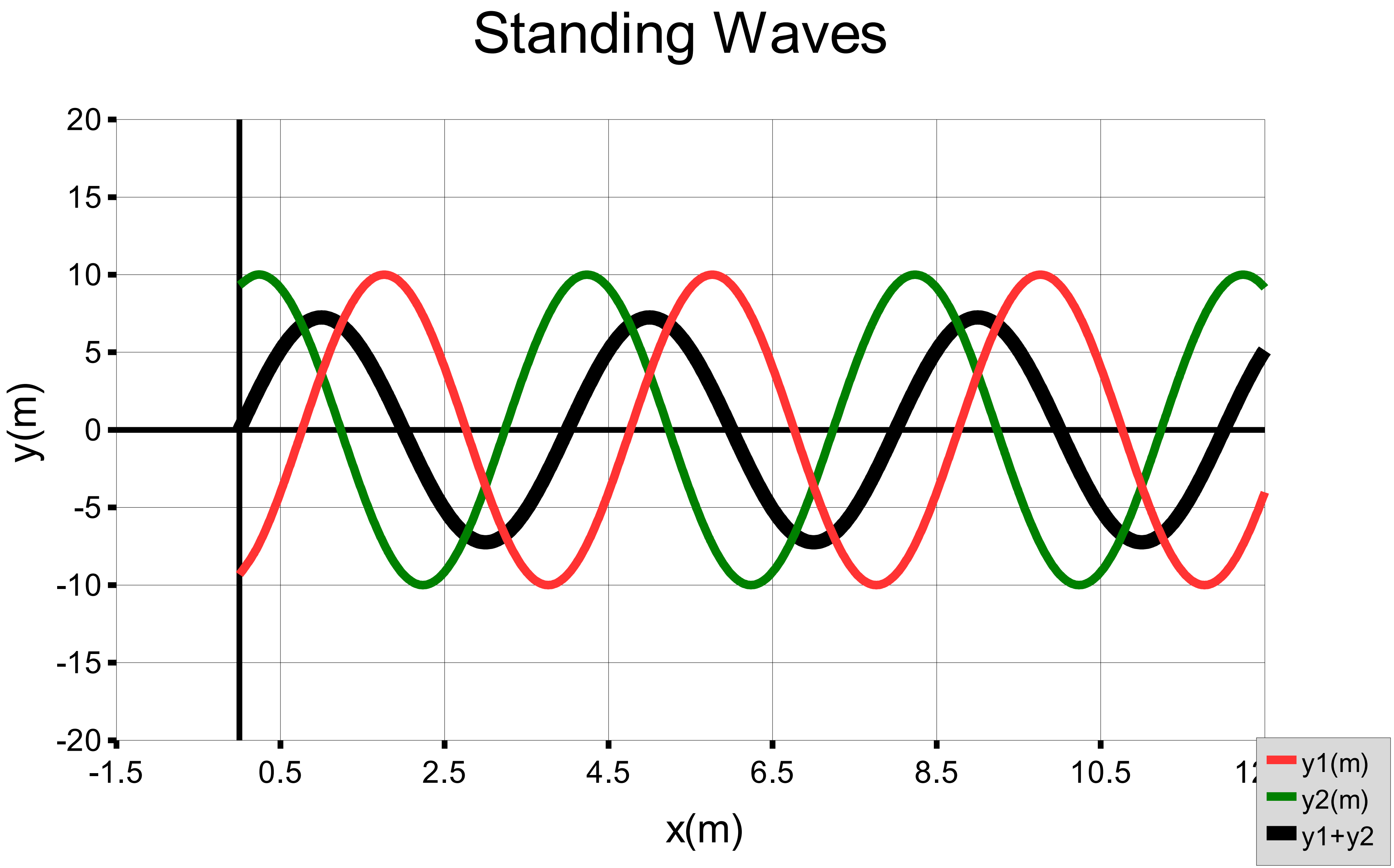


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

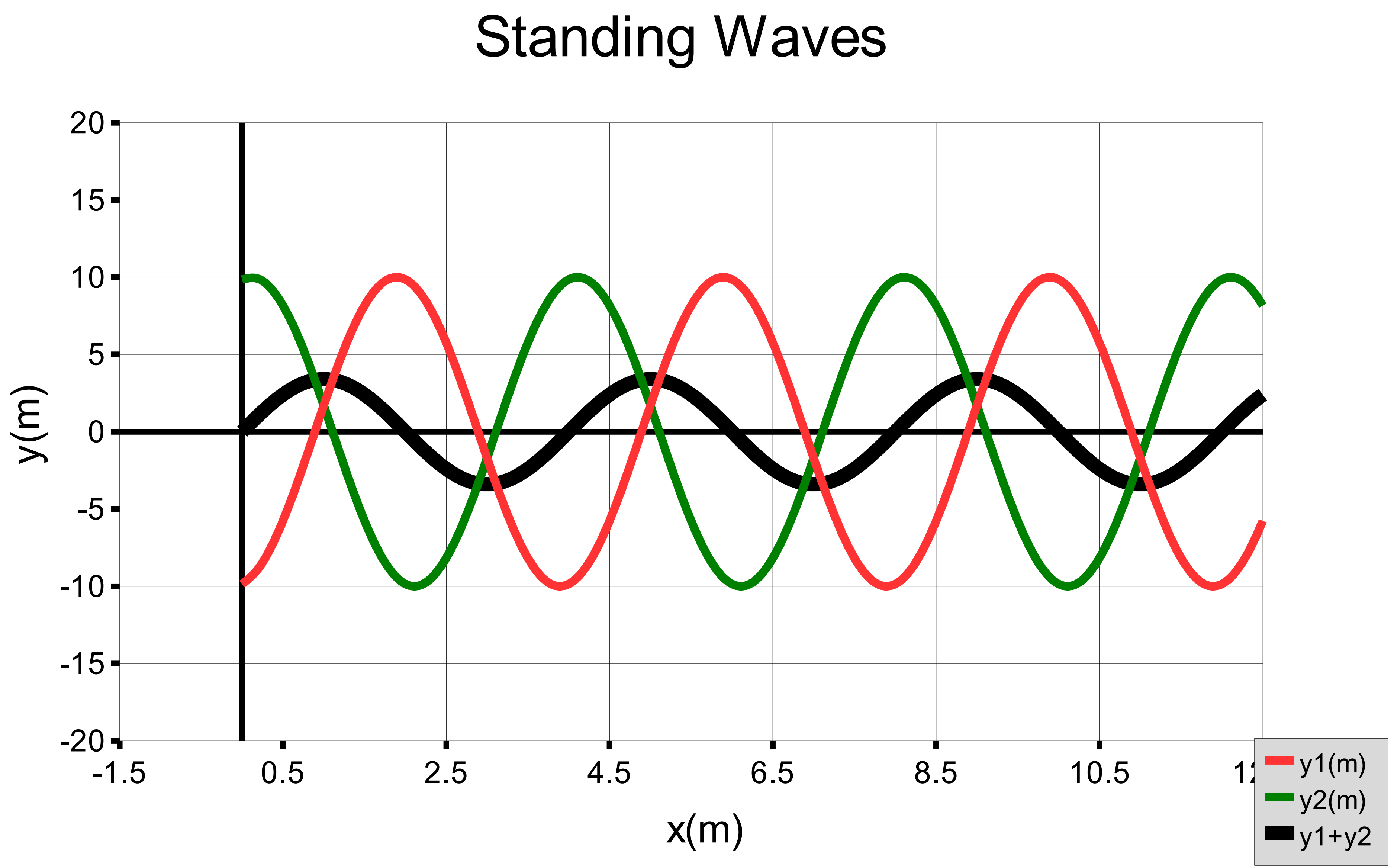
t=0.5 s


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.6 s

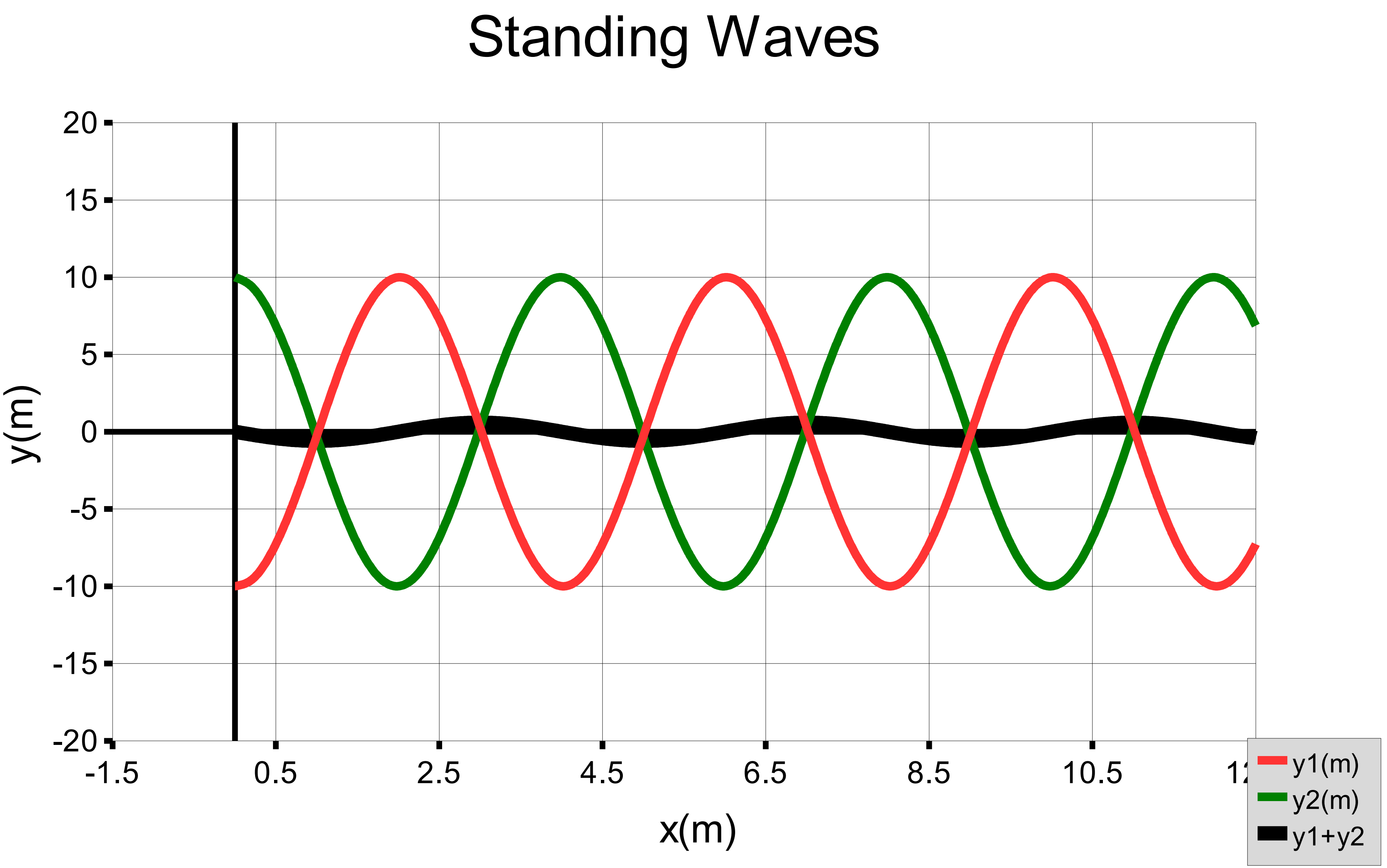

Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.7 s



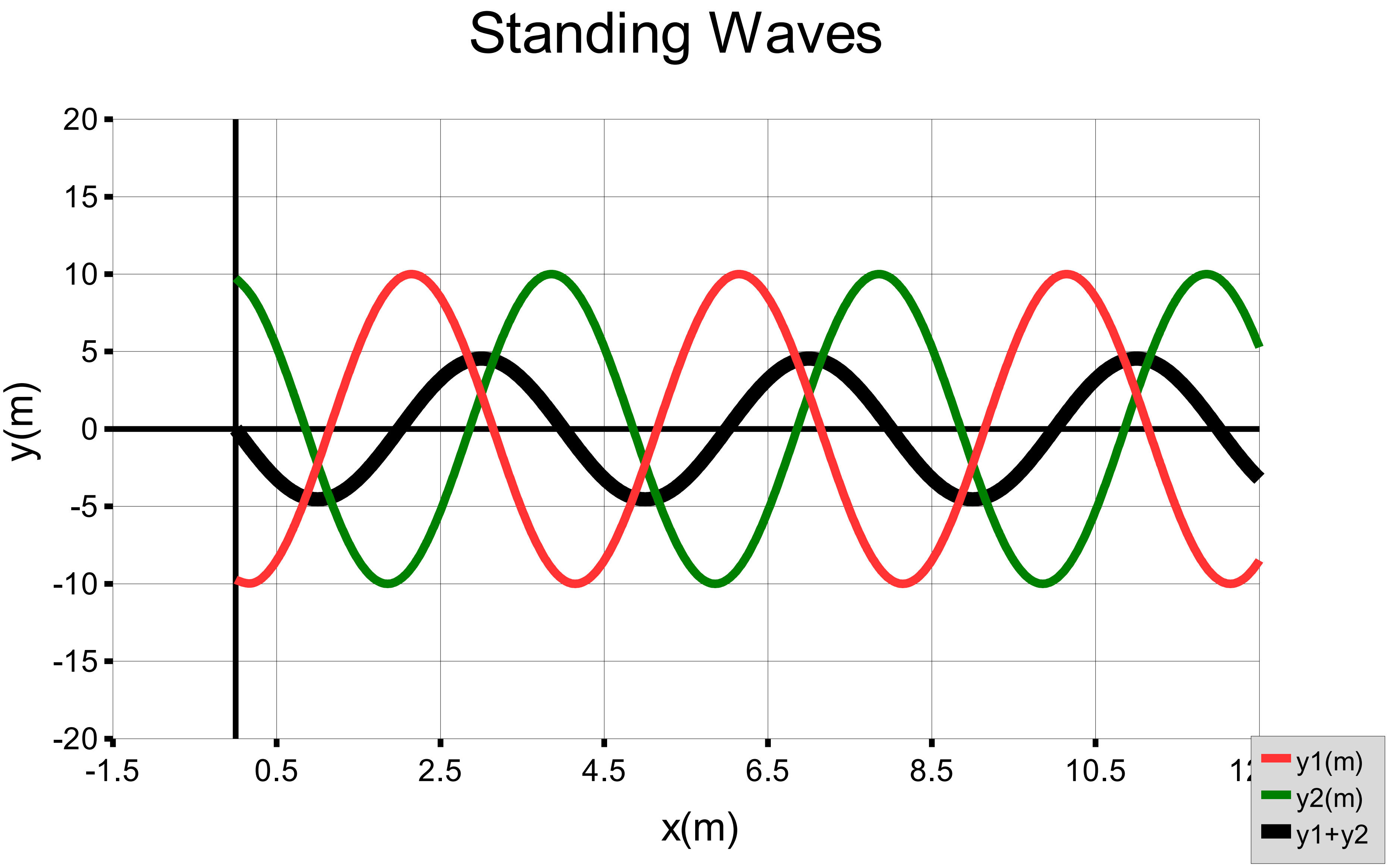
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.8 s

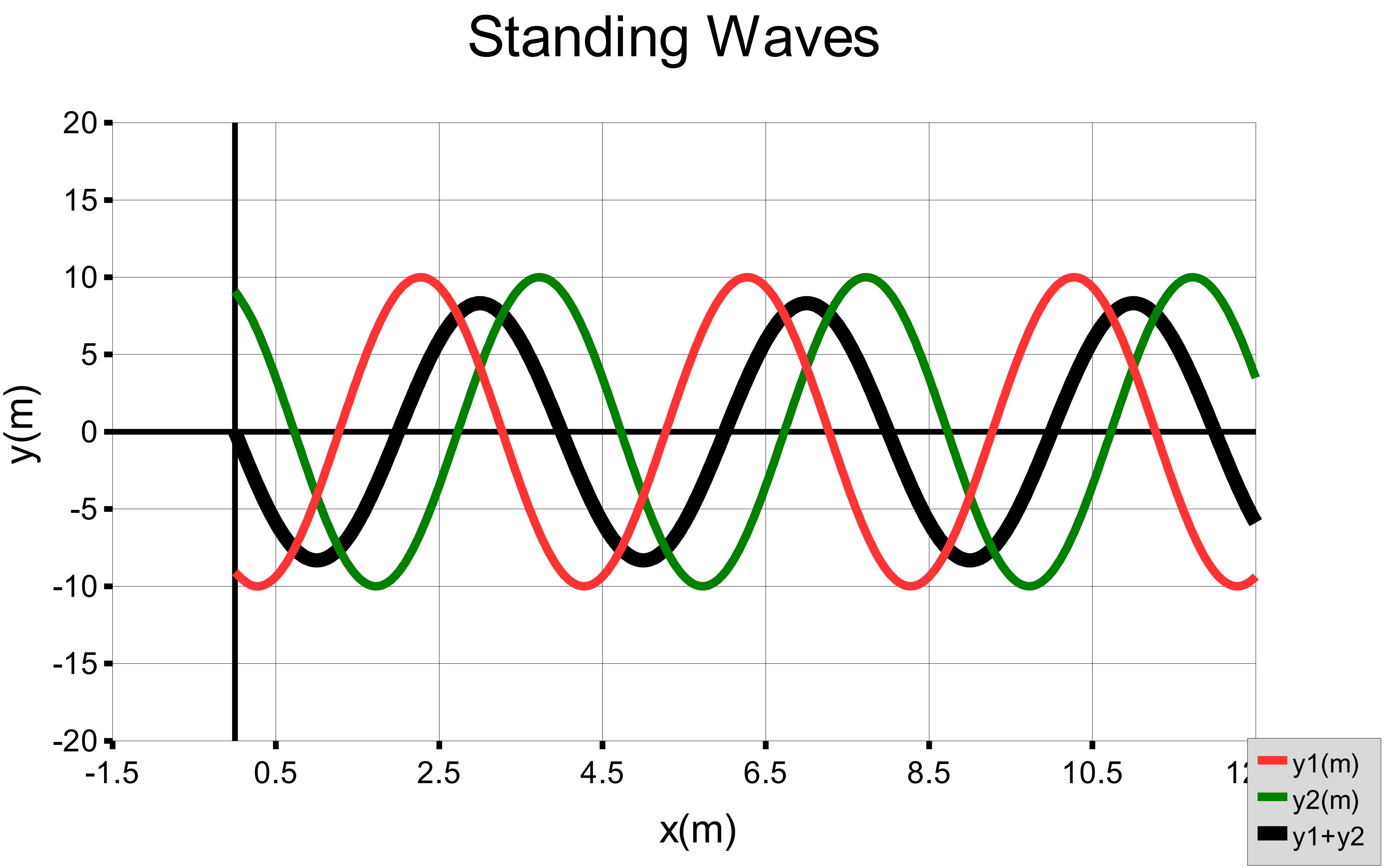


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=0.9 s

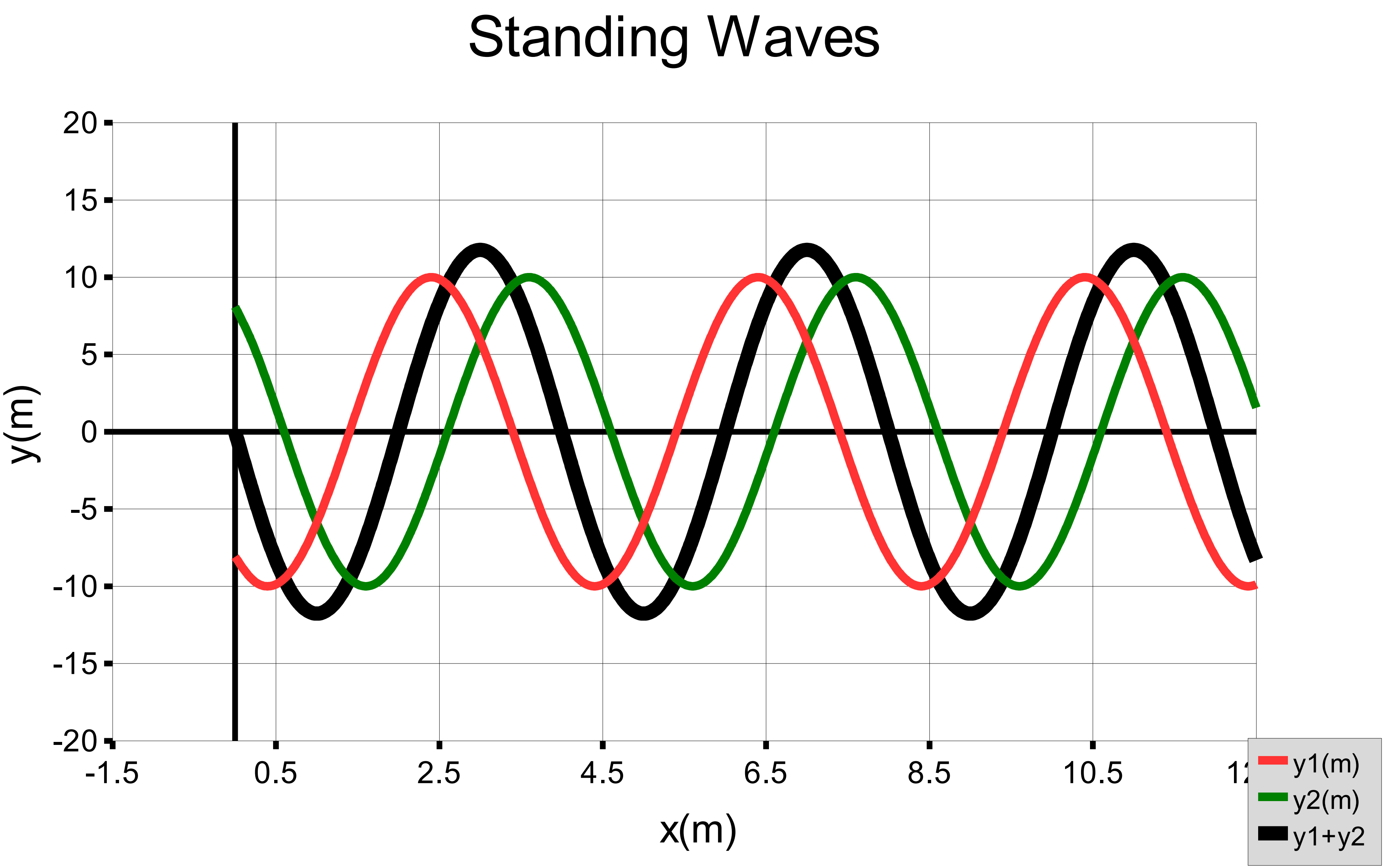

Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.0 s

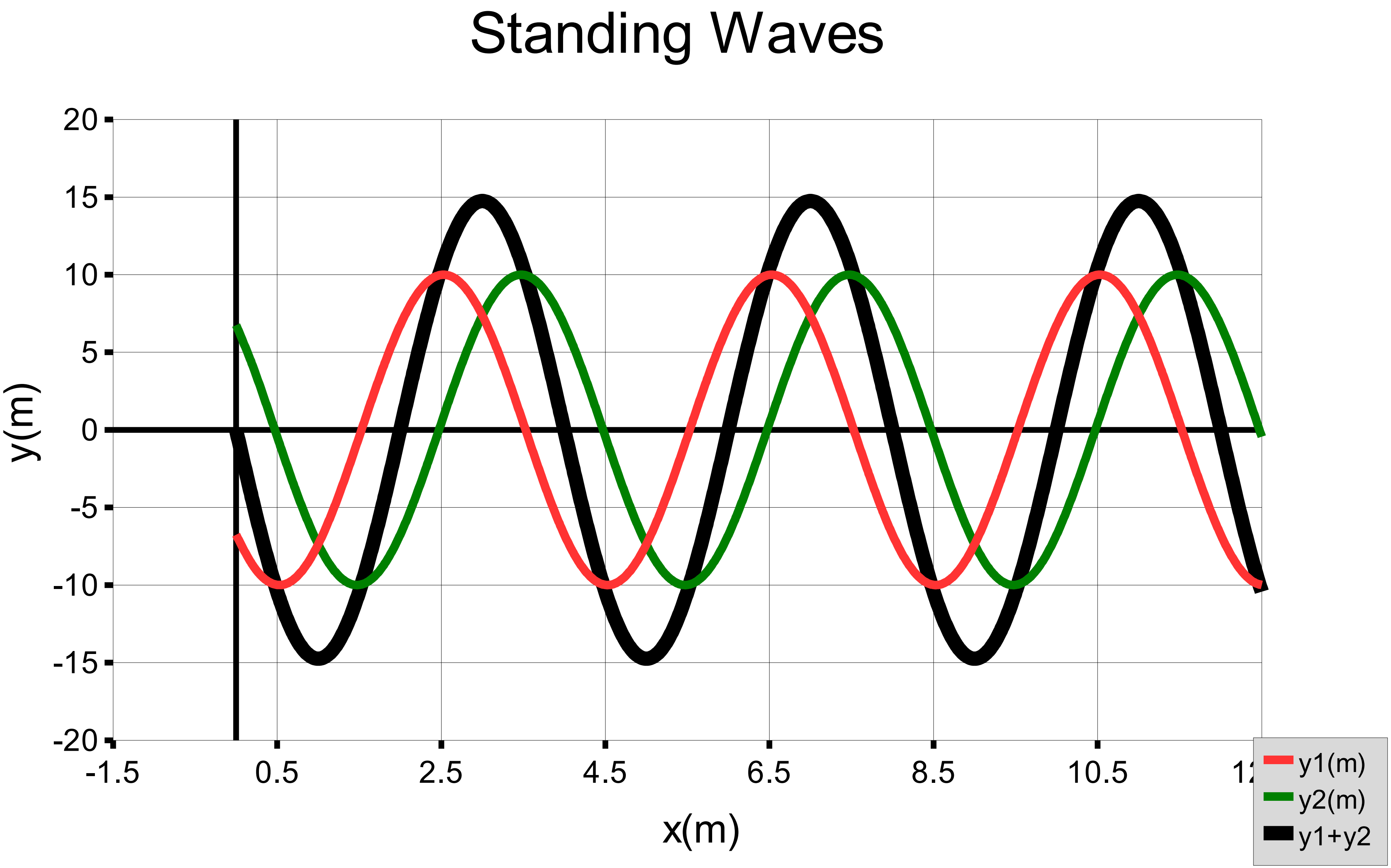


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.1 s

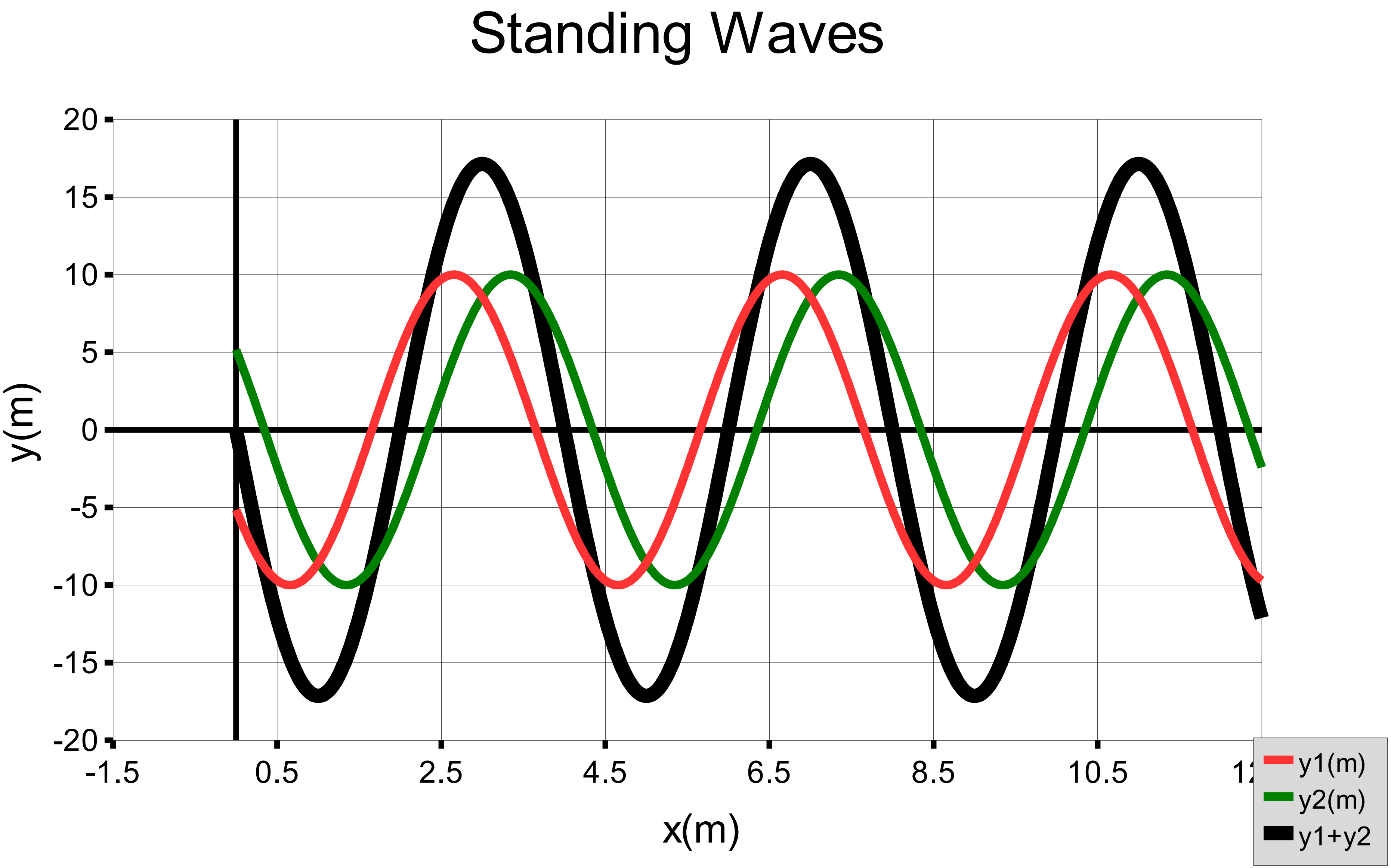

Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.2 s

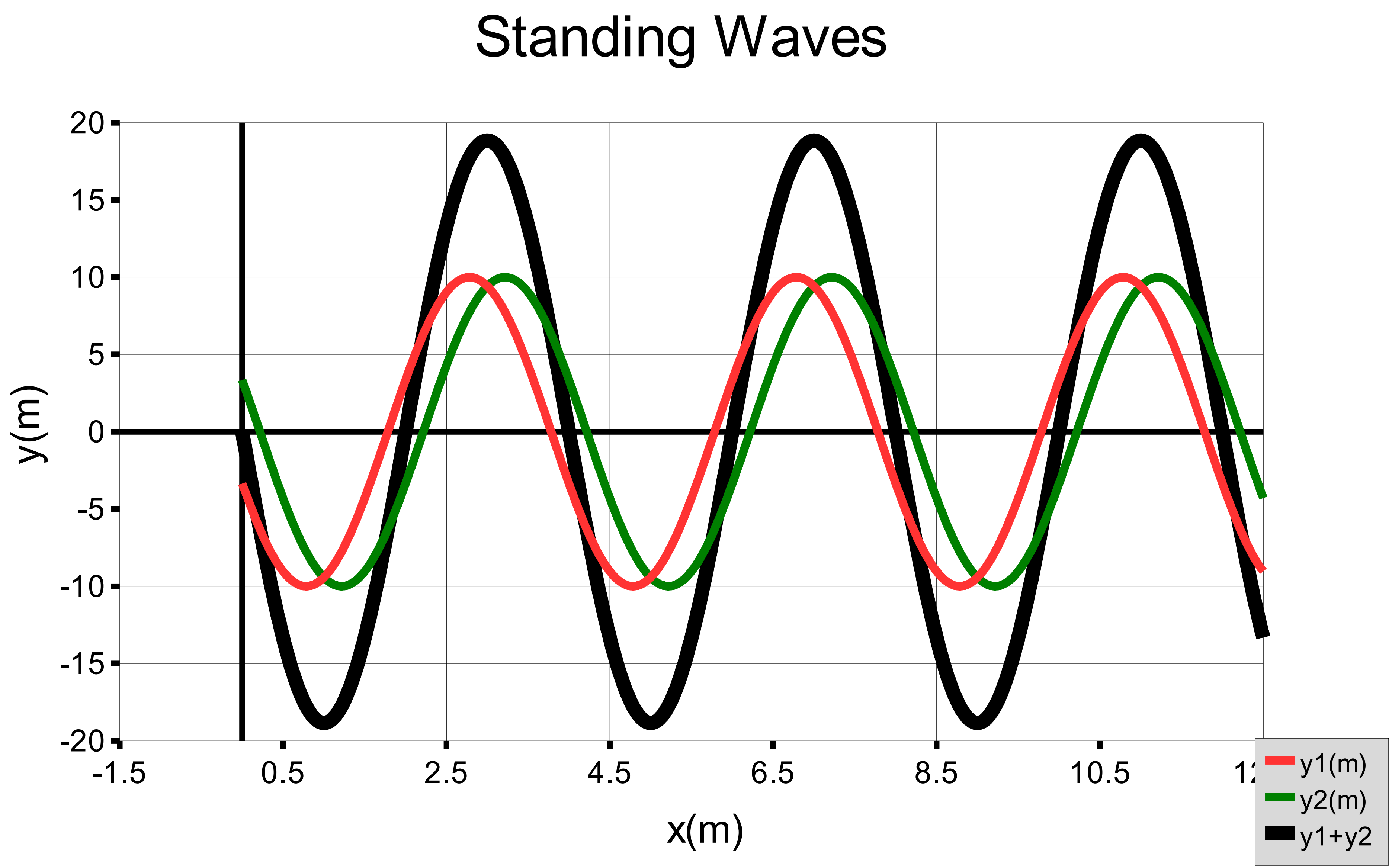


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.3 s


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.4 s


Standing Waves
20
15
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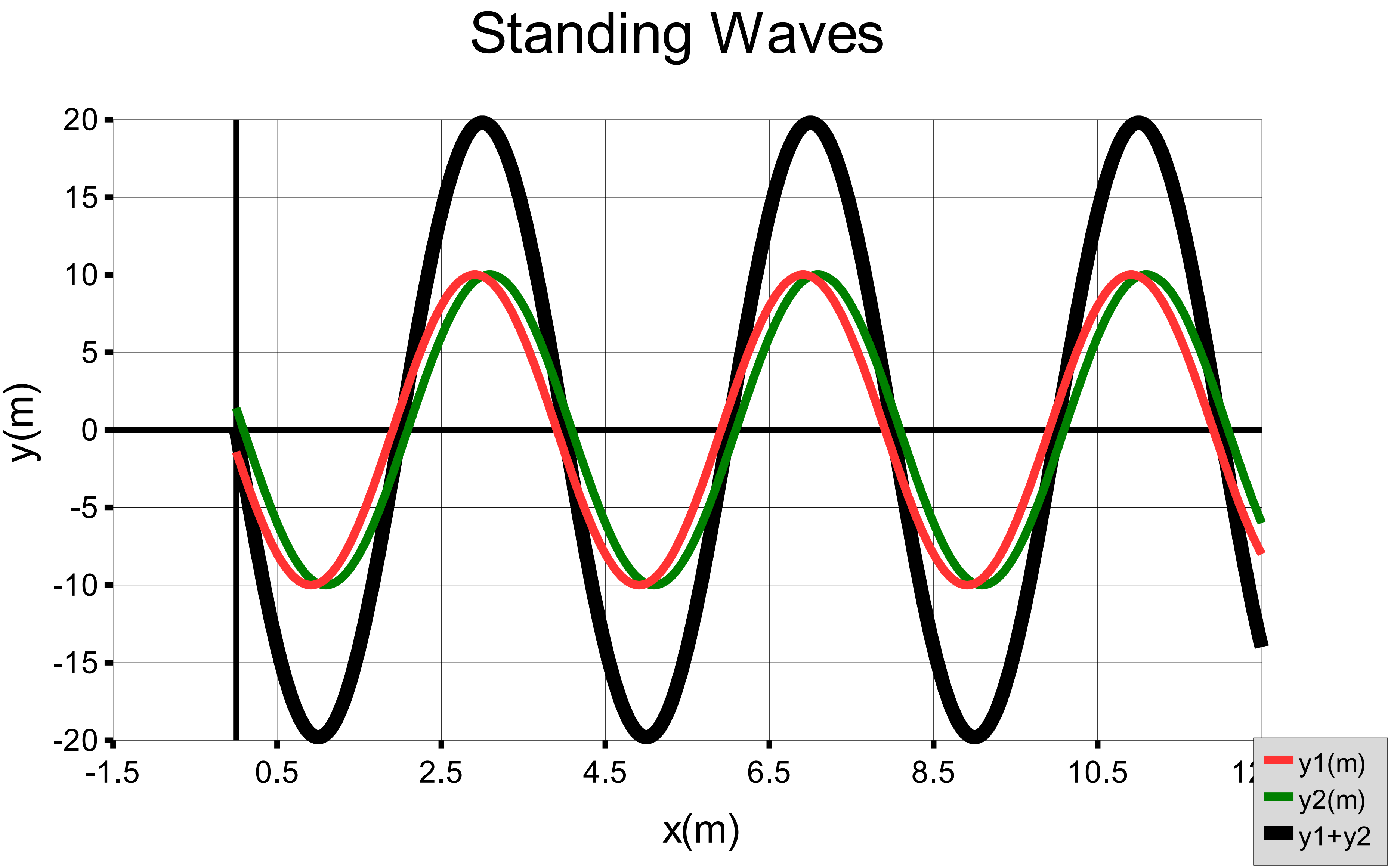
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.5 s



Standing Waves
20
15
10
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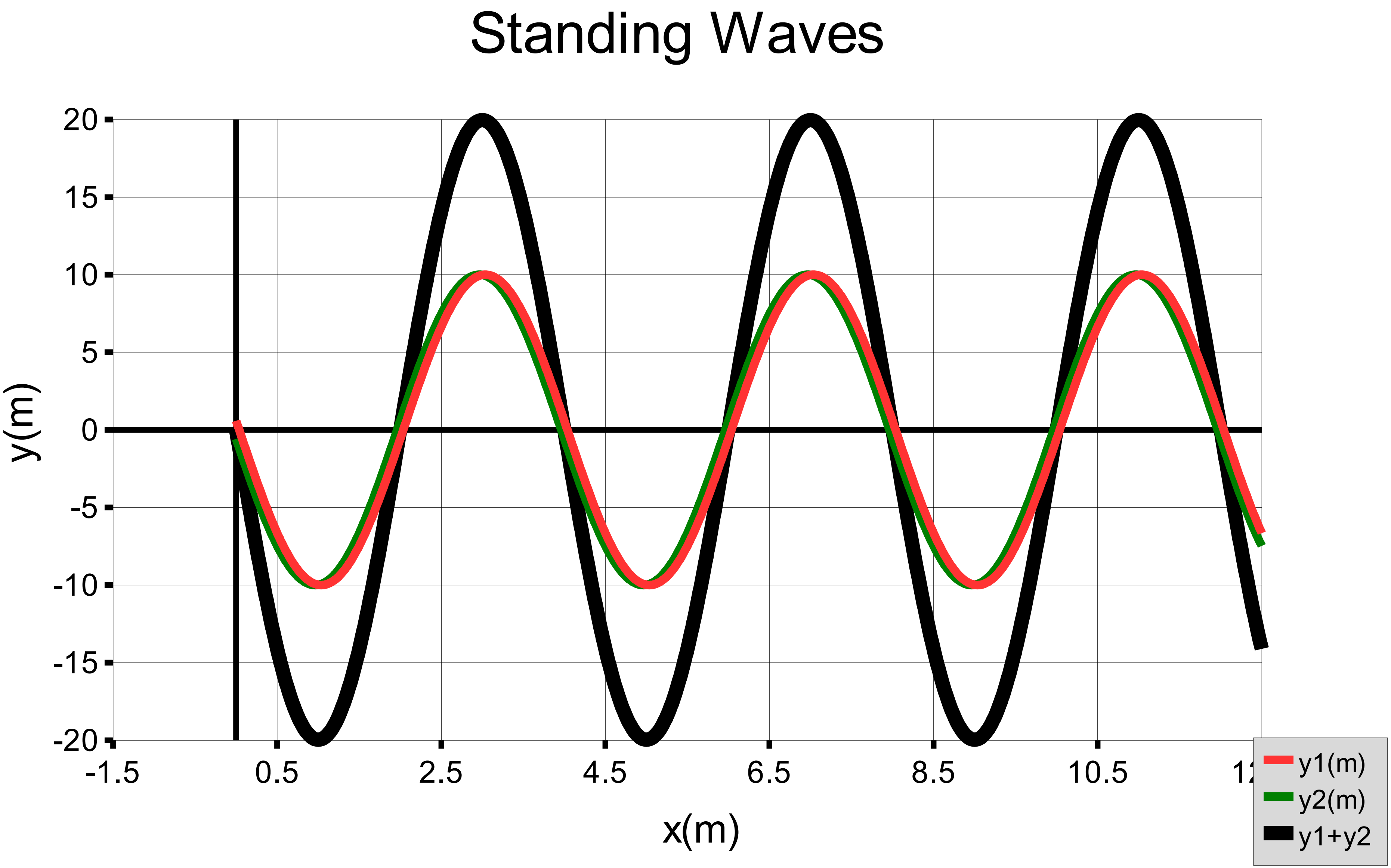
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.6 s



Standing Waves
20
15
10
5
0
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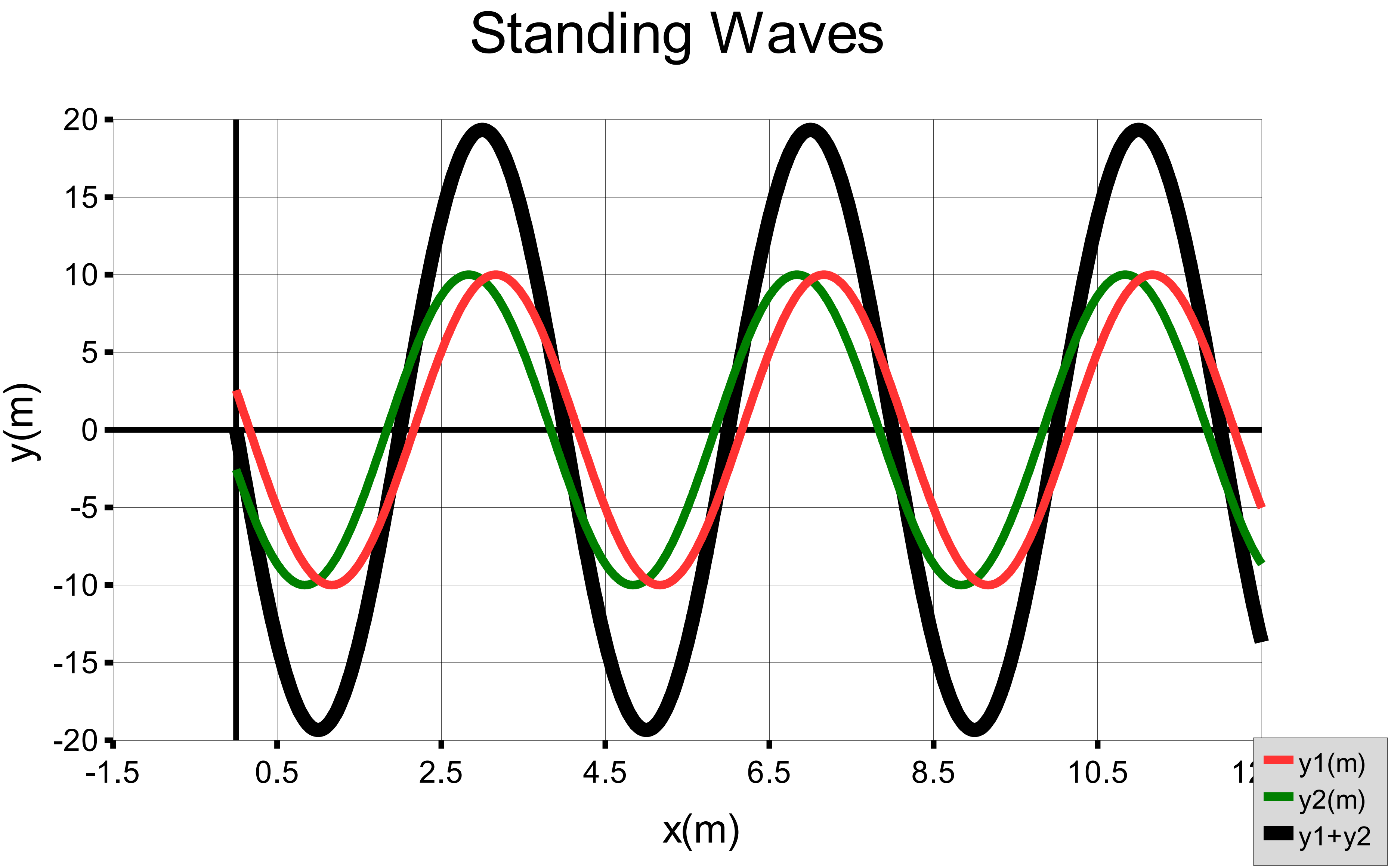
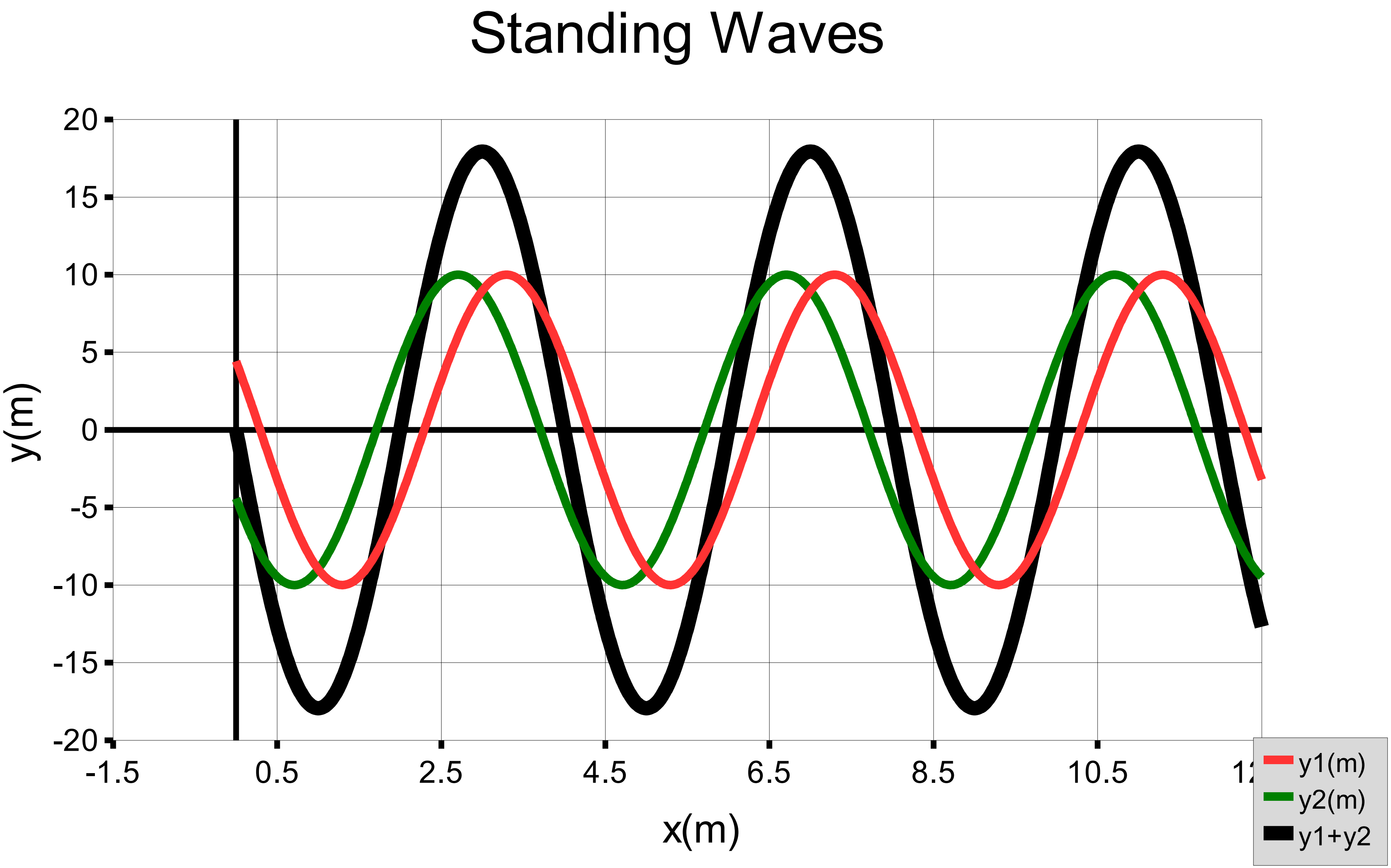
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.7 s



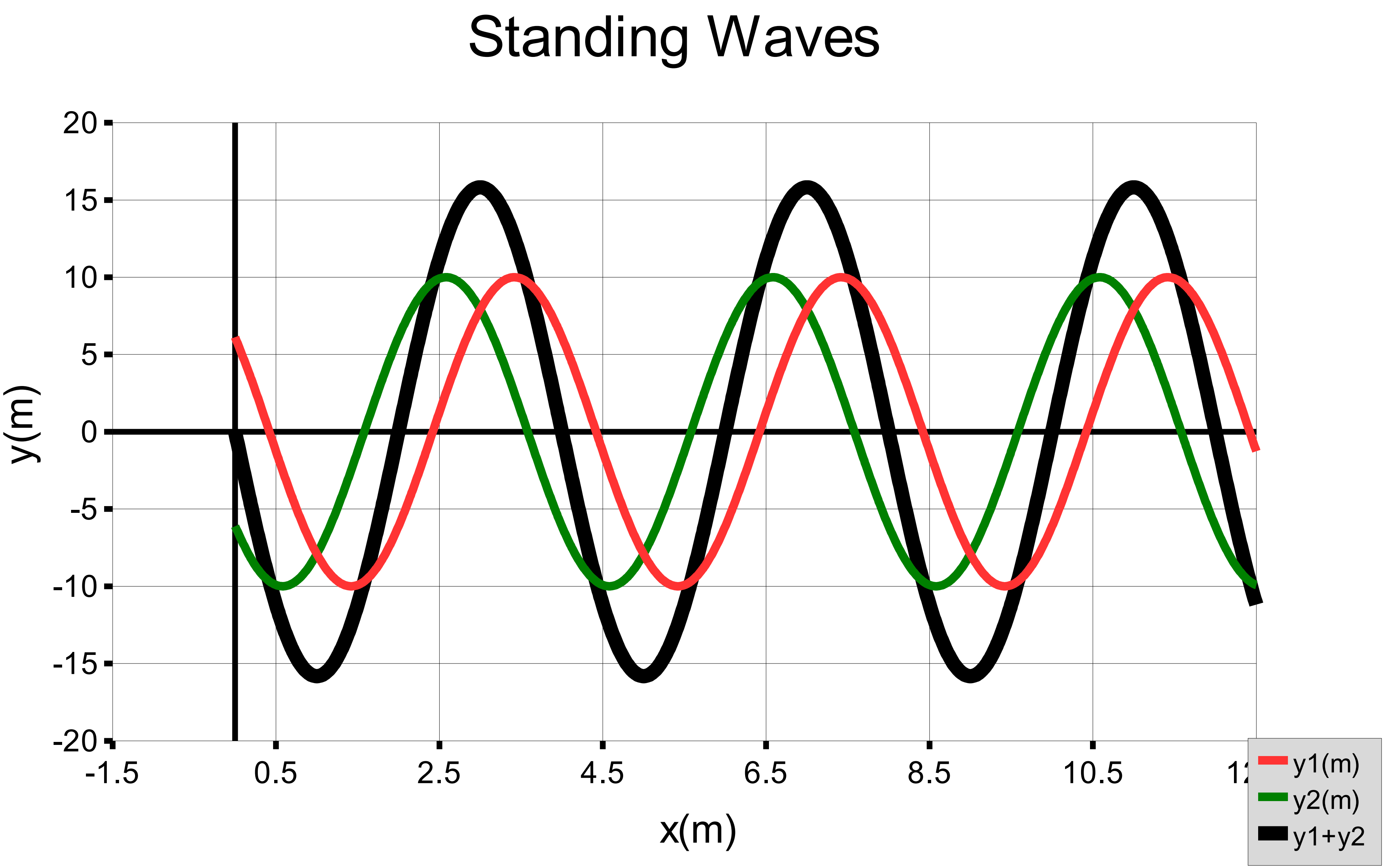
Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.8 s


Standing Waves
20
15
10
5
0
-5
-10
-15
-20
y(m)
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

t=1.9 s

STOP

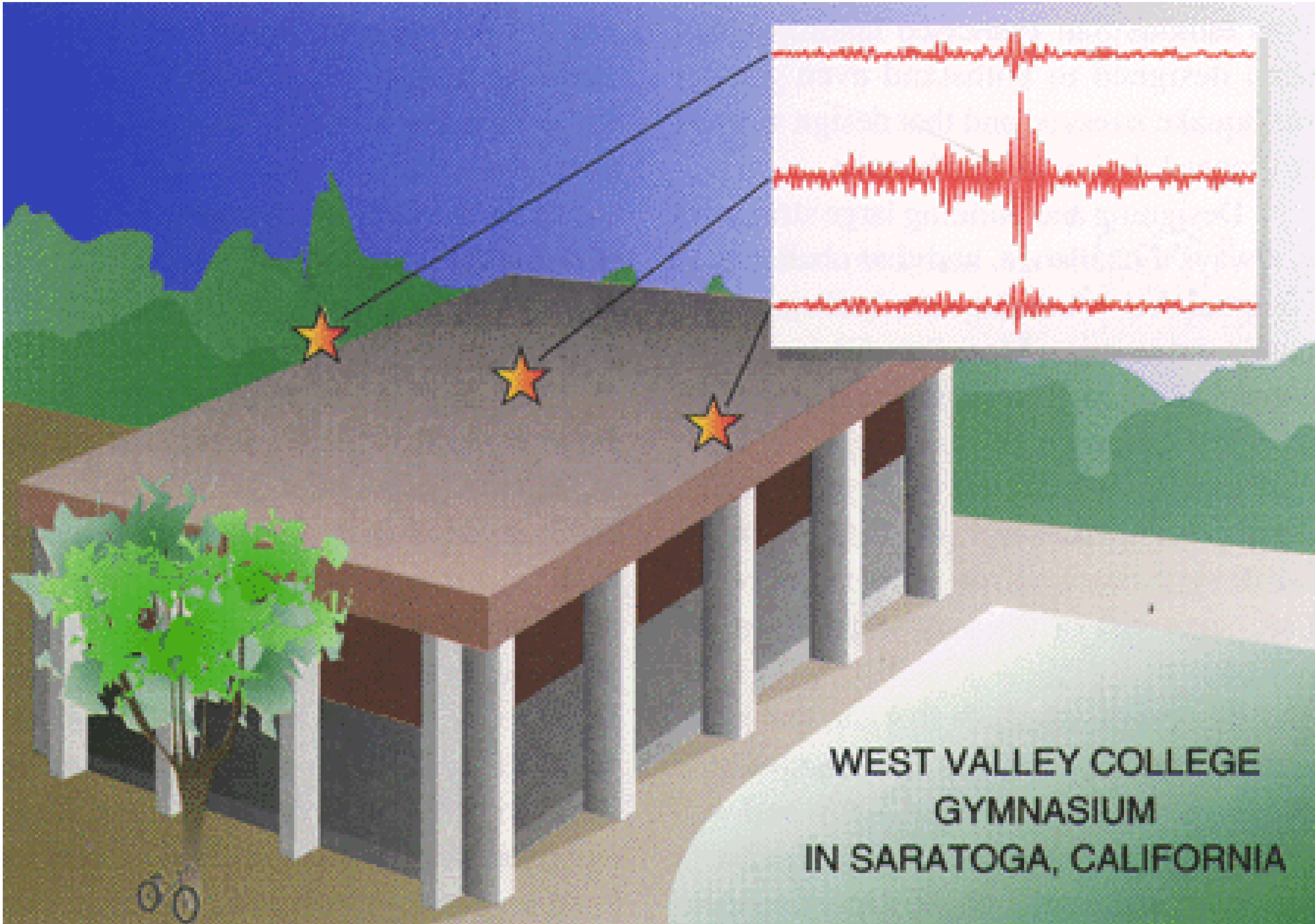


Standing Waves
y(m)
20
15
10
5
0
-5
-10
-15
-20
-1.5
0.5
2.5
4.5
6.5
8.5
10.5
x(m)
y1(m)
y2(m)
y1+y2

Resonance – Small amplitude driving force produces large amplitude waves.

Oscillations in Buildings.

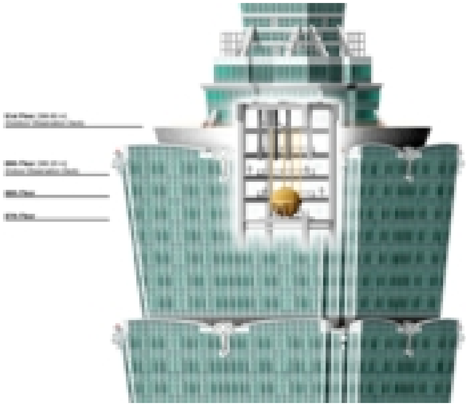
Seismic records from a 1984 earthquake show how much stress different parts of a roof may endure.



Collapse of Masonry Church

On December 7, 1988, a magnitude 6.9 earthquake shook northwestern Armenia, and was followed four minutes later by a magnitude 5.8 aftershock. The earthquakes affected an area 80 km in diameter. This earthquake devastated the cities of Spitak and Leninakan, where 25,000 deaths occurred. This photo illustrates the collapse of an old stone masonry Armenian church in Leninakan. Churches are vulnerable to earthquake damage because of their high, unsupported roofs. Many such historical buildings either collapsed totally or sustained severe damage.

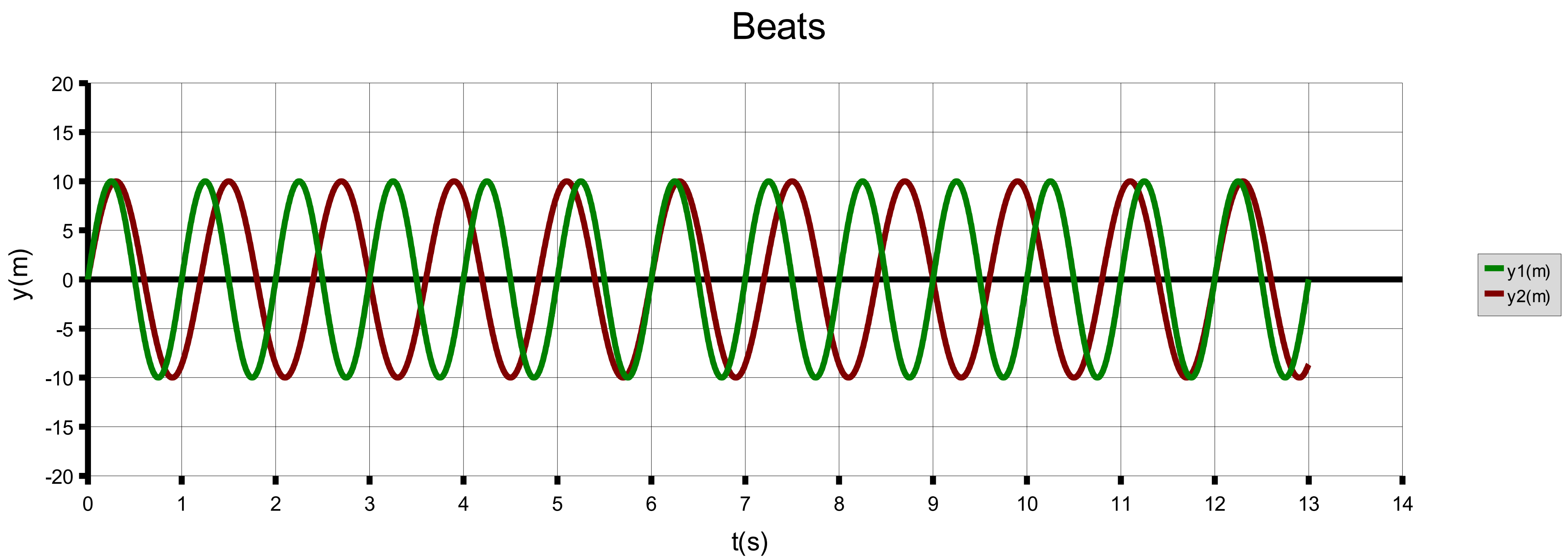
Taipei, Taiwan



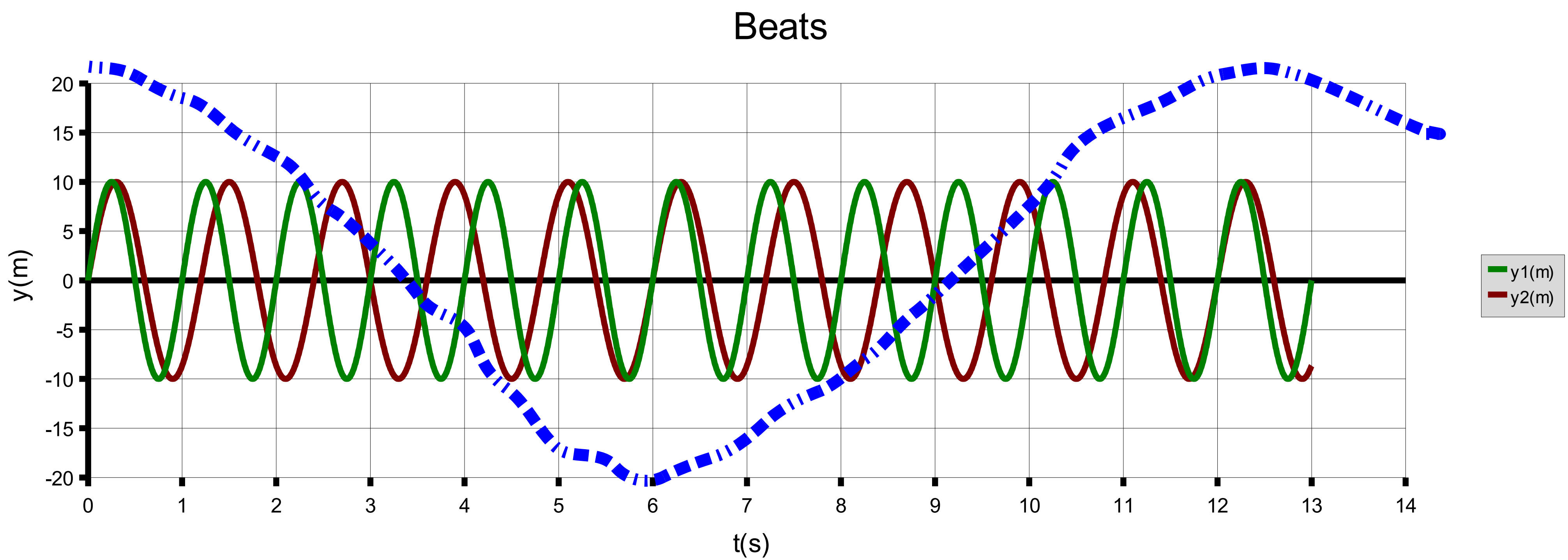
Beats:

Two Waves with different frequencies.

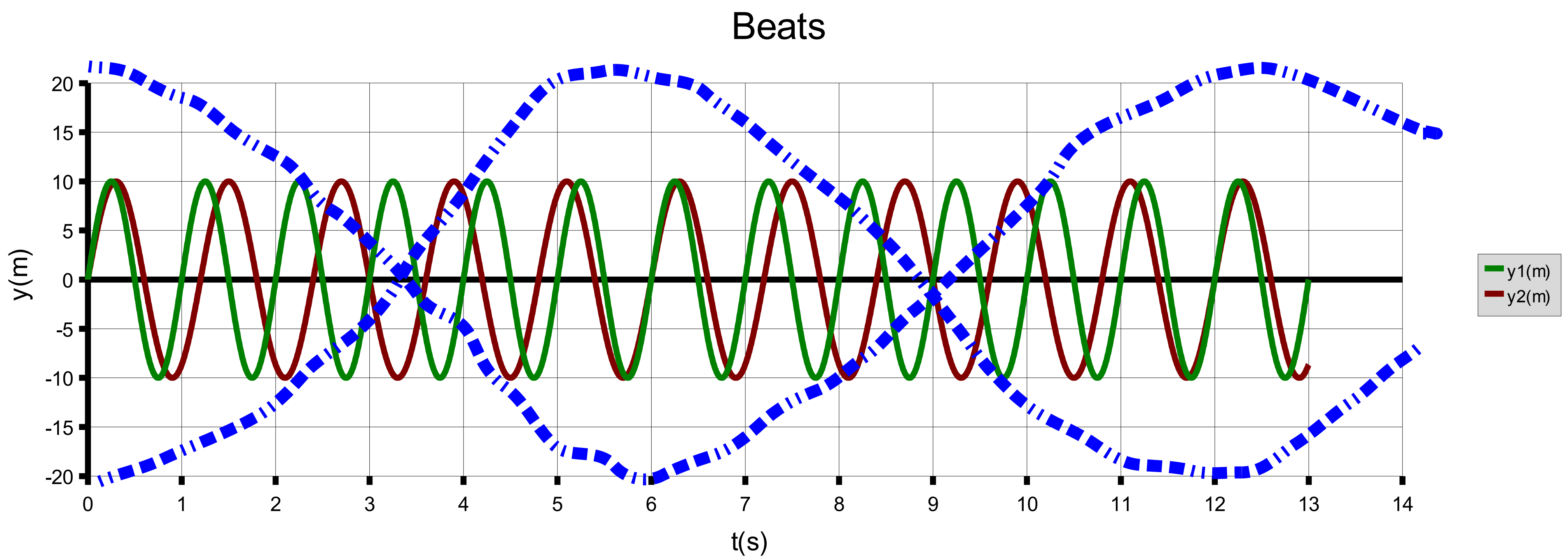
Two Waves with different frequencies.



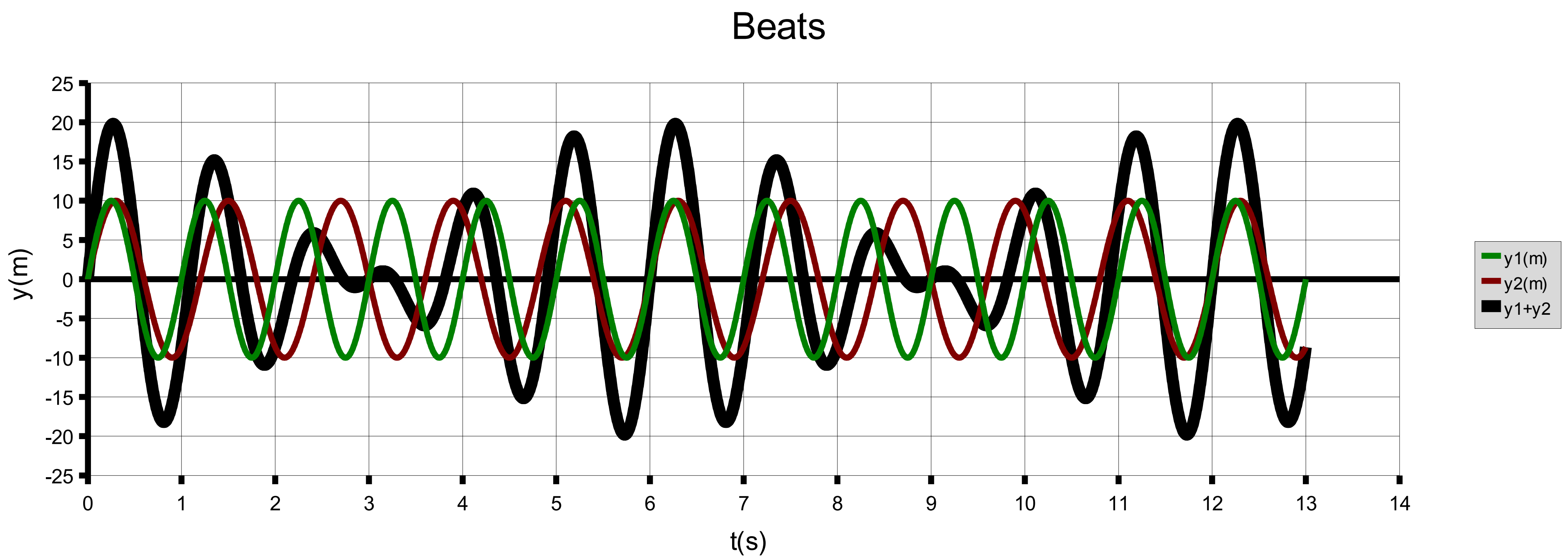
Two Waves with different frequencies.



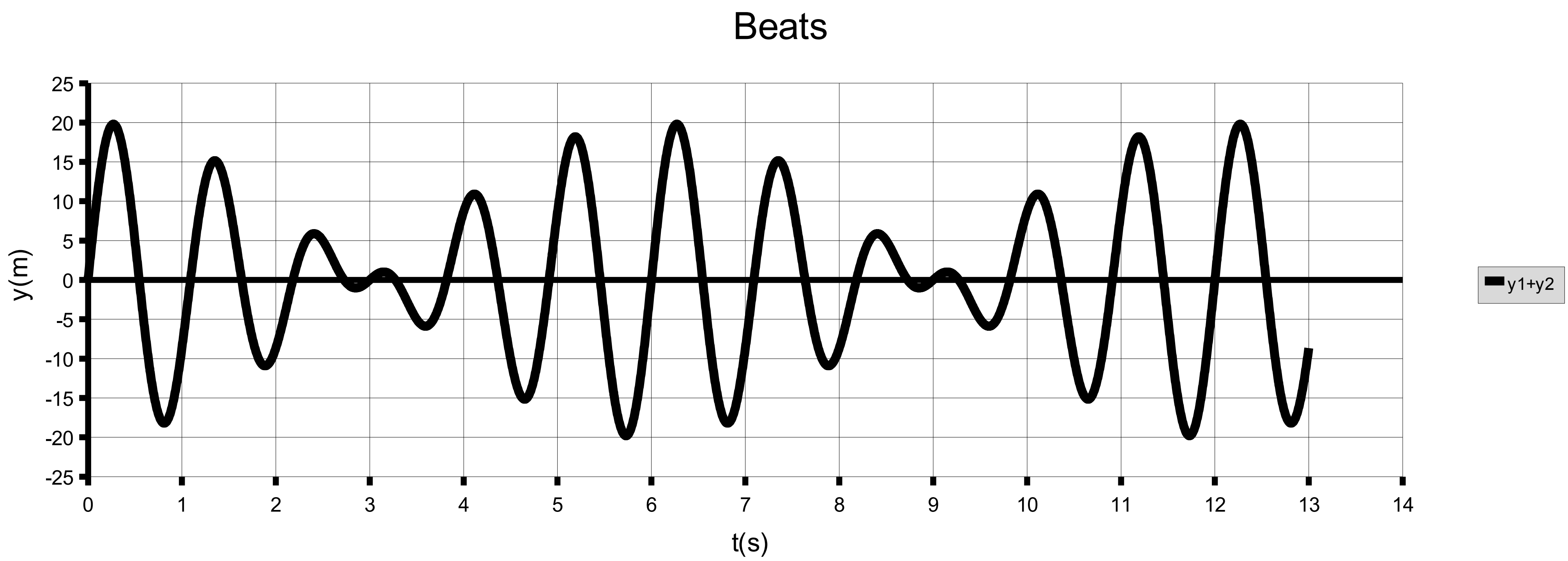
Two Waves with different frequencies.



Two Waves with different frequencies.



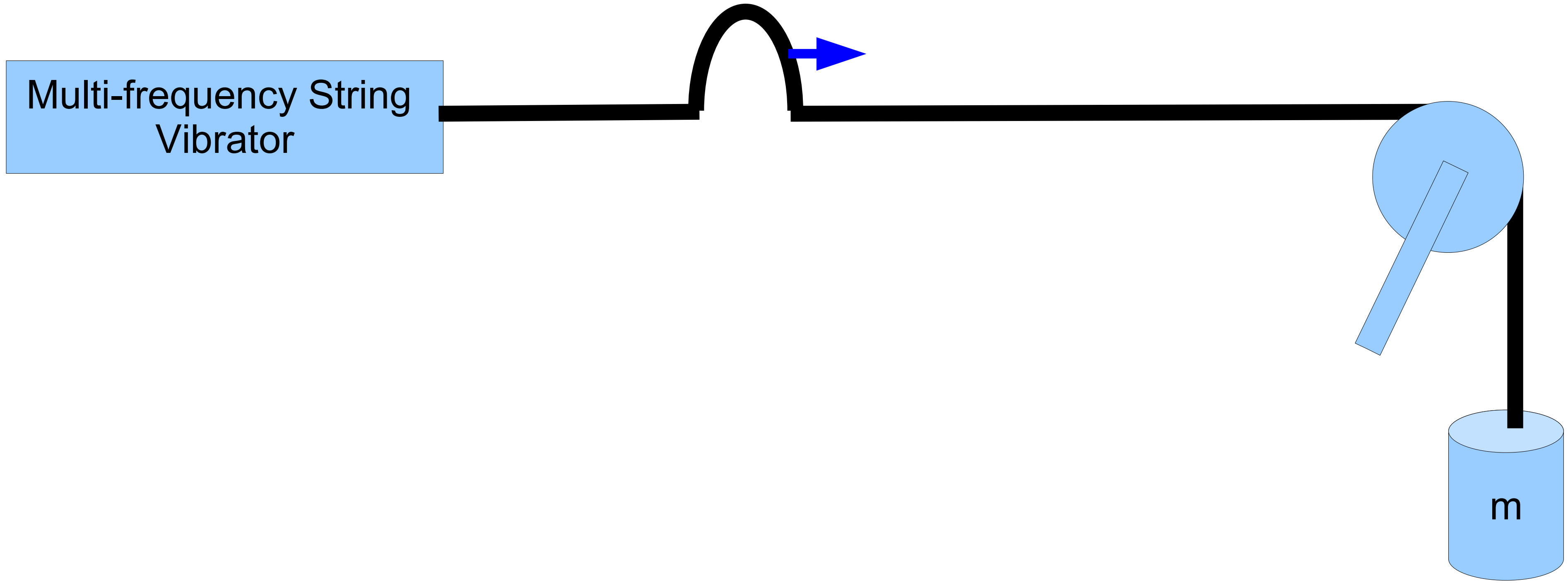
Two Waves with different frequencies.



More on Standing Waves:

Two Identical Waves moving in two different directions.

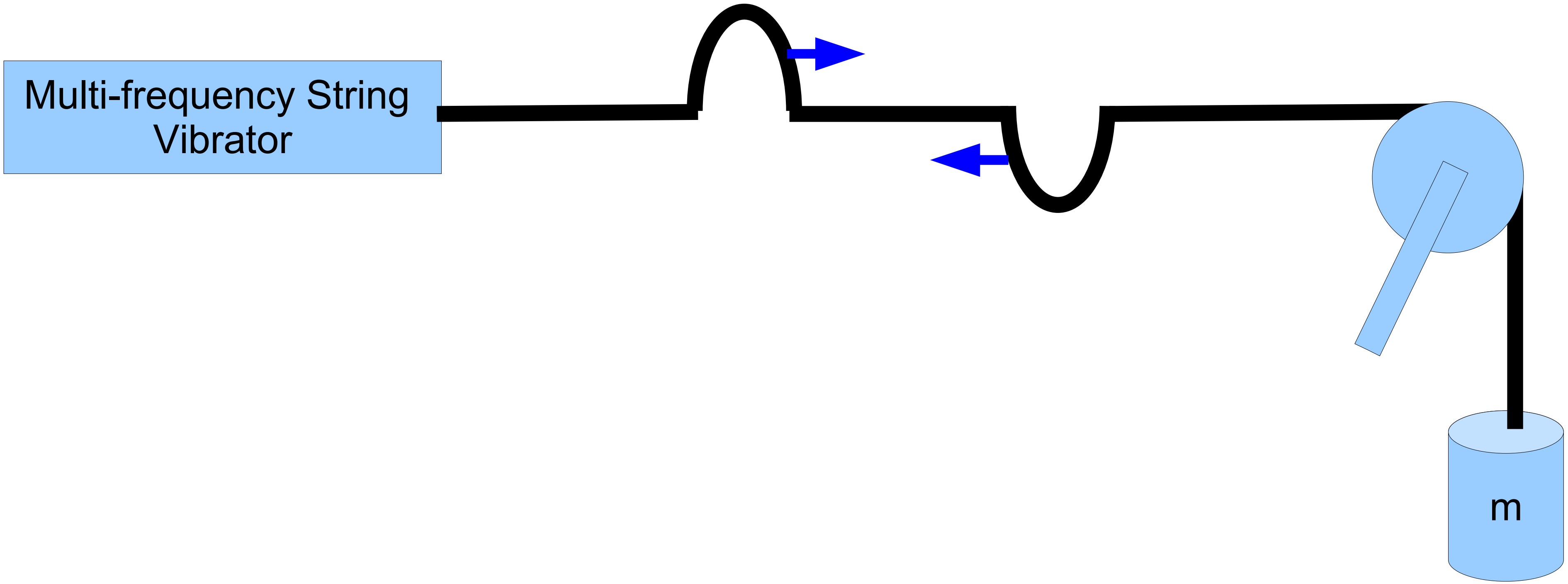
What is the speed of a pulse or wave on a spring?



Linear Density

$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

What is the speed of a pulse or wave on a spring?



Linear Density

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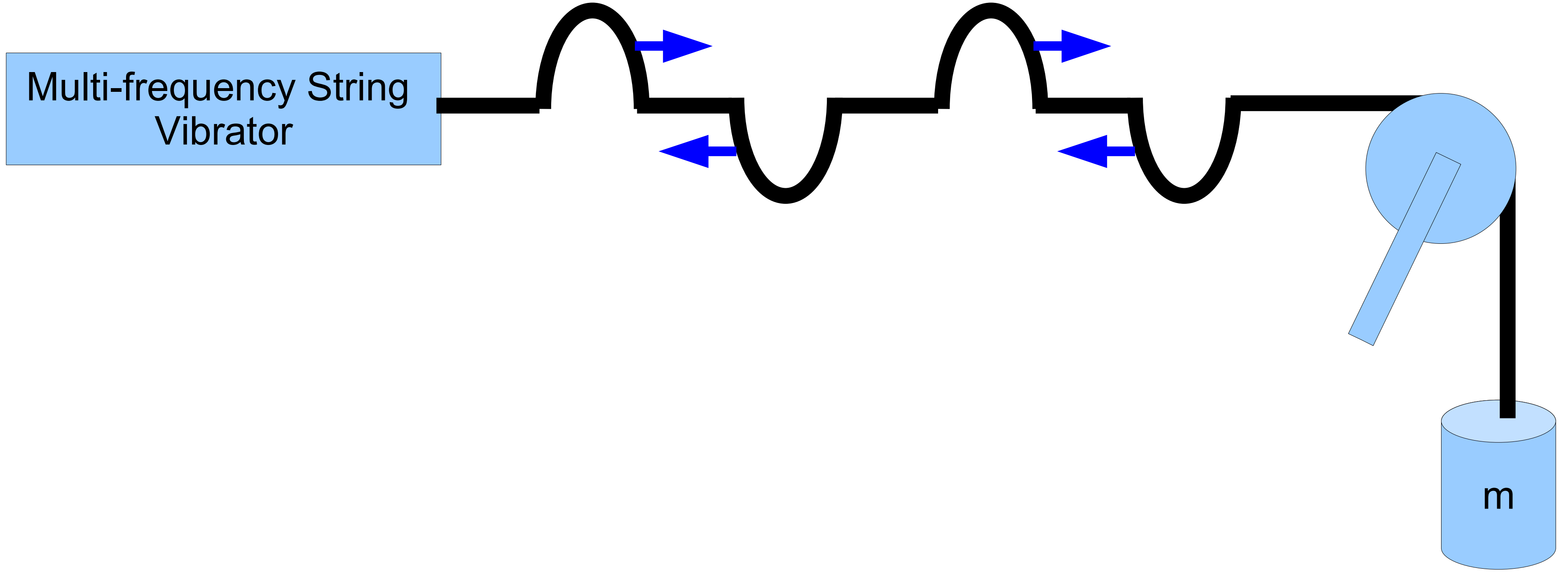
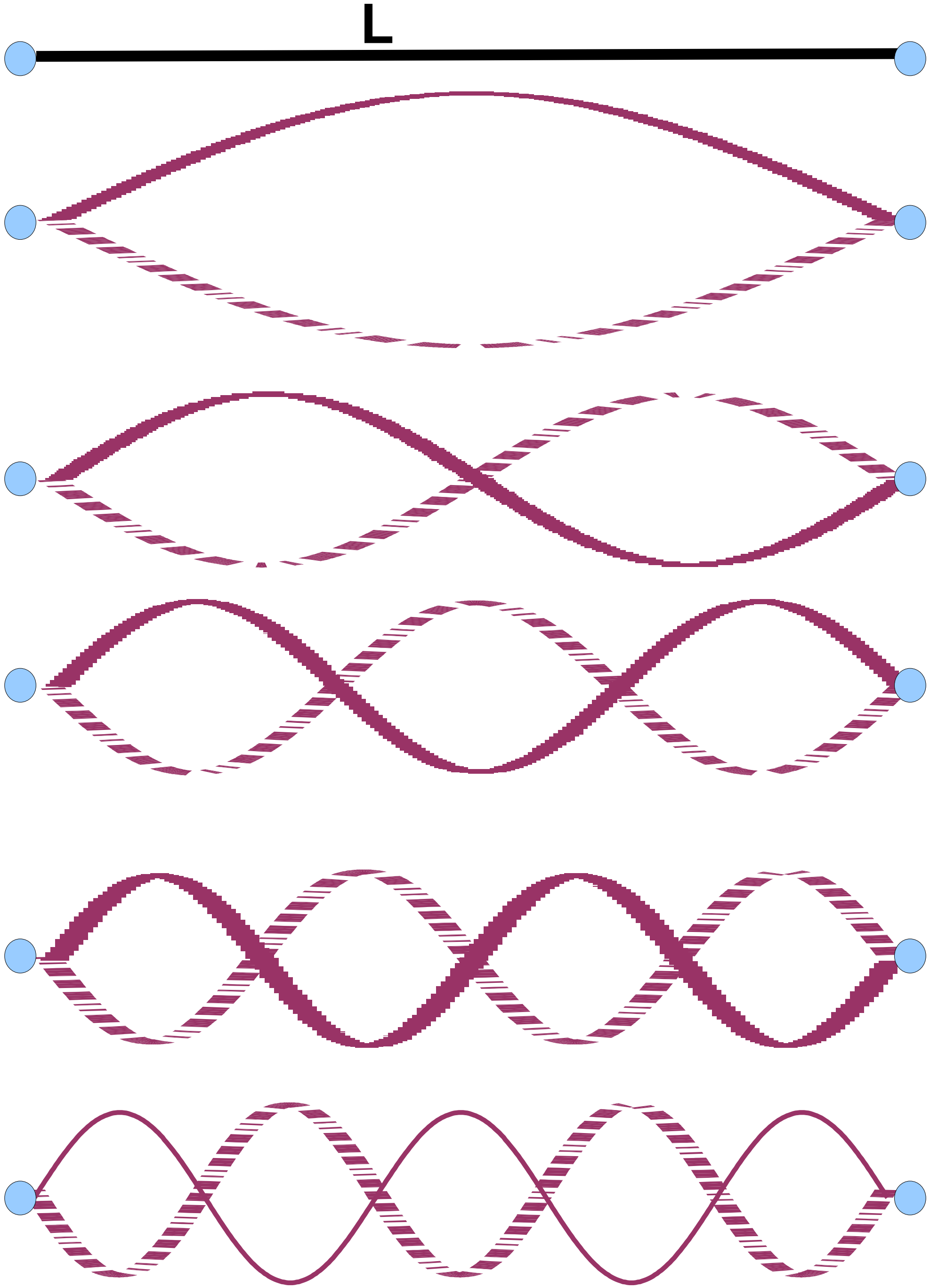


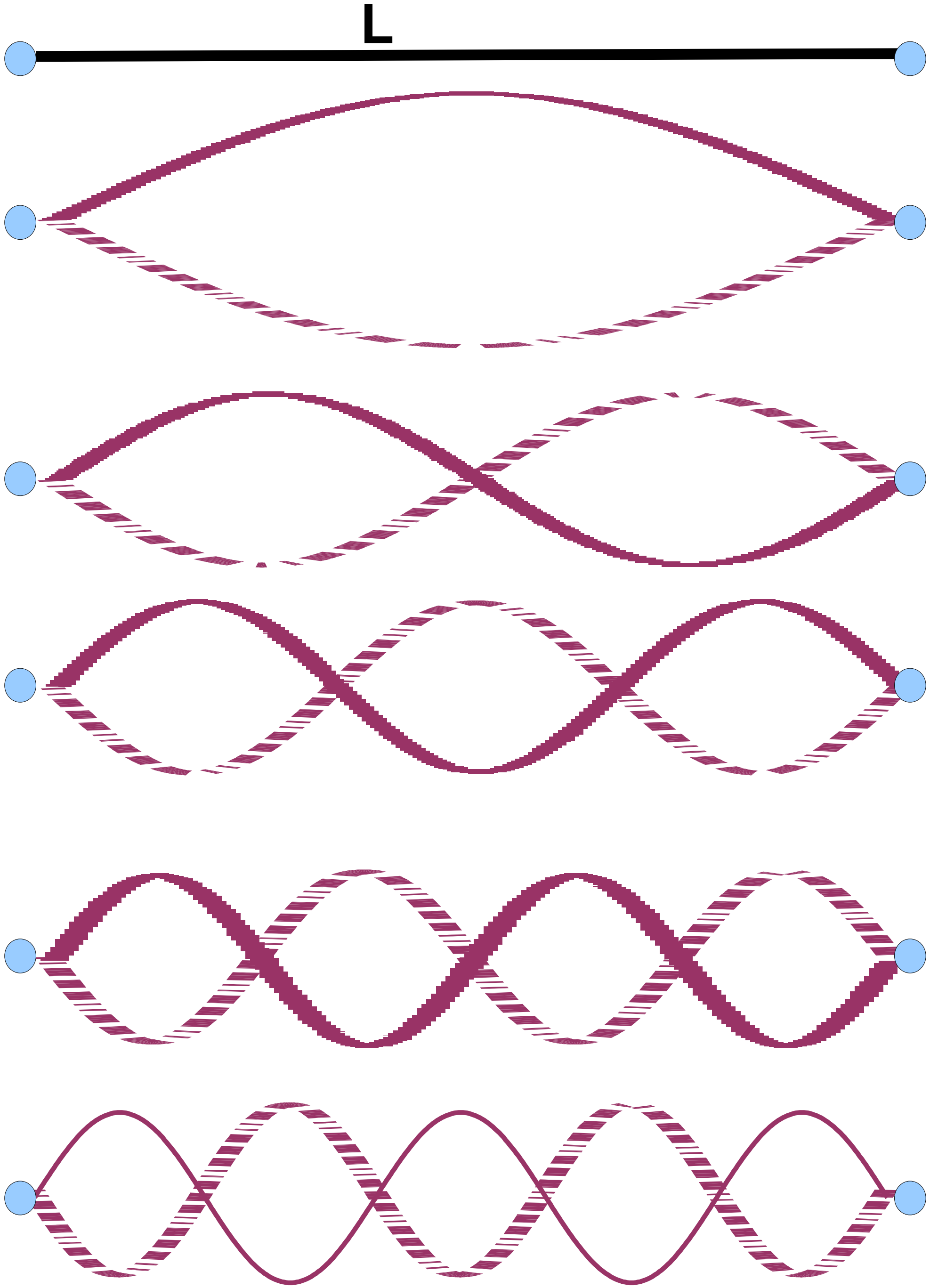
Linear Density

$$\mu \equiv \frac{\Delta m}{\Delta l} = \frac{m_{total}}{l_{total}}$$

<u>Boundary Conditions</u> – Endpoints MUST be nodes.



mode		
$n=1$	$\frac{1}{2}\lambda_1 = L$	$\lambda_1 = 2\mathrm{L}$
$n=2$	$\lambda_2 = L$	$\lambda_2 = L$
$n=3$	$\frac{3}{2}\lambda_3 = L$	$\lambda_3 = \frac{2}{3}L$
$n=4$	$2\lambda_4 = L$	$\lambda_4 = \frac{1}{2}L$
$n=5$	$\frac{5}{2}\lambda_5 = L$	$\lambda_5 = \frac{2}{5}L$



mode

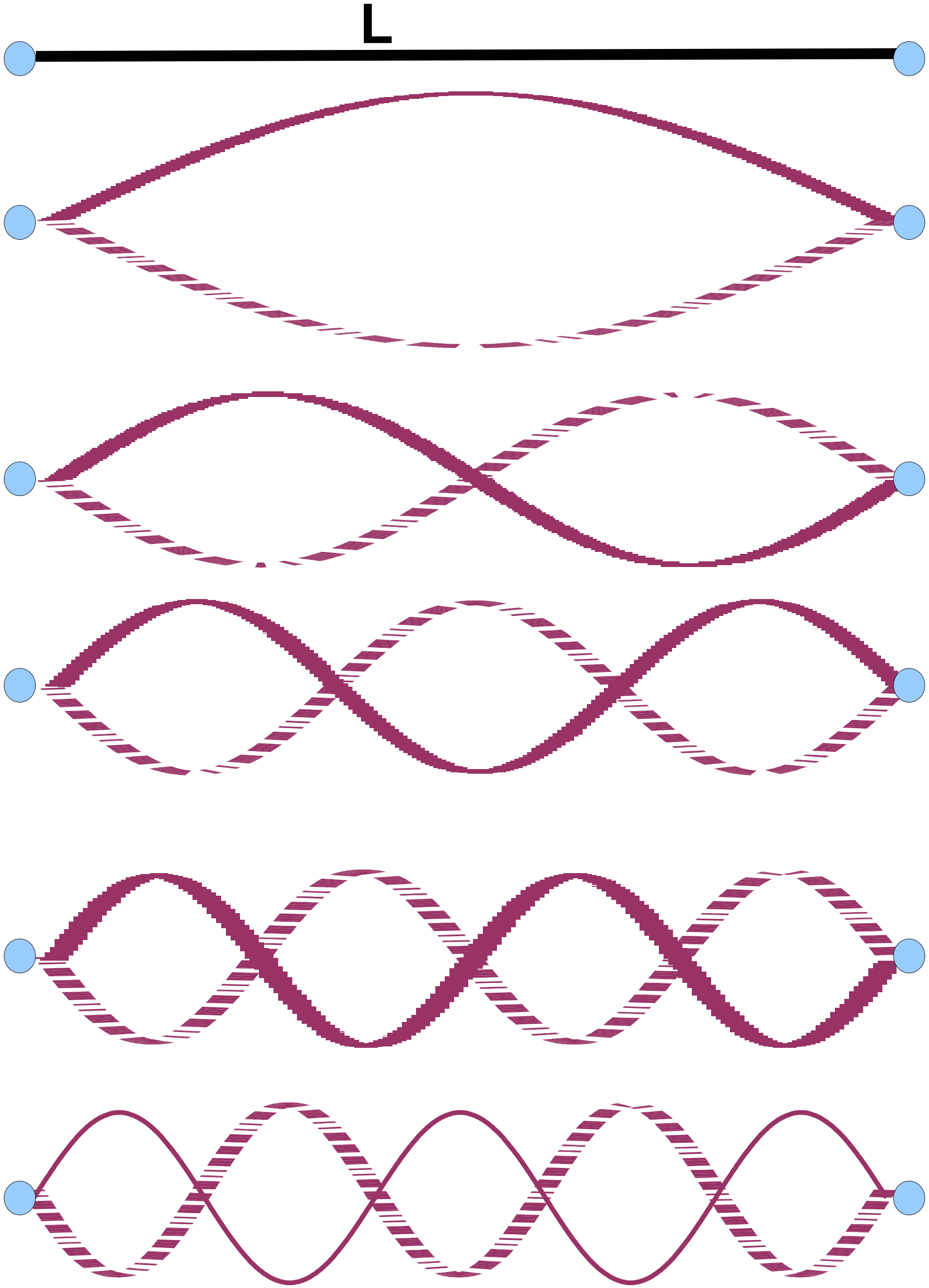
$n=1$ $\qquad \lambda_1 = 2L = \frac{2}{1}L$

$n=2$ $\qquad \lambda_2 = L = \frac{2}{2}L$

$n=3$ $\qquad \lambda_3 = \frac{2}{3}L$

$n=4$ $\qquad \lambda_4 = \frac{1}{2}L = \frac{2}{4}L$

$n=5$ $\qquad \lambda_5 = \frac{2}{5}L$



mode

$n=1$ $\lambda_1=\frac{2}{1}L$

$n=2$ $\lambda_2=\frac{2}{2}L$

$n=3$ $\lambda_3=\frac{2}{3}L$

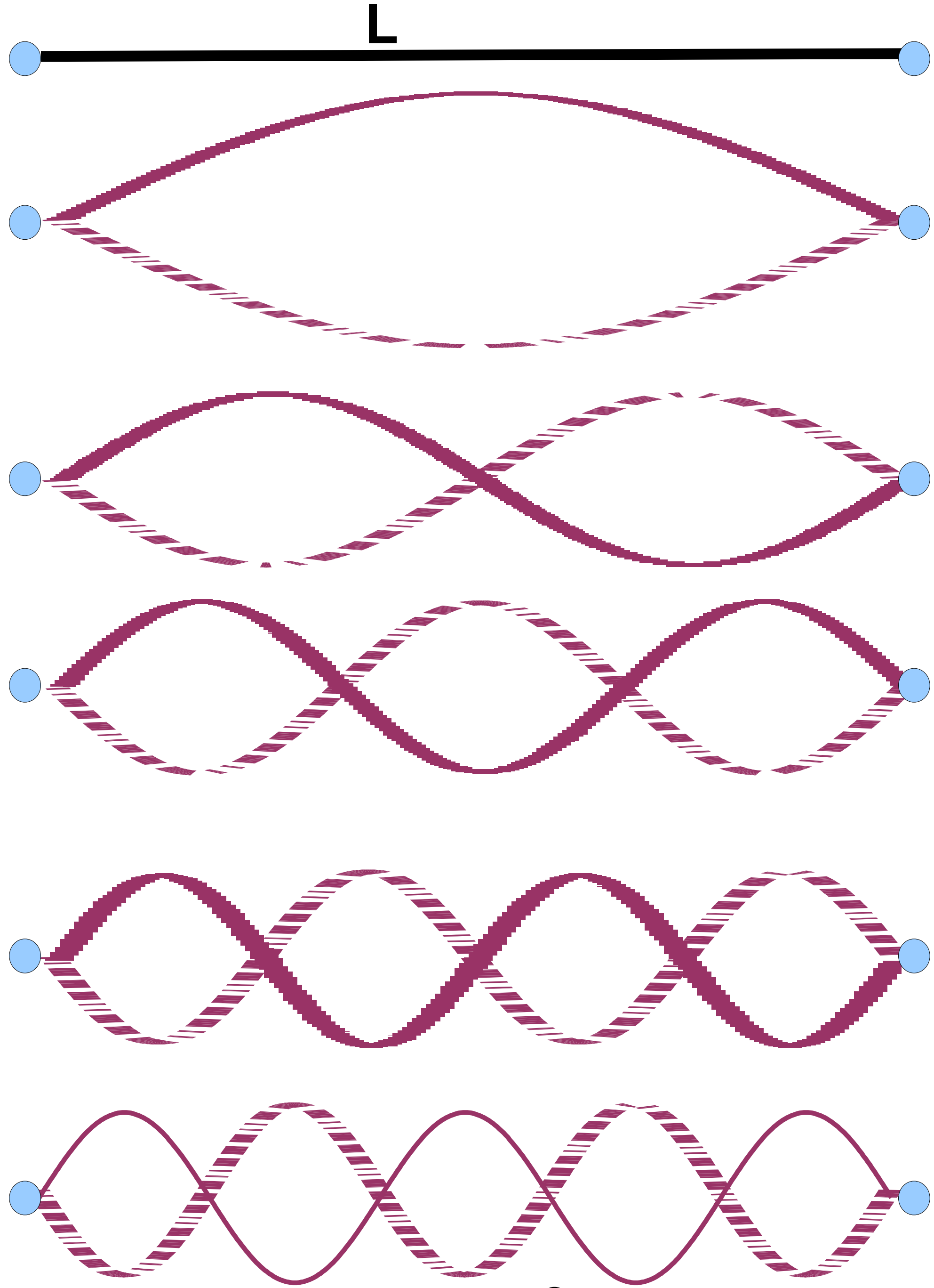
$n=4$ $\lambda_4=\frac{2}{4}L$

$n=5$ $\lambda_5=\frac{2}{5}L$

Recursion Relationship $\lambda_n=\frac{2}{n}L$

$$v = \frac{distance}{time} = \frac{\lambda}{T} = \lambda f$$

$$f = \frac{v}{\lambda}$$



mode

$n=1 \quad \lambda_1=\frac{2}{1}L \quad f_1=\frac{v}{\lambda_1}=\frac{v}{2L}$

$n=2 \quad \lambda_2=\frac{2}{2}L \quad f_2=\frac{v}{\lambda_2}=\frac{2v}{2L}$

$n=3 \quad \lambda_3=\frac{2}{3}L \quad f_3=\frac{v}{\lambda_3}=\frac{3v}{2L}$

$n=4 \quad \lambda_4=\frac{2}{4}L \quad f_4=\frac{v}{\lambda_4}=\frac{4v}{2L}$

$n=5 \quad \lambda_5=\frac{2}{5}L \quad f_5=\frac{v}{\lambda_5}=\frac{5v}{2L}$

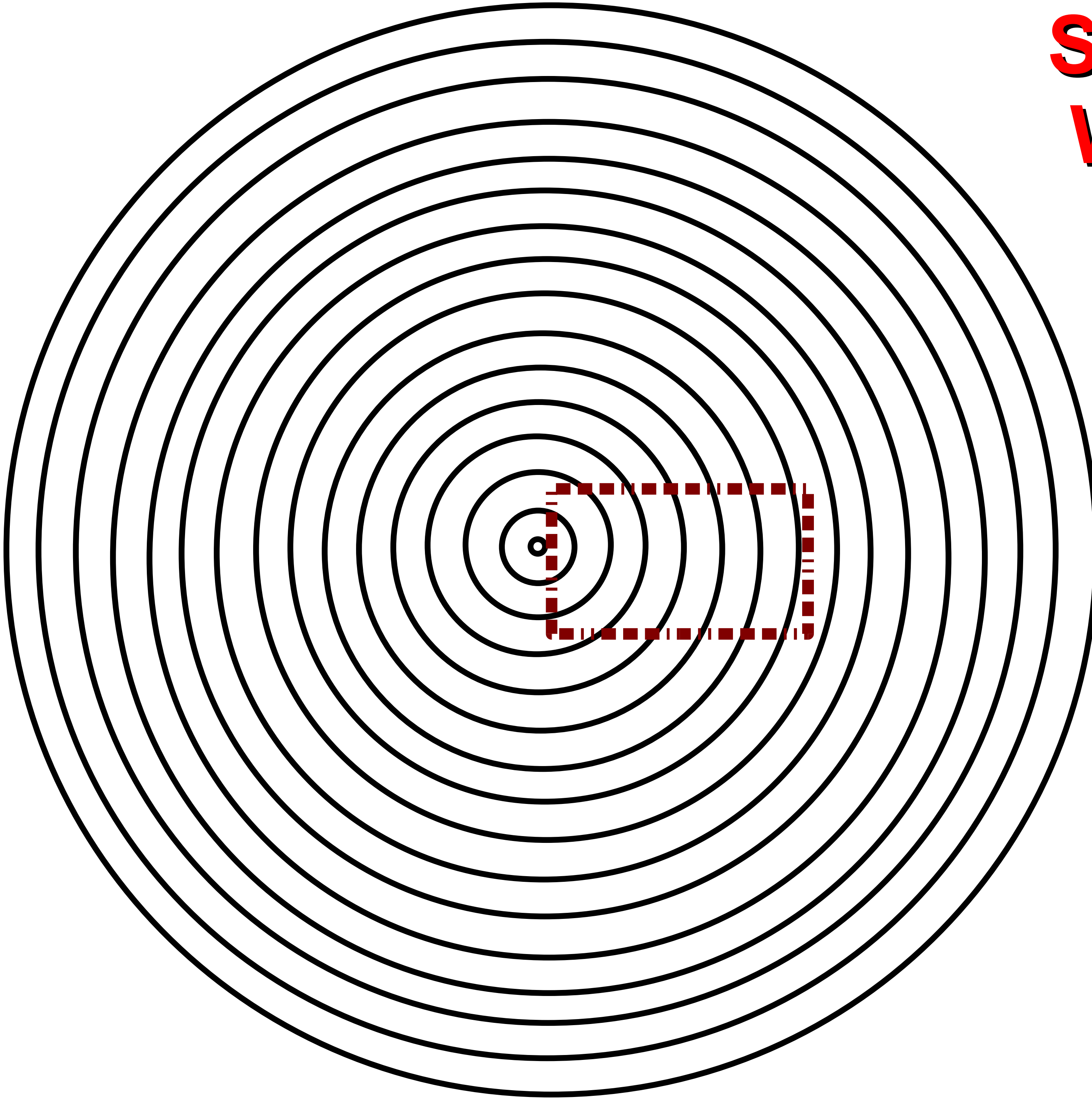
Recursion Relationship $\lambda_n=\frac{2}{n}L \quad f_n=\frac{v}{\lambda_n}=\frac{nv}{2L}=n f_1 \quad n=1,2,3,4,5\ldots$

Do example of:

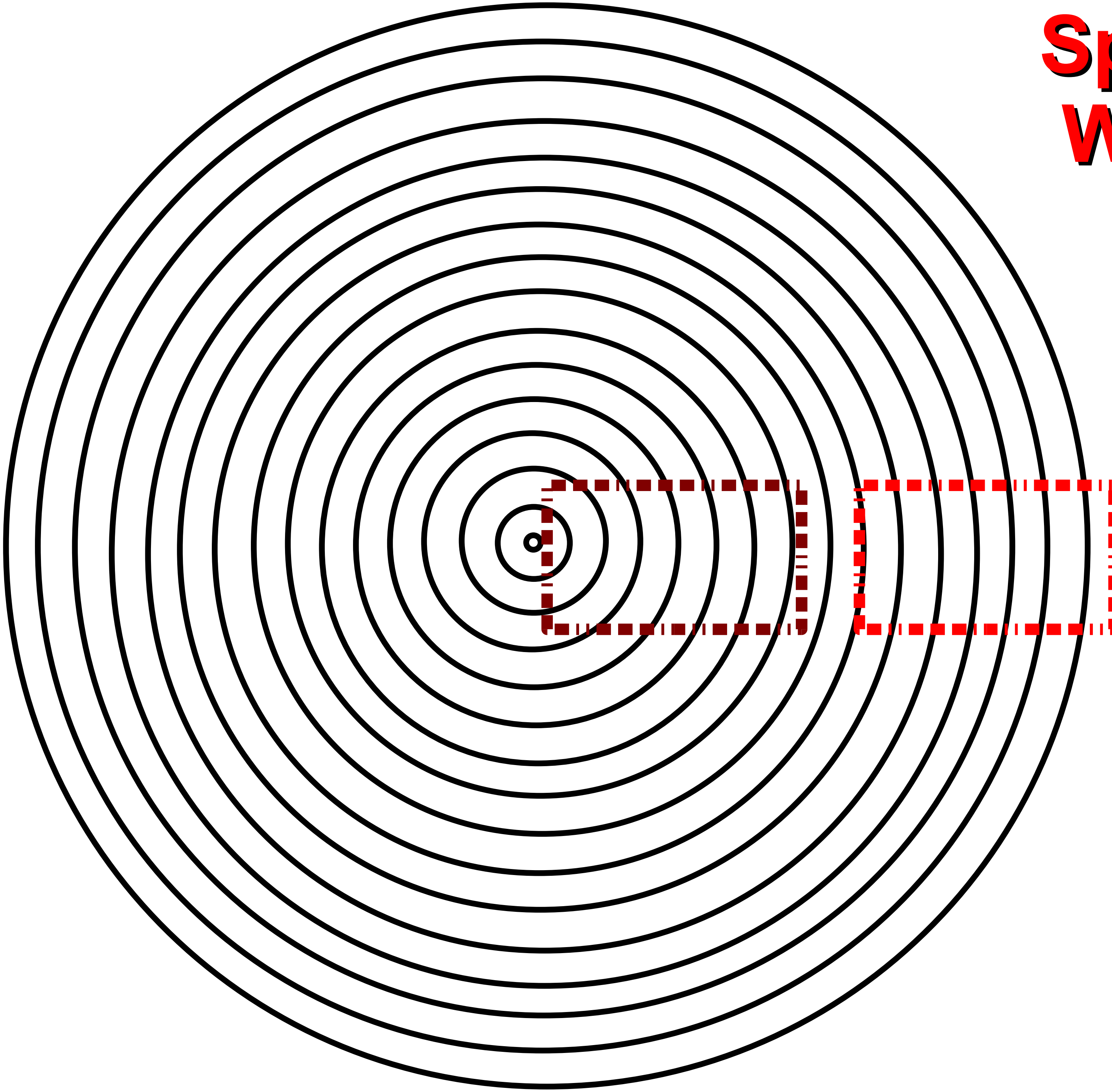
Standing sound waves in a tube open on one end, closed on other end.

Refraction

Spherical Waves

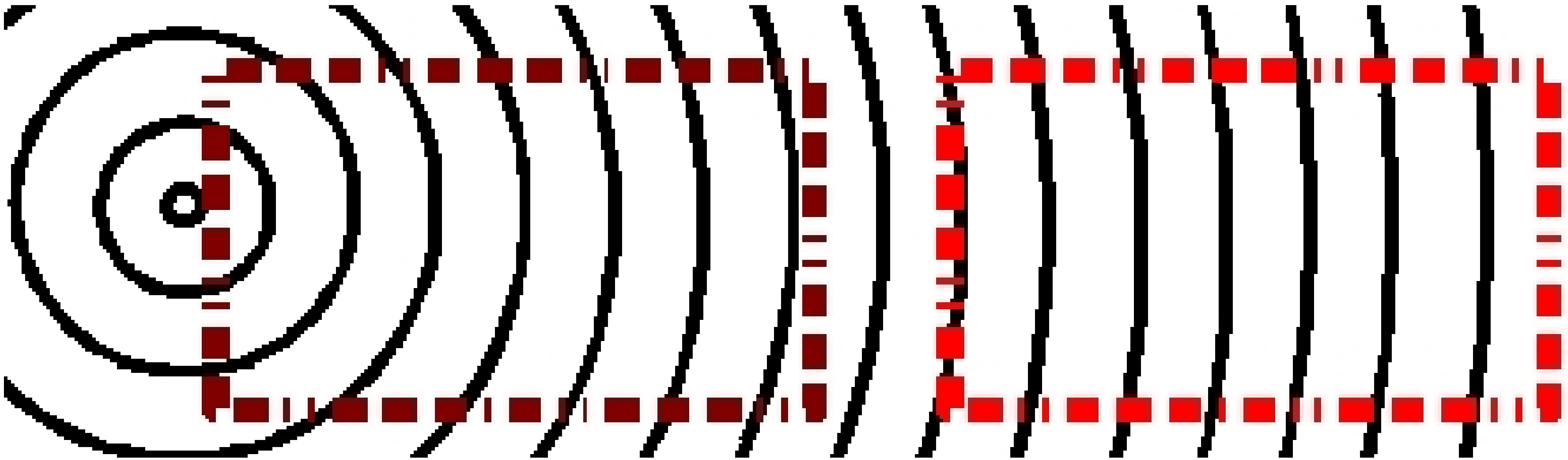


Spherical Waves



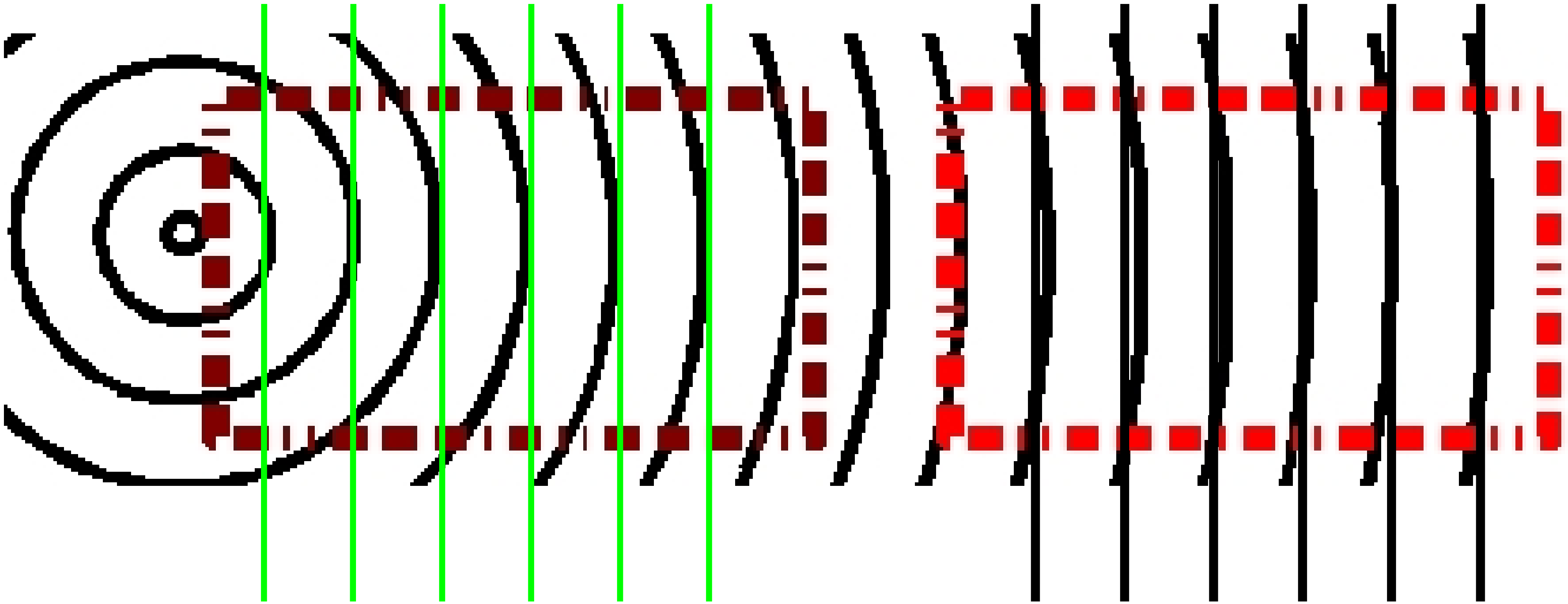
Spherical Waves

Plane Waves

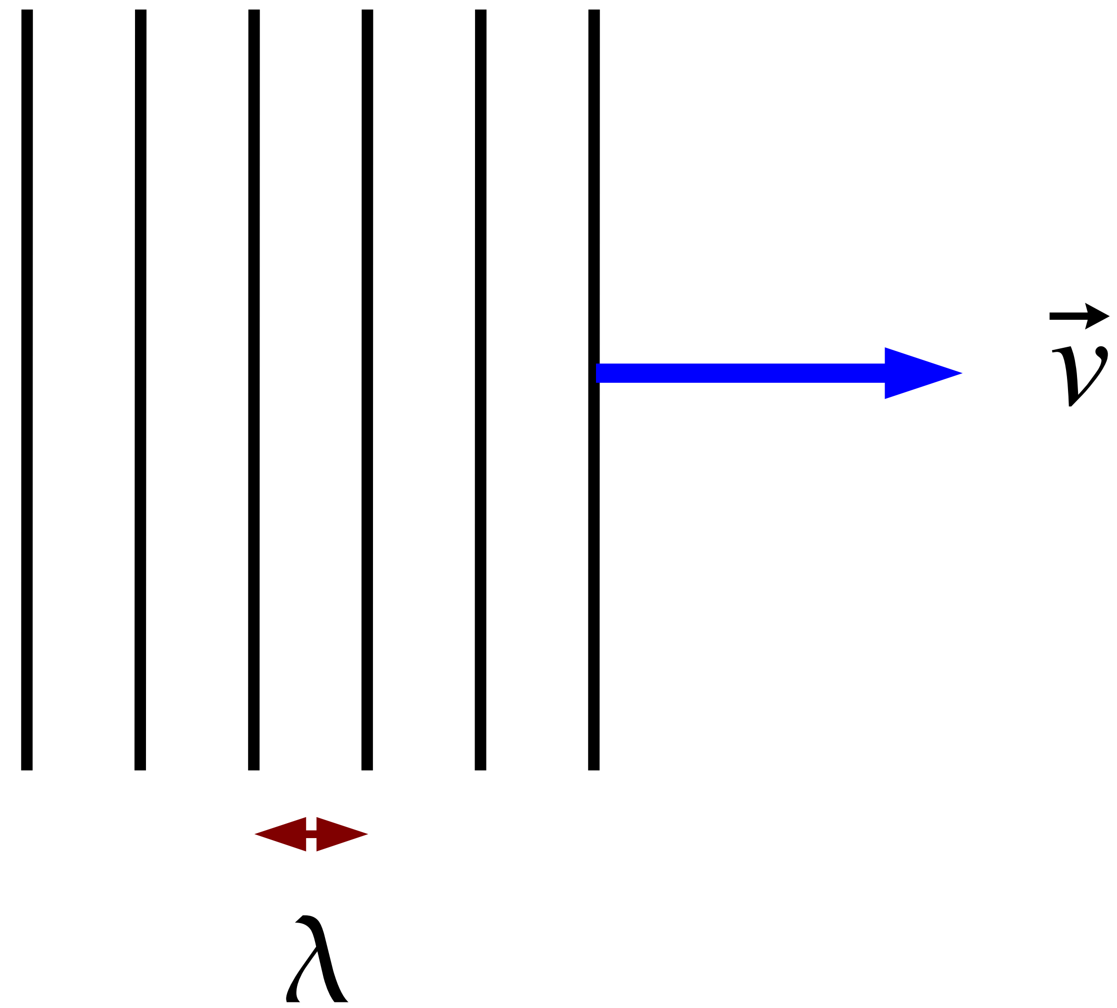


Spherical Waves

Plane Waves



Plane Waves

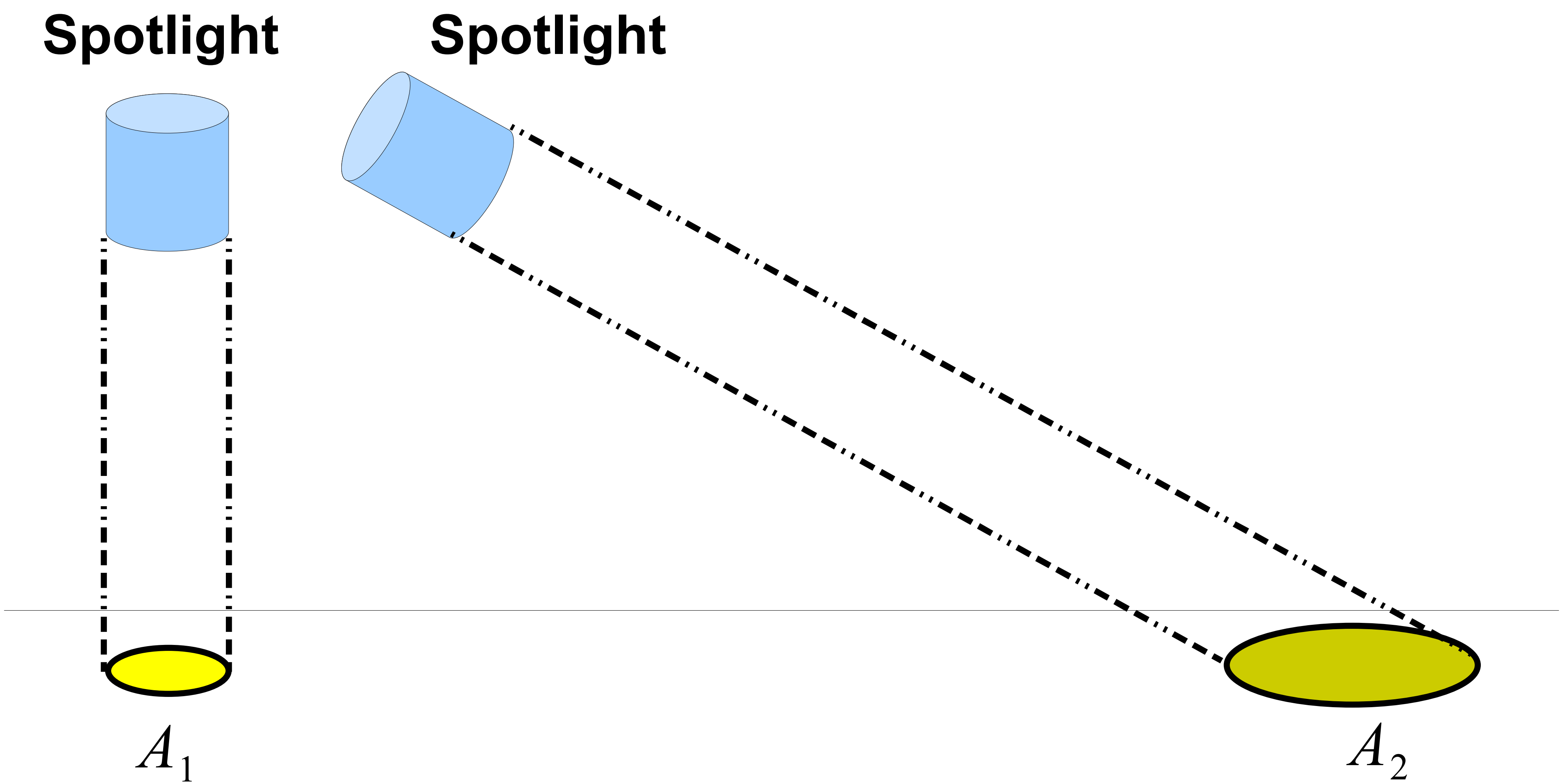


Power (P) – Energy per time.

$$P = \frac{dE}{dt}$$

Intensity (I) – power per unit area.

$$I = \frac{P}{A}$$



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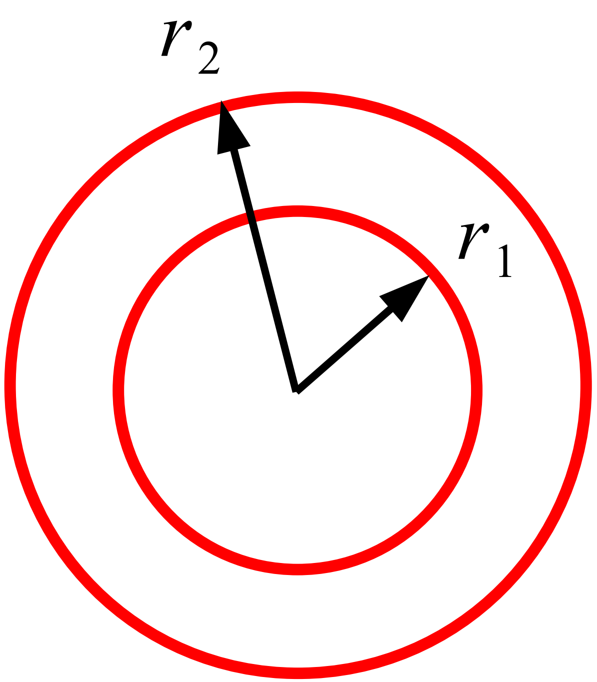


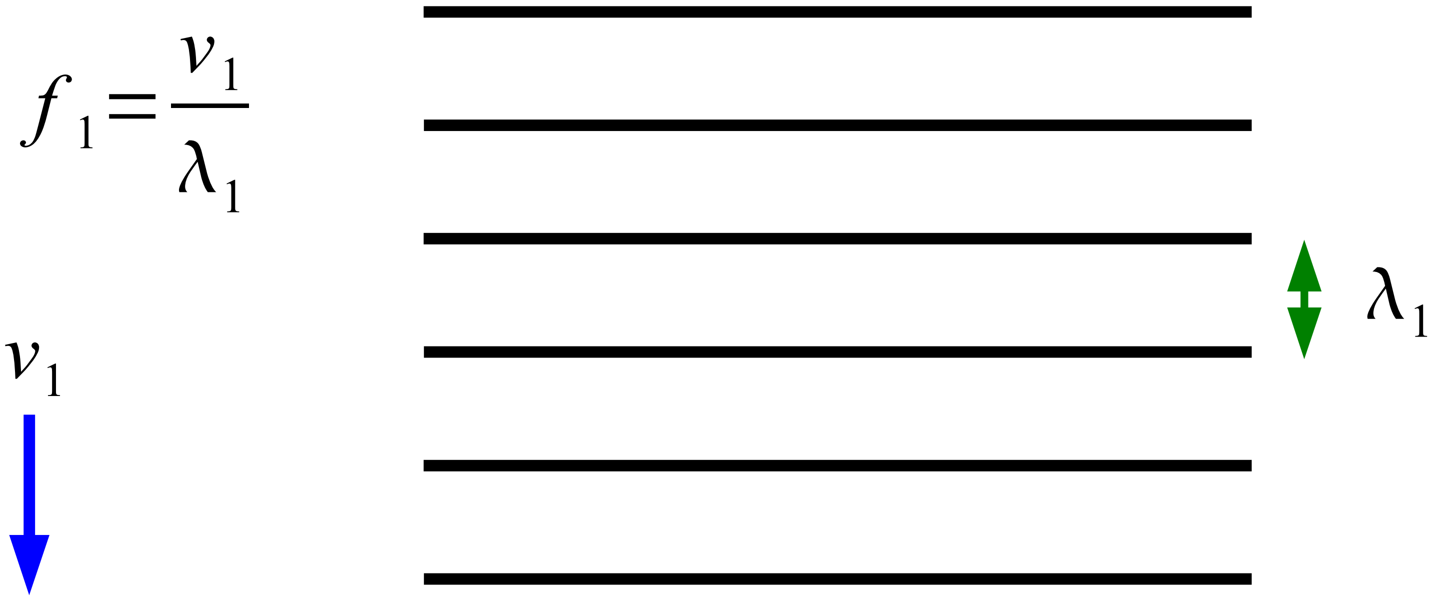
$$P_1 = P_2$$

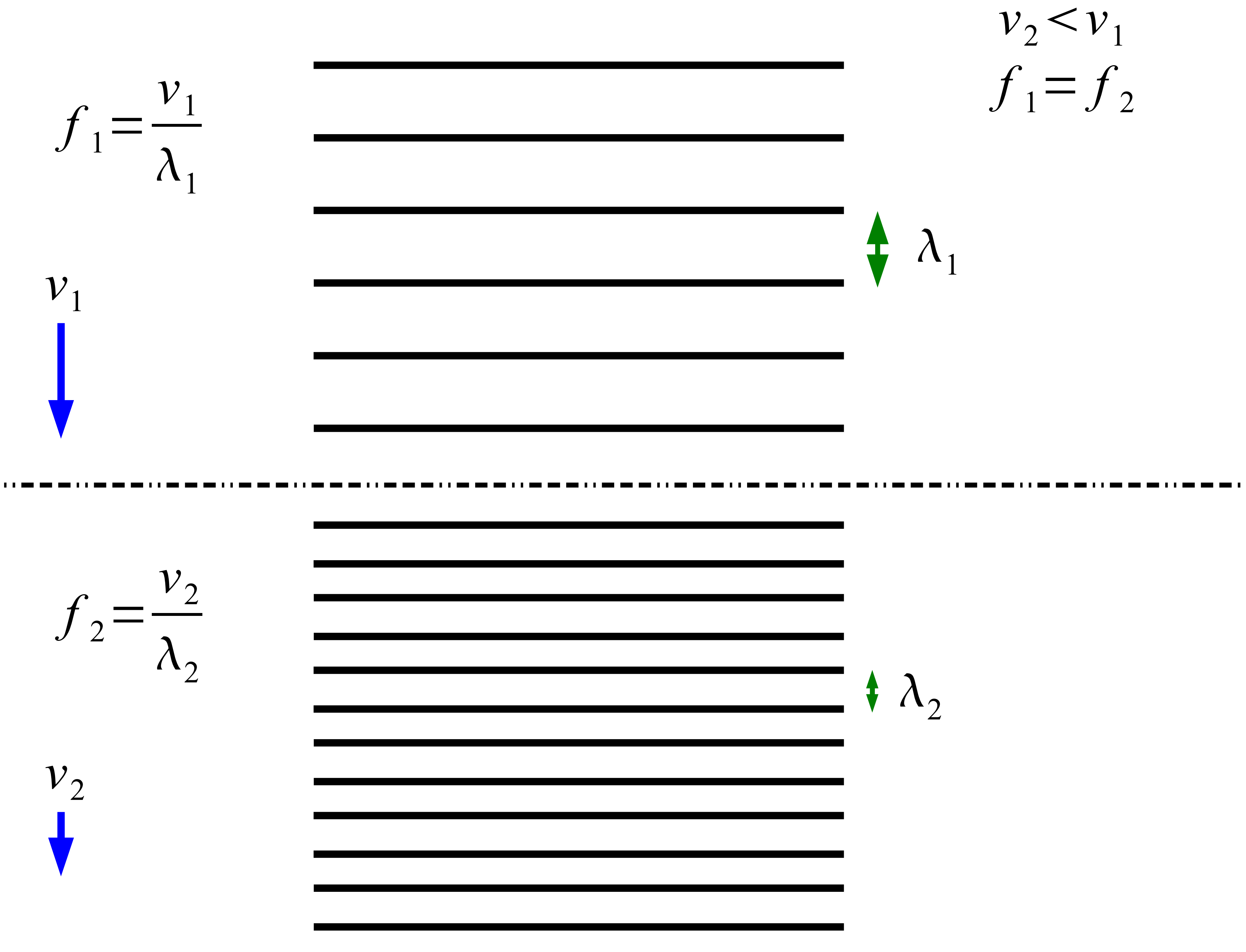
$$I_1 A_1 = I_2 A_2$$

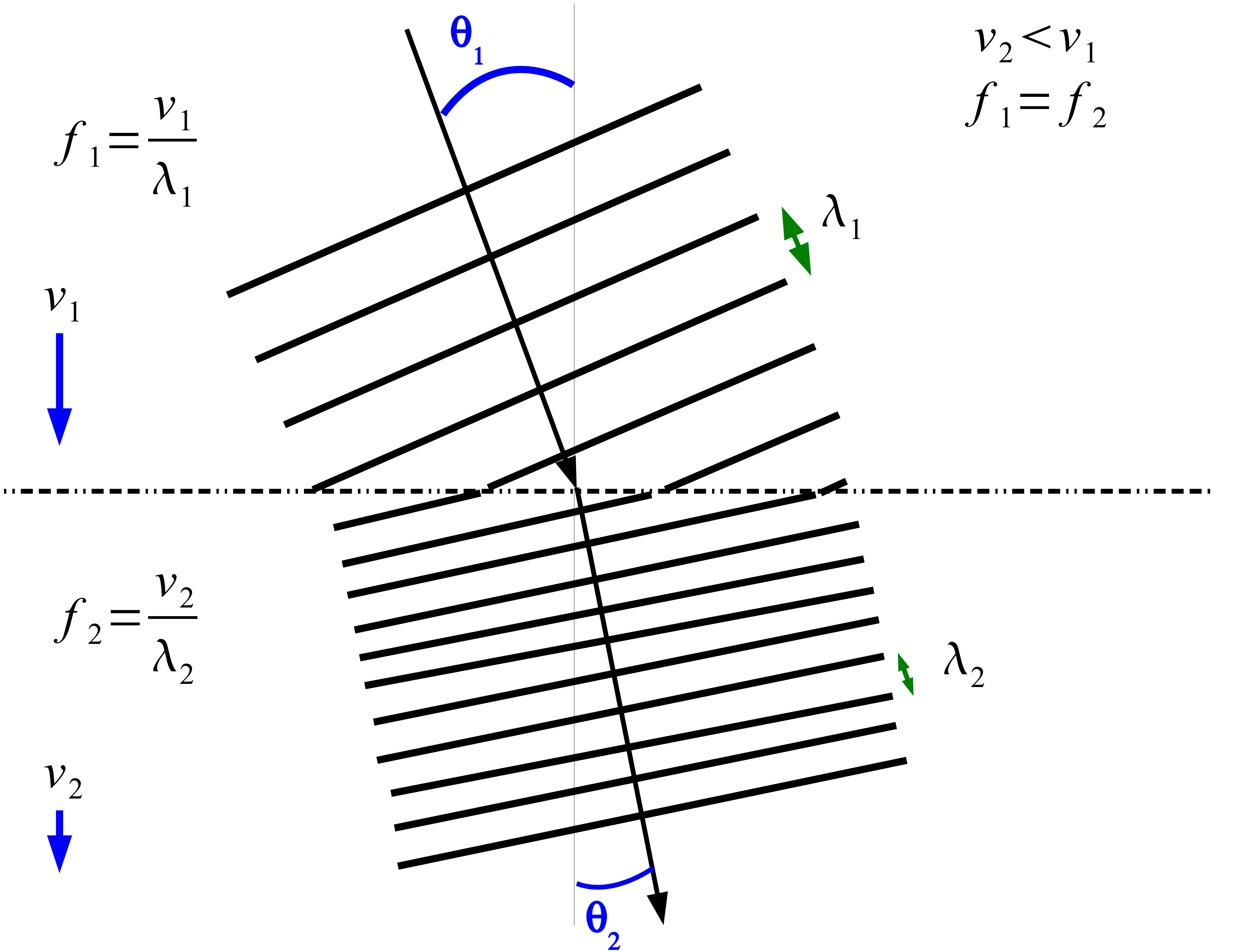
$$I_1 \left(4\pi r_1^2\right) = I_2 \left(4\pi r_2^2\right)$$

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1$$

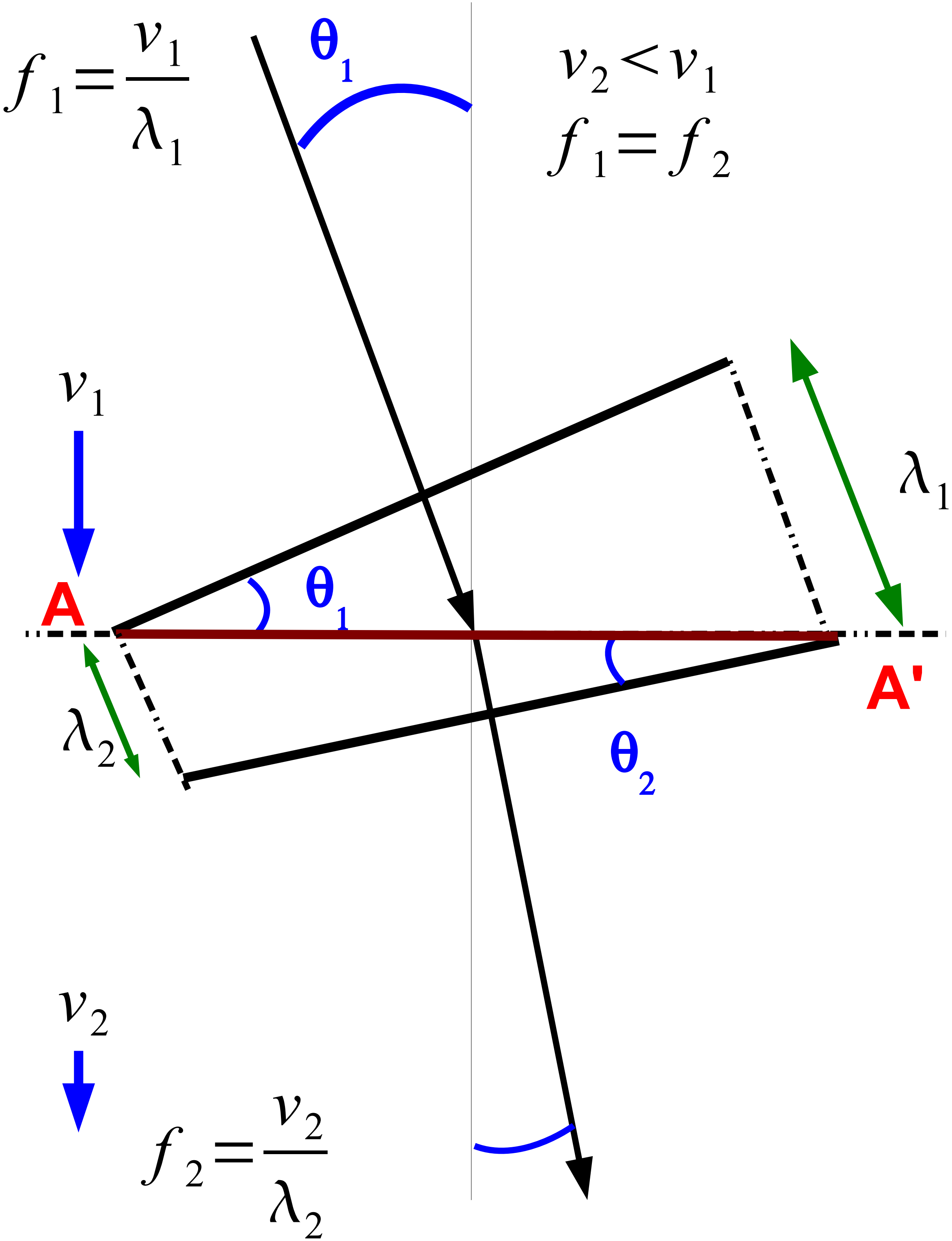
$$f_1 = \frac{v_1}{\lambda_1}$$




$v_2 < v_1$
$f_1 = f_2$
$f_1 = \frac{v_1}{\lambda_1}$
λ_1
v_1
$f_2 = \frac{v_2}{\lambda_2}$
λ_2
v_2



θ_1
$v_2 < v_1$
$f_1 = f_2$
$f_1 = \frac{v_1}{\lambda_1}$
λ_1
v_1
$f_2 = \frac{v_2}{\lambda_2}$
λ_2
v_2
θ_2



$$\sin\theta_1 = \frac{\lambda_1}{\overline{AA'}}$$

$$\sin\theta_2 = \frac{\lambda_2}{\overline{AA'}}$$

$$\frac{1}{\lambda_2}\sin\theta_2 = \frac{1}{\lambda_1}\sin\theta_1$$

$$T_1 = T_2$$

$$\frac{T}{\lambda_2}\sin\theta_2 = \frac{T}{\lambda_1}\sin\theta_1$$

$$\frac{1}{v_2}\sin\theta_2 = \frac{1}{v_1}\sin\theta_1$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

Questions????

Mass on a Spring	Simple Pendulum
$m a_x = -kx$	$m a_x = -mg\left(\frac{x}{l}\right)$
$a_x = -\left(\frac{k}{m}\right)x$	$a_x = -\left(\frac{g}{l}\right)x$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{l}}$
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{l}{g}}$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

$$x = l\theta$$

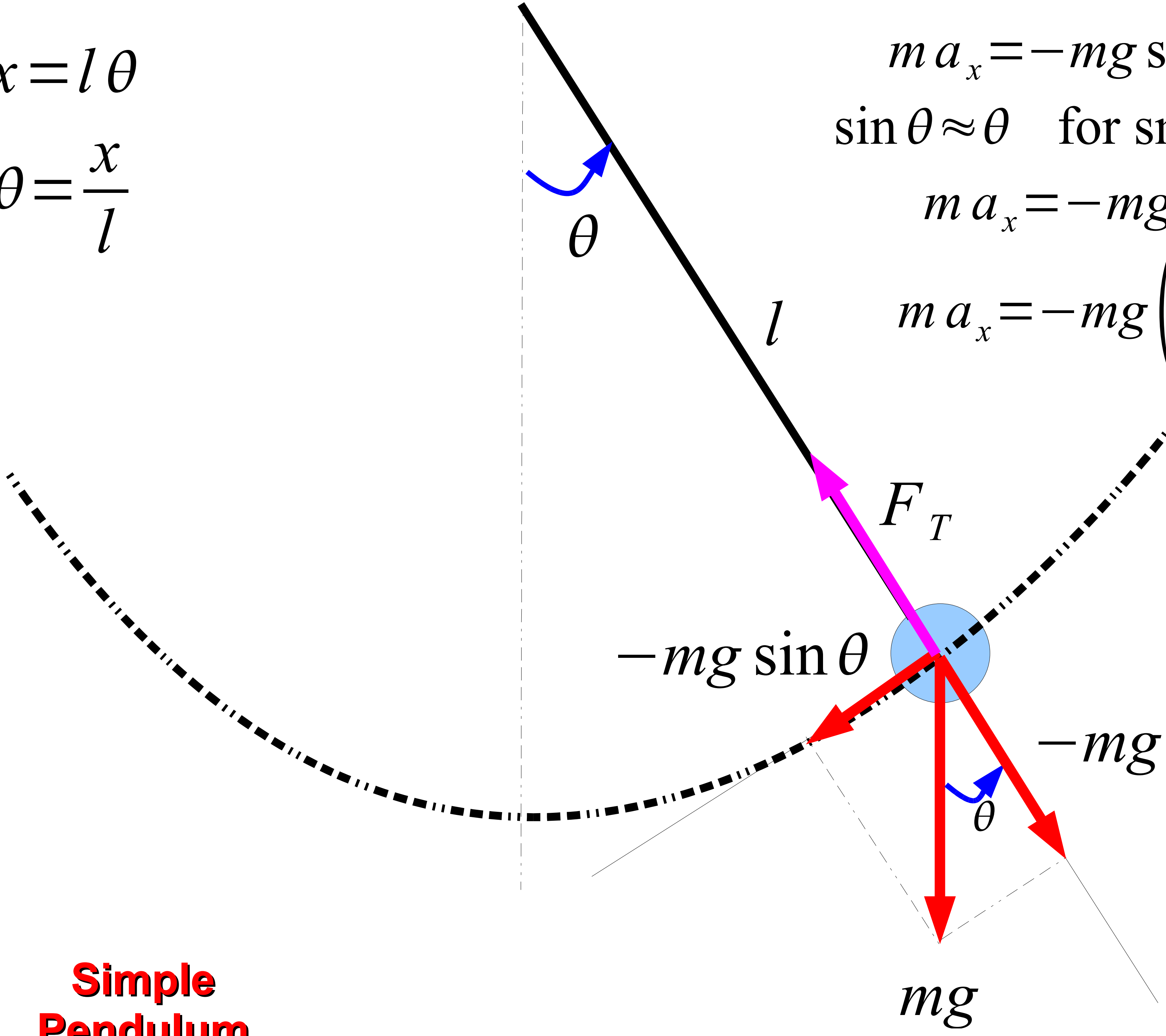
$$\theta = \frac{x}{l}$$

$$m a_x = -mg\sin\theta$$

$$\sin\theta \approx \theta \quad \text{for small } \theta$$

$$m a_x = -mg\theta$$

$$m a_x = -mg\left(\frac{x}{l}\right)$$



Simple Pendulum