Physics 281
Physics III

Week One
Chapter 13

Oscillations

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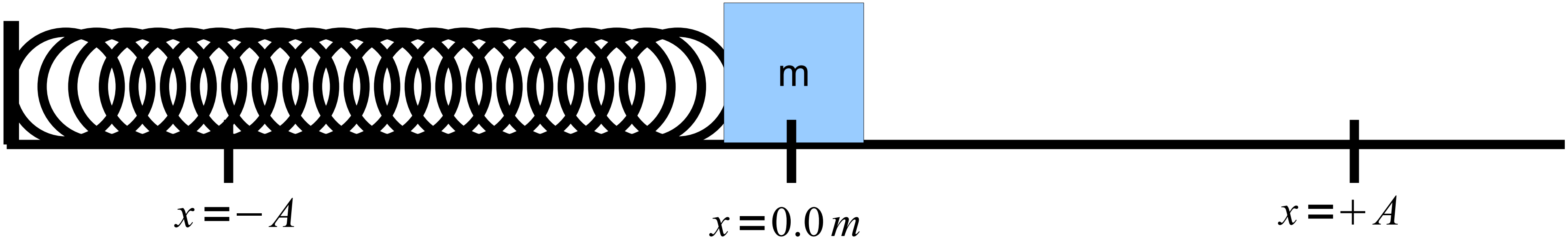
SHM

Simple Harmonic Motion

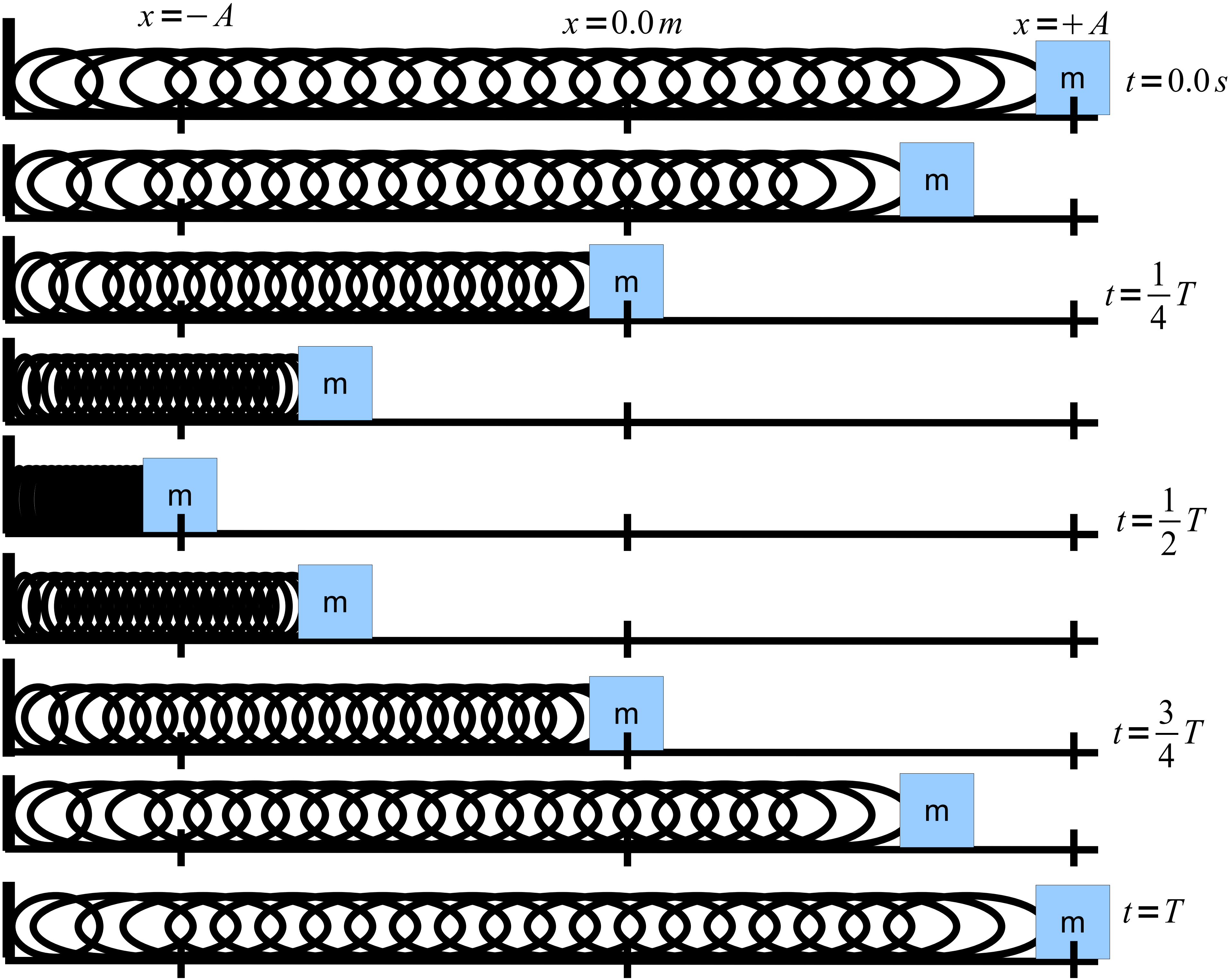
Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



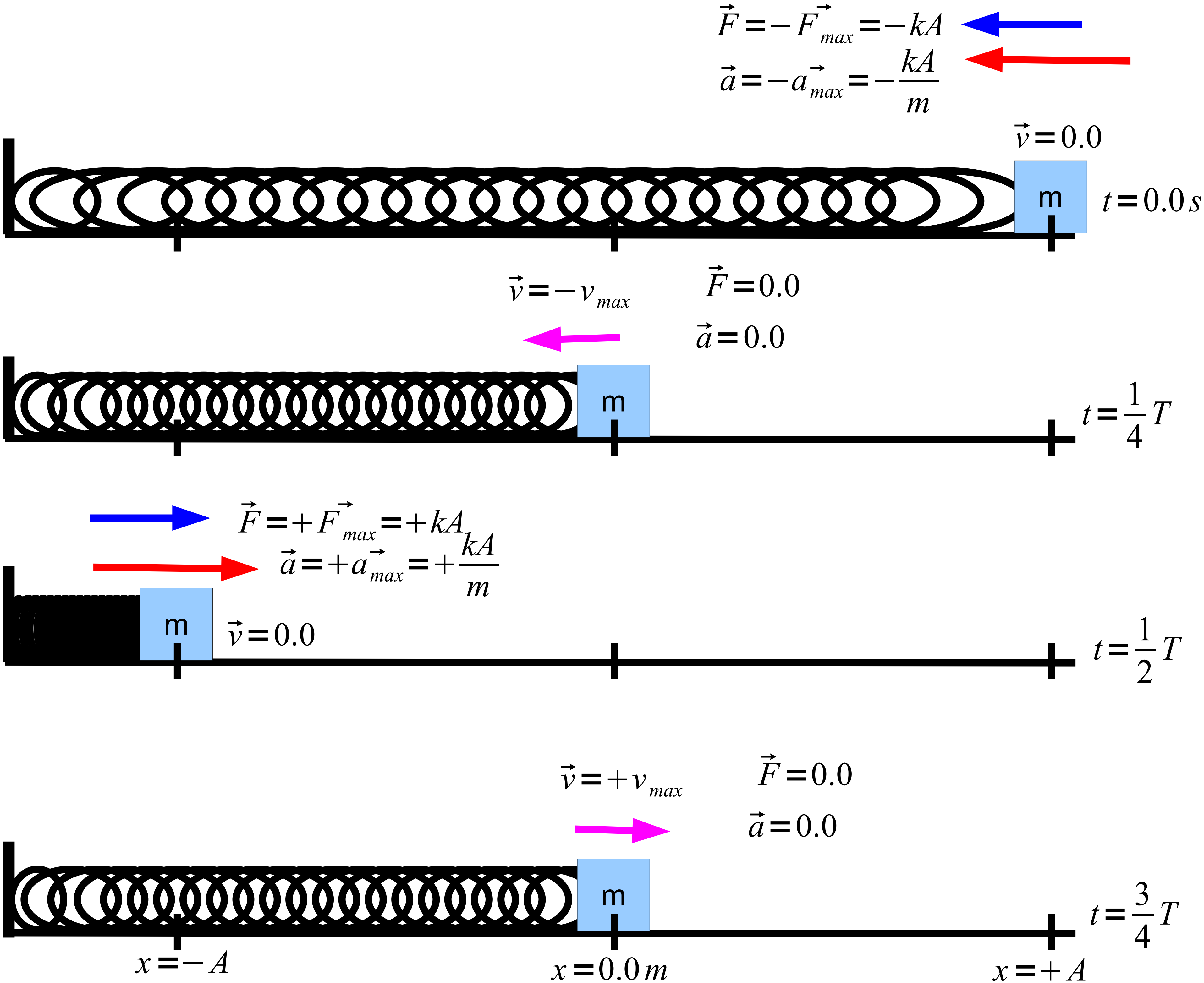
PERIOD (T) – Time to complete one oscillation. Measured in seconds.

FREQUENCY (f) – Number of oscillations per seconds. Measured in Hertz (1 Hz = 1 1/s).

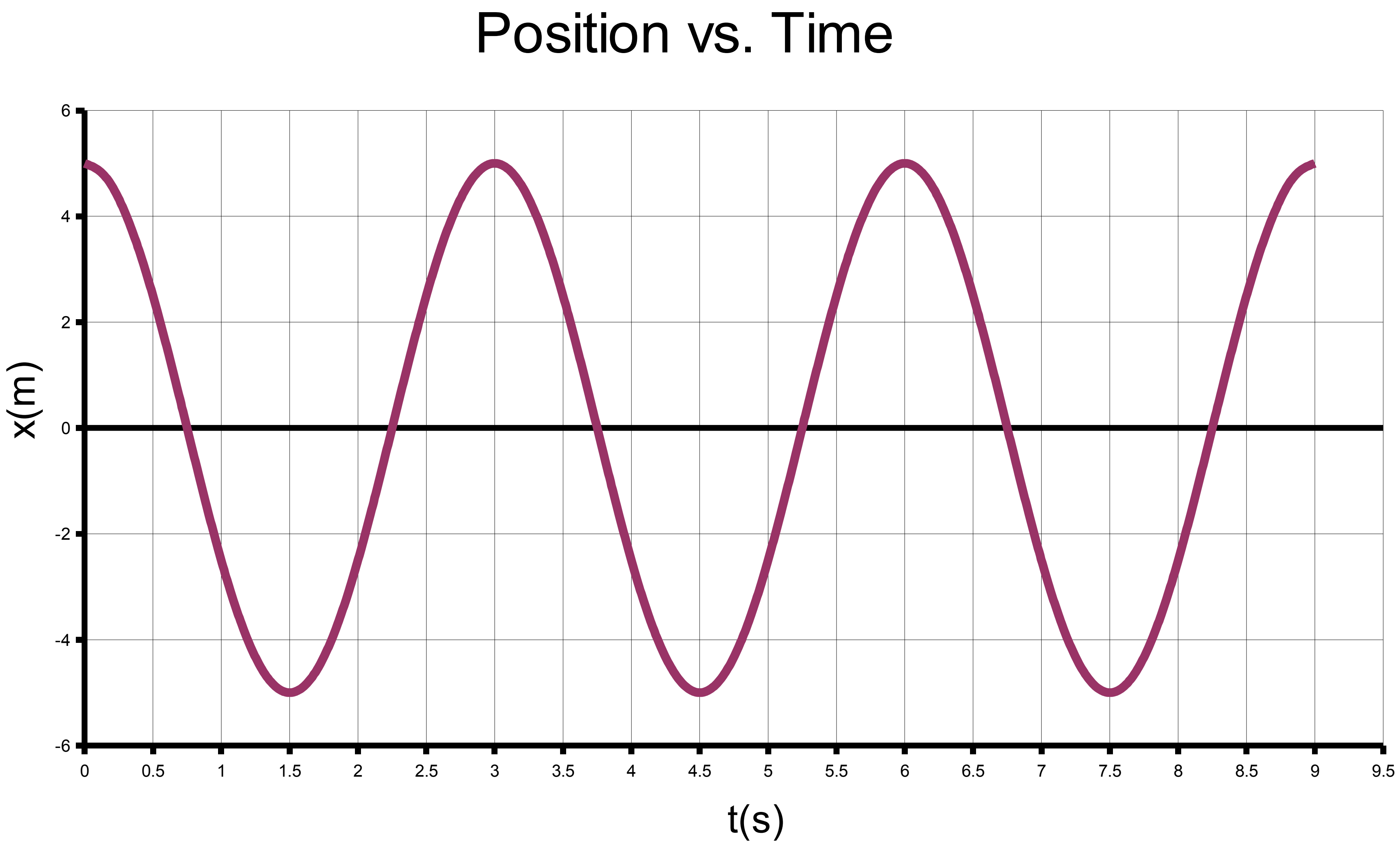
$$f = \frac{1}{T}$$



$x = -A$
$x = 0.0\,m$
$x = +A$
m
$t = 0.0\,s$
m
m
$t = \frac{1}{4}T$
m
m
$t = \frac{1}{2}T$
m
m
$t = \frac{3}{4}T$
m
m
$t = T$



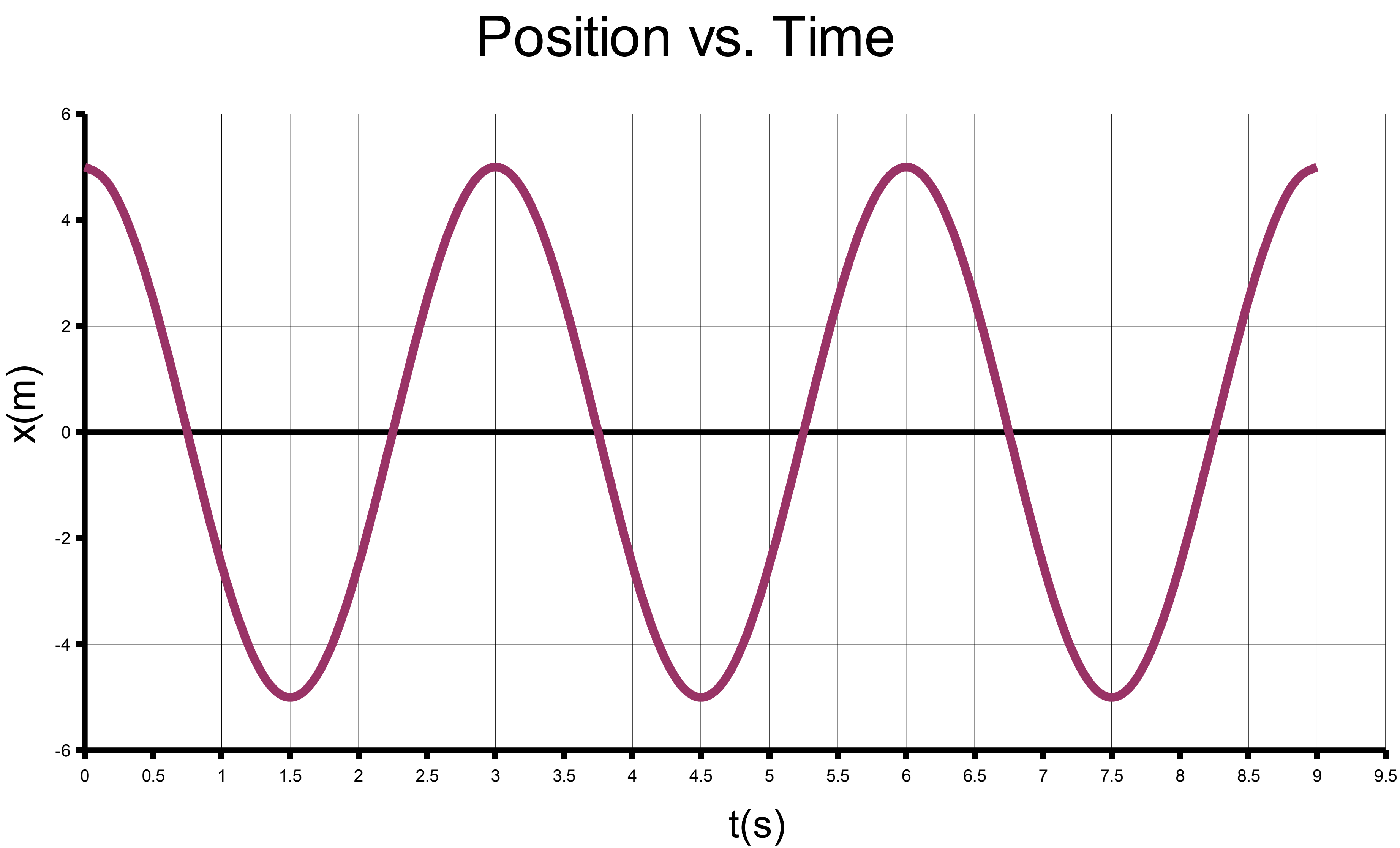
$\vec{F} = -\vec{F}_{max} = -kA$
$\vec{a} = -\vec{a}_{max} = -\frac{kA}{m}$
$\vec{v} = 0.0$
m
$t = 0.0\,s$
$\vec{v} = -v_{max}$
$\vec{F} = 0.0$
$\vec{a} = 0.0$
m
$t = \frac{1}{4}T$
$\vec{F} = +\vec{F}_{max} = +kA$
$\vec{a} = +\vec{a}_{max} = +\frac{kA}{m}$
m
$\vec{v} = 0.0$
$t = \frac{1}{2}T$
$\vec{v} = +v_{max}$
$\vec{F} = 0.0$
$\vec{a} = 0.0$
m
$t = \frac{3}{4}T$
$x = -A$
$x = 0.0\,m$
$x = +A$



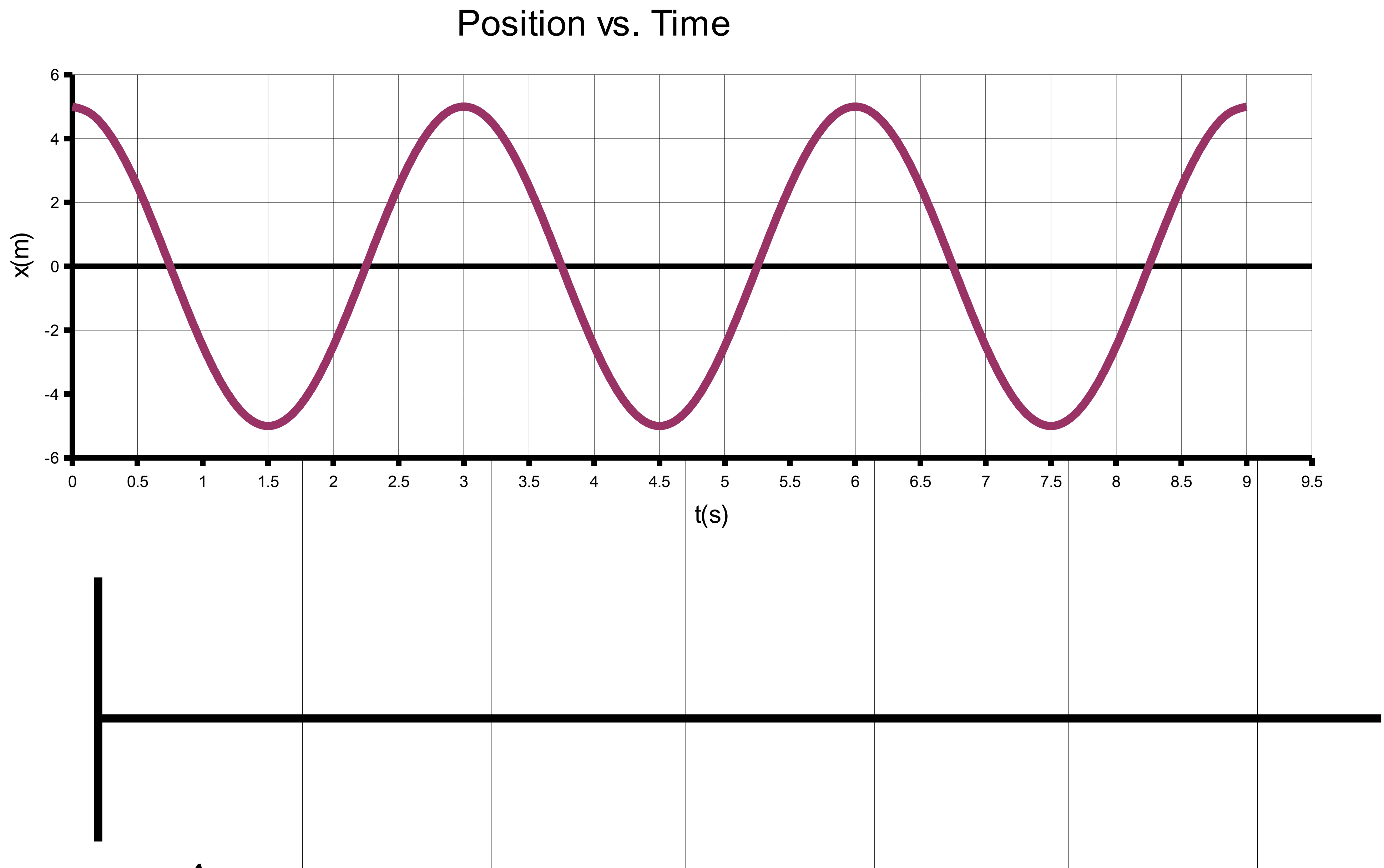
$$x(t) = A\cos(\theta)$$

Angular Frequency

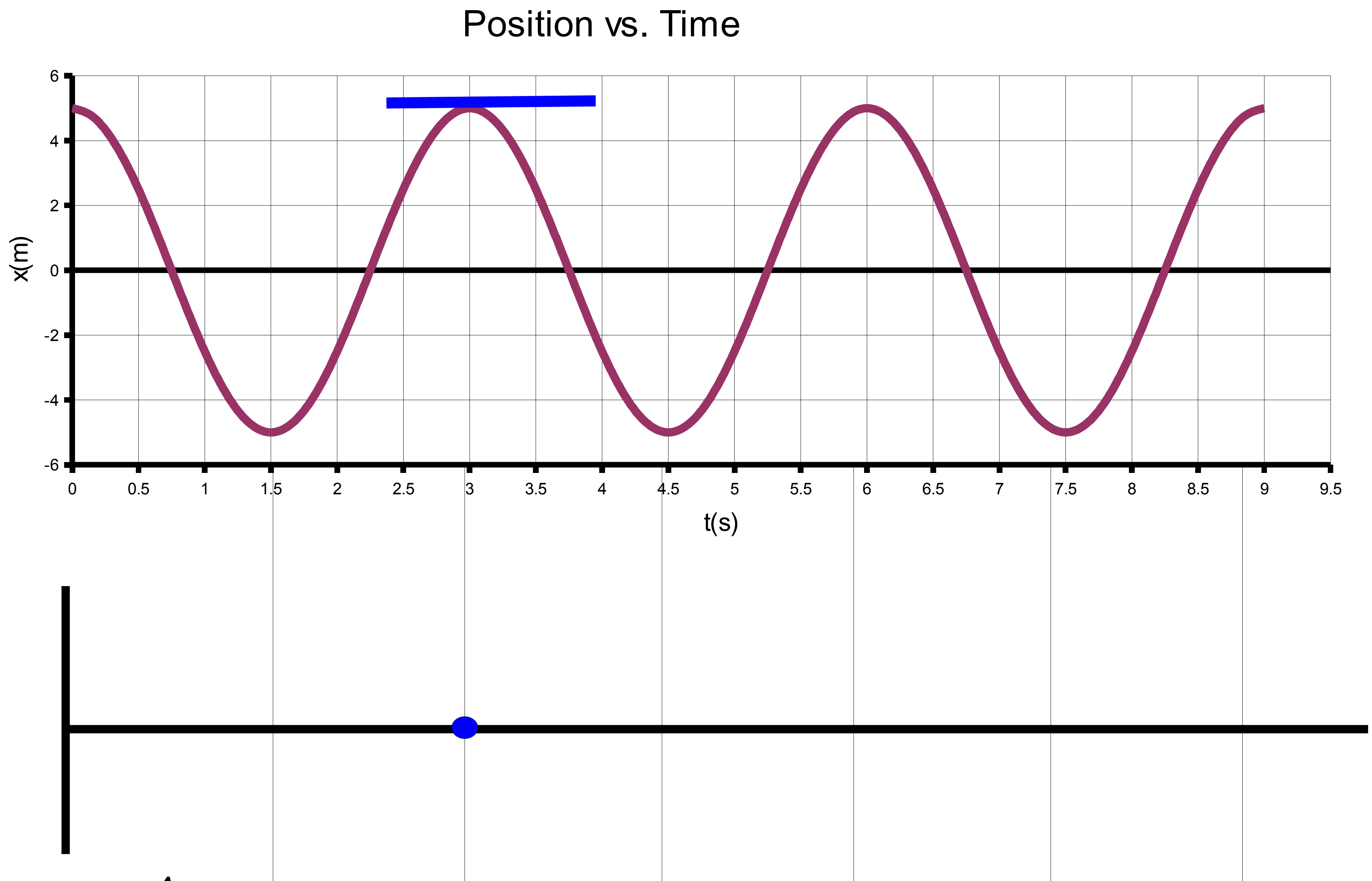
$$\omega = \frac{2\pi}{T}$$



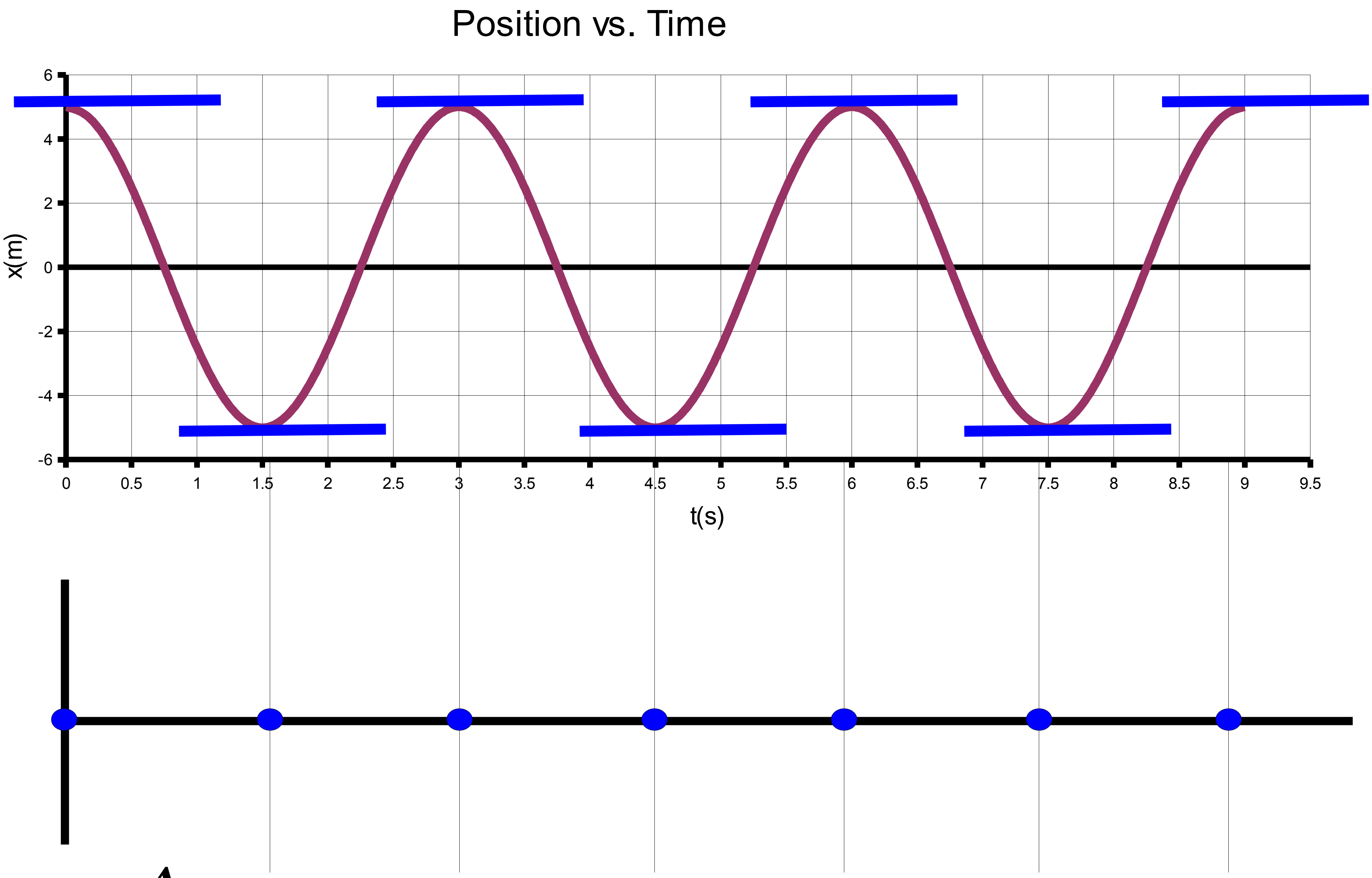
$$x(t) = A\cos(\omega t)$$



$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

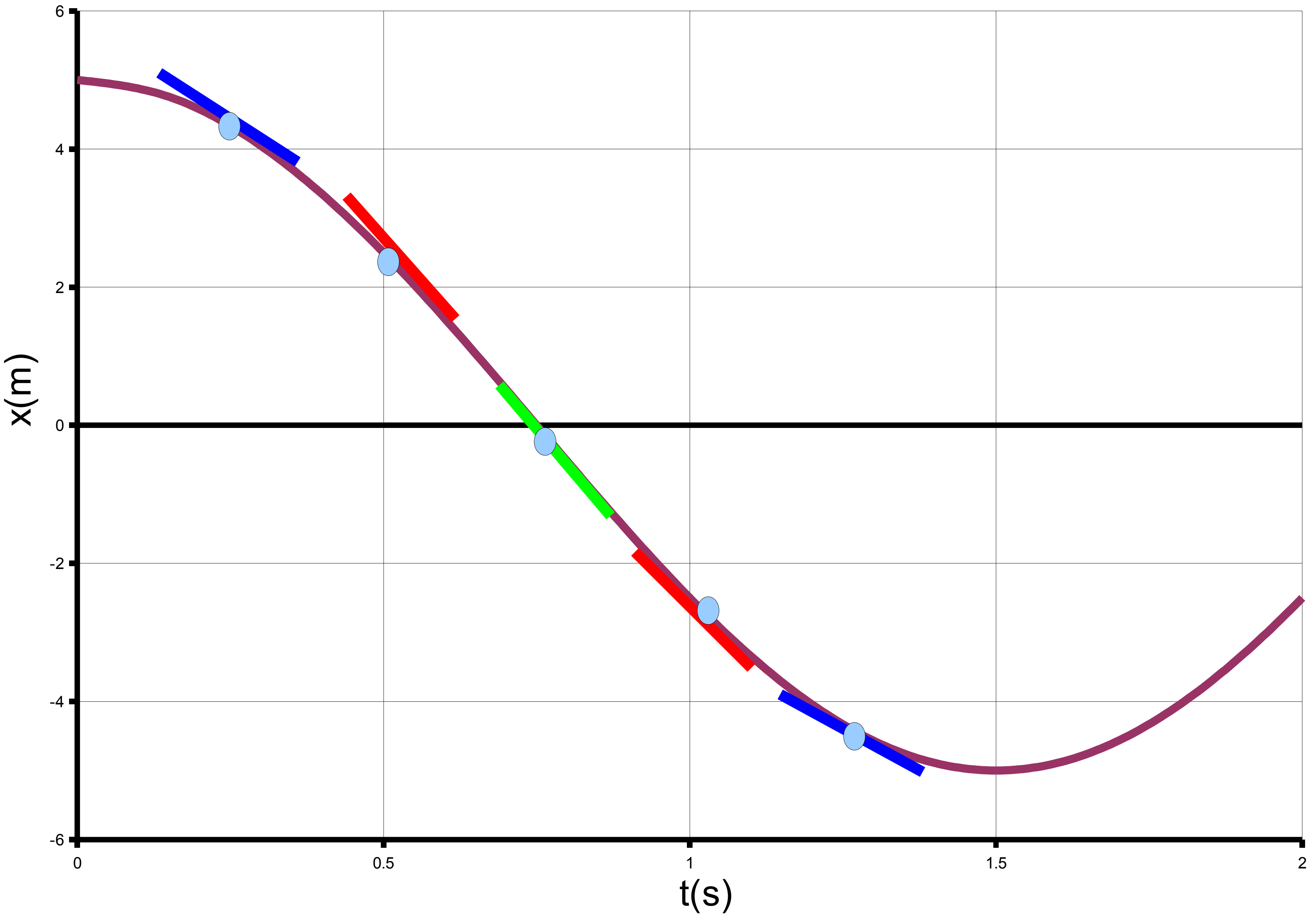


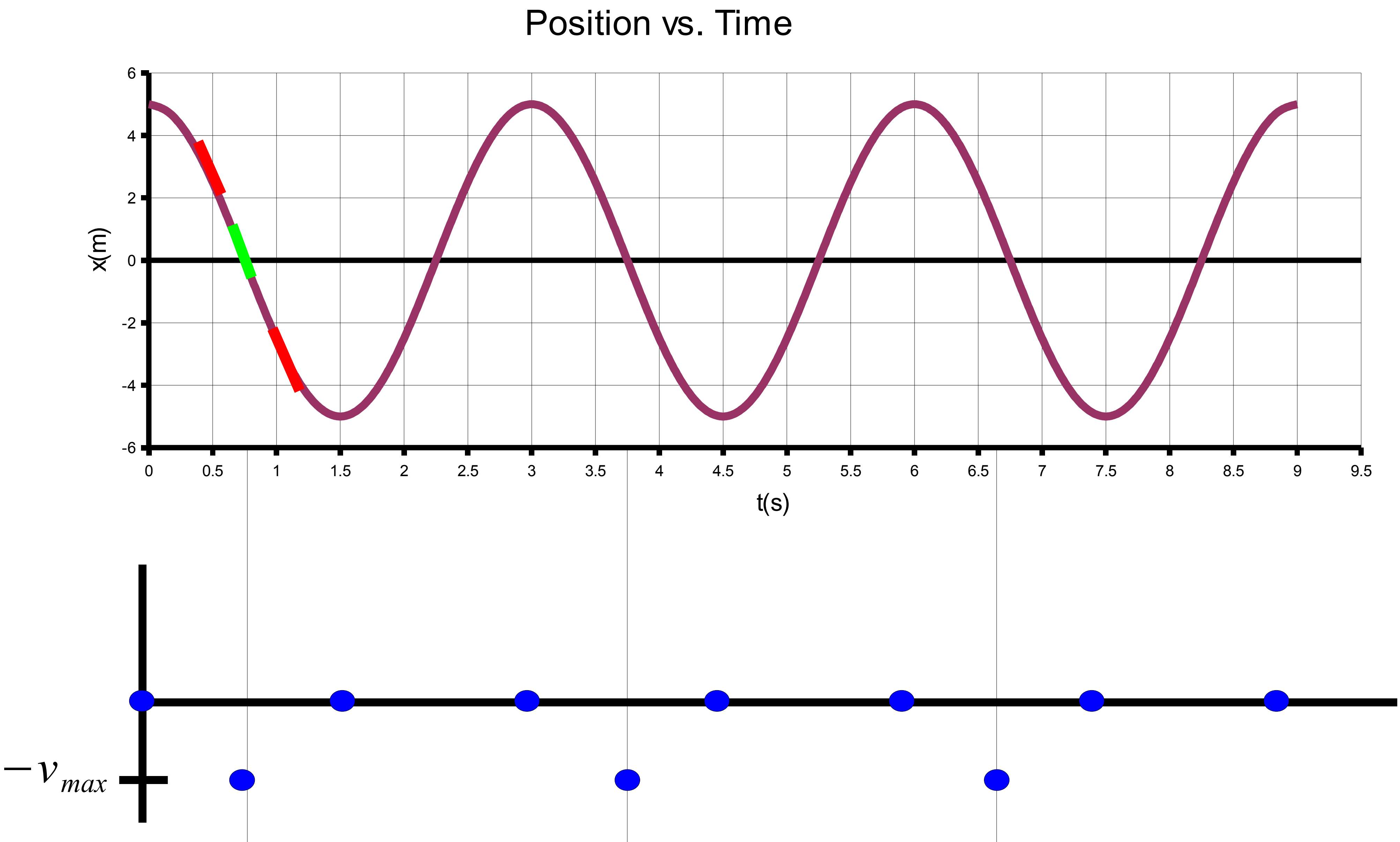
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$



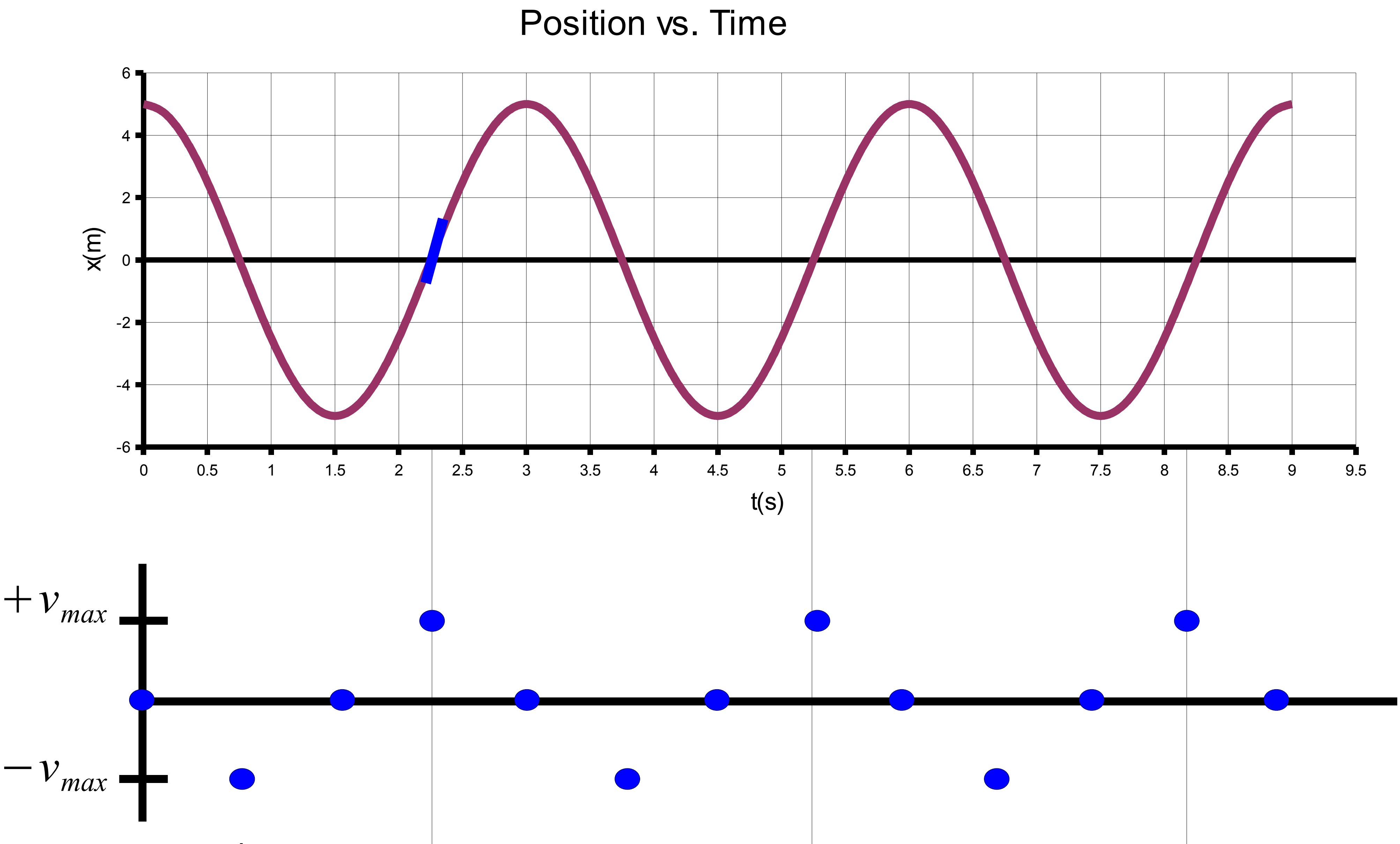
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

Position vs. Time

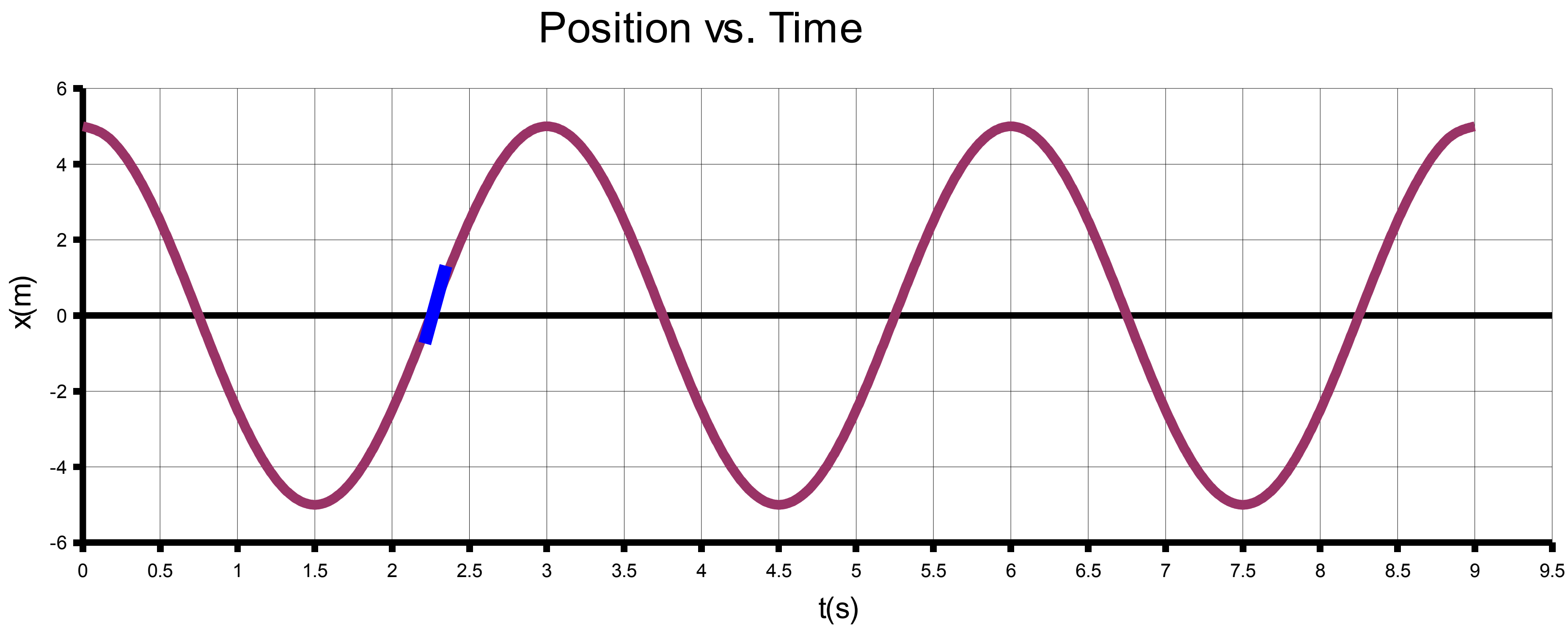


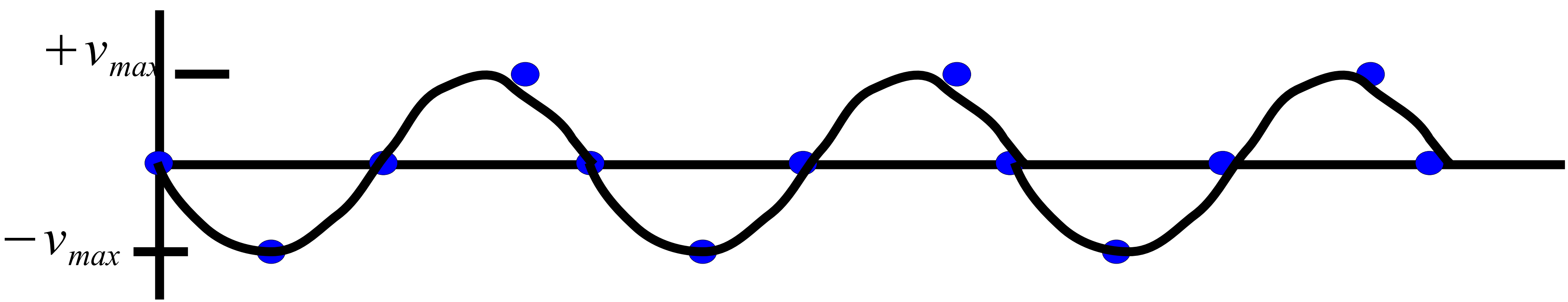


$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$



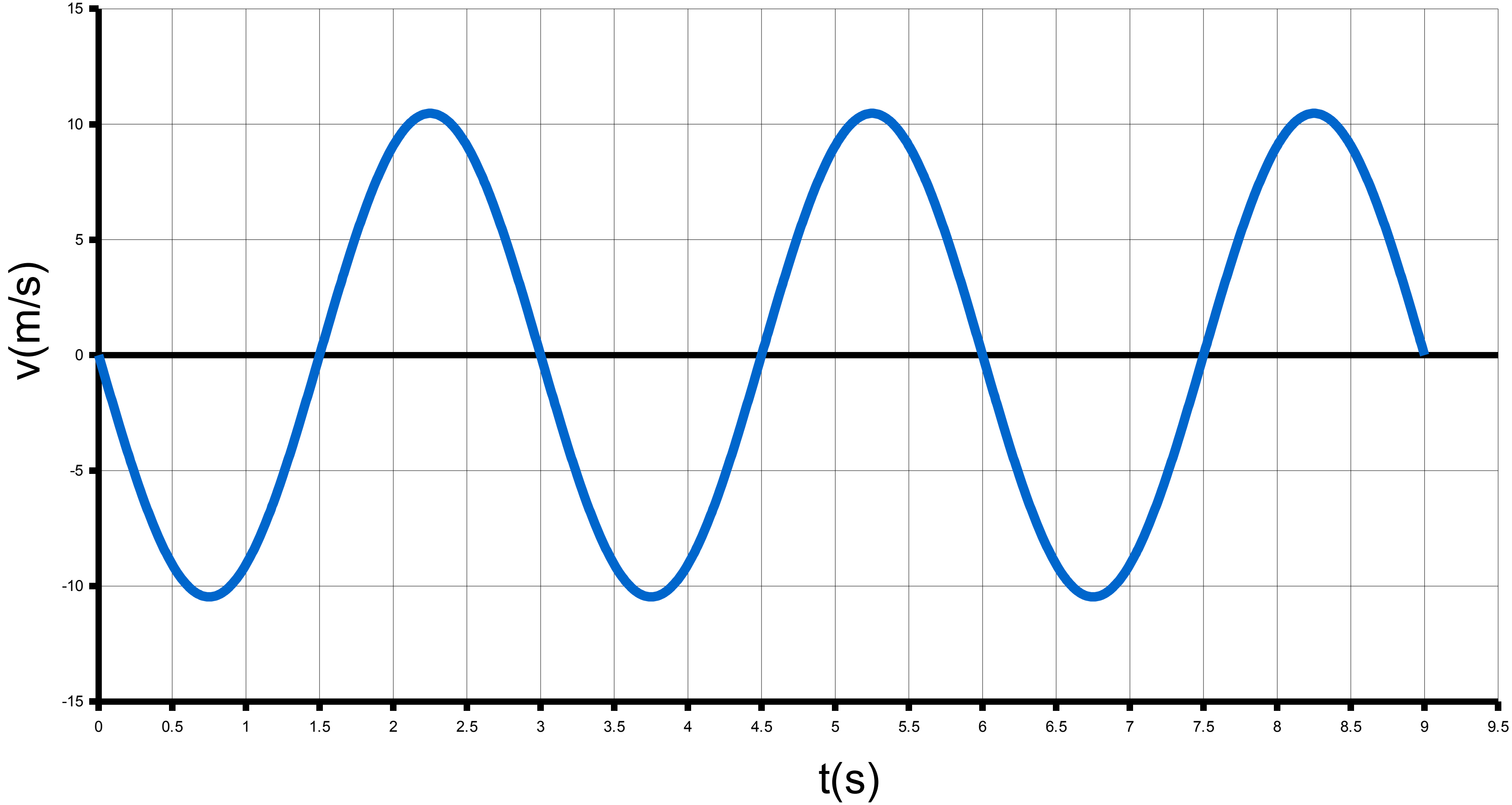
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$





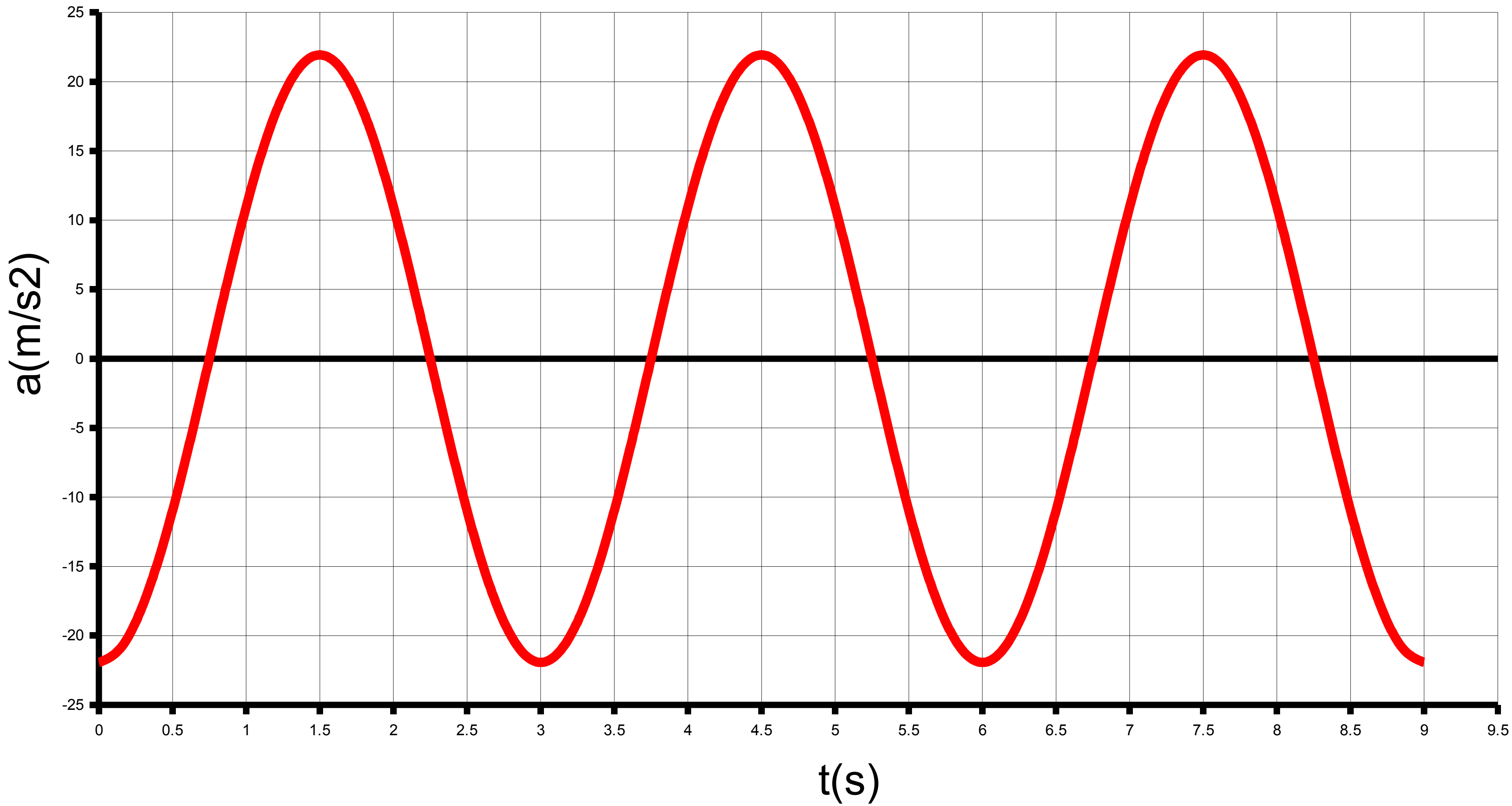
$$v_{avg} = \frac{\Delta x}{\Delta t} = \text{Slope of x vs. t plot.}$$

Velocity vs. Time



$$v(t) = -v_{max} \sin(\omega t)$$

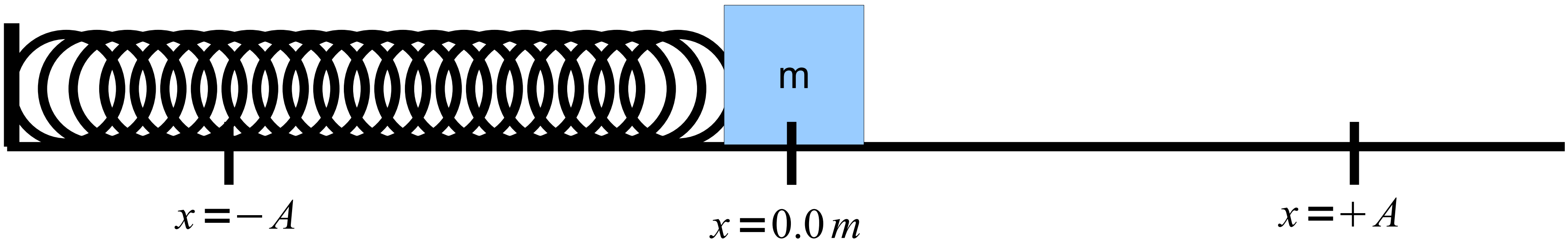
Acceleration vs. Time



$$a(t) = -a_{max}\cos(\omega t)$$

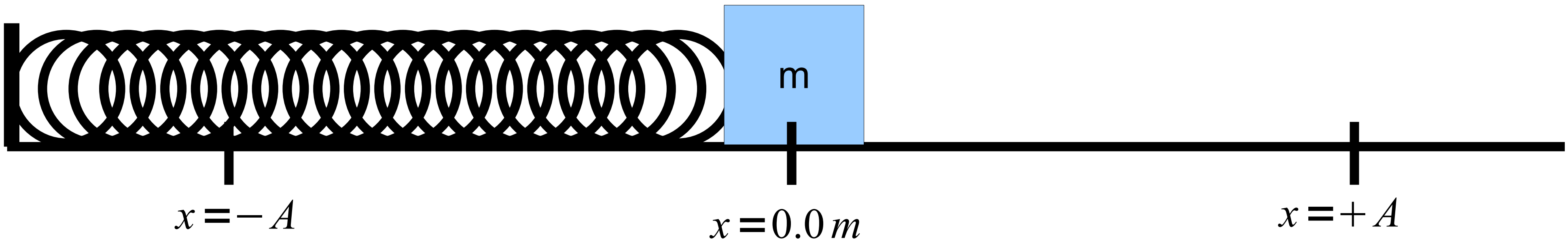
A Different Approach….

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.

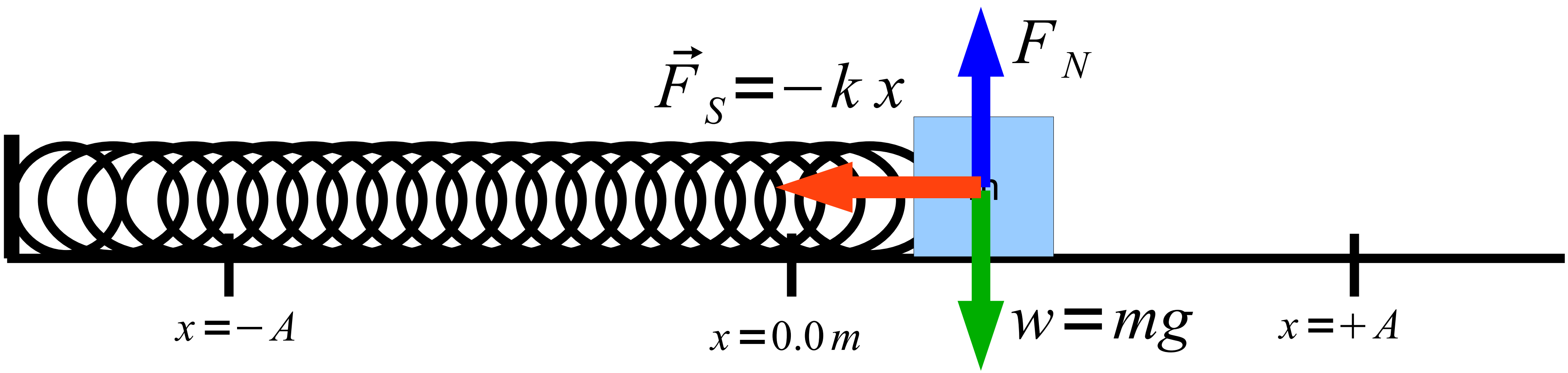


$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

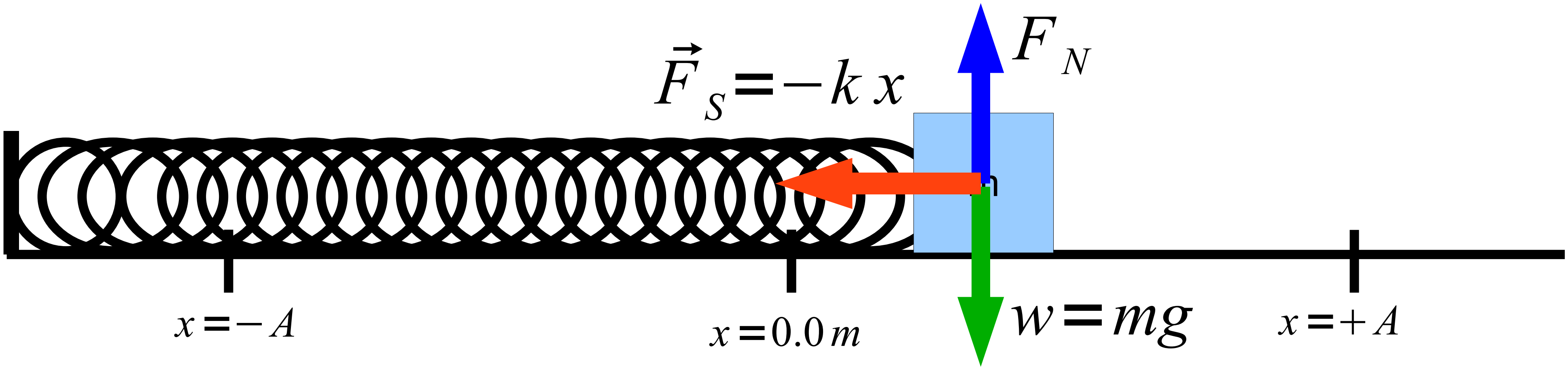
$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

$$\sum F_y = m a_y$$

$$F_N - mg = 0$$

$$F_N = mg$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi)$$

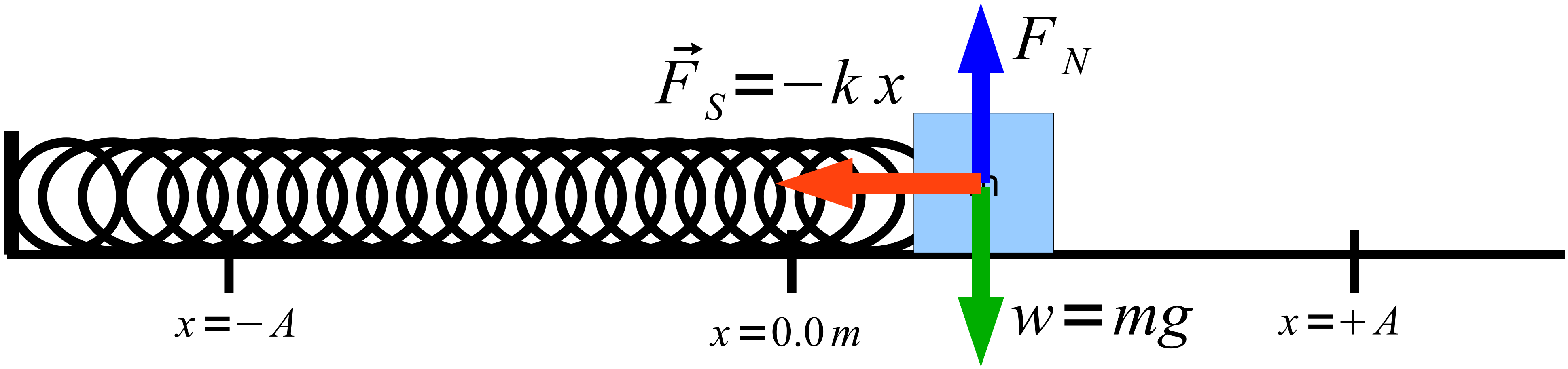
$$\sum F_x = m a_x$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-k\left[A\cos(\omega t)\right] = m\left[-A\omega^2\cos(\omega t)\right]$$

$$k = m\omega^2$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\,\omega\sin(\omega t + \phi)$$

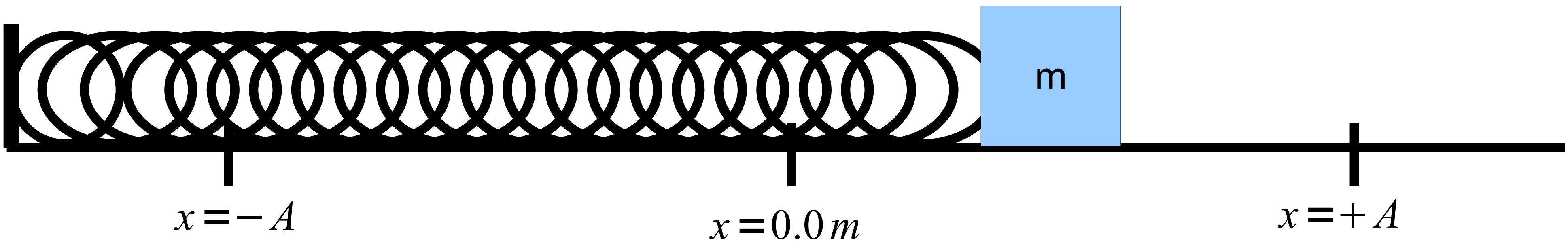
$$a(t) = \frac{dv}{dt} = -A\,\omega^2\cos(\omega t + \phi)$$

$$k = m\,\omega^2$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Consider a mass attached to a spring with a spring constant, k, at rest at x=0.0m, on a frictionless table. Then pull mass out to x = +A and release from rest.



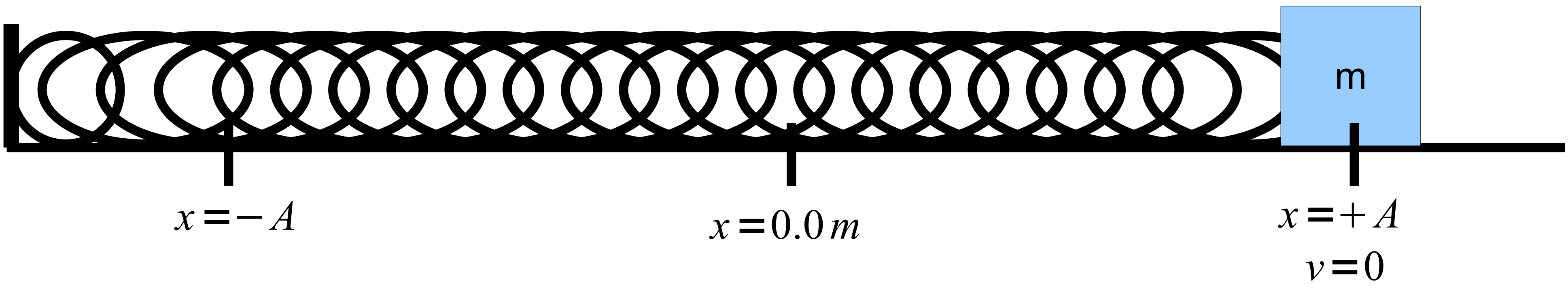
$$E_{tot} = U_s + K$$

$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

When the mass is at x = A, the velocity is zero.

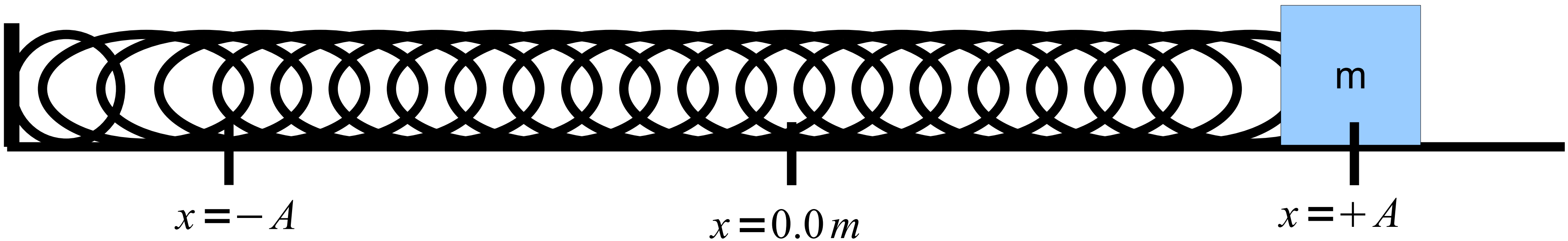
$$E_{tot} = U_s + K$$

$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$



$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \frac{1}{2} k A^2 + 0 = \frac{1}{2} k A^2$$

When the mass is at x = A, the velocity is zero.



$$E_{tot} = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

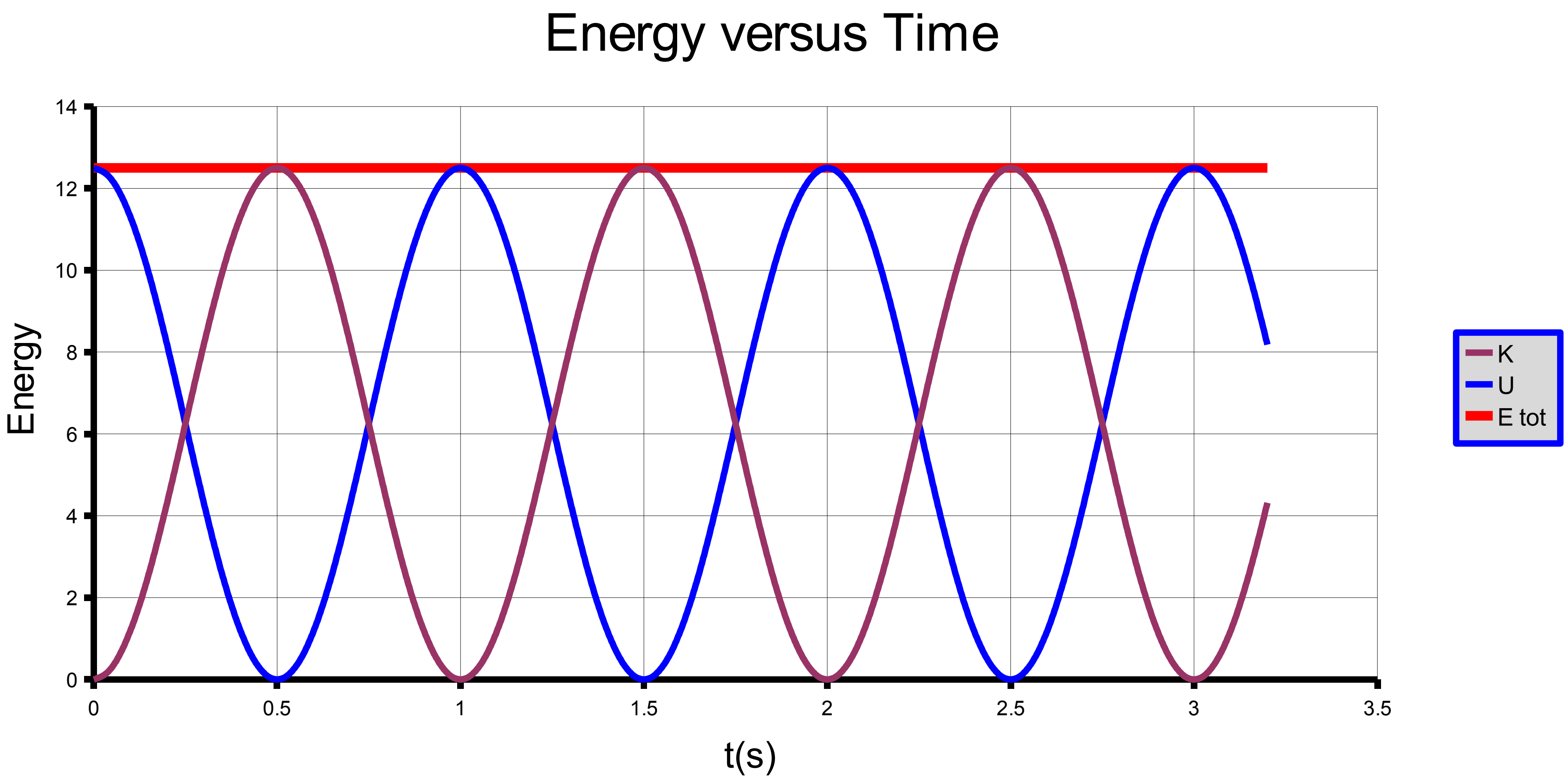
$$E_{tot} = \frac{1}{2} k \left(A \cos(\omega t) \right)^2 + \frac{1}{2} m \left(-A \omega \sin(\omega t) \right)^2$$

$$E_{tot} = \frac{1}{2} k A^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$E_{tot} = \frac{1}{2} k A^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \left(\frac{k}{m} \right) \sin^2(\omega t)$$

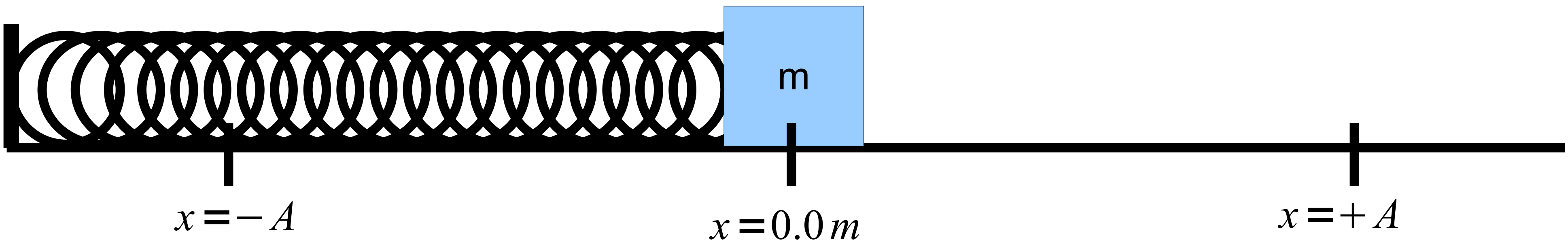
$$E_{tot} = \frac{1}{2} k A^2 \left(\cos^2(\omega t) + \sin^2(\omega t) \right) \qquad \sin^2\theta + \cos^2\theta = 1$$

$$E_{tot} = \frac{1}{2} k A^2$$


Energy versus Time
Energy
14
12
10
8
6
4
2
0
0
0.5
1
1.5
2
2.5
3
3.5
t(s)
K
U
E tot

When the mass is at x = 0.0 m, there is no energy stored in the spring, and the velocity is equal to the maximum velocity.

$$\frac{1}{2}k\,x^2+\frac{1}{2}m\,v^2=\frac{1}{2}k\,A^2$$

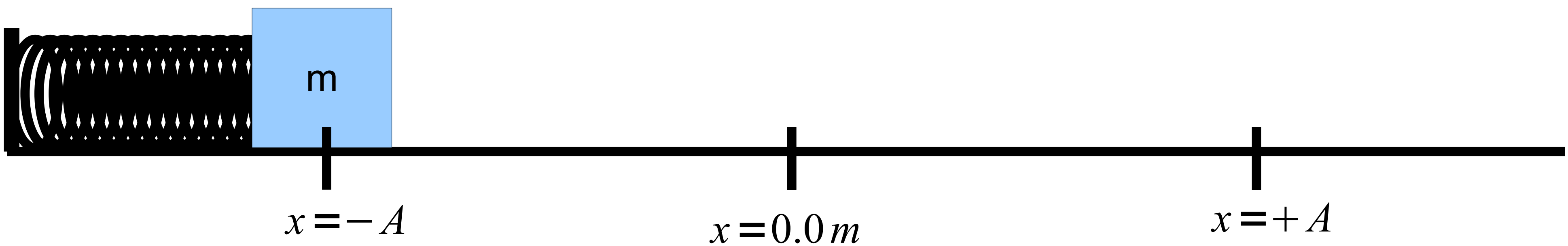


$$0+\frac{1}{2}m\,v_{max}^2=\frac{1}{2}k\,A^2$$

$$v_{max}=A\sqrt{\frac{k}{m}}$$

When the mass is at x = 0.0 m, there is no energy stored in the spring, and the velocity is equal to the maximum velocity.

$$F_s = -k\,x$$

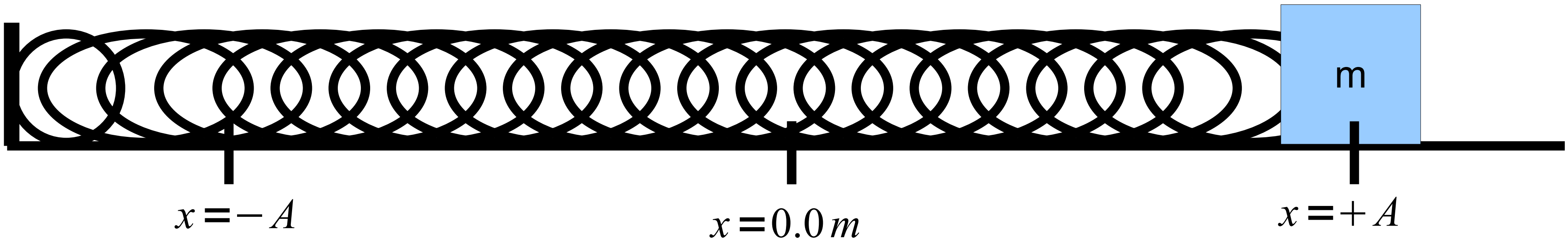


$$F_{net} = F_s$$

$$ma = -kx$$

$$a_{max} = -\left(\frac{k}{m}\right)\left(-A\right) = \left(\frac{k}{m}\right)A$$

Energy can be used to find a function for the magnitude of the velocity at any position (x).



$$\frac{1}{2}k\,x^2+\frac{1}{2}m\,v^2=\frac{1}{2}k\,A^2$$

$$\frac{1}{2}m\,v^2=\frac{1}{2}k\,A^2-\frac{1}{2}k\,x^2$$

$$m\,v^2=k\,A^2-k\,x^2$$

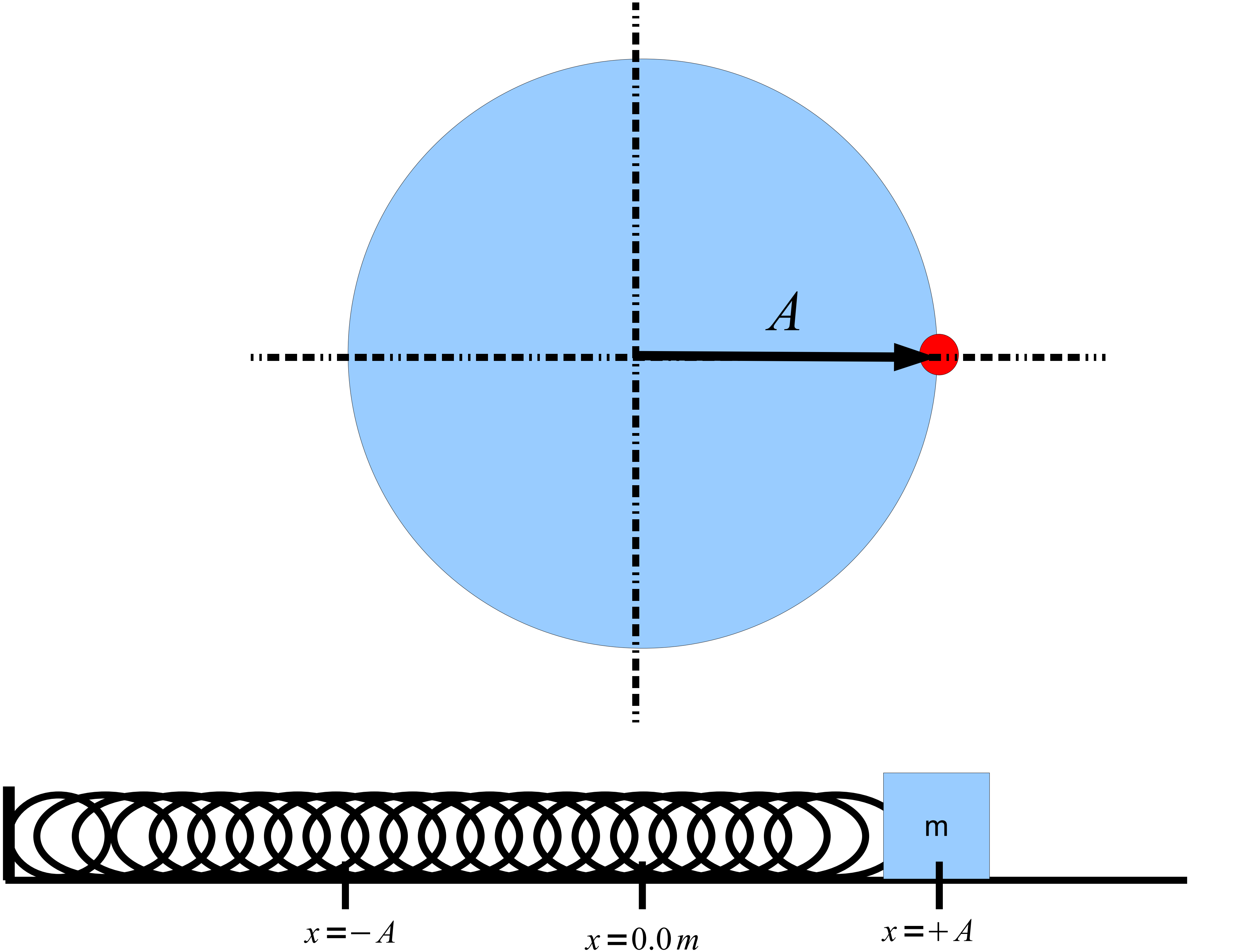
$$m\,v^2=k\left(A^2-x^2\right)$$

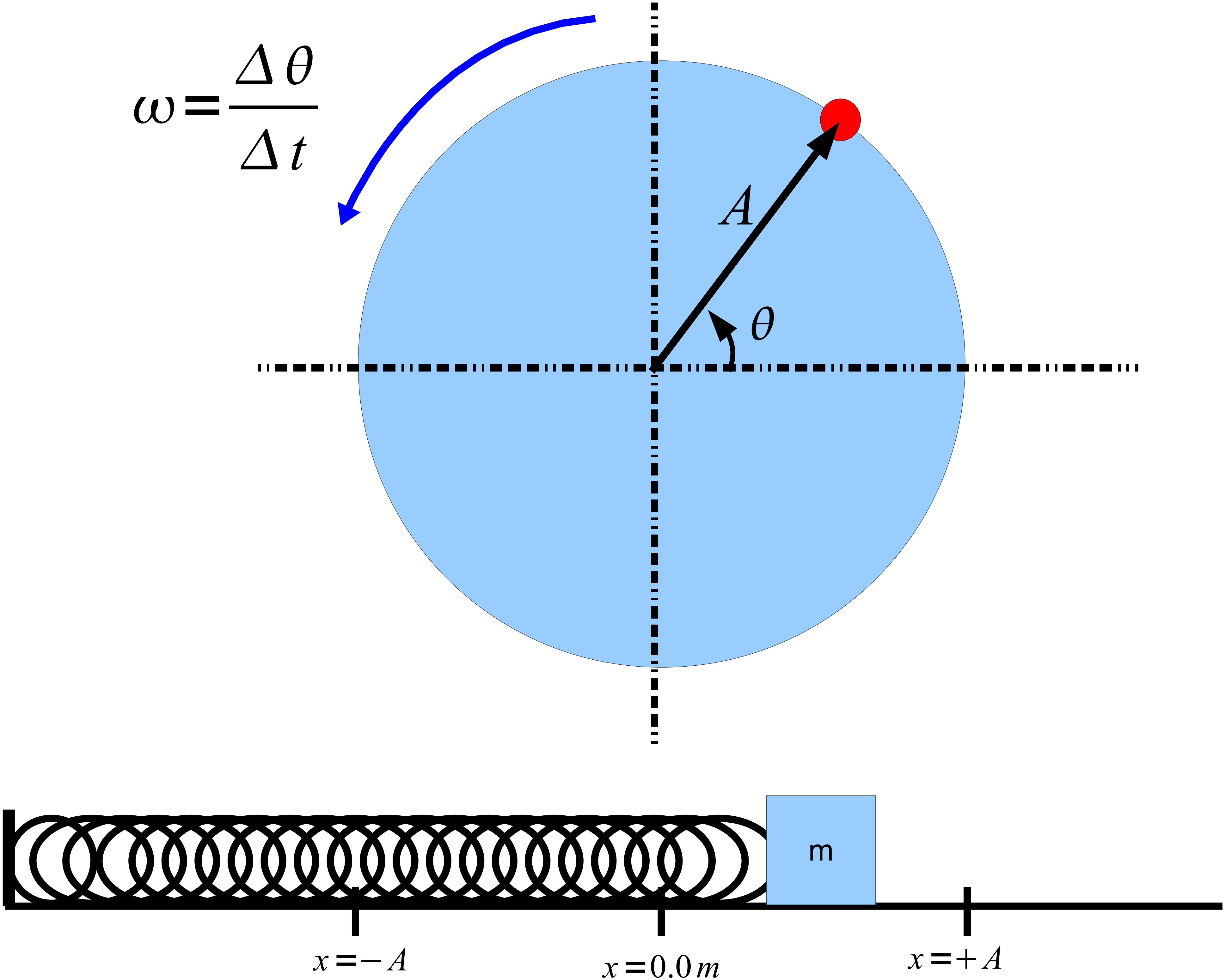
$$v^2=\frac{k}{m}\left(A^2-x^2\right)$$

$$|v|=\sqrt{\frac{k}{m}\left(A^2-x^2\right)}$$

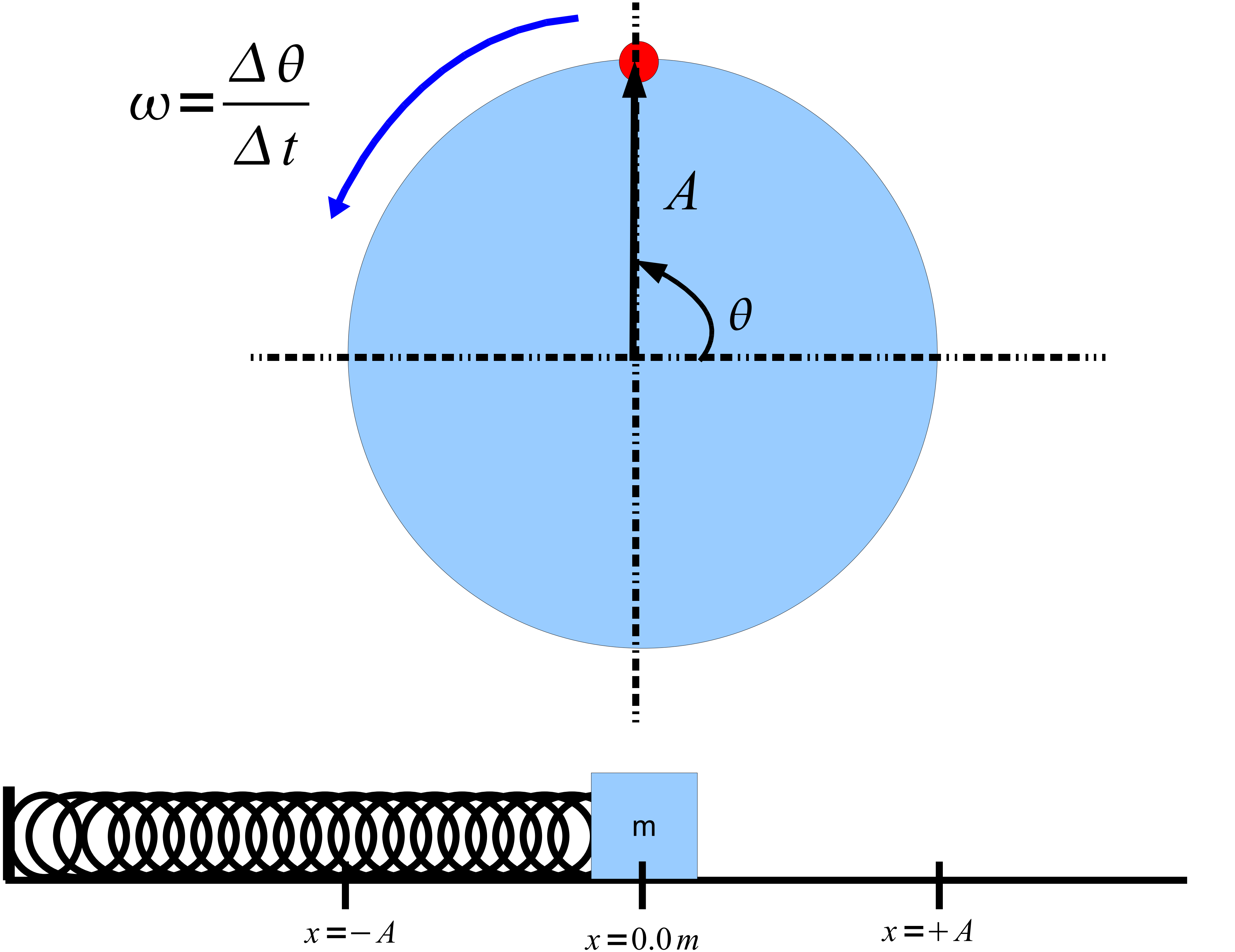
$$|v(x)|=\sqrt{\frac{k}{m}\left(A^2-x^2\right)}$$

Simple Harmonic Motion and Circular Motion

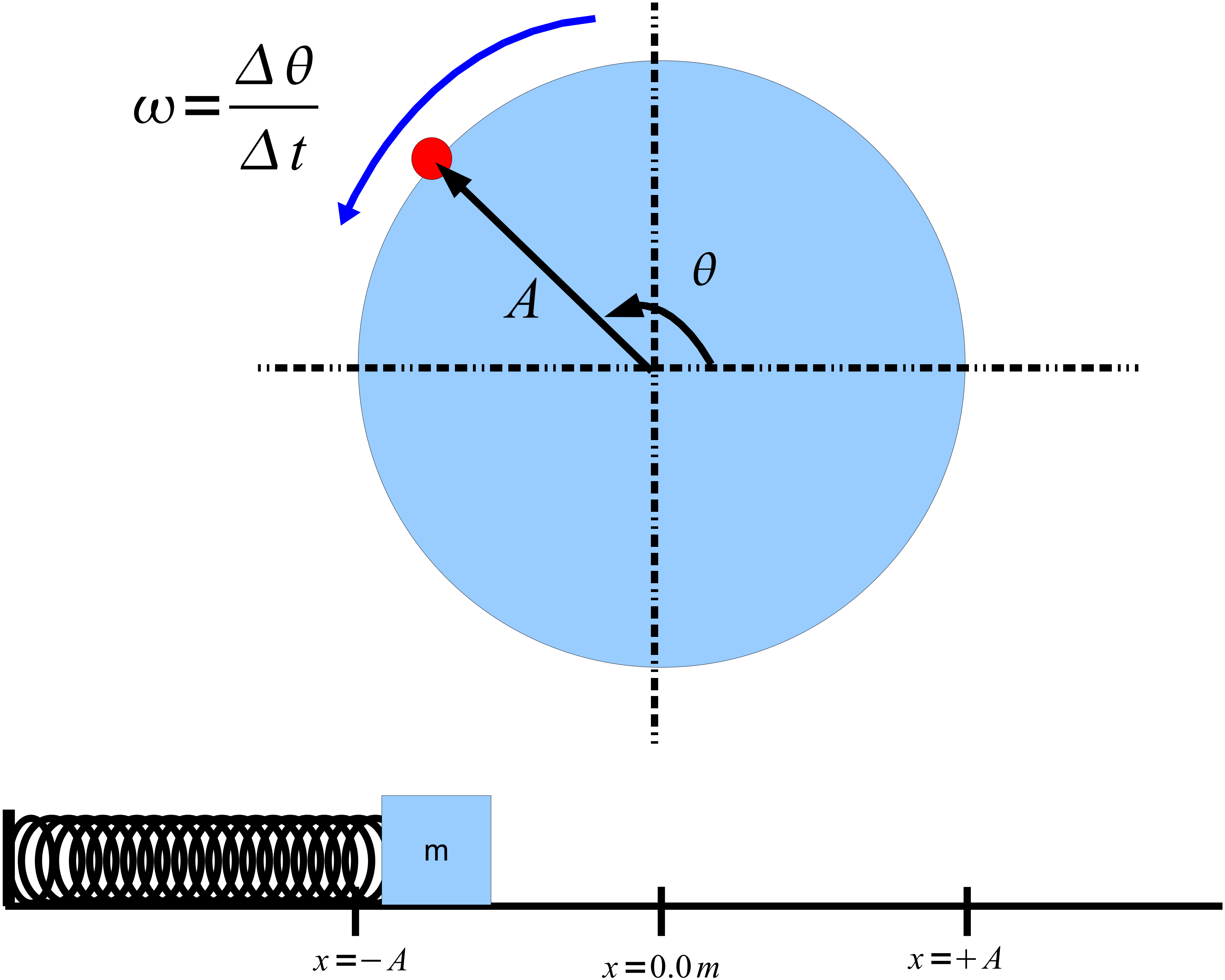

A
m
x = −A
x = 0.0 m
x = +A



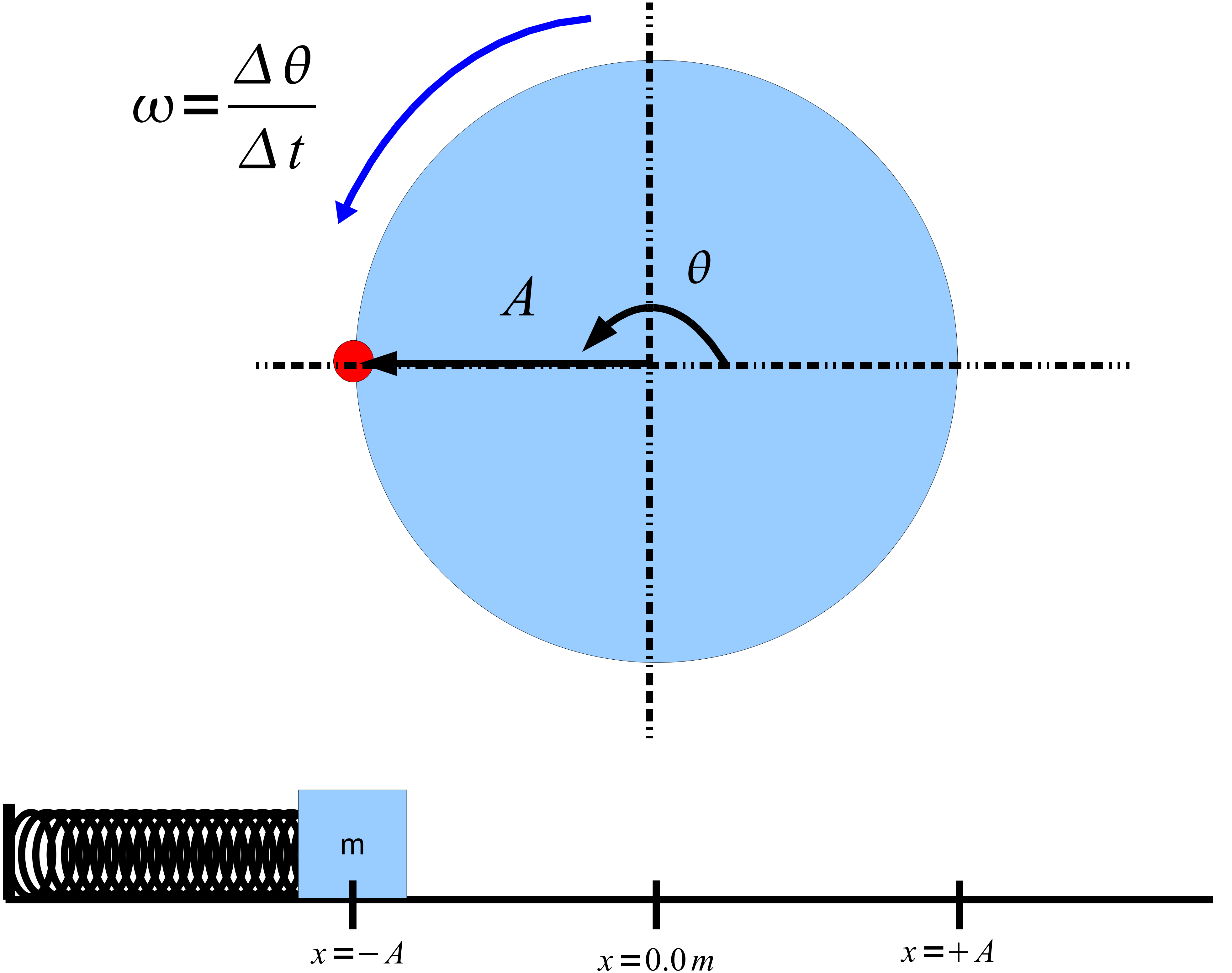
$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$



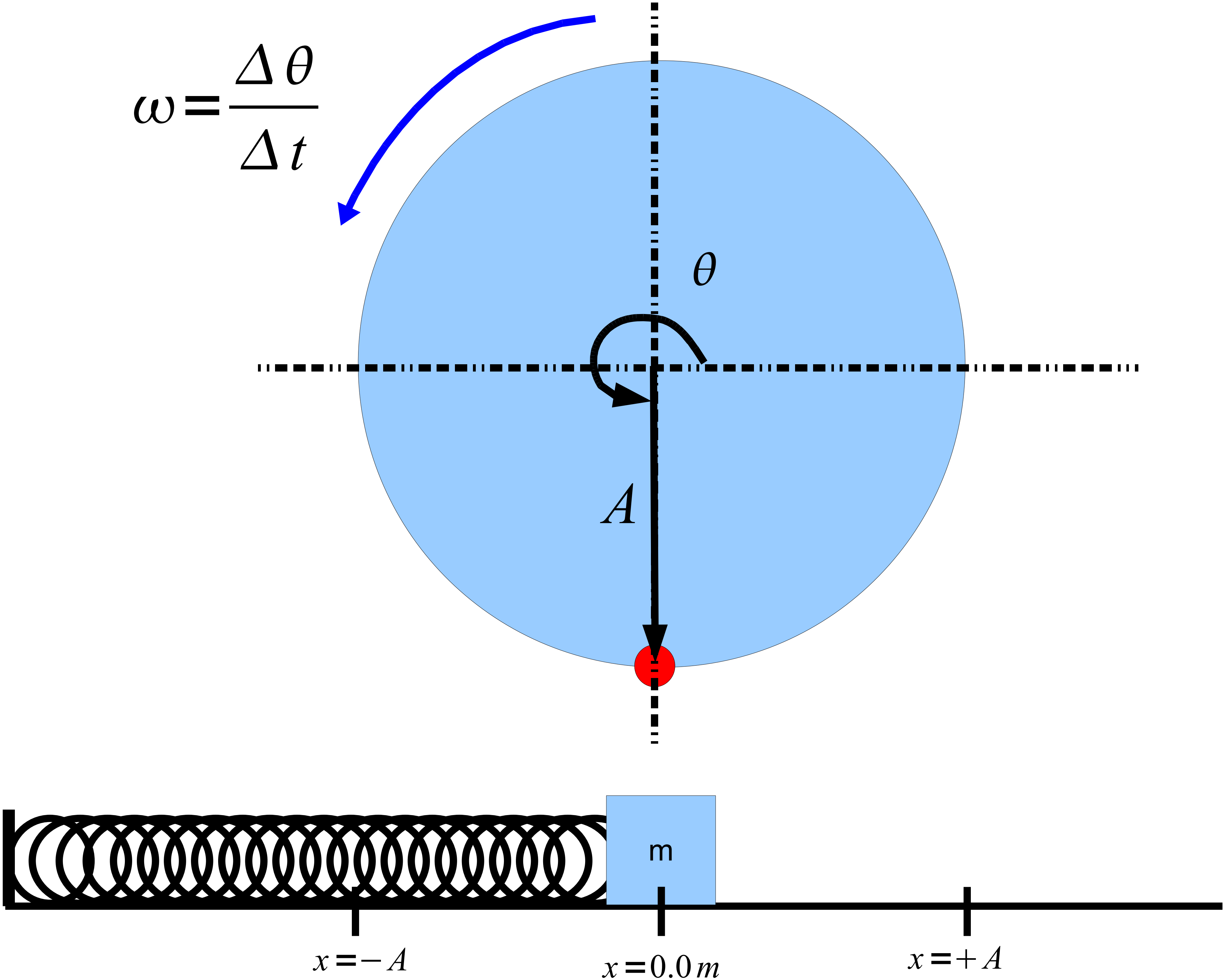
$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$



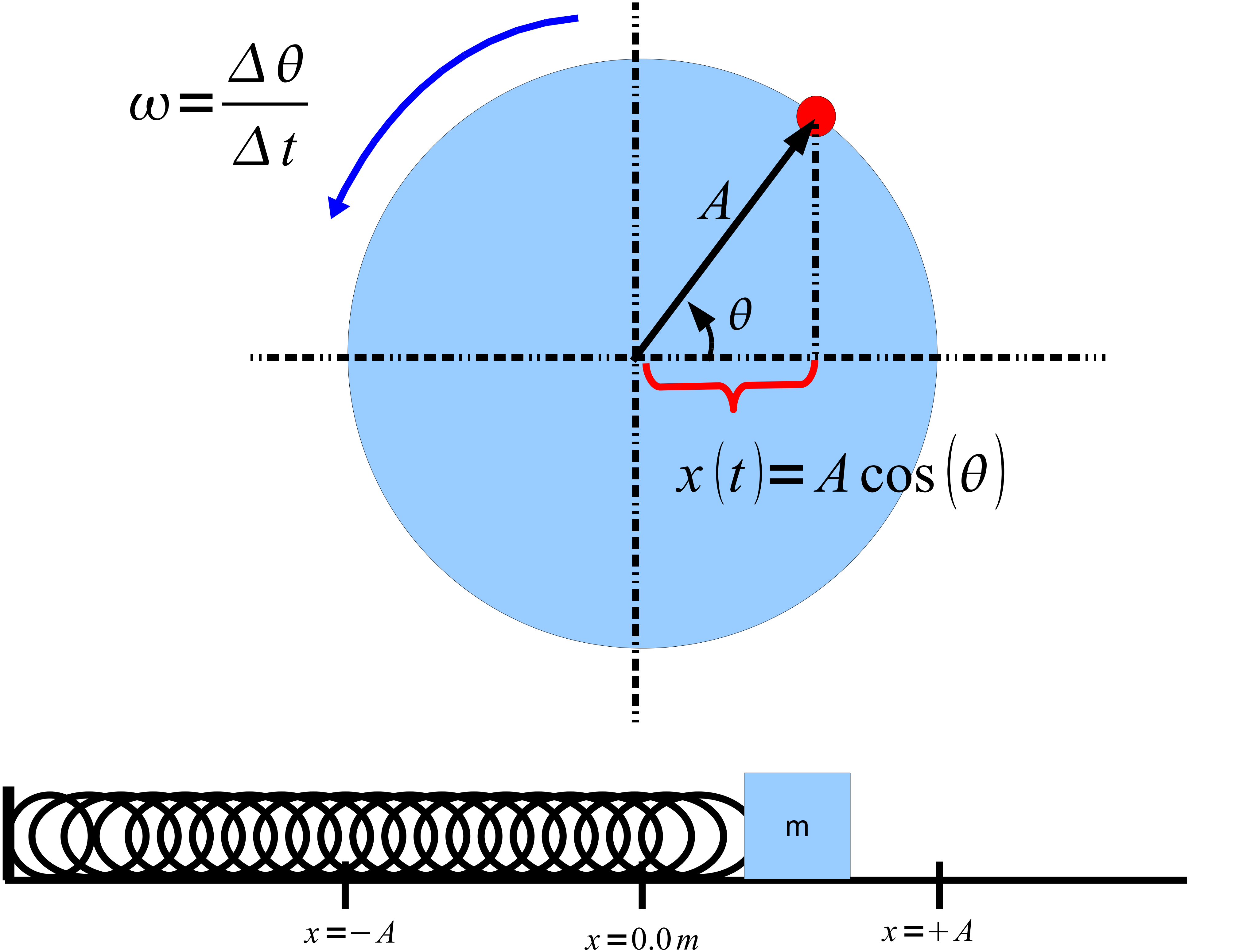
$\omega = \frac{\Delta \theta}{\Delta t}$
A
θ
m
$x = -A$
$x = 0.0\,m$
$x = +A$



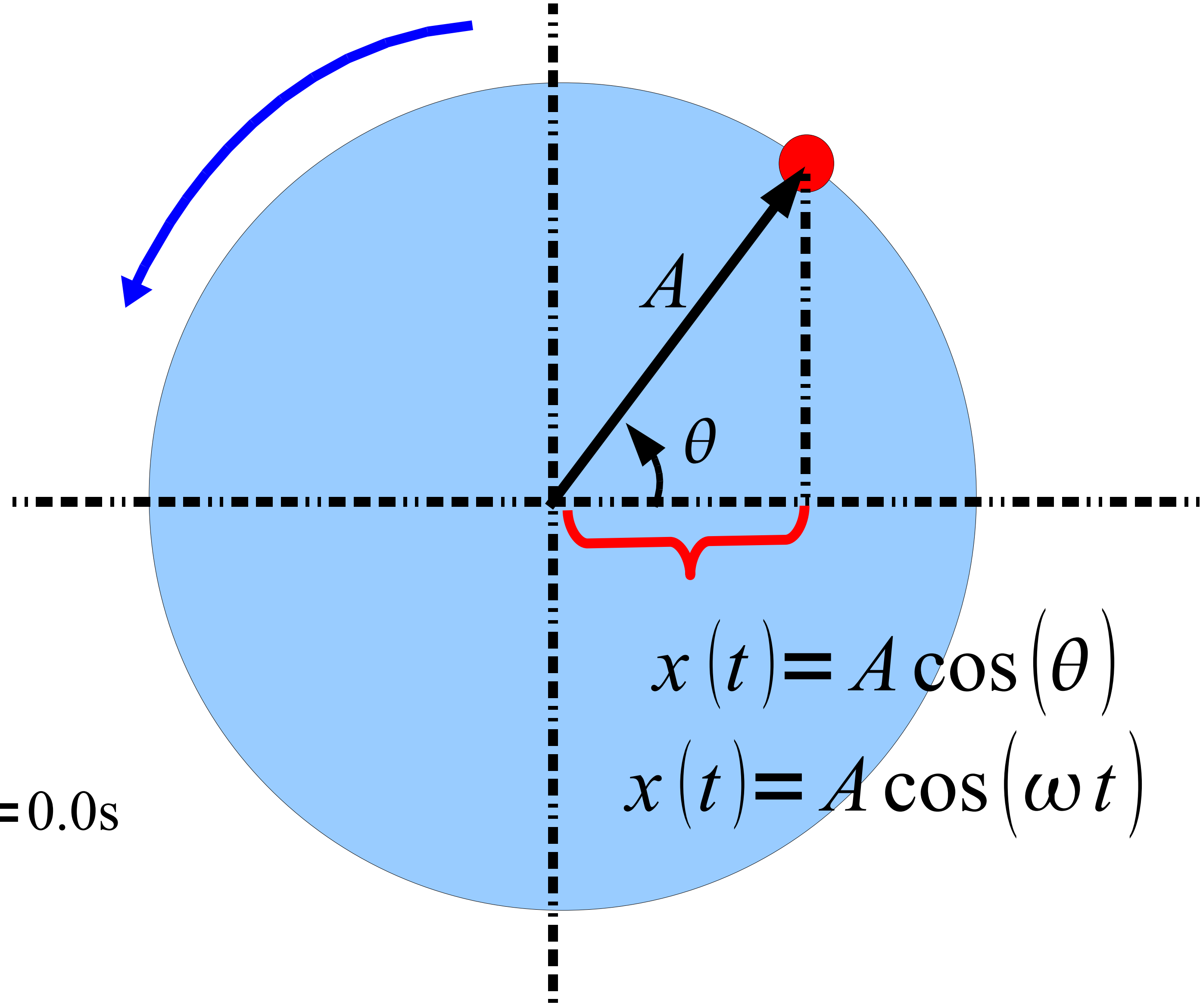
$\omega = \frac{\Delta\theta}{\Delta t}$
θ
A
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$\omega = \frac{\Delta\theta}{\Delta t}$
θ
A
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$\omega = \frac{\Delta\theta}{\Delta t}$
A
θ
$x(t) = A\cos(\theta)$
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$$x(t) = A\cos(\theta)$$

$$x(t) = A\cos(\omega t)$$

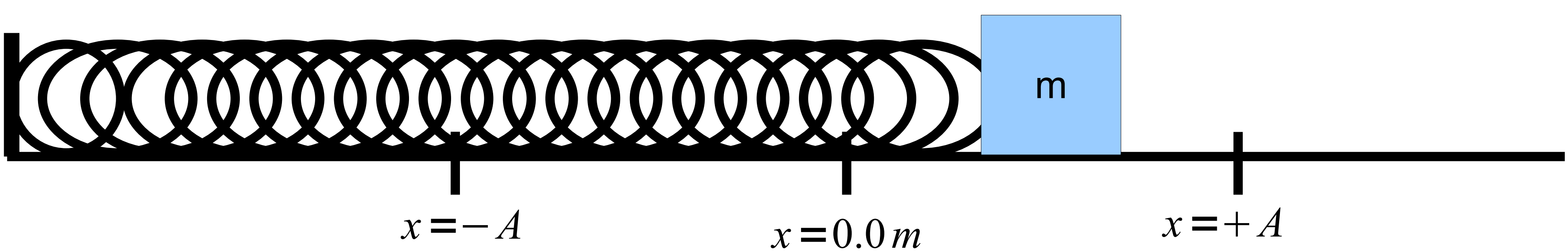
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \frac{\theta - \theta_o}{t - t_o}$$

$$\theta_o = 0^o \text{ when } t = 0.0\text{s}$$

$$\omega = \frac{\theta}{t}$$

$$\theta = \omega t$$



$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

Solutions must be cos, sin, or exponential.

$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

$$|v_{max}| = A\omega$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

$$|a_{max}| = A\omega^2$$

$$F_{net} = ma$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)x = \frac{d^2x}{dt^2}$$

$$-\left(\frac{k}{m}\right)\left(A\cos\left(\omega t + \phi\right)\right) = -A\omega^2\cos\left(\omega t + \phi\right)$$

$$\frac{k}{m} = \omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos\left(\omega t + \phi\right)$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin\left(\omega t + \phi\right)$$

$$\left|v_{max}\right| = A\omega$$

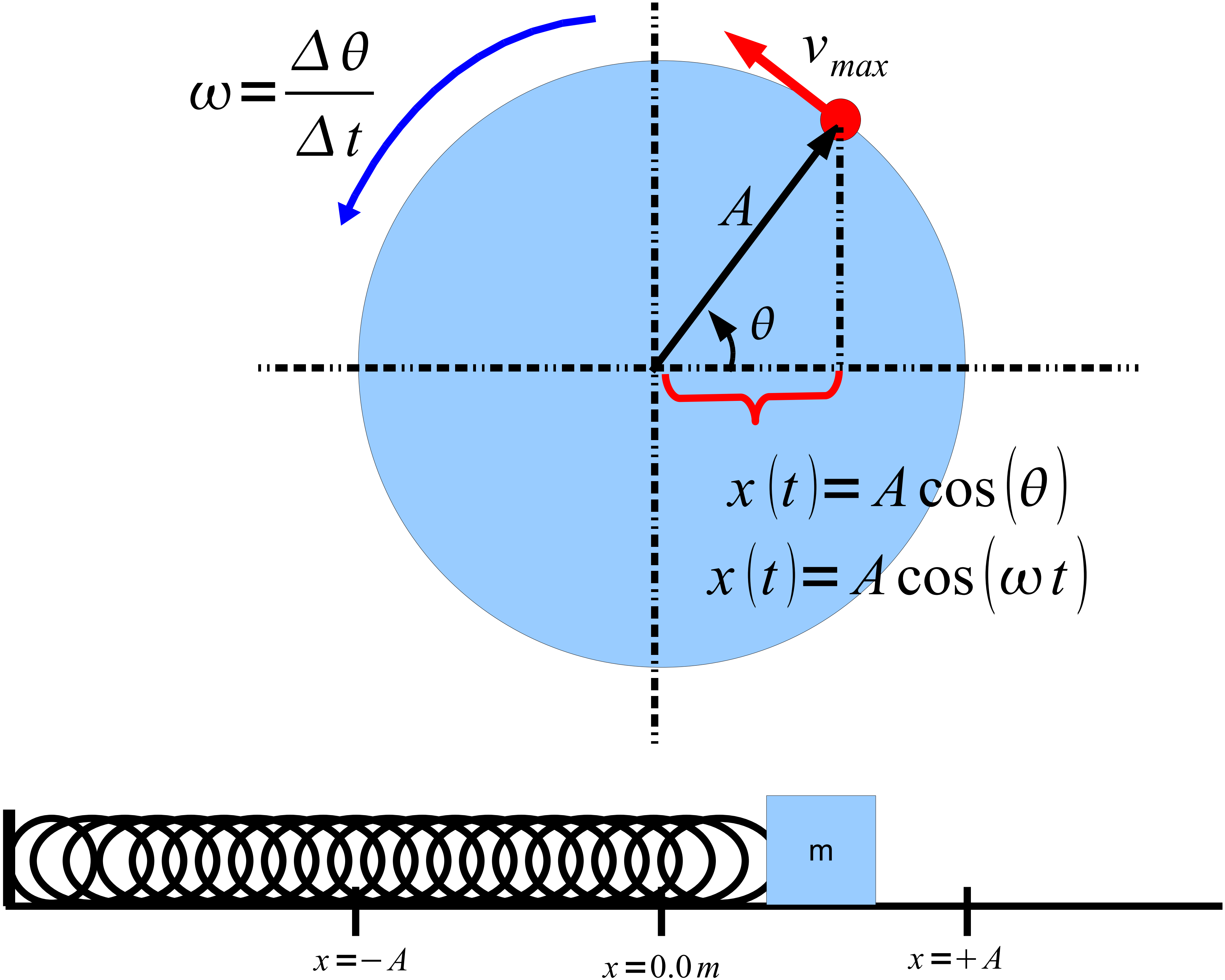
$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2\sin\left(\omega t + \phi\right)$$

$$\left|a_{max}\right| = A\omega^2$$

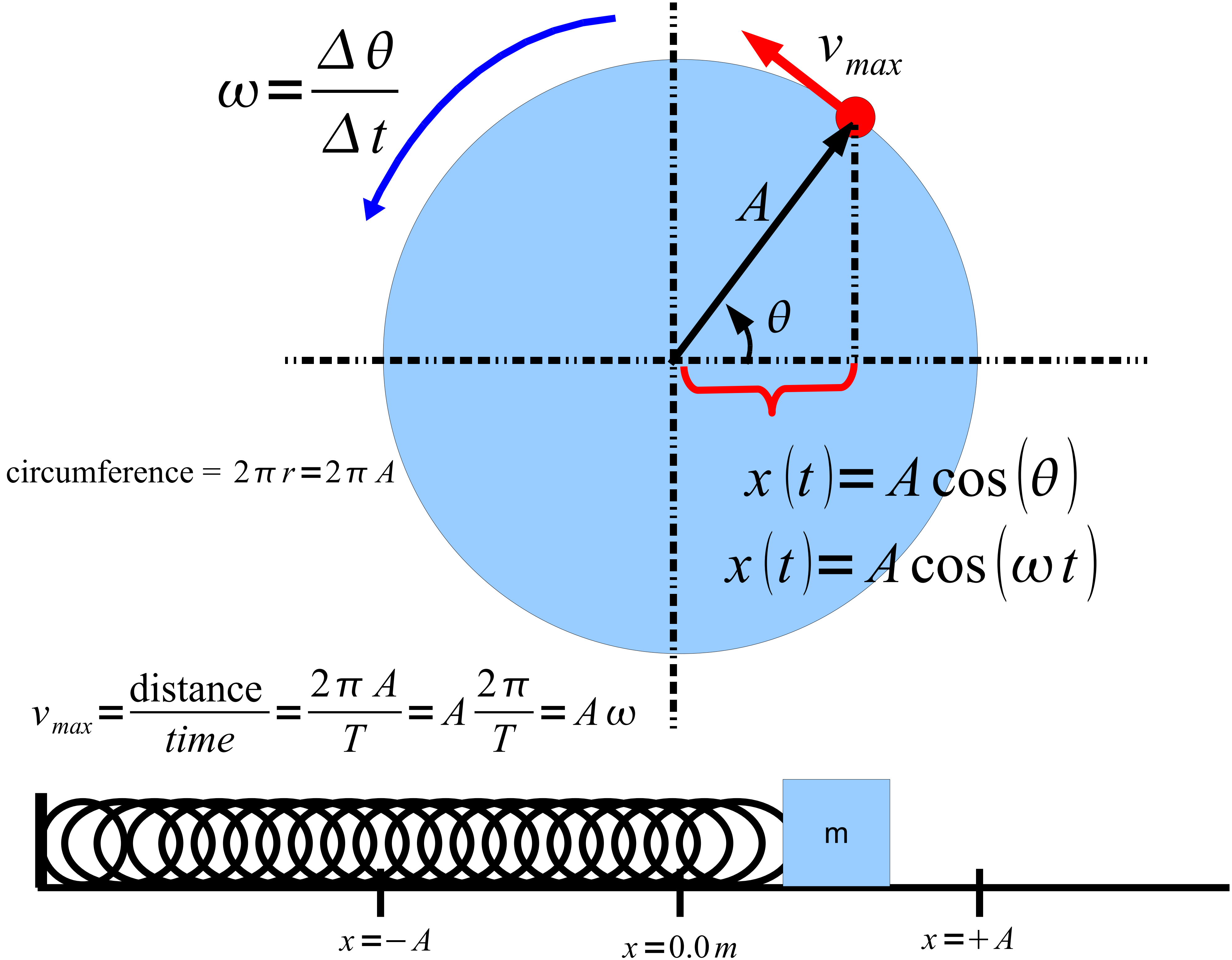
$$x(t) = A\cos(\omega t)$$

$$v(t) = -v_{max}\sin(\omega t) = -A\omega\sin(\omega t)$$

$$a(t) = -a_{max}\cos(\omega t) = -A\omega^2\cos(\omega t)$$



$\omega = \frac{\Delta\theta}{\Delta t}$
v_{max}
A
θ
$x(t) = A\cos(\theta)$
$x(t) = A\cos(\omega t)$
m
$x = -A$
$x = 0.0\,m$
$x = +A$



$\omega = \frac{\Delta\theta}{\Delta t}$
v_{max}
A
θ
$x(t) = A\cos(\theta)$
$x(t) = A\cos(\omega t)$
circumference = $2\pi r = 2\pi A$
$v_{max} = \frac{\text{distance}}{time} = \frac{2\pi A}{T} = A\frac{2\pi}{T} = A\omega$
m
$x = -A$
$x = 0.0\,m$
$x = +A$

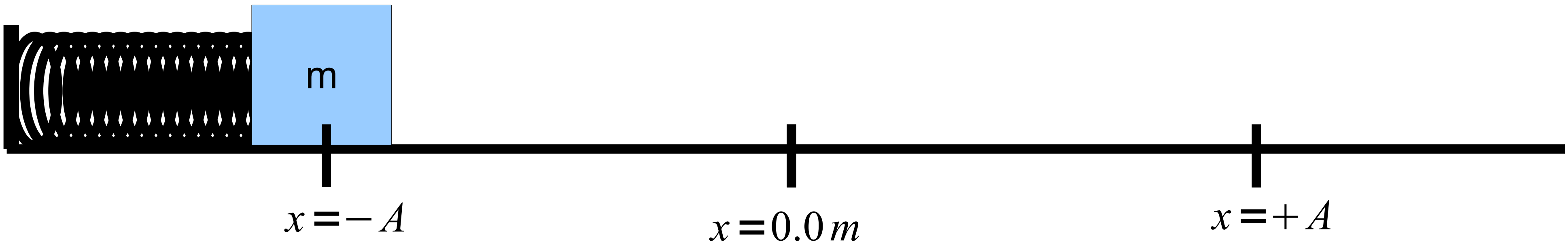
Recall:

$$0 + \frac{1}{2} m v_{max}^2 = \frac{1}{2} k A^2$$

$$v_{max} = A \sqrt{\frac{k}{m}}$$

$$v_{max} = A \sqrt{\frac{k}{m}} = A \omega$$

$$\omega = \sqrt{\frac{k}{m}}$$



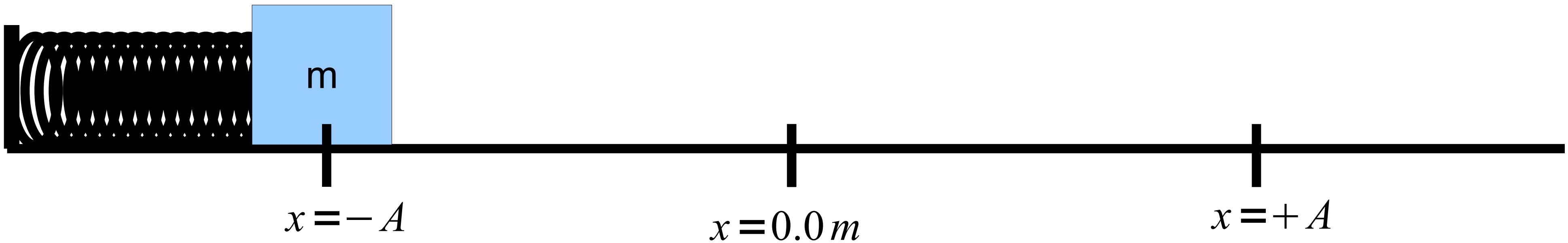
$$F_{net} = F_s$$

$$ma = -k\,x$$

$$m\left(-A\,\omega^2 \cos\left(\omega\,t\right)\right) = -k\left(A\,\omega \cos\left(\omega\,t\right)\right)$$

$$m = k\,\omega$$

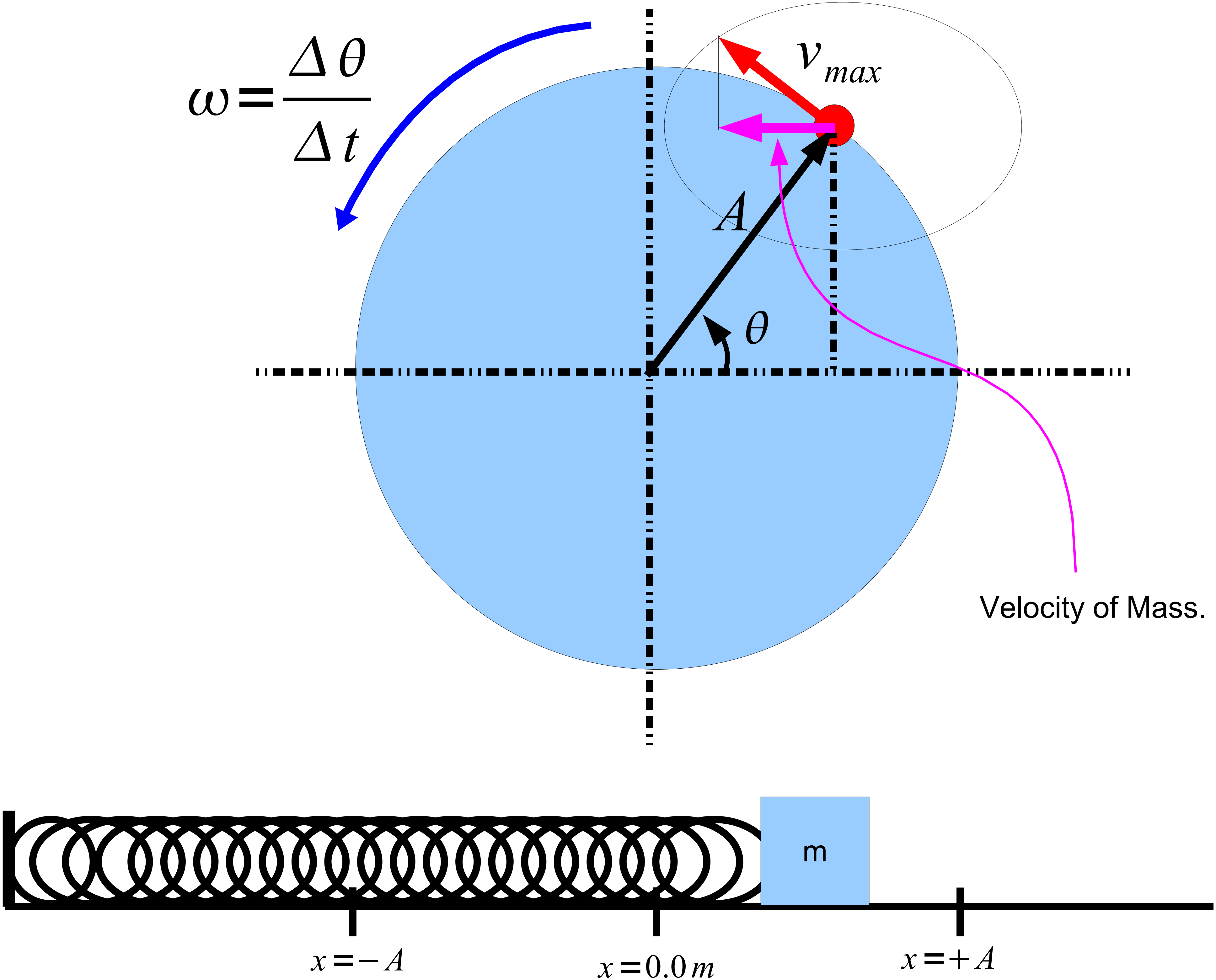
$$\omega = \sqrt{\frac{k}{m}}$$



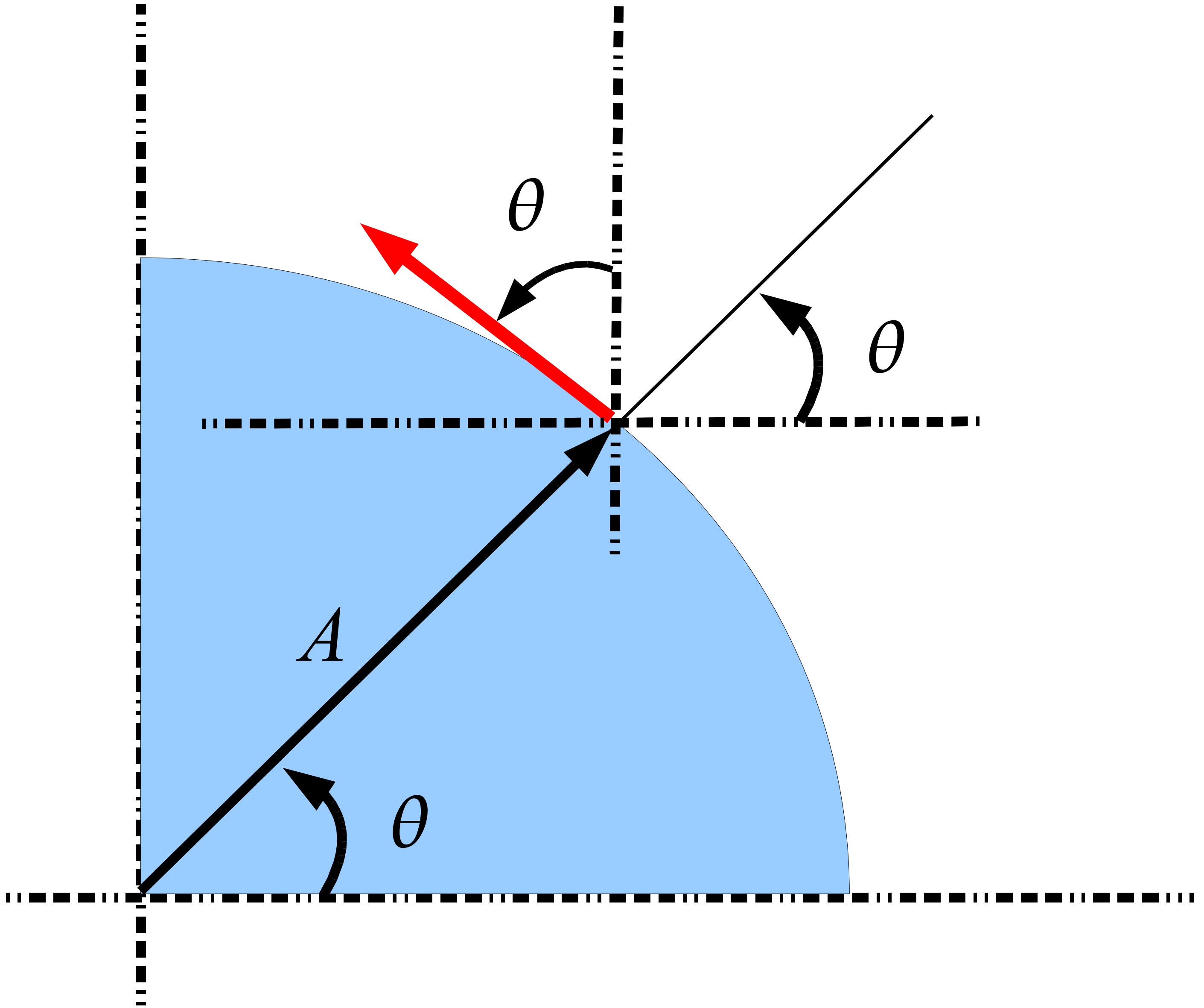
$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \frac{2\pi}{T}$$

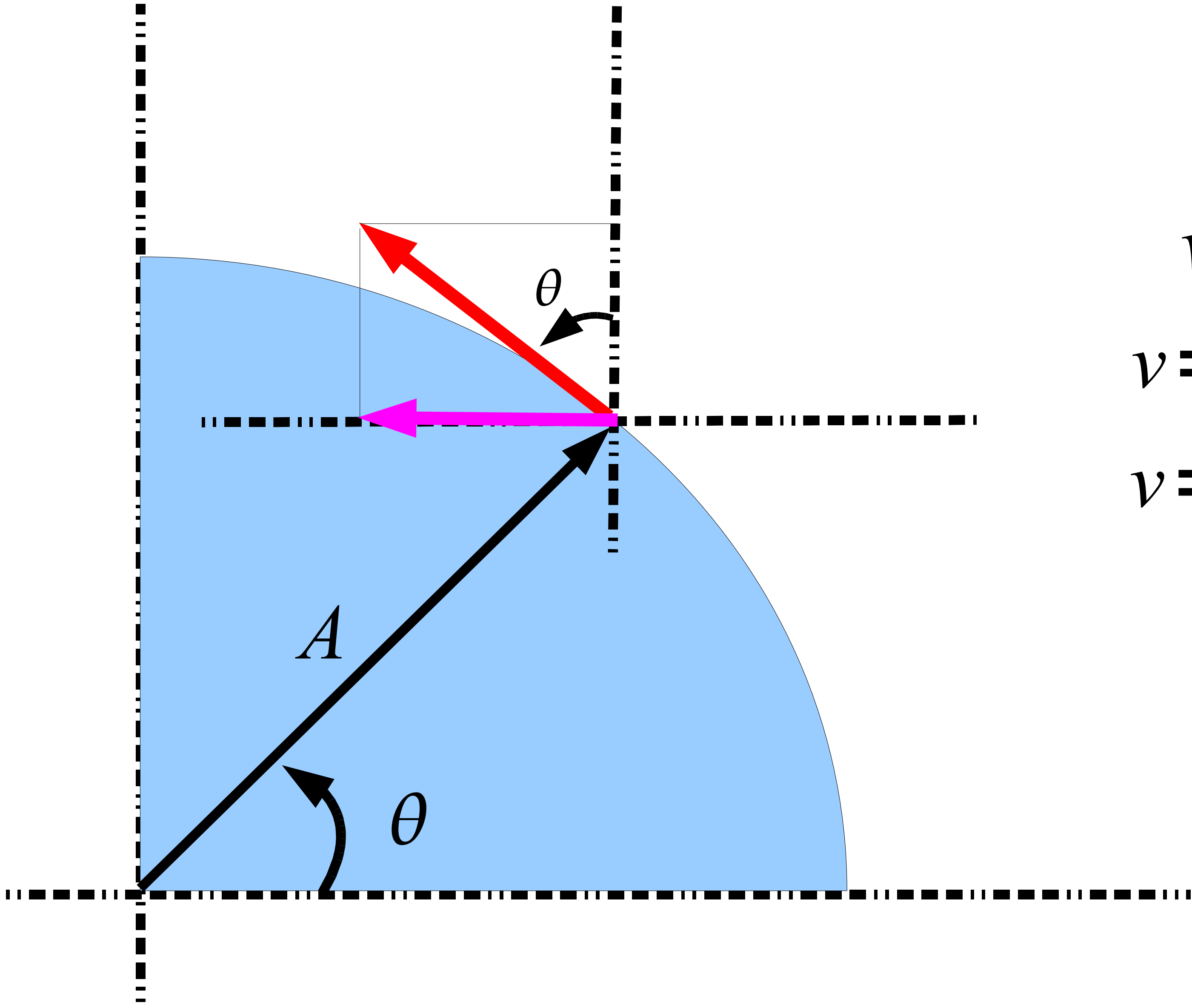
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$



$\omega = \frac{\Delta\theta}{\Delta t}$
v_{max}
A
θ
Velocity of Mass.
m
$x = -A$
$x = 0.0\,m$
$x = +A$

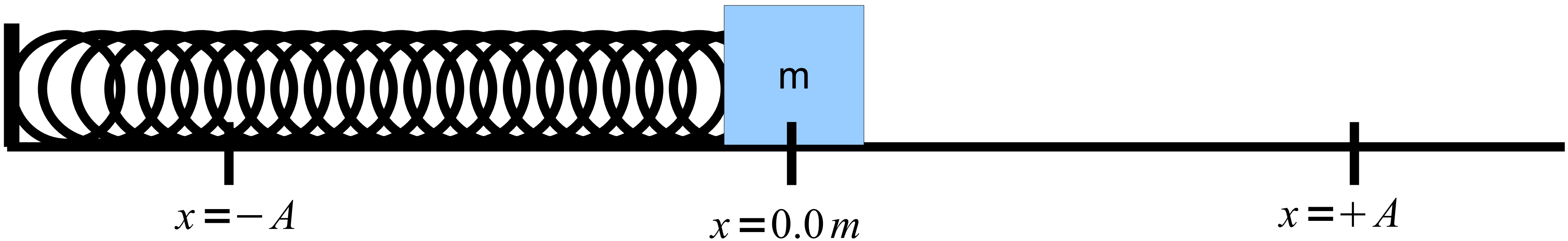


θ
θ
A
θ



$$v = -v_{max} \sin\theta$$
$$v = -v_{max} \sin(\omega t)$$
$$v = -A\omega \sin(\omega t)$$

So what we know so far ....



$$x(t) = A\cos(\omega t)$$

$$v(t) = -v_{max}\sin(\omega t) = -A\omega\sin(\omega t)$$

$$a(t) = -a_{max}\cos(\omega t) = -A\omega^2\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$|v_{max}| = A\omega = A\sqrt{\frac{k}{m}}$$

$$|a_{max}| = A\omega^2 = A\left(\frac{k}{m}\right)$$

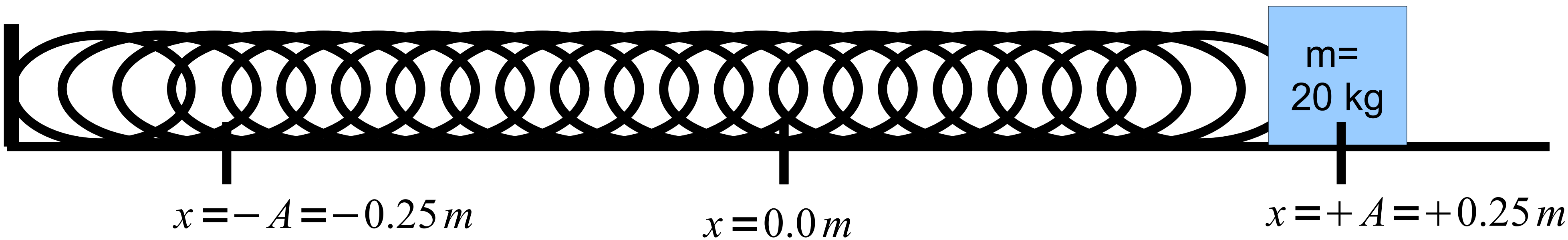
$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v(x) = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$k = 100\,N/m$ **Mass is released from rest at x = +A = 0.25 m**



$$A = 0.25\,m$$

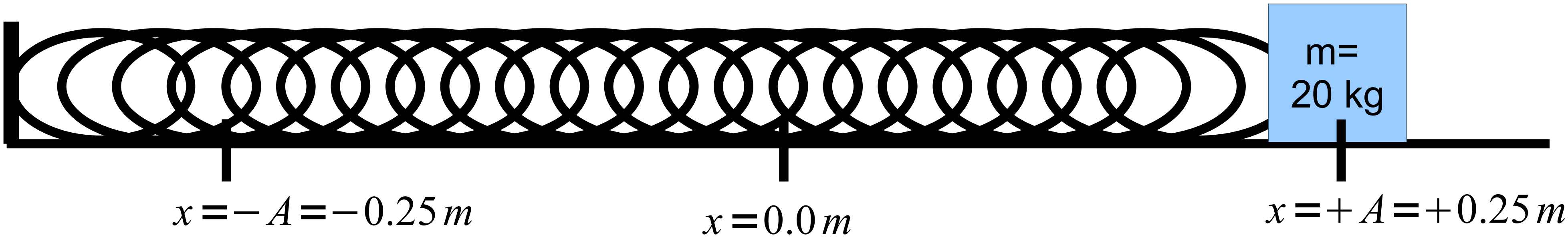
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100\,N/m}{20\,kg}} = 2.24\,s^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.24\,s^{-1}} = 2.81\,s$$

$$v_{max} = A\omega = (0.25\,m)(2.24\,s^{-1}) = 0.56\,m/s$$

$$a_{max} = A\omega^2 = (0.25\,m)(2.24\,s^{-1})^2 = 1.25\,m/s^2$$

$k = 100\, N/m$ **Mass is released from rest at x = +A = 0.25 m**



$$A = 0.25\, m$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100\, N/m}{20\, kg}} = 2.24\, s^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.24\, s^{-1}} = 2.81\, s$$

$$v_{max} = A\omega = (0.25\, m)(2.24\, s^{-1}) = 0.56\, m/s$$

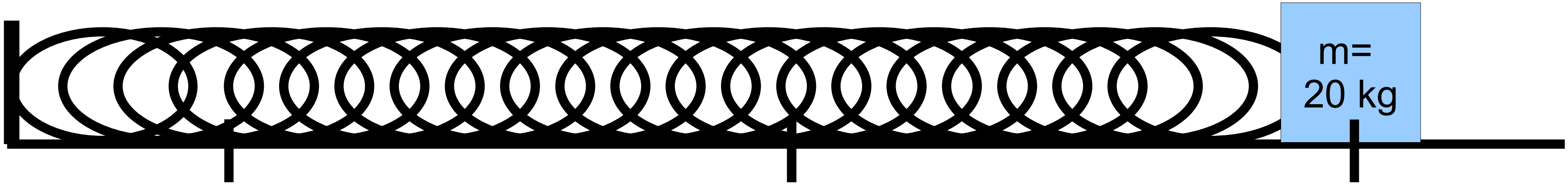
$$a_{max} = A\omega^2 = (0.25\, m)(2.24\, s^{-1})^2 = 1.25\, m/s^2$$

Check:

$$v_{max} = A\sqrt{\frac{k}{m}} = 0.25\, m\sqrt{\frac{100\, N/m}{20\, kg}} = 0.56\, m/s$$

$$a_{max} = A\frac{k}{m} = (0.25\, m)\frac{100\, N/m}{20\, kg} = 1.25\, m/s^2$$

$k = 100\, N/m$ **Mass is released from rest at x = +A = 0.25 m**



$x = -A = -0.25\, m$ $x = 0.0\, m$ $x = +A = +0.25\, m$

$$A = 0.25\, m$$

$$\omega = \sqrt{\frac{k}{m}} = 2.24\, s^{-1}$$

$$T = \frac{2\pi}{\omega} = 2.81\, s$$

$$v_{max} = A\,\omega = 0.56\, m/s$$

$$a_{max} = A\,\omega^2 = 1.25\, m/s^2$$

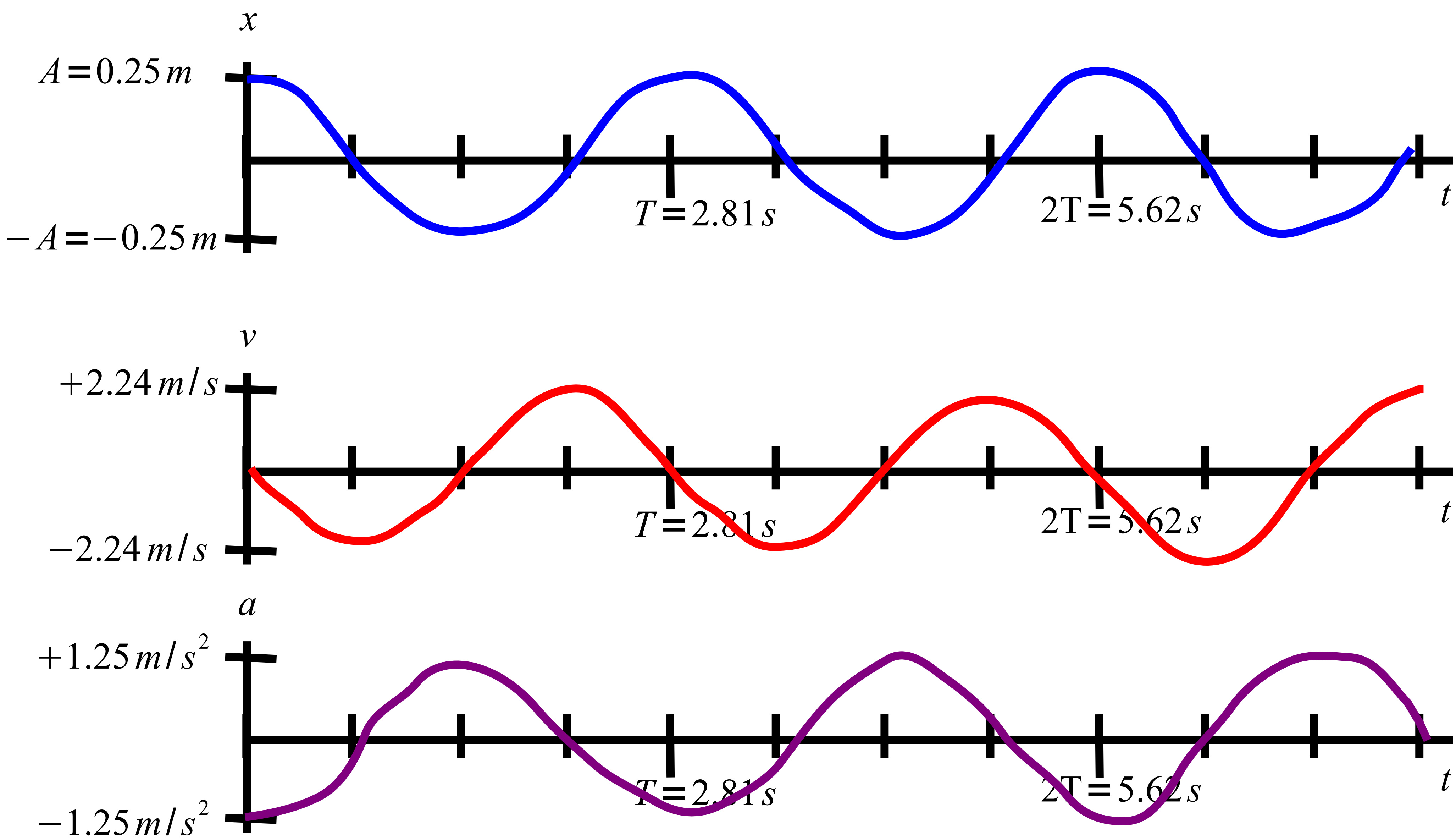
$$x(t) = A\cos(\omega t) = 0.25\, m \cos\left(2.24\, s^{-1}\, t\right)$$

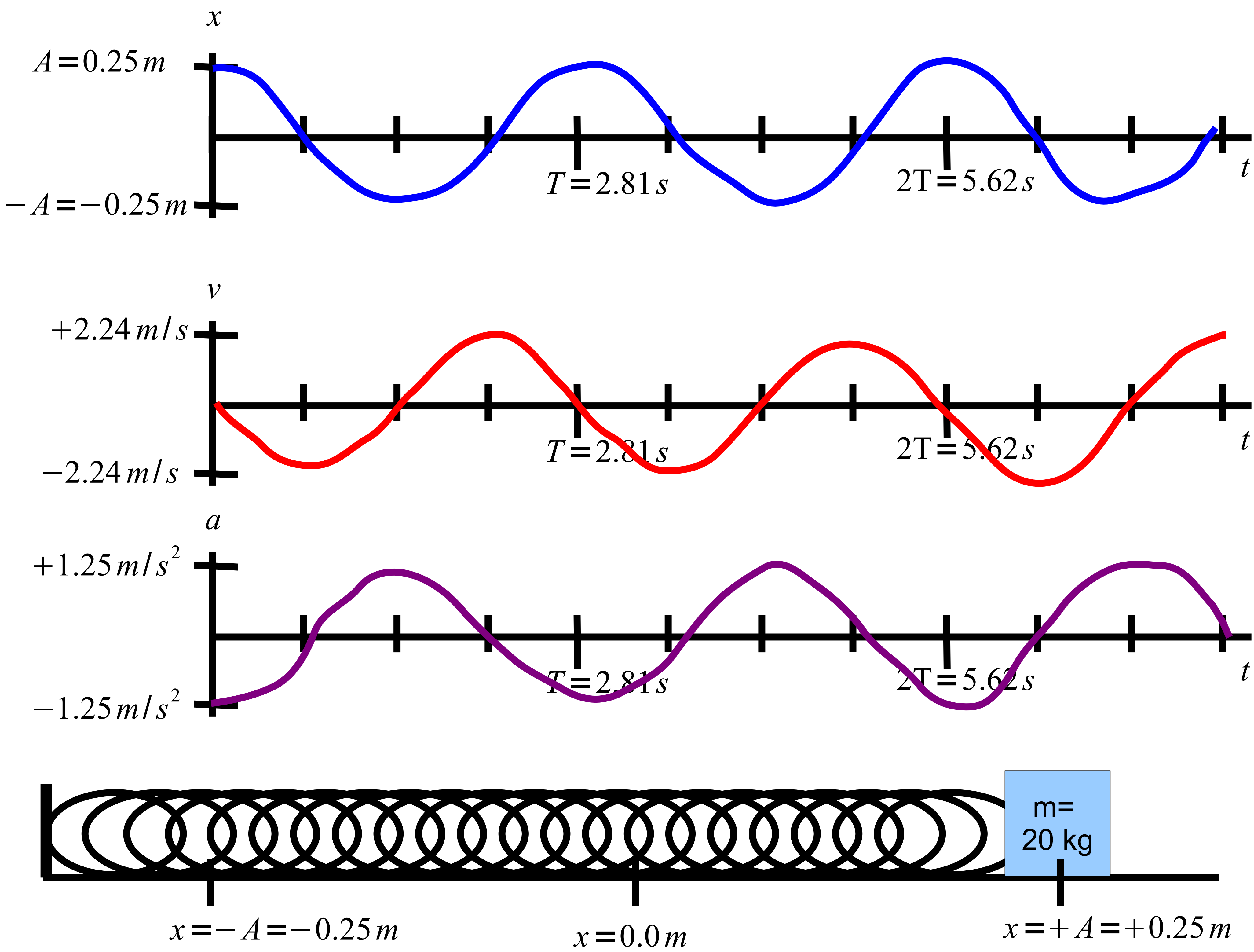
$$v(t) = -A\,\omega \sin(\omega t) = -0.56\, m/s \sin\left(2.24\, s^{-1}\, t\right)$$

$$a(t) = -A\,\omega^2 \cos(\omega t) = -1.25\, m/s^2 \cos\left(2.24\, s^{-1}\, t\right)$$

$$x(t) = A\cos(\omega t) = 0.25\,m\cos(2.24\,s^{-1}\,t)$$
$$v(t) = -A\omega\sin(\omega t) = -0.56\,m/s\sin(2.24\,s^{-1}\,t)$$
$$a(t) = -A\omega^2\cos(\omega t) = -1.25\,m/s^2\cos(2.24\,s^{-1}\,t)$$

You should be able to roughly draw out plots.

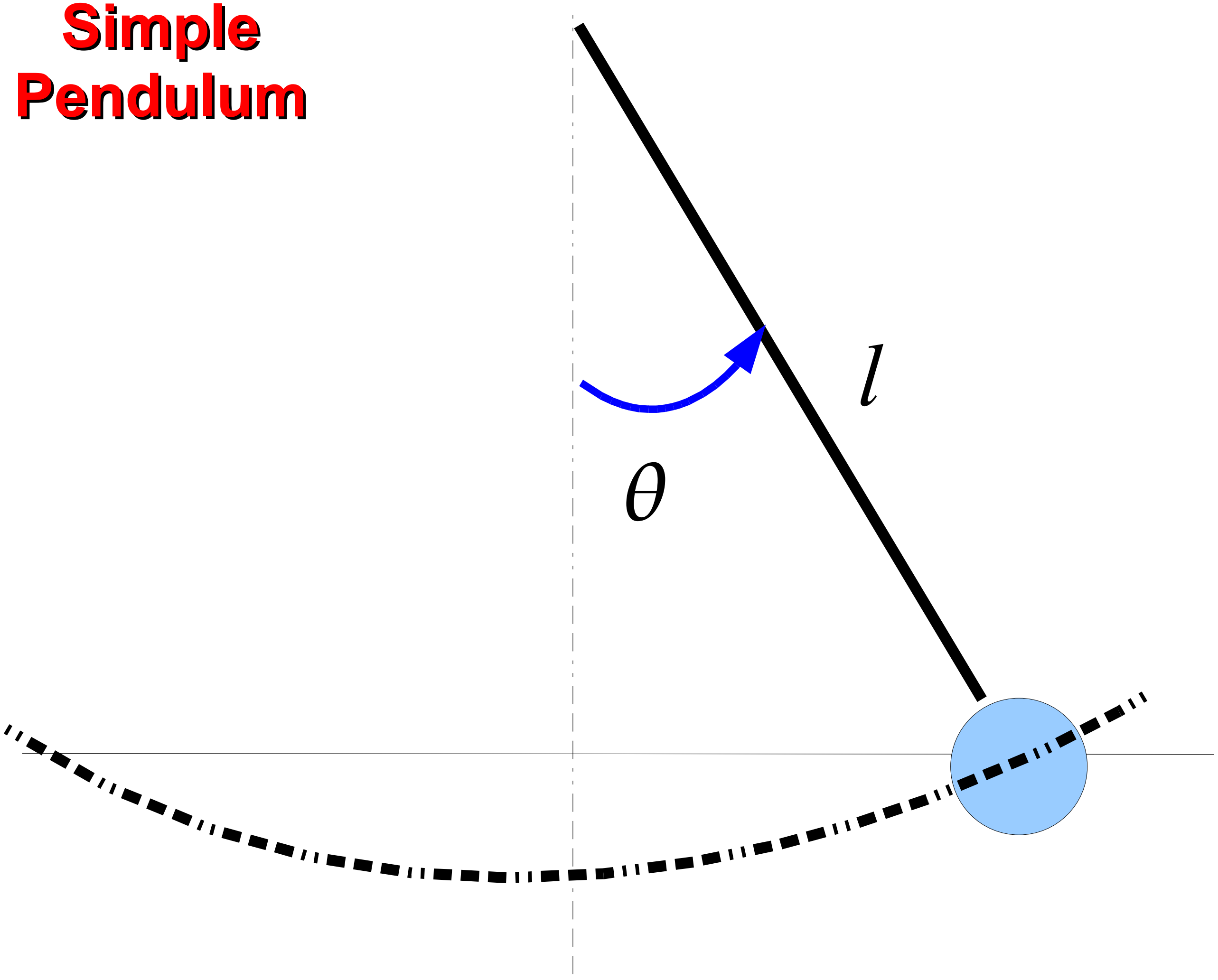



x
$A = 0.25\,m$
$-A = -0.25\,m$
$T = 2.81\,s$
$2T = 5.62\,s$
t
v
$+2.24\,m/s$
$-2.24\,m/s$
$T = 2.81\,s$
$2T = 5.62\,s$
t
a
$+1.25\,m/s^2$
$-1.25\,m/s^2$
$T = 2.81\,s$
$2T = 5.62\,s$
t
m=
20 kg
$x = -A = -0.25\,m$
$x = 0.0\,m$
$x = +A = +0.25\,m$

Group Quiz

Simple Pendulum

Simple Pendulum

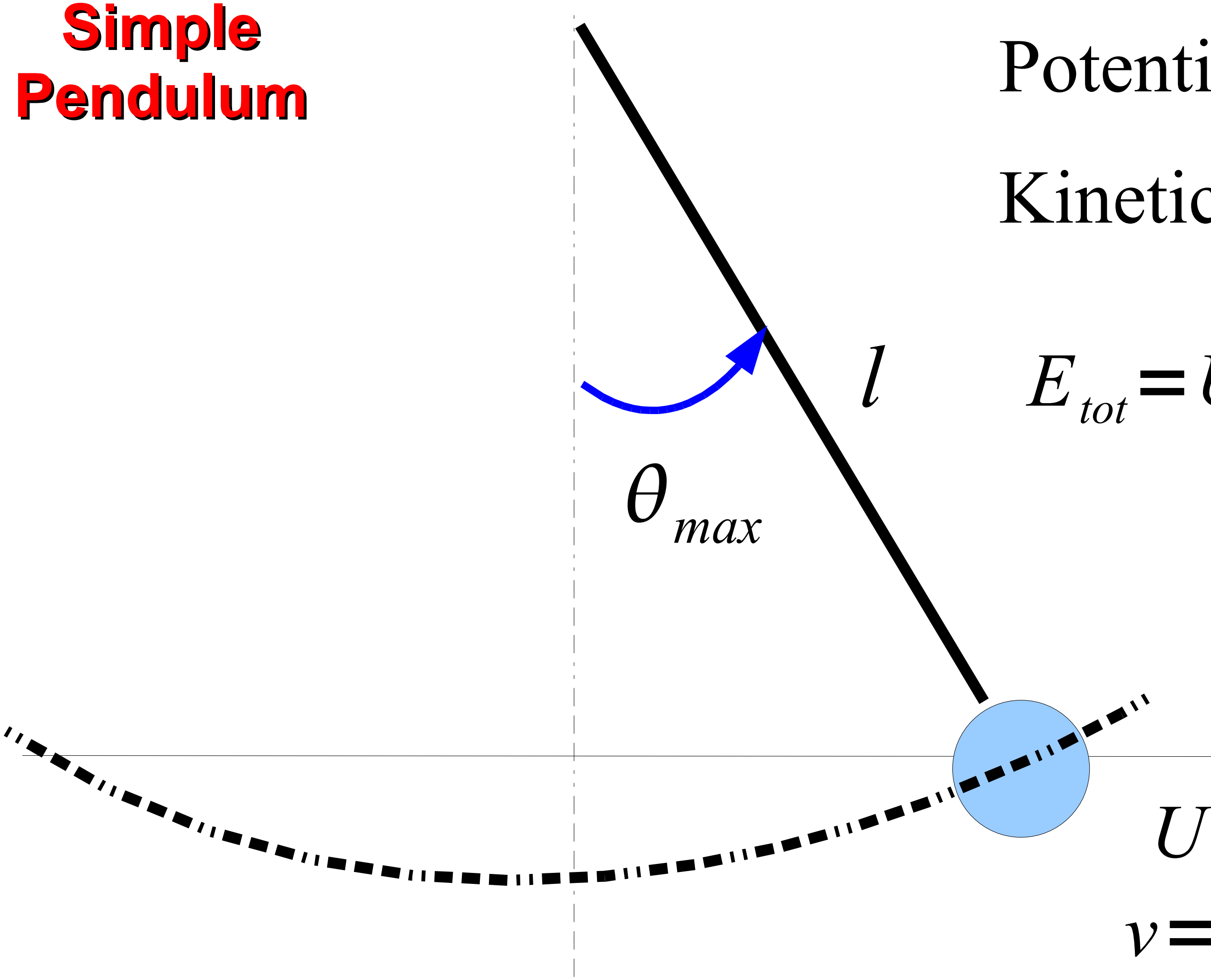


Simple Pendulum

Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$



$U = U_{max}$

$v = 0.0\, m/s$

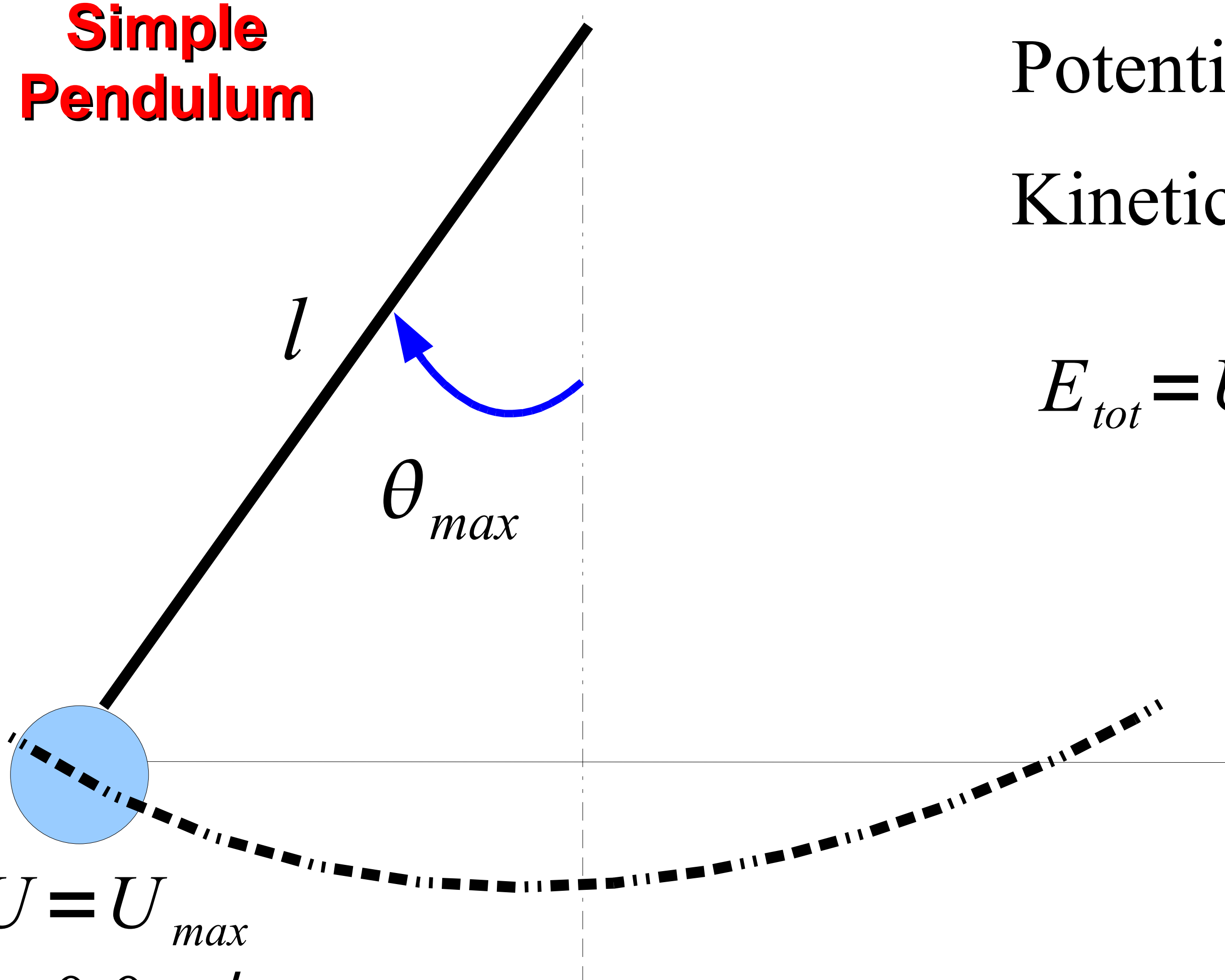
$K = 0.0\, J$

Simple Pendulum

Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$



$U = U_{max}$

$v = 0.0\, m/s$

$K = 0.0\, J$

Simple Pendulum

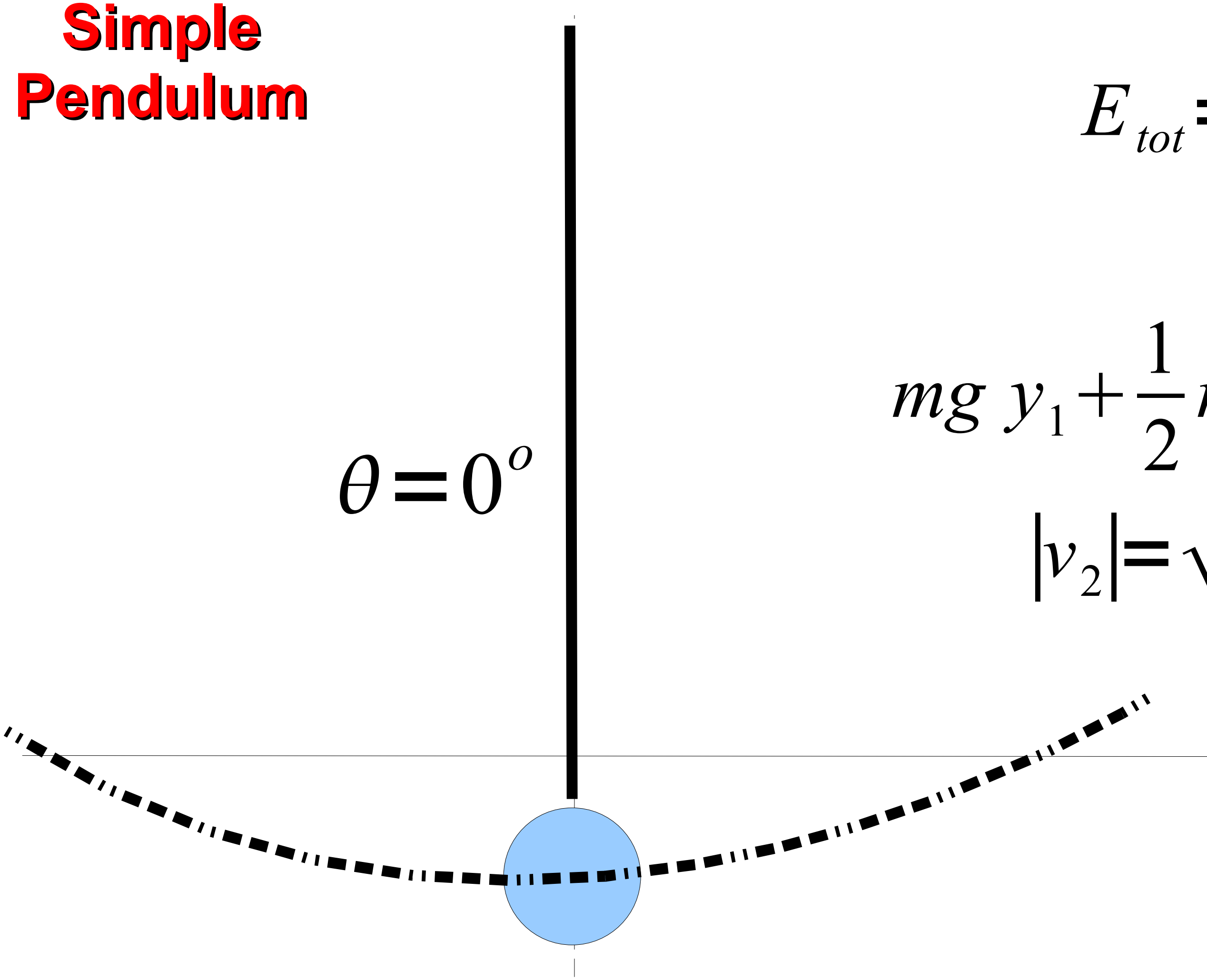
$$E_{tot} = mgy + \frac{1}{2} m v^2$$

$$E_1 = E_2$$

$$mg\, y_1 + \frac{1}{2} m v_1^2 = mg\, y_2 + \frac{1}{2} m v_2^2$$

$$|v_2| = \sqrt{v_1^2 + 2g\left(y_1 - y_2\right)}$$

$$\theta = 0^o$$

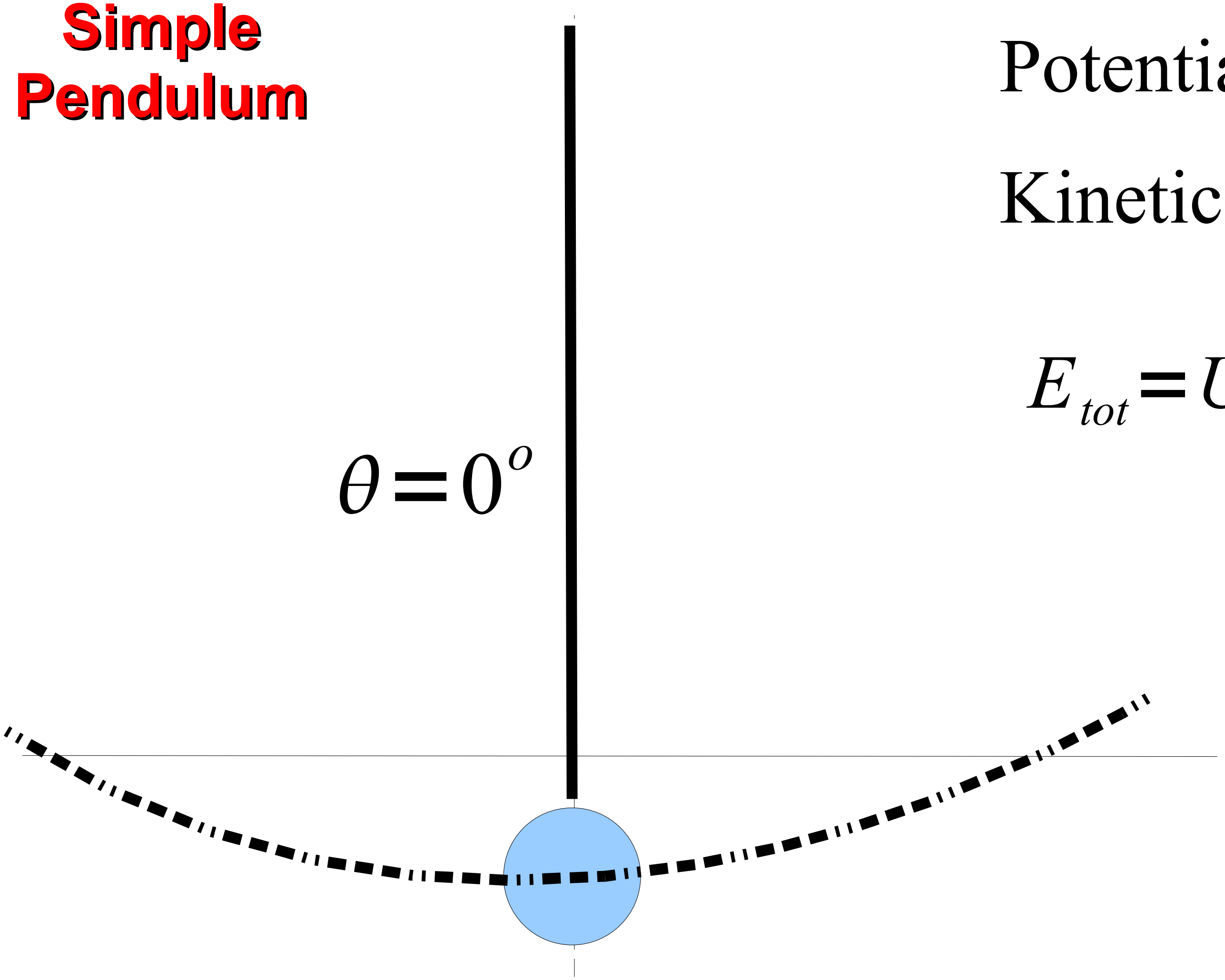


$$U = U_{min}$$

$$v = v_{max}$$

$$K = \frac{1}{2} m v_{max}^2$$

Simple Pendulum

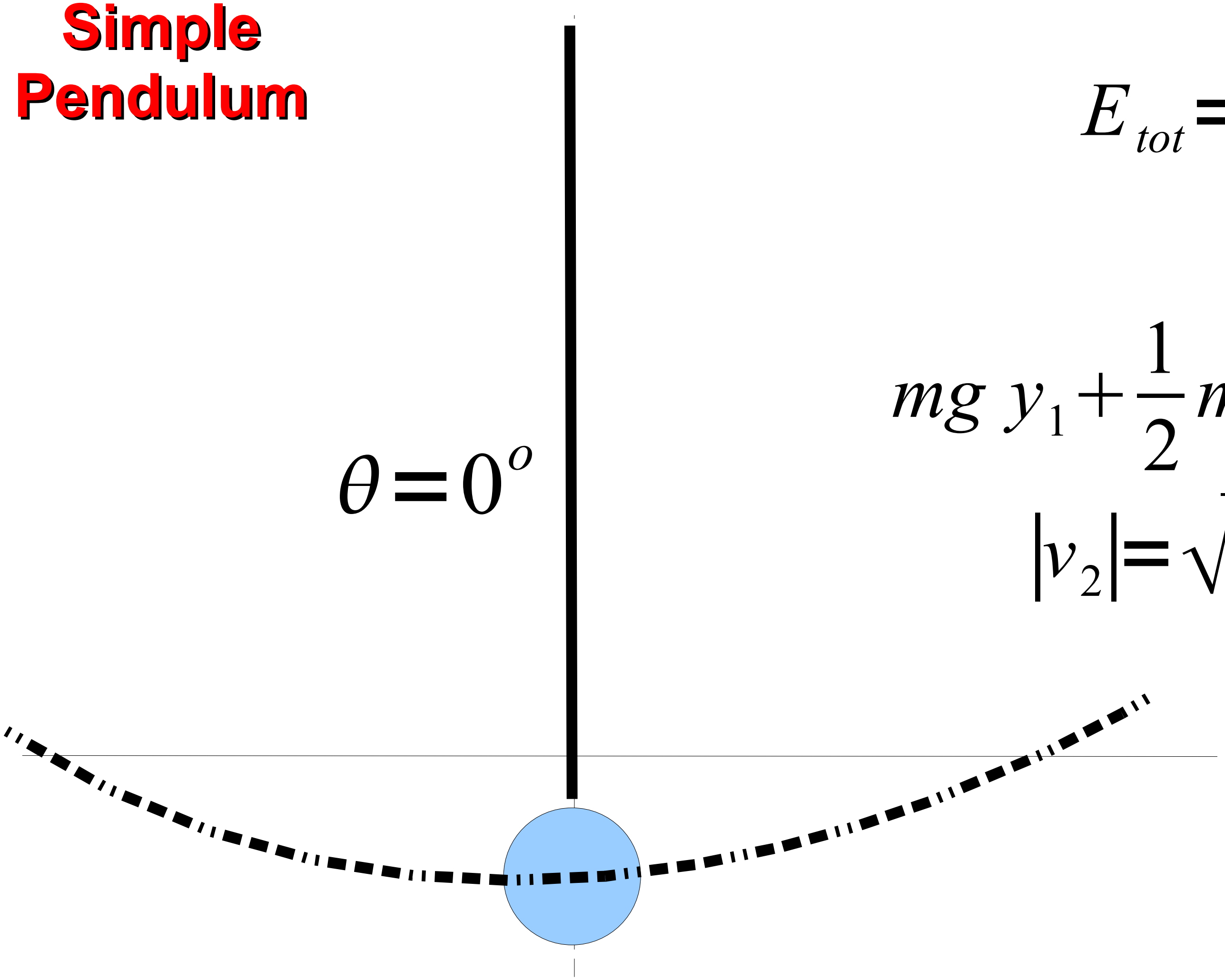


Potential Energy $U = mgy$

Kinetic Energy $K = \frac{1}{2} m v^2$

$$E_{tot} = U + K = mgy + \frac{1}{2} m v^2$$

Simple Pendulum



$$E_{tot} = mgy + \frac{1}{2} m v^2$$

$$E_1 = E_2$$

$$mg\, y_1 + \frac{1}{2} m v_1^2 = mg\, y_2 + \frac{1}{2} m v_2^2$$

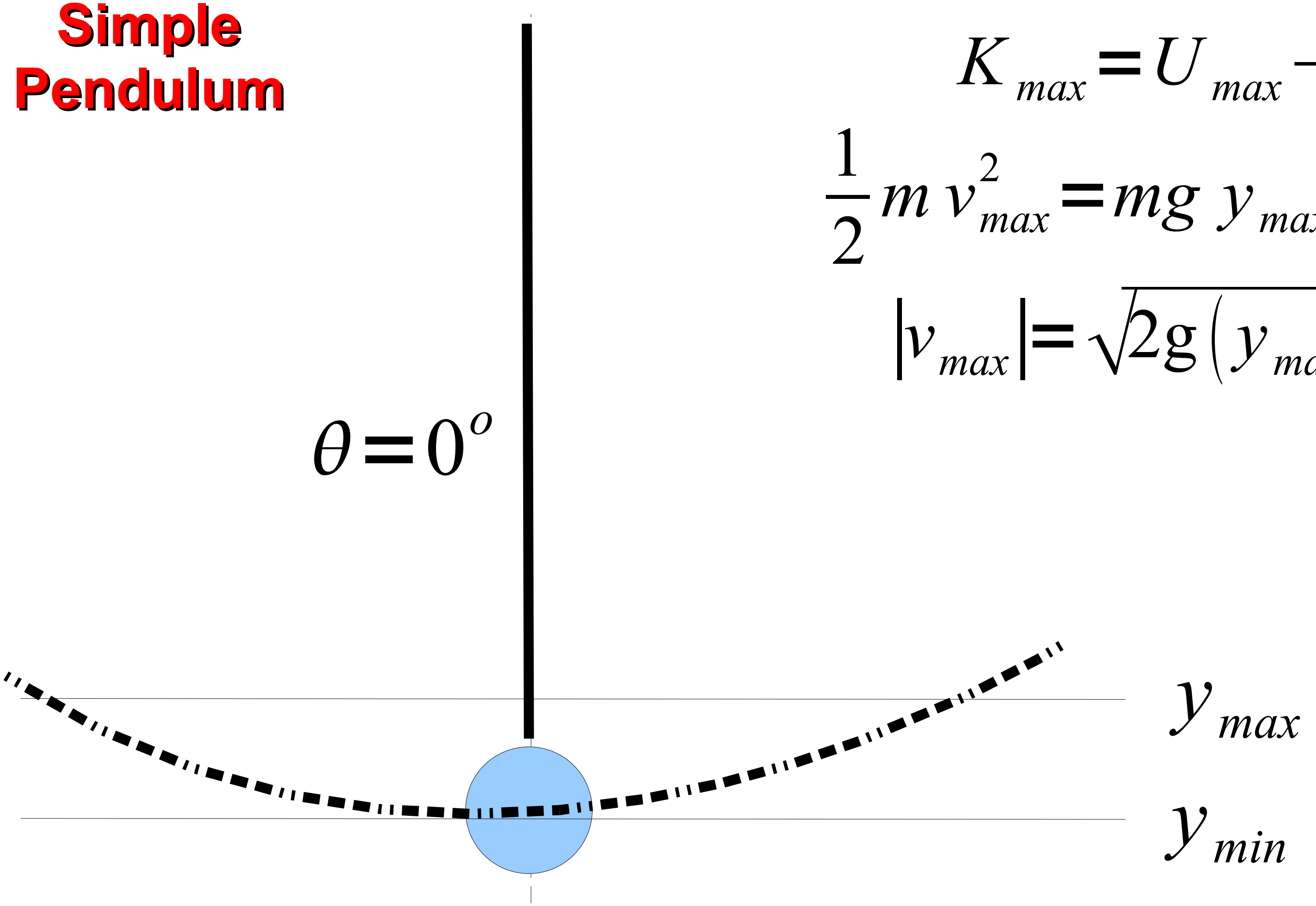
$$|v_2| = \sqrt{v_1^2 + 2g(y_1 - y_2)}$$

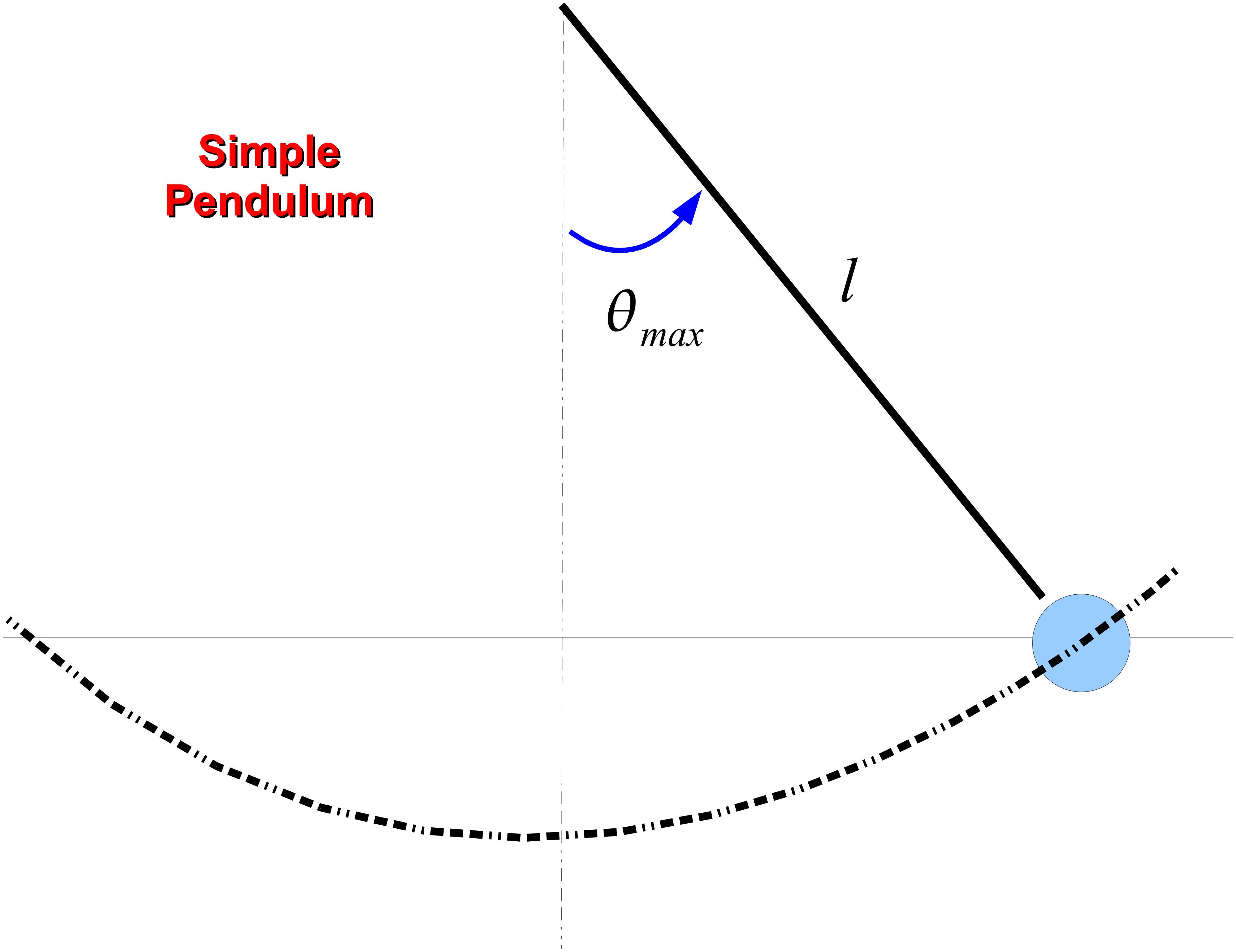
Simple Pendulum

$$K_{max} = U_{max} - U_{min}$$

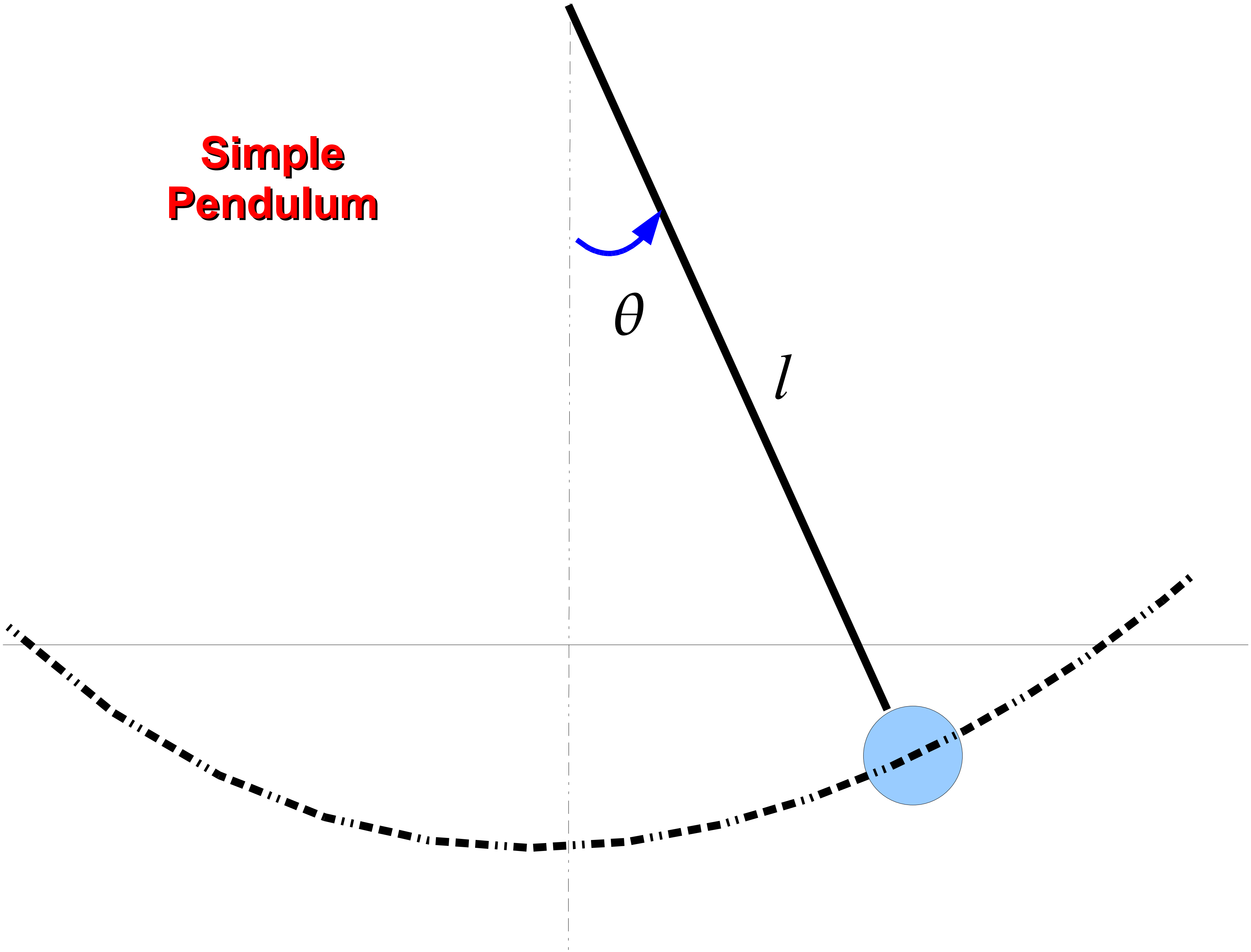
$$\frac{1}{2} m v_{max}^2 = mg\, y_{max} - mg\, y_{min}$$

$$|v_{max}| = \sqrt{2g\left(y_{max} - y_{min}\right)}$$



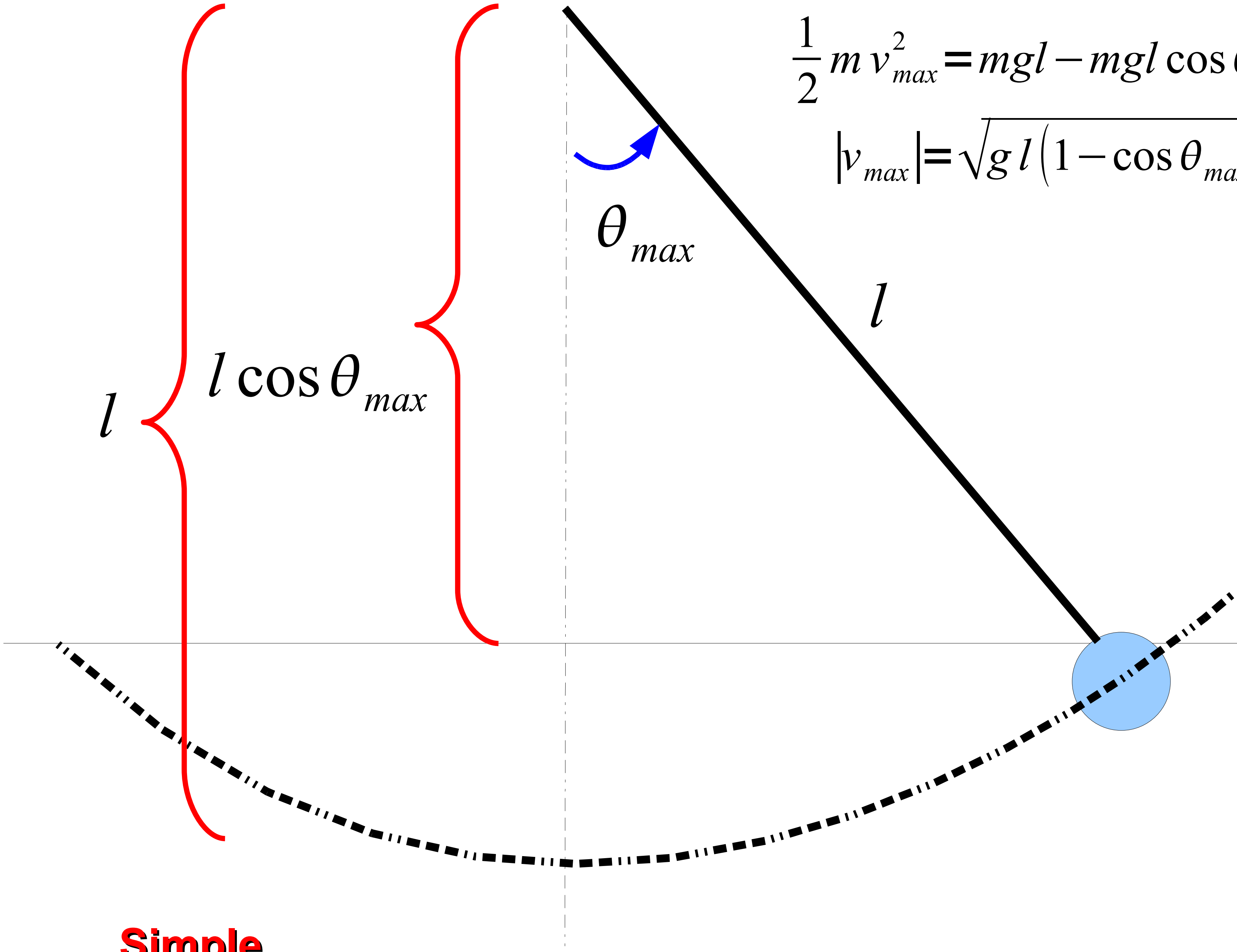

Simple Pendulum
θ_{max}
l

Simple Pendulum

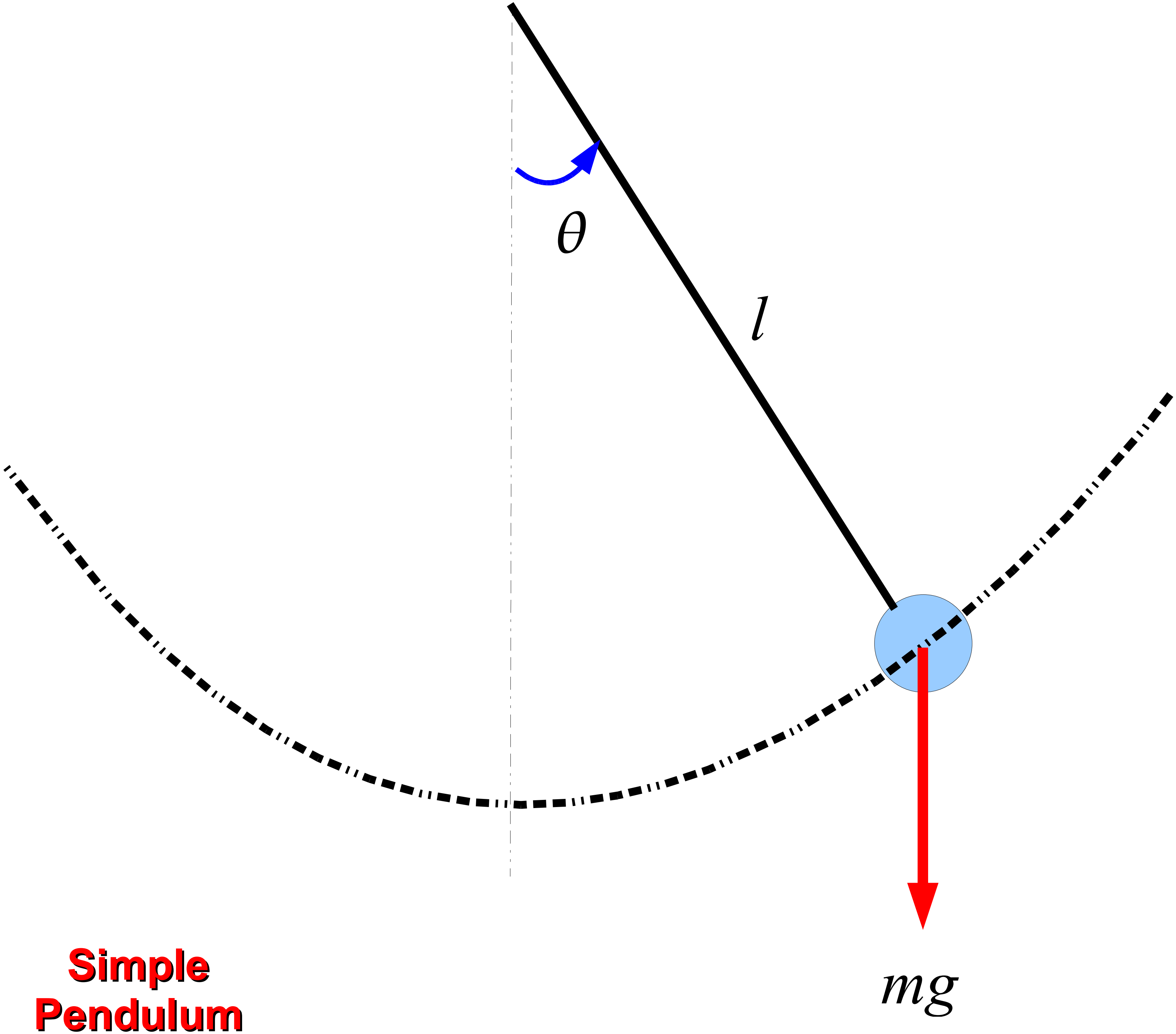


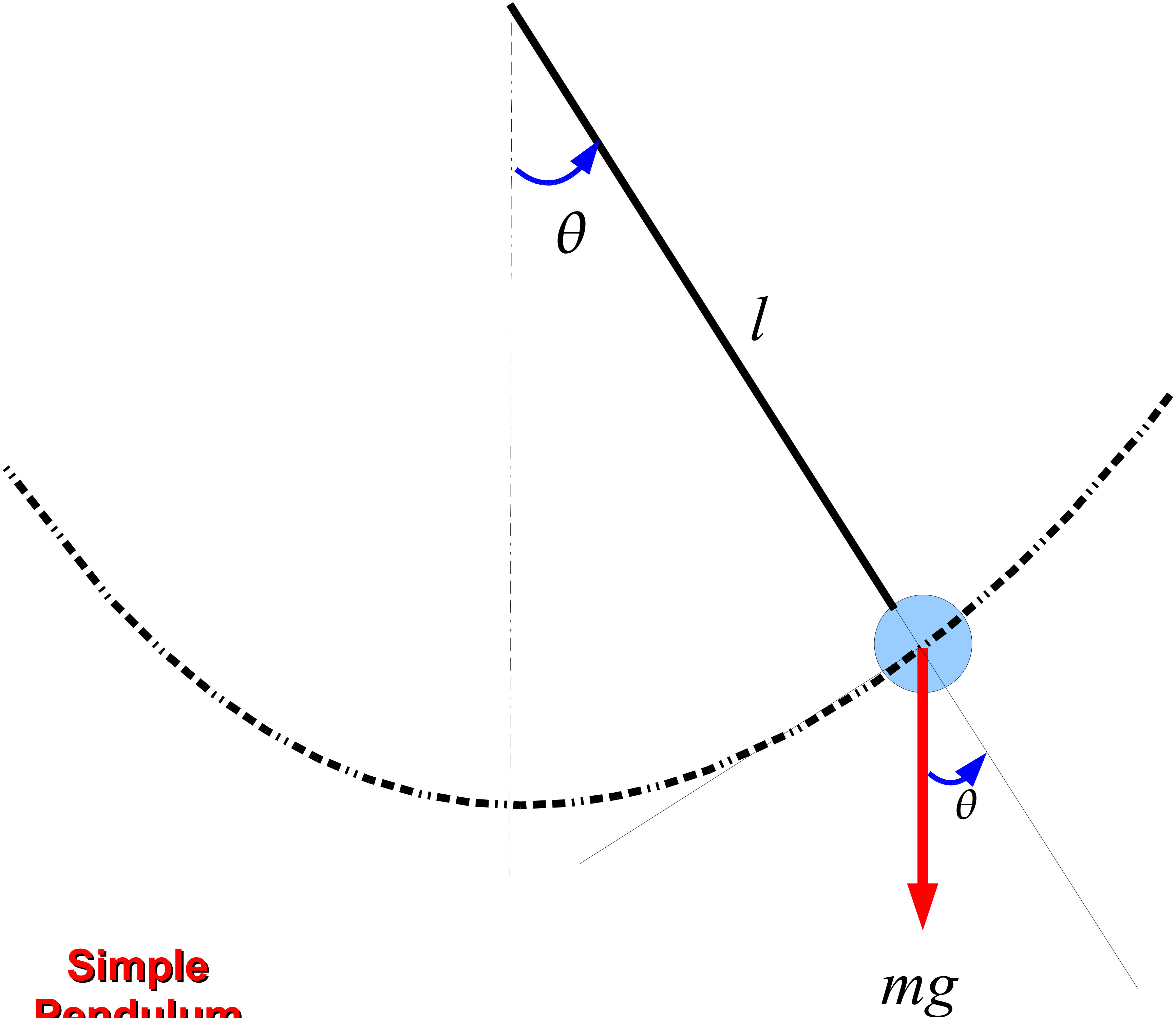
$$\frac{1}{2} m v_{max}^2 = mgl - mgl\cos\theta_{max}$$

$$|v_{max}| = \sqrt{g\,l\left(1 - \cos\theta_{max}\right)}$$

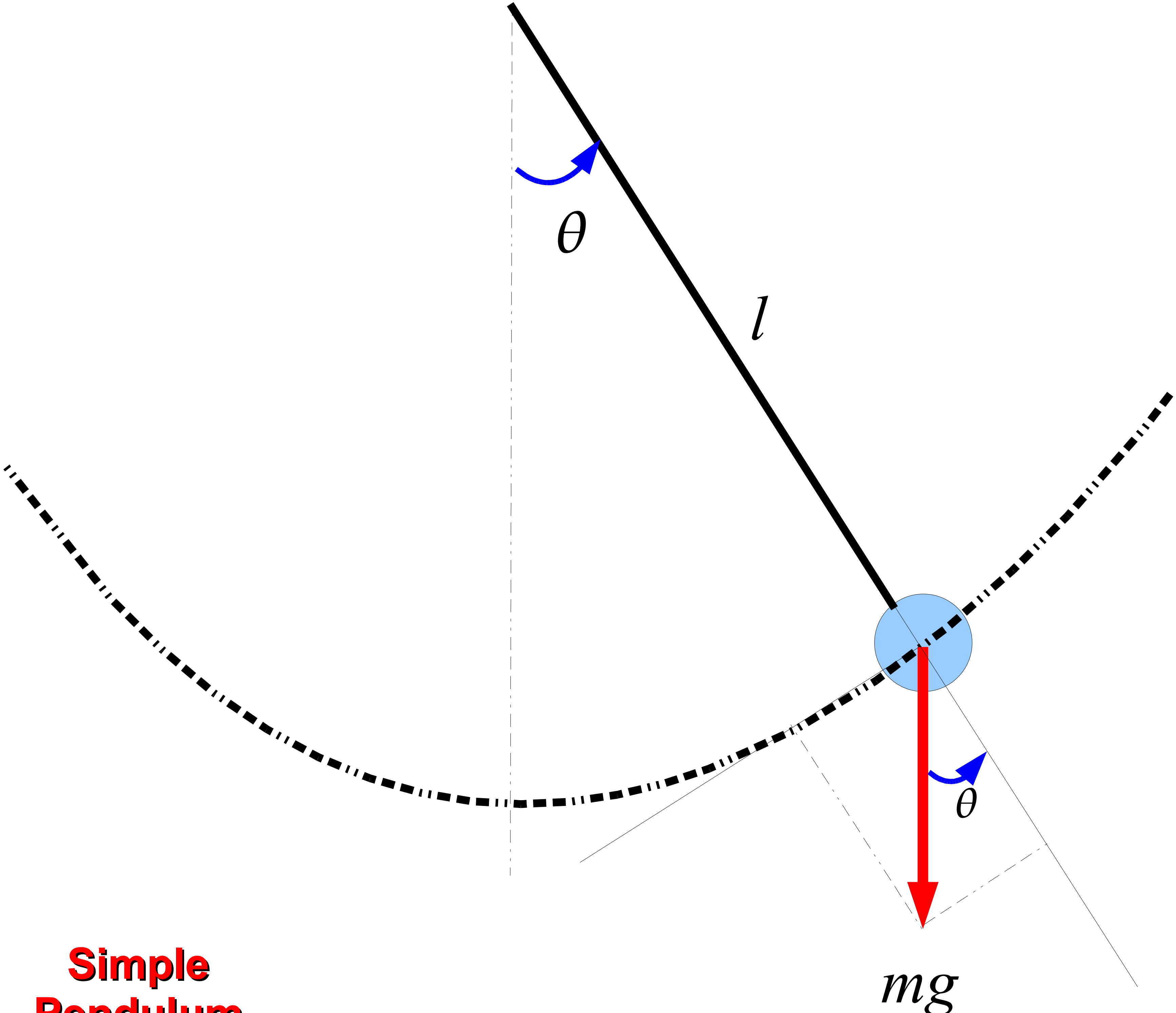


Simple Pendulum


θ
l
mg
Simple
Pendulum



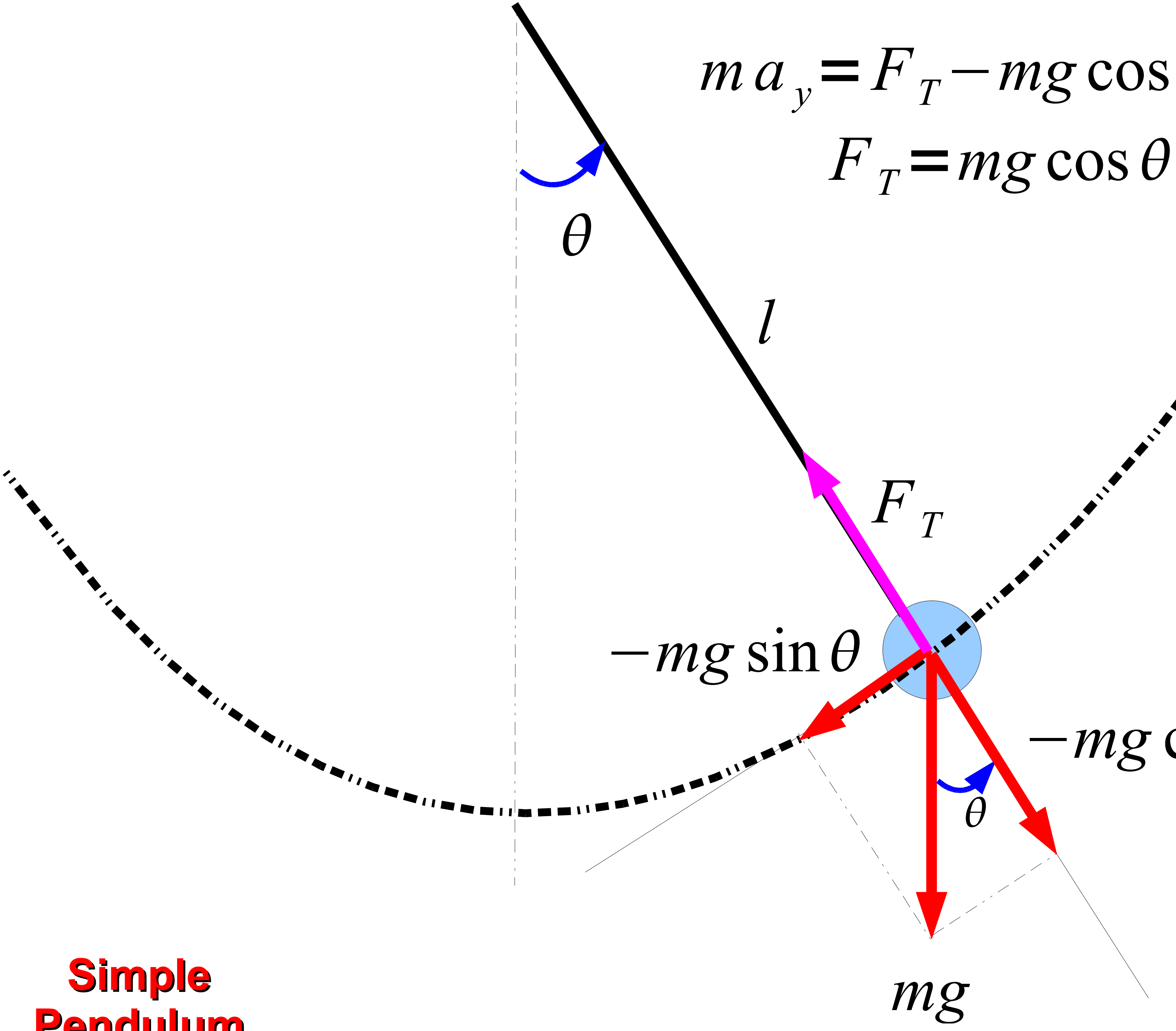
Simple Pendulum



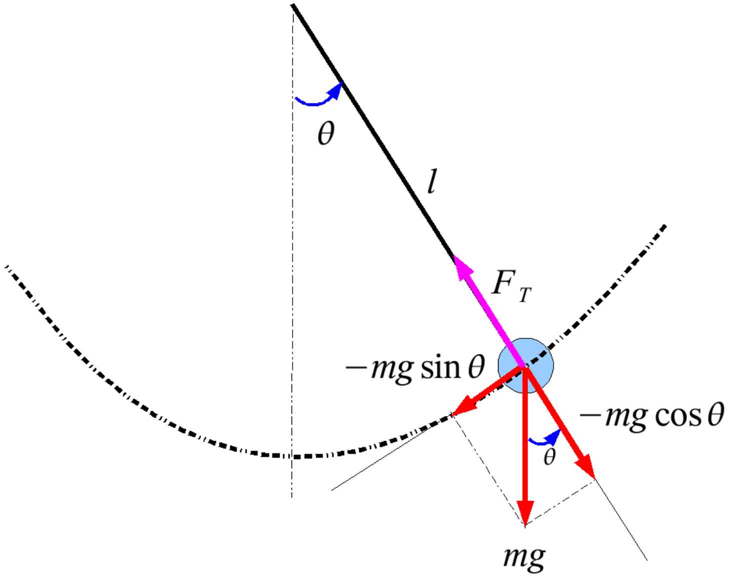
Simple Pendulum

$$m a_y = F_T - mg\cos\theta = 0$$

$$F_T = mg\cos\theta$$



Simple Pendulum



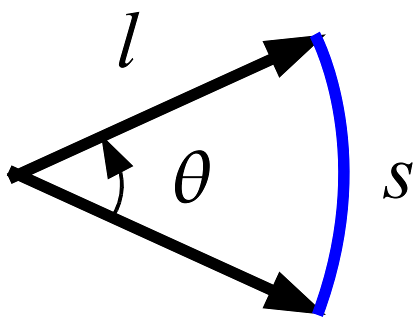
$$F_{s_{net}} = m a_s$$

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

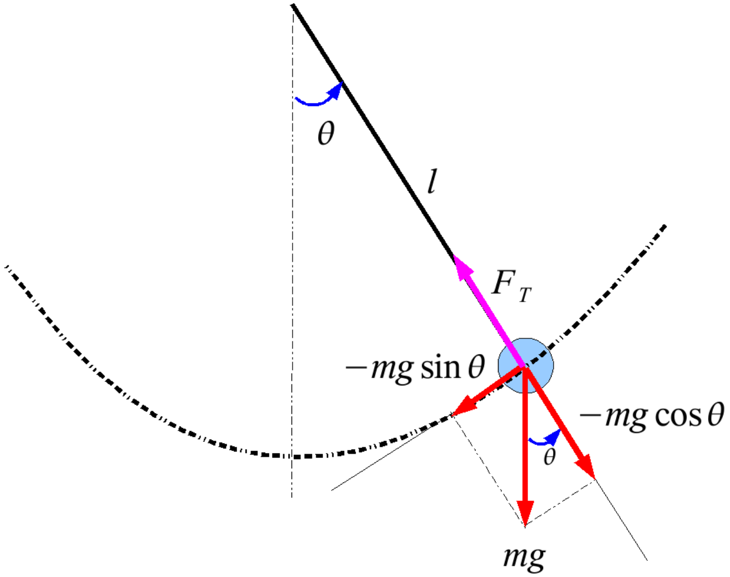
$$\sin\theta \approx \theta$$

$$s = l\theta \qquad \theta = \frac{s}{l}$$

$$-g\theta = \frac{d^2 s}{dt^2}$$



Simple Pendulum



$$F_{net} = m\,a$$

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

$$\sin\theta \approx \theta$$

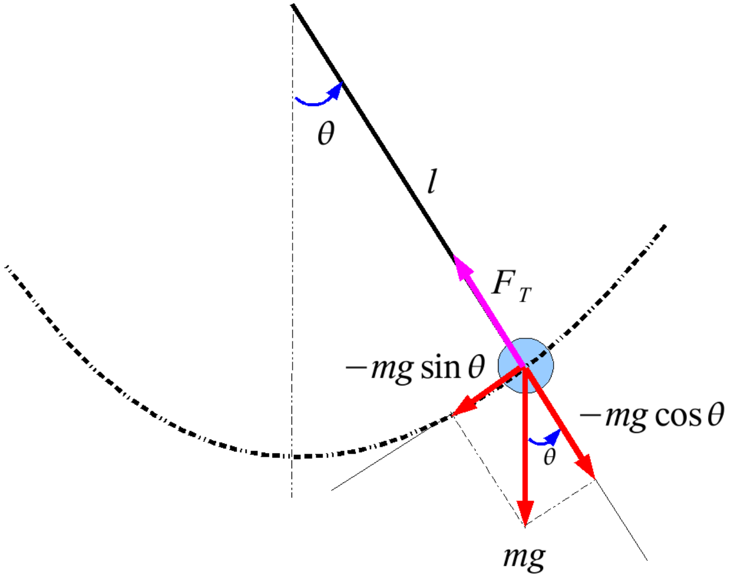
$$-g\,\theta = \frac{d^2 s}{dt^2}$$

$$s = l\,\theta$$

$$\frac{ds}{dt} = l\,\frac{d\,\theta}{dt}$$

$$\frac{d^2 s}{dt^2} = l\,\frac{d^2\theta}{dt^2}$$

Simple Pendulum



$$F_{net} = m\,a$$

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

$$\sin\theta \approx \theta$$

$$-g\,\theta = \frac{d^2 s}{dt^2}$$

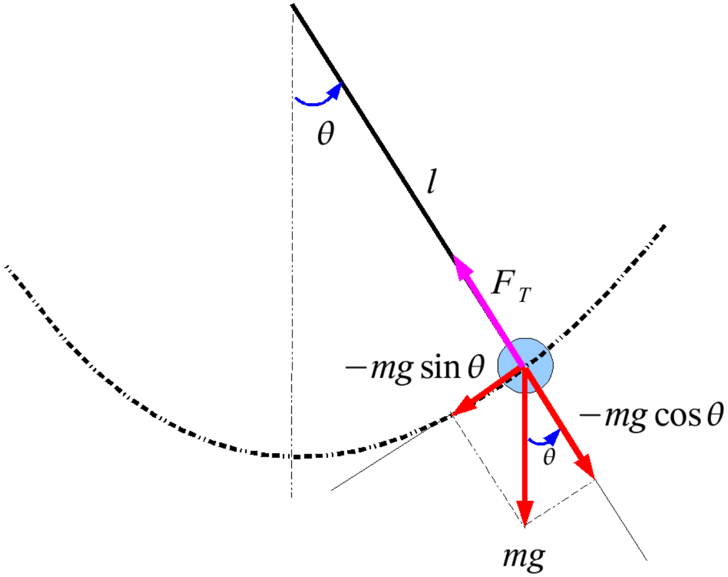
$$-g\,\theta = l\frac{d^2\theta}{dt^2}$$

$$\theta = \theta_{max}\cos(\omega t)$$

$$\frac{d\theta}{dt} = -\theta_{max}\,\omega\sin(\omega t)$$

$$\frac{d^2\theta}{dt^2} = -\theta_{max}\,\omega^2\cos(\omega t)$$

Simple Pendulum



$$-g\theta = l\frac{d^2\theta}{dt^2}$$

$$-g\theta_{max}\cos(\omega t) = l\left(\theta_{max}\omega^2\cos(-\omega t)\right)$$

$$g = l\omega^2$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\theta = \theta_{max}\cos(\omega t)$$

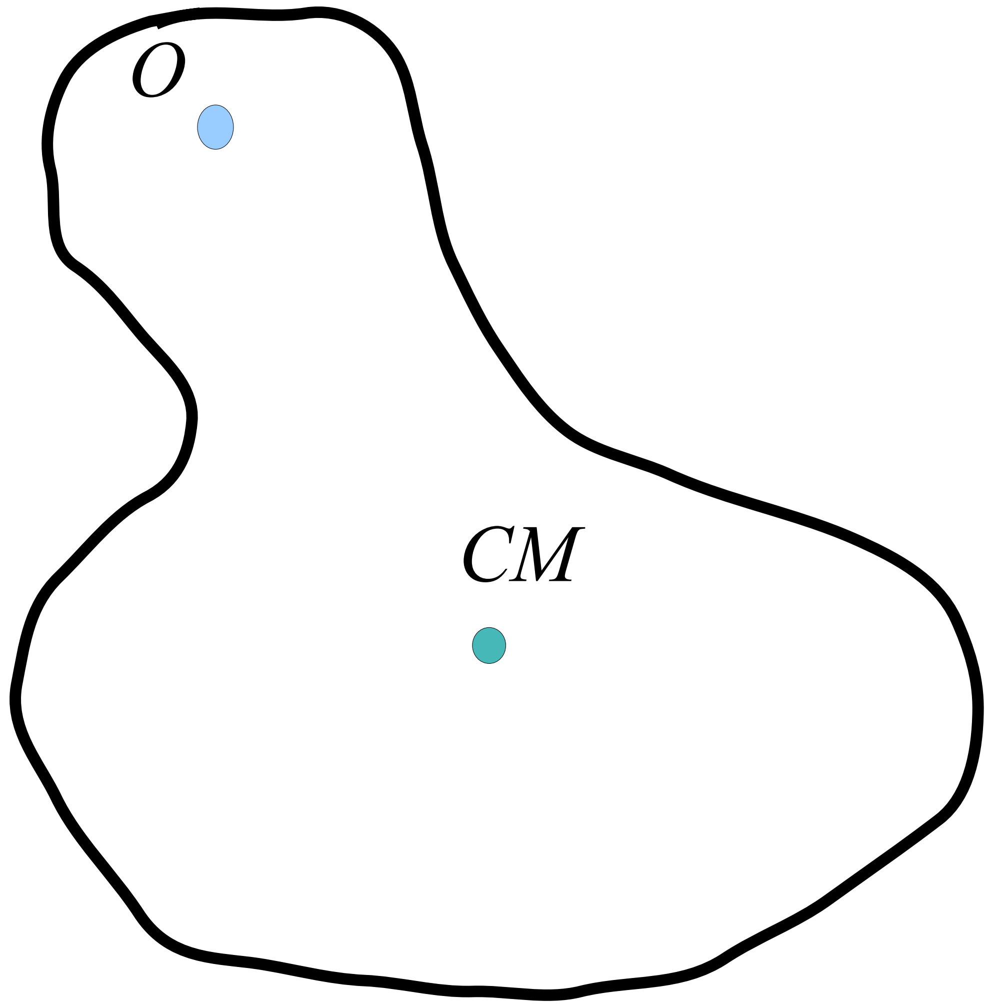
$$\frac{d\theta}{dt} = -\theta_{max}\omega\sin(\omega t)$$

$$\frac{d^2\theta}{dt^2} = -\theta_{max}\omega^2\cos(\omega t)$$

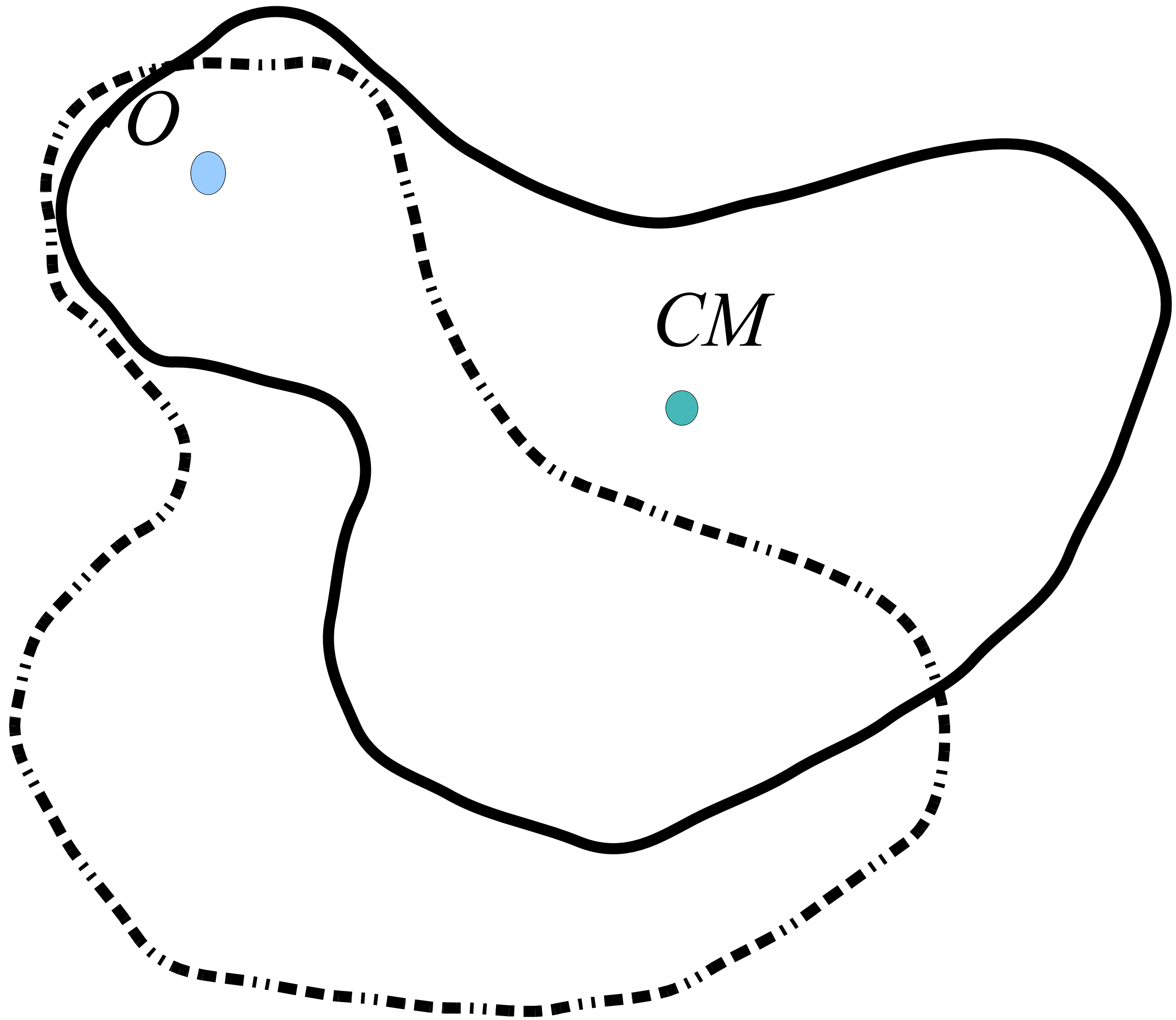
Simple Pendulum

Physical Pendulum

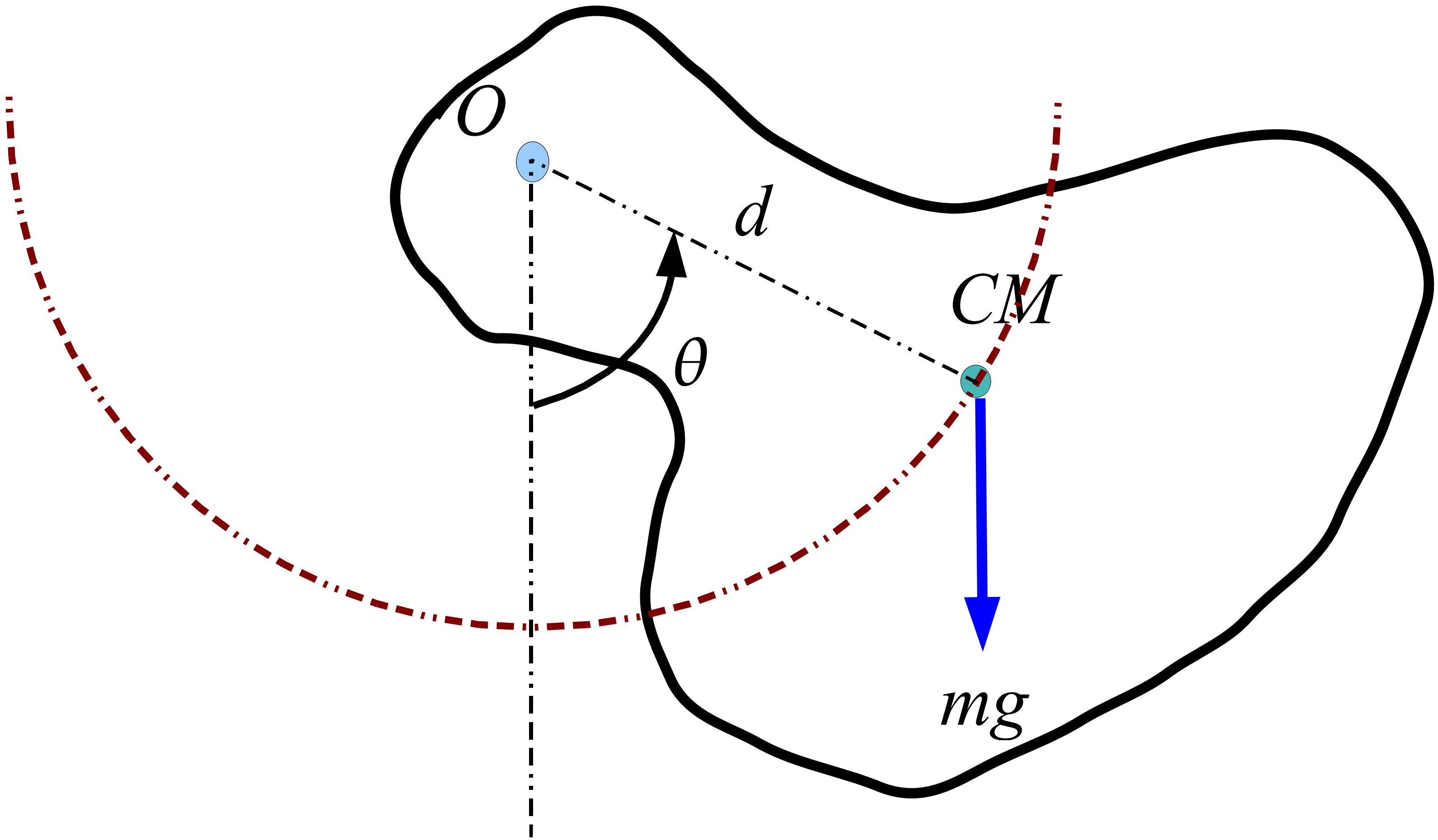
Physical Pendulum



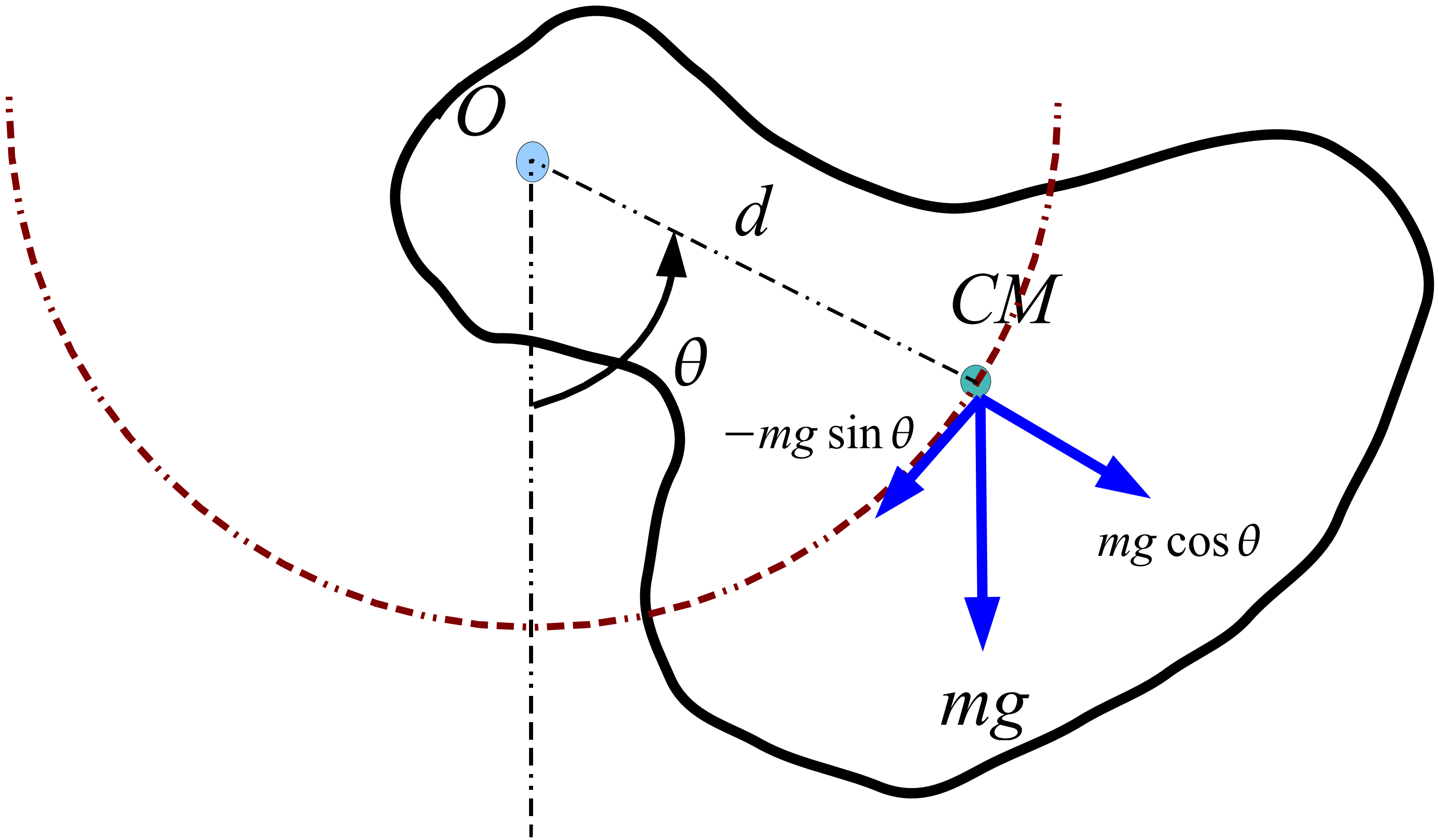
Physical Pendulum



Physical Pendulum



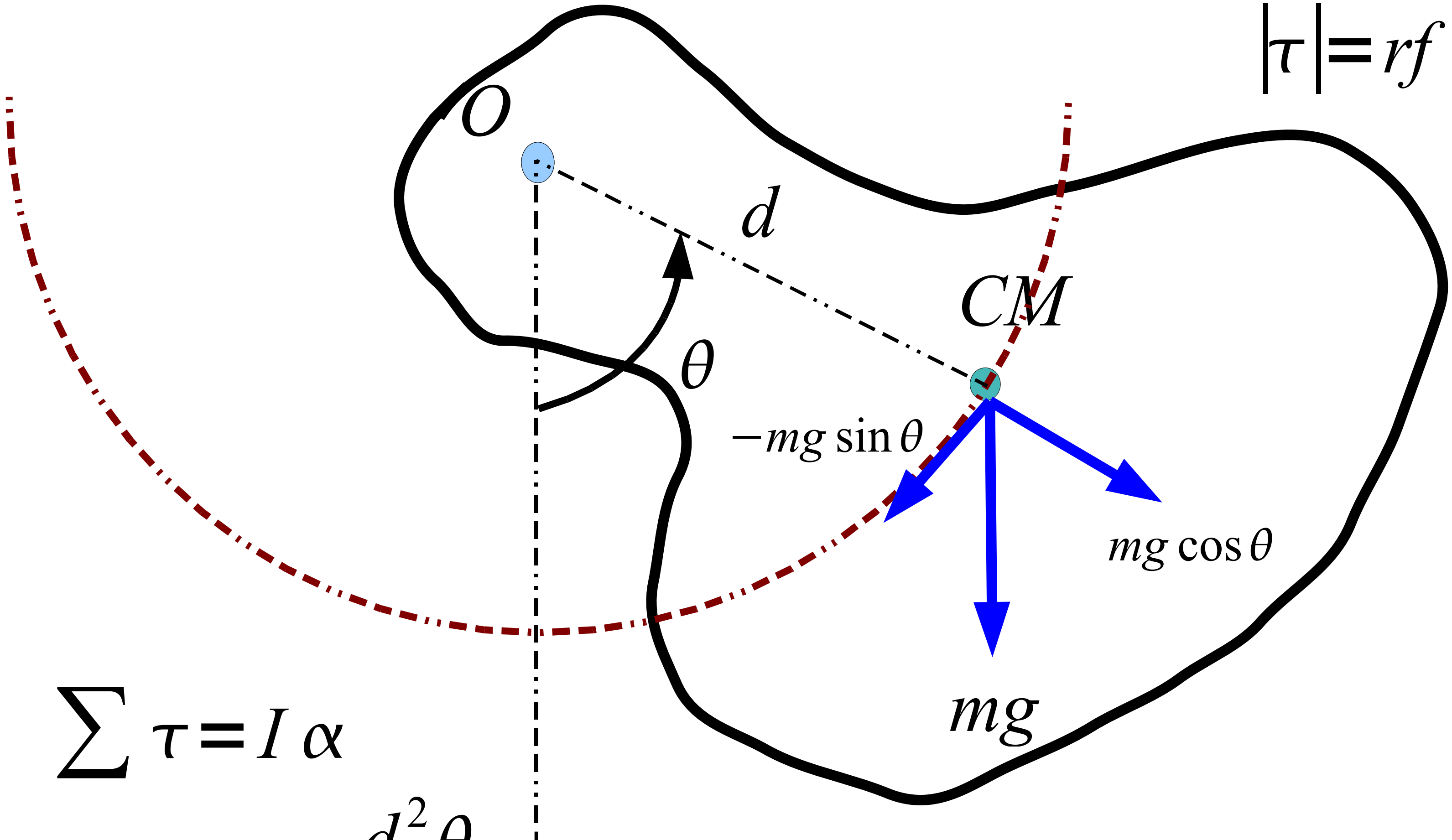
Physical Pendulum



Physical Pendulum

$$\vec{\tau} = \vec{r} \; X \; \vec{F}$$

$$|\tau| = rf \sin\theta$$



$$\sum \tau = I\alpha$$

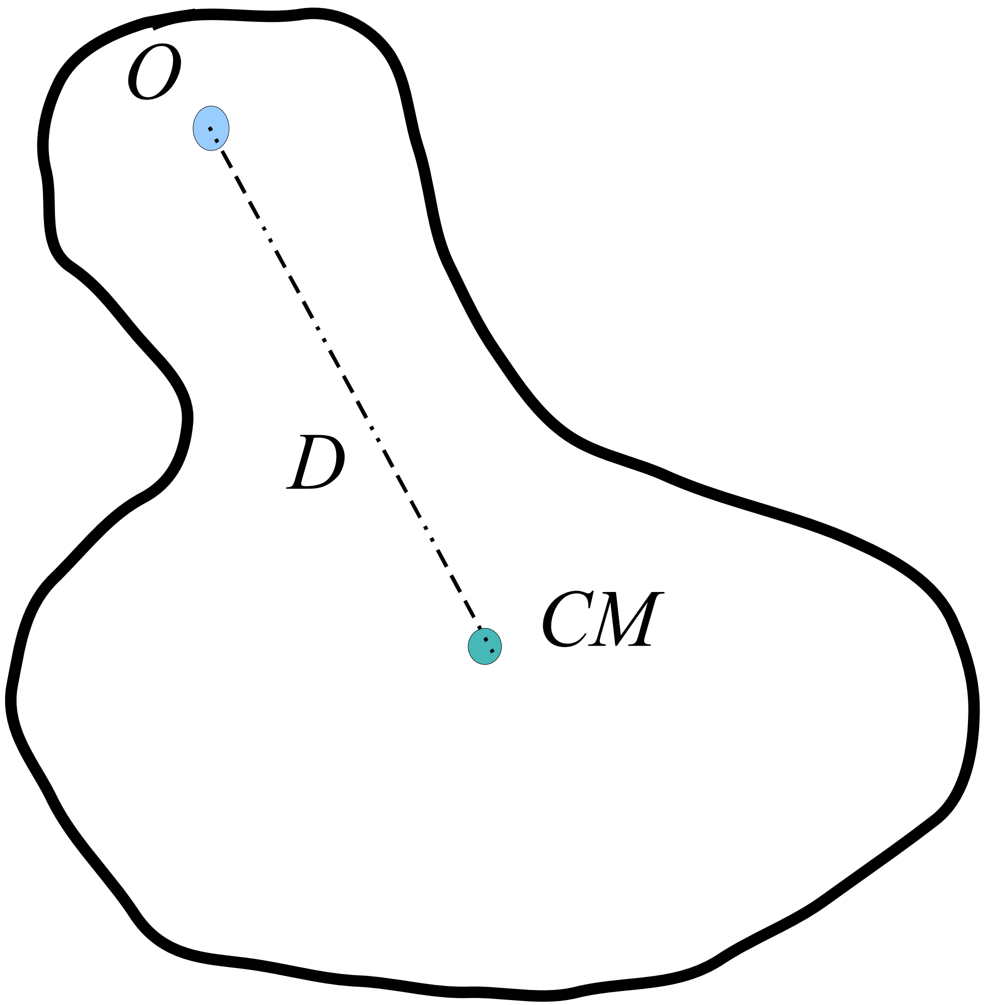
$$-mgd \sin\theta = I \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} \approx -\left(\frac{mgd}{I}\right)\theta$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

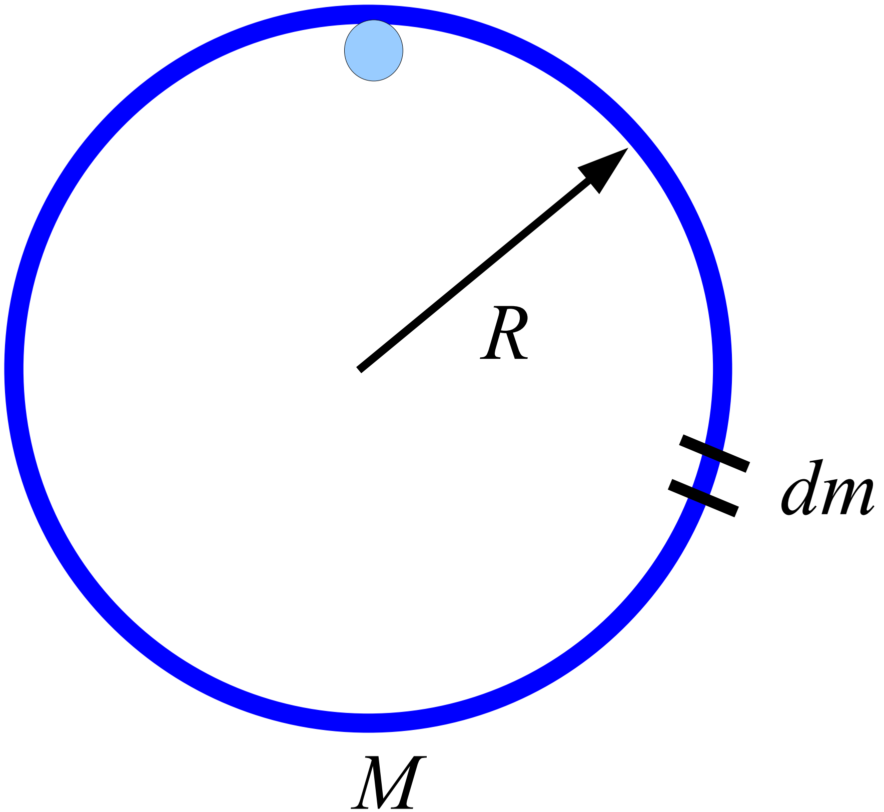
RECALL:



$$I_{CM} = \int r^2 \, dm$$

$$I = I_{CM} + MD^2$$

Hula Hoop on a Peg



$$I_{CM} = \int r^2 \, dm$$

$$I_{CM} = R^2 \int_0^M dm$$

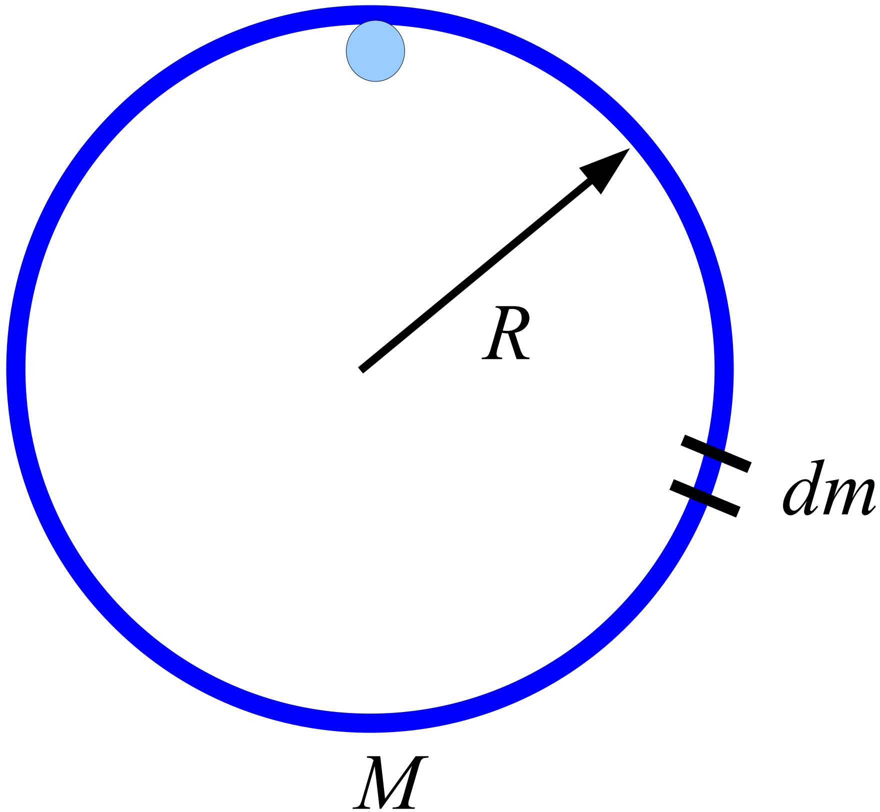
$$I_{CM} = MR^2$$

$$I = I_{CM} + MD^2$$

$$I = M\,R^2 + M\,R^2$$

$$I = 2\mathrm{MR}^2$$

Hula Hoop on a Peg



$$I_{CM} = \int r^2 \, dm$$

$$I_{CM} = R^2 \int_0^M dm$$

$$I_{CM} = MR^2$$

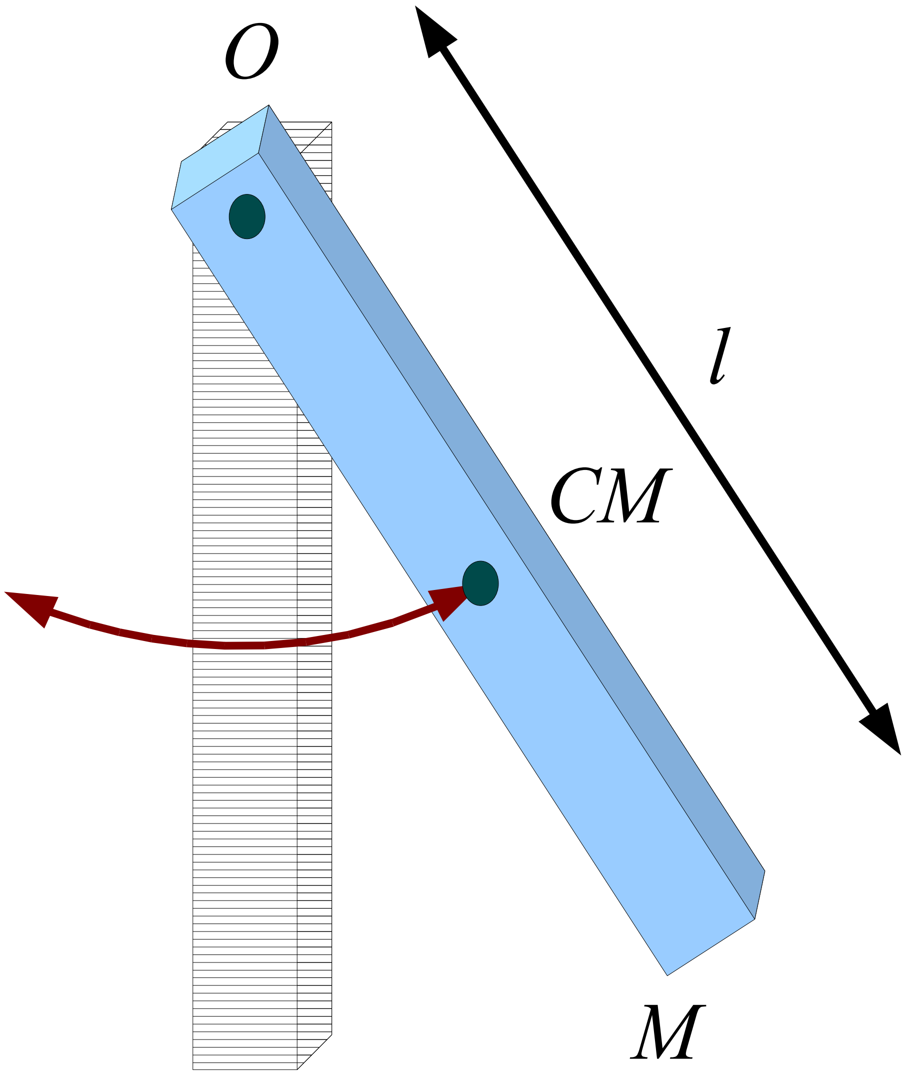
$$I = I_{CM} + MD^2$$

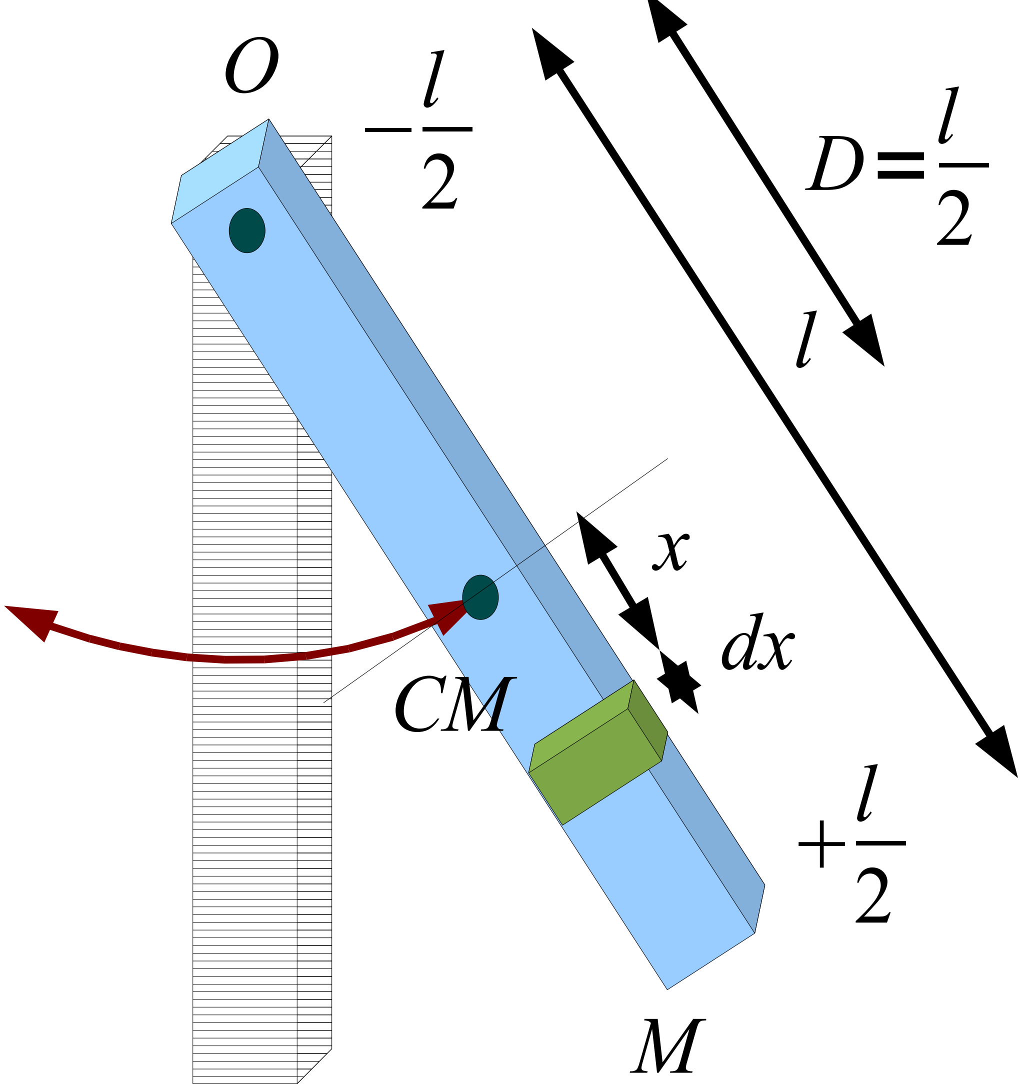
$$I = M\,R^2 + M\,R^2$$

$$I = 2\mathrm{MR}^2$$

$$T = 2\pi \sqrt{\frac{I}{mgD}}$$

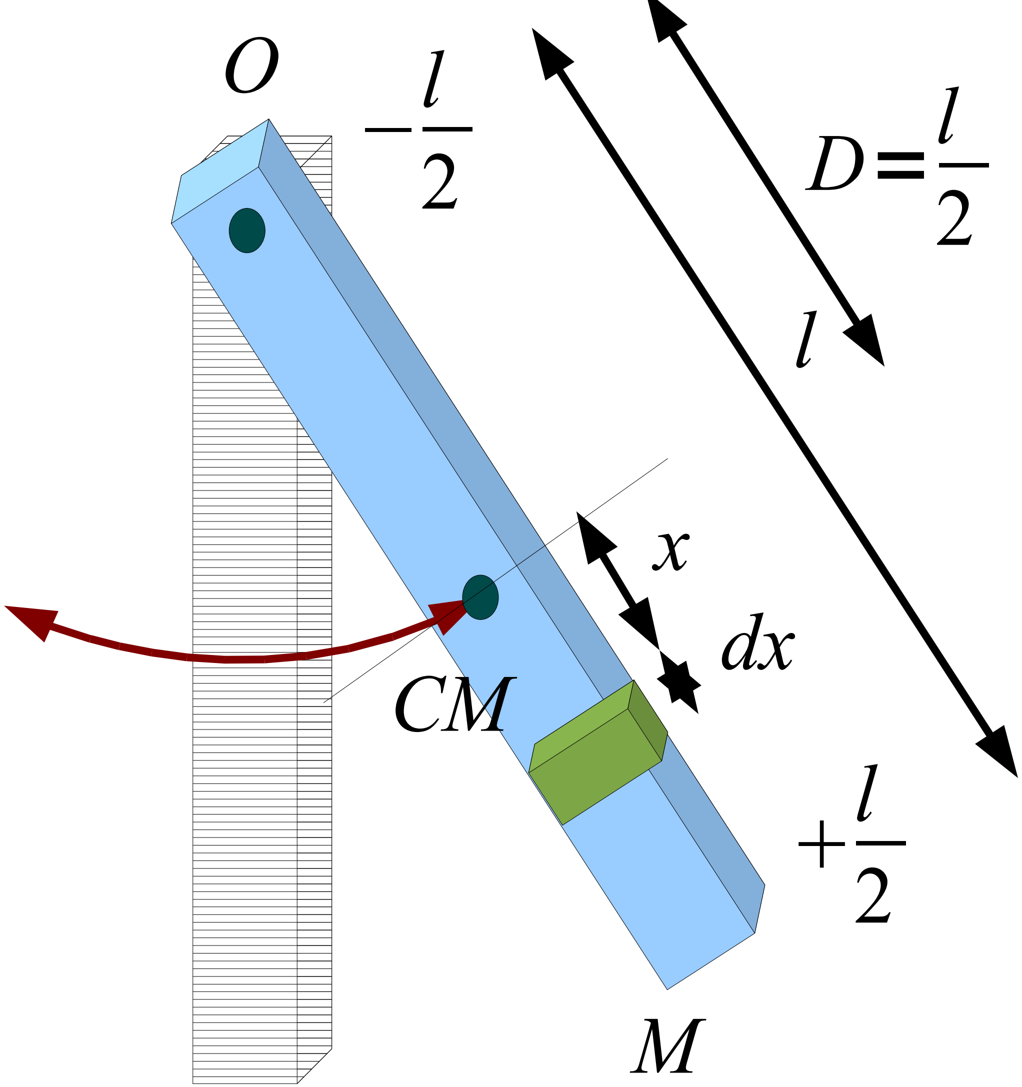
$$T = 2\pi \sqrt{\frac{2\mathrm{MR}^2}{MgR}} = 2\pi \sqrt{\frac{2\mathrm{R}}{g}}$$


O
l
CM
M



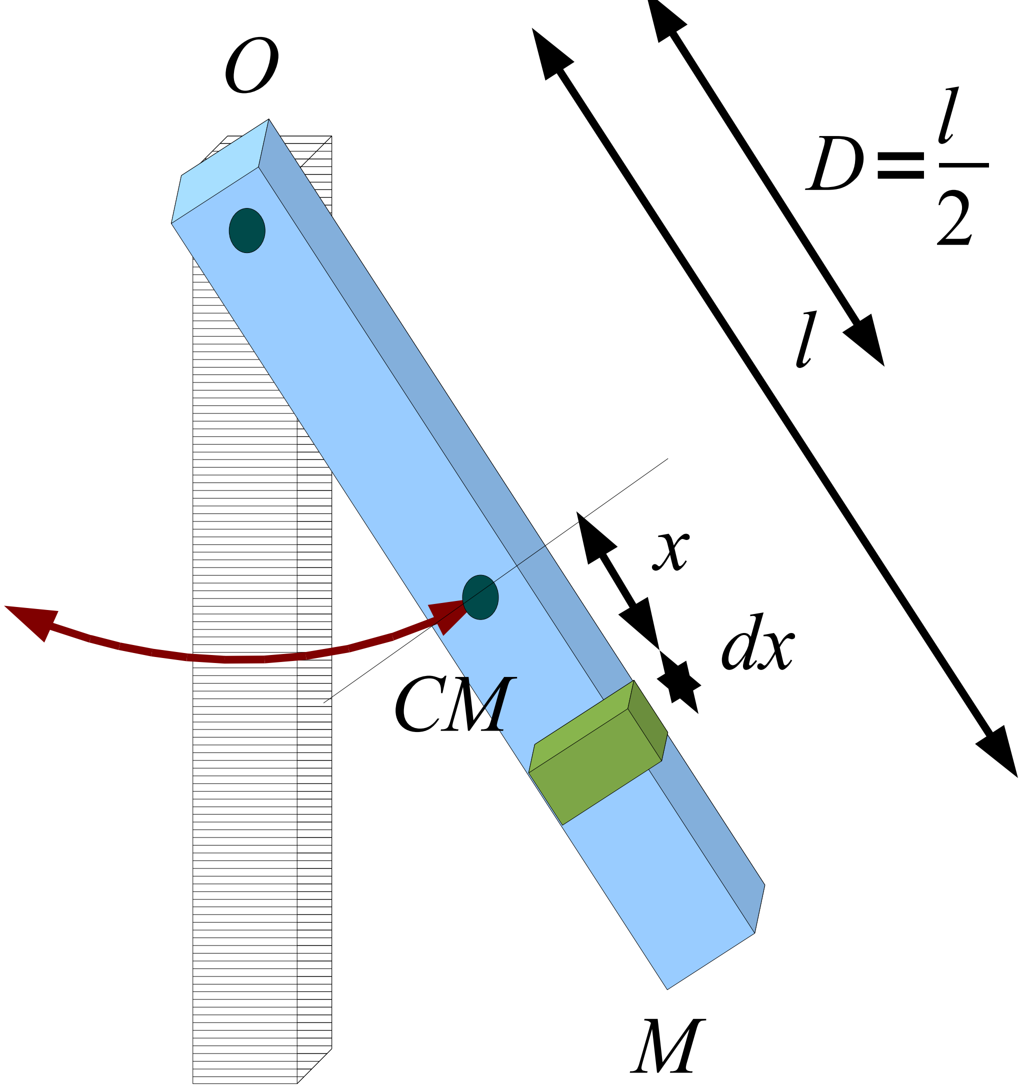
$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$

$$I_{CM} = \int r^2\, dm$$

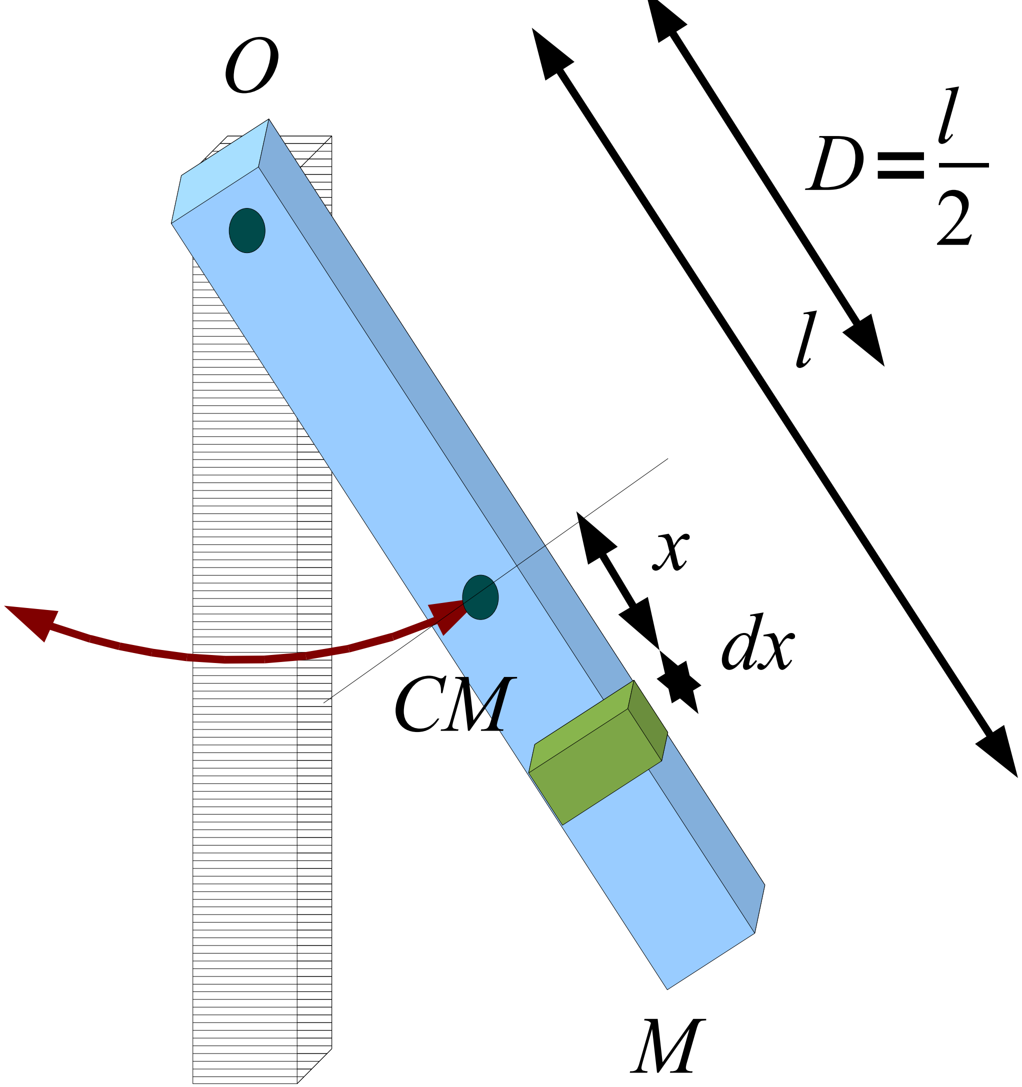
$$I_{CM} = \int_{-l/2}^{+l/2} x^2 (\lambda\, dx)$$

$$I_{CM} = \lambda \int_{-l/2}^{+l/2} x^2\, dx$$

$$I_{CM} = \lambda \left[\frac{1}{3} x^3 \right]_{-l/2}^{+l/2}$$

$$I_{CM} = \lambda \left(\frac{1}{3} \frac{l^3}{8} - \left(-\frac{1}{3} \frac{l^3}{8} \right) \right)$$

$$I_{CM} = \lambda \left(\frac{l^3}{12} \right) = \frac{M}{l} \left(\frac{l^3}{12} \right) = \frac{1}{12} M\, l^2$$



$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$

$$dm = \lambda\, dx$$

$$I_{CM} = \int r^2\, dm$$

$$I_{CM} = \int_{-l/2}^{+l/2} x^2 \left(\lambda\, dx\right)$$

$$I_{CM} = \lambda \int_{-l/2}^{+l/2} x^2\, dx$$

$$I_{CM} = \lambda \left[\frac{1}{3} x^3\right]_{-l/2}^{+l/2}$$

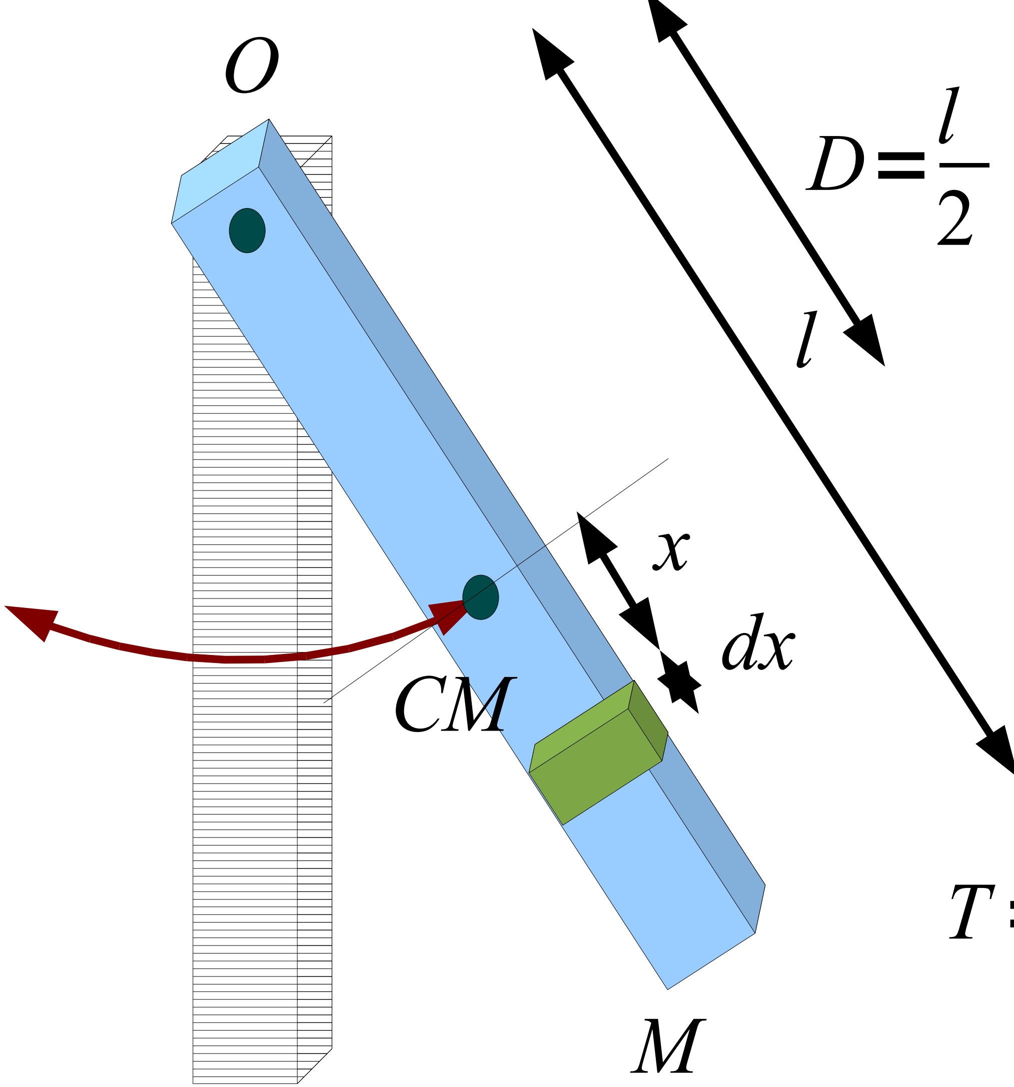
$$I_{CM} = \lambda \left(\frac{1}{3}\frac{l^3}{8} - \left(-\frac{1}{3}\frac{l^3}{8}\right)\right)$$

$$I_{CM} = \lambda \left(\frac{l^3}{12}\right) = \frac{M}{l}\left(\frac{l^3}{12}\right) = \frac{1}{12} M\, l^2$$

$$I = I_{CM} + MD^2$$

$$I = \frac{1}{12} M\, l^2 + M\left(\frac{l^2}{4}\right)$$

$$I = \frac{1}{12} M\, l^2 + \frac{3}{12} M\left(l^2\right) = \frac{4}{12} M\, l^2 = \frac{1}{3} M\, l^2$$



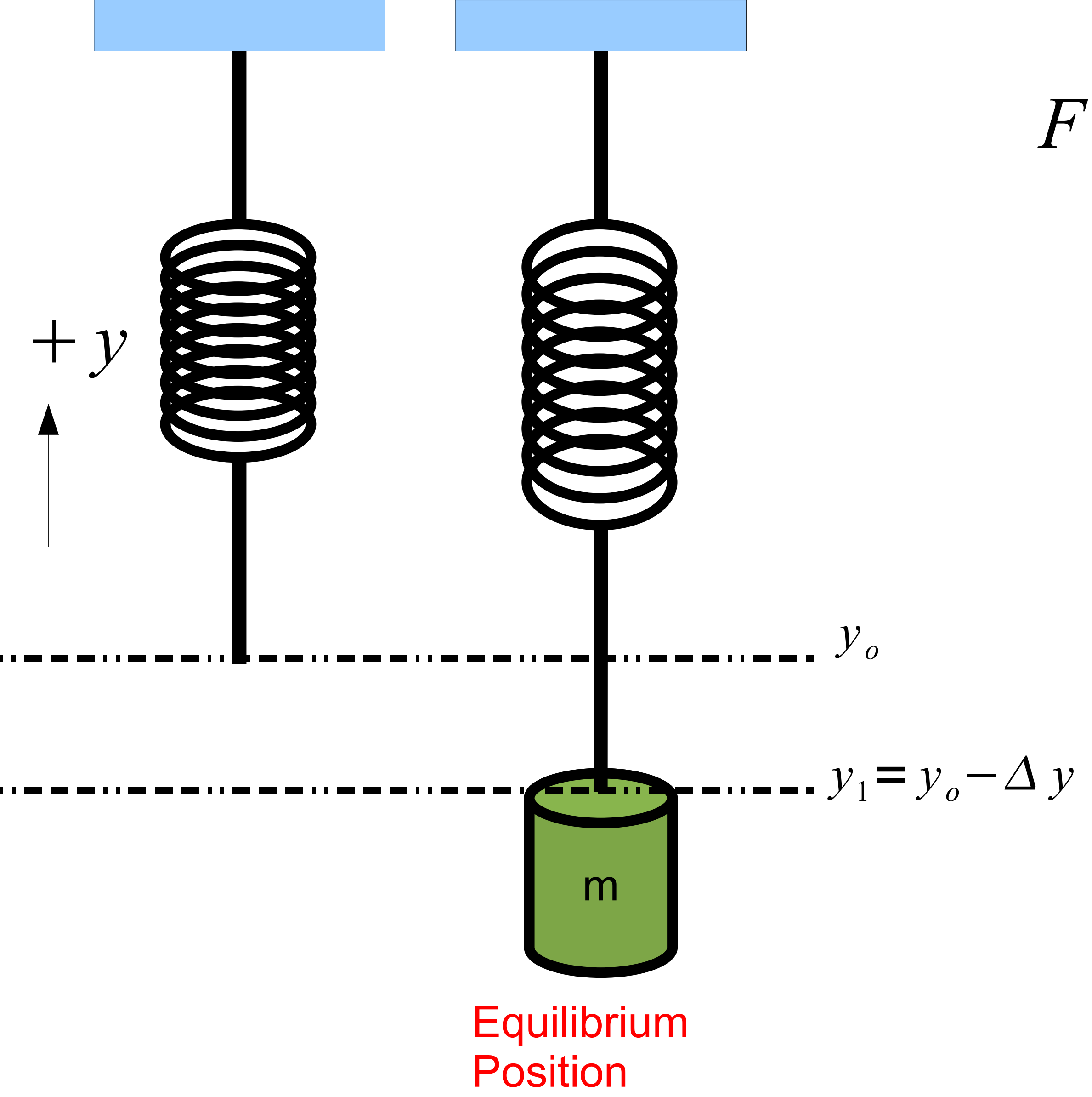
$$T = 2\pi\sqrt{\frac{I}{mgD}}$$

$$T = 2\pi\sqrt{\frac{1/3\,M\,l^2}{Mg\left(l/2\right)}} = 2\pi\sqrt{\frac{2}{3}\,\frac{l}{g}}$$

$$I = I_{CM} + MD^2$$

$$I = \frac{1}{12}\,M\,l^2 + M\left(\frac{l^2}{4}\right)$$

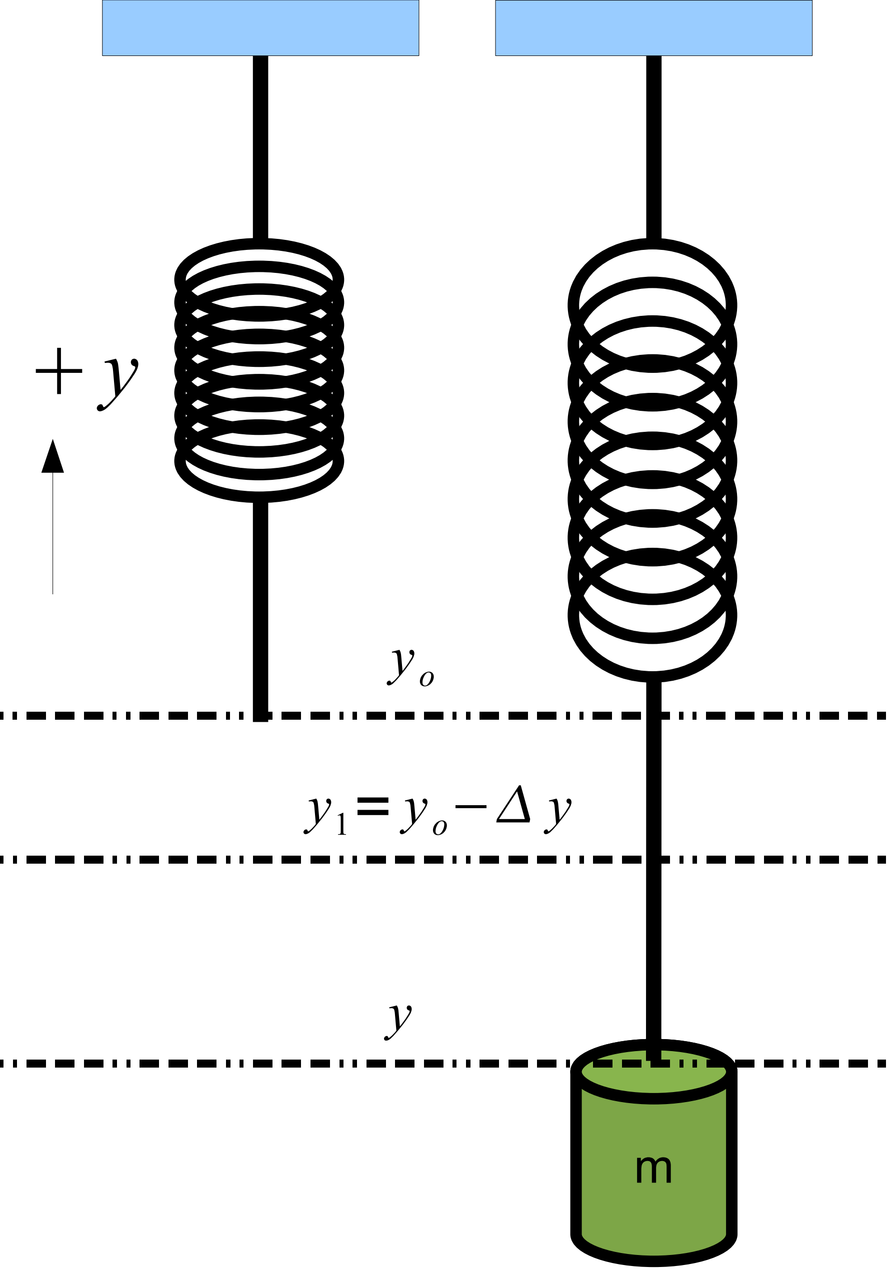
$$I = \frac{1}{12}\,M\,l^2 + \frac{3}{12}\,M\left(l^2\right) = \frac{4}{12}\,M\,l^2 = \frac{1}{3}\,M\,l^2$$



$$F_{net} = F_s - w = k\,\Delta y - mg = 0$$

$$k\,\Delta y = mg$$

$$k\left(y_o - y_1\right) = mg$$



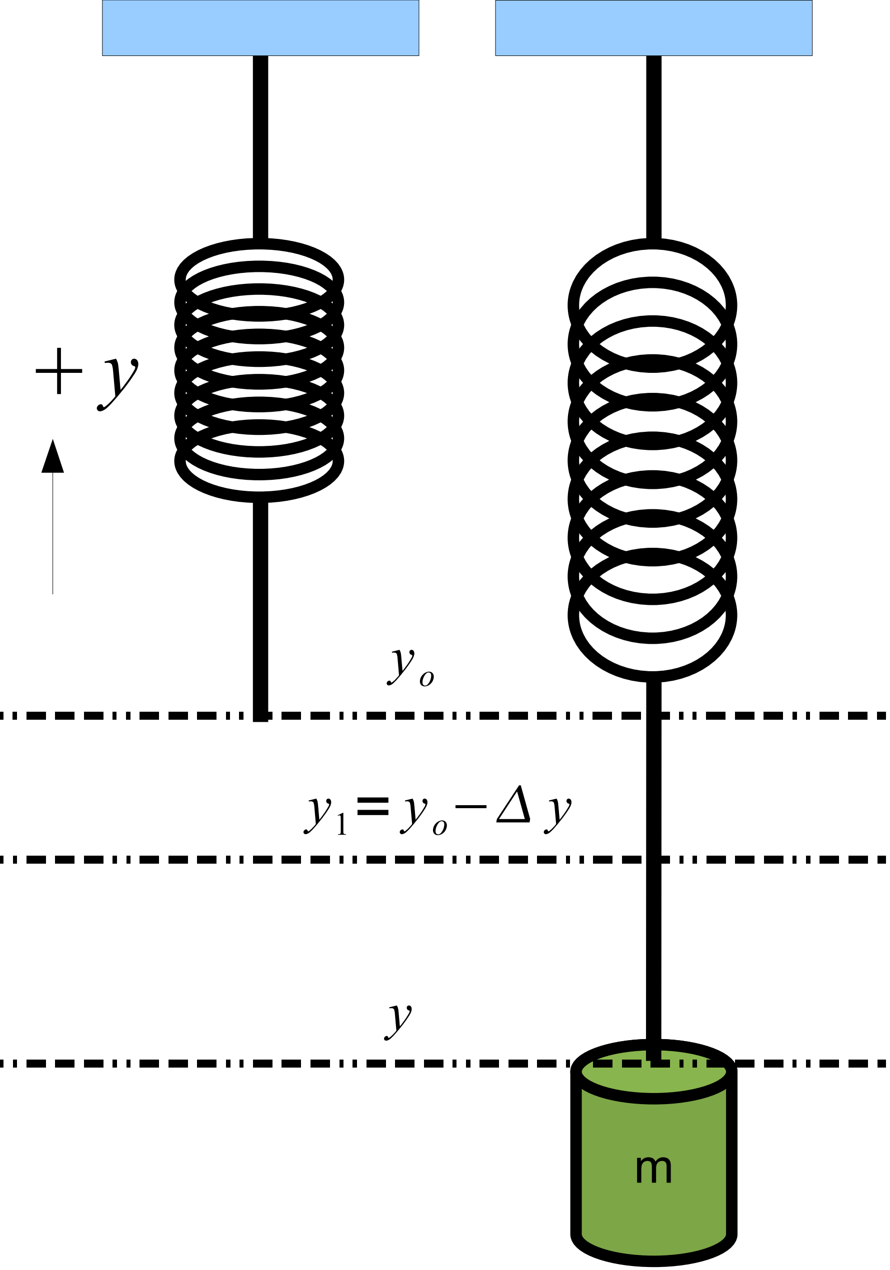
$$F_{net} = F_s - w = k\,\Delta y - mg = 0$$

$$k\,\Delta y = mg$$

$$k\left(y_o - y_1\right) = mg$$

$$F_{net} = -k\left(y - y_o\right) - mg$$

$$ma = -k\left(y - y_o\right) - mg$$



$$F_{net} = F_s - w = k\Delta y - mg = 0$$

$$k\Delta y = mg$$

$$k(y_o - y_1) = mg$$

$$F_{net} = -k(y - y_o) - mg$$

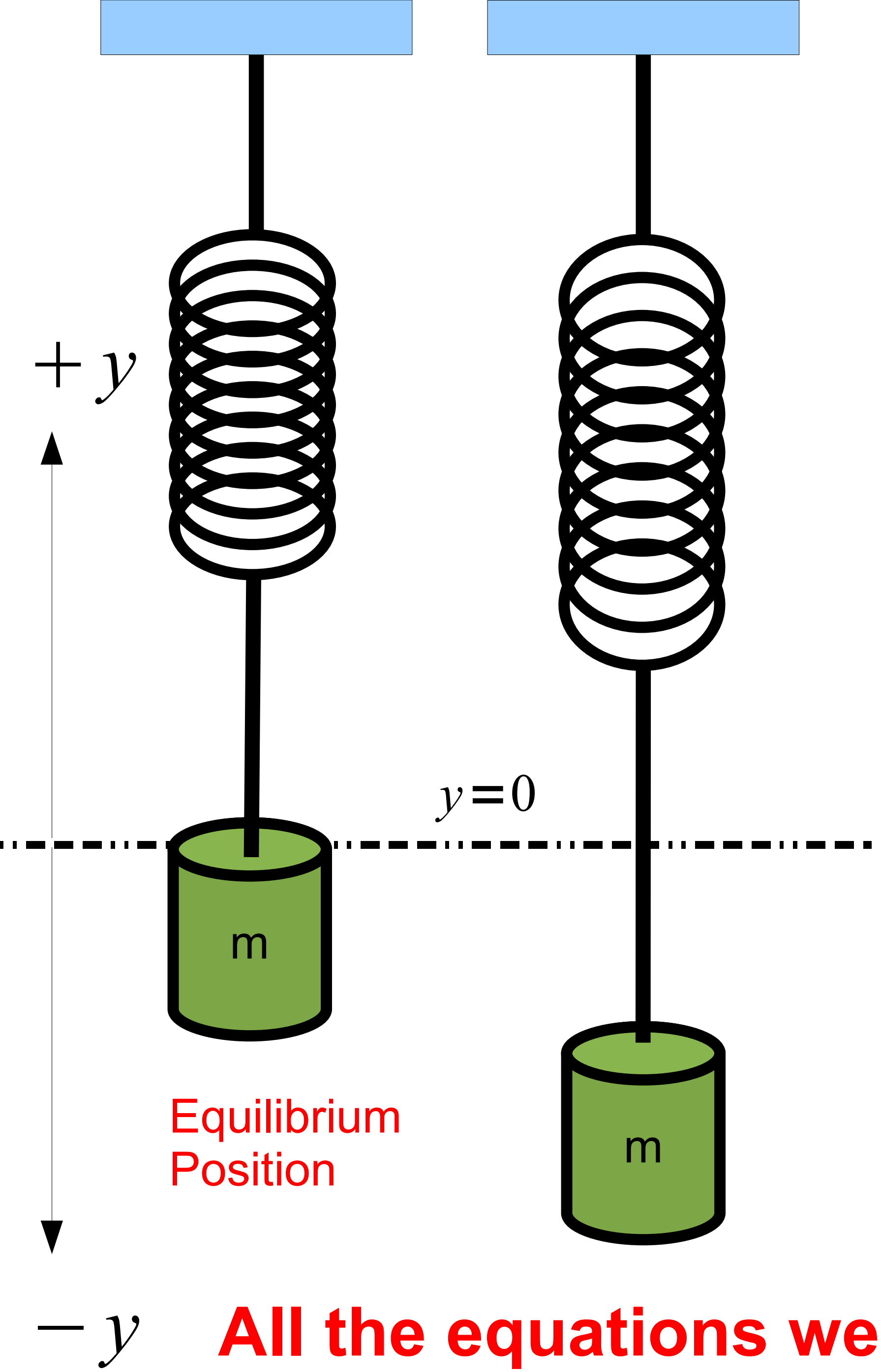
$$ma = -k(y - y_o) - mg$$

$$ma = -k(y - y_o) - k(y_o - y_1)$$

$$ma = -(ky - ky_o) - (ky_o - ky_1)$$

$$ma = -ky + ky_o - ky_o + ky_1$$

$$ma = -k(y - y_1)$$



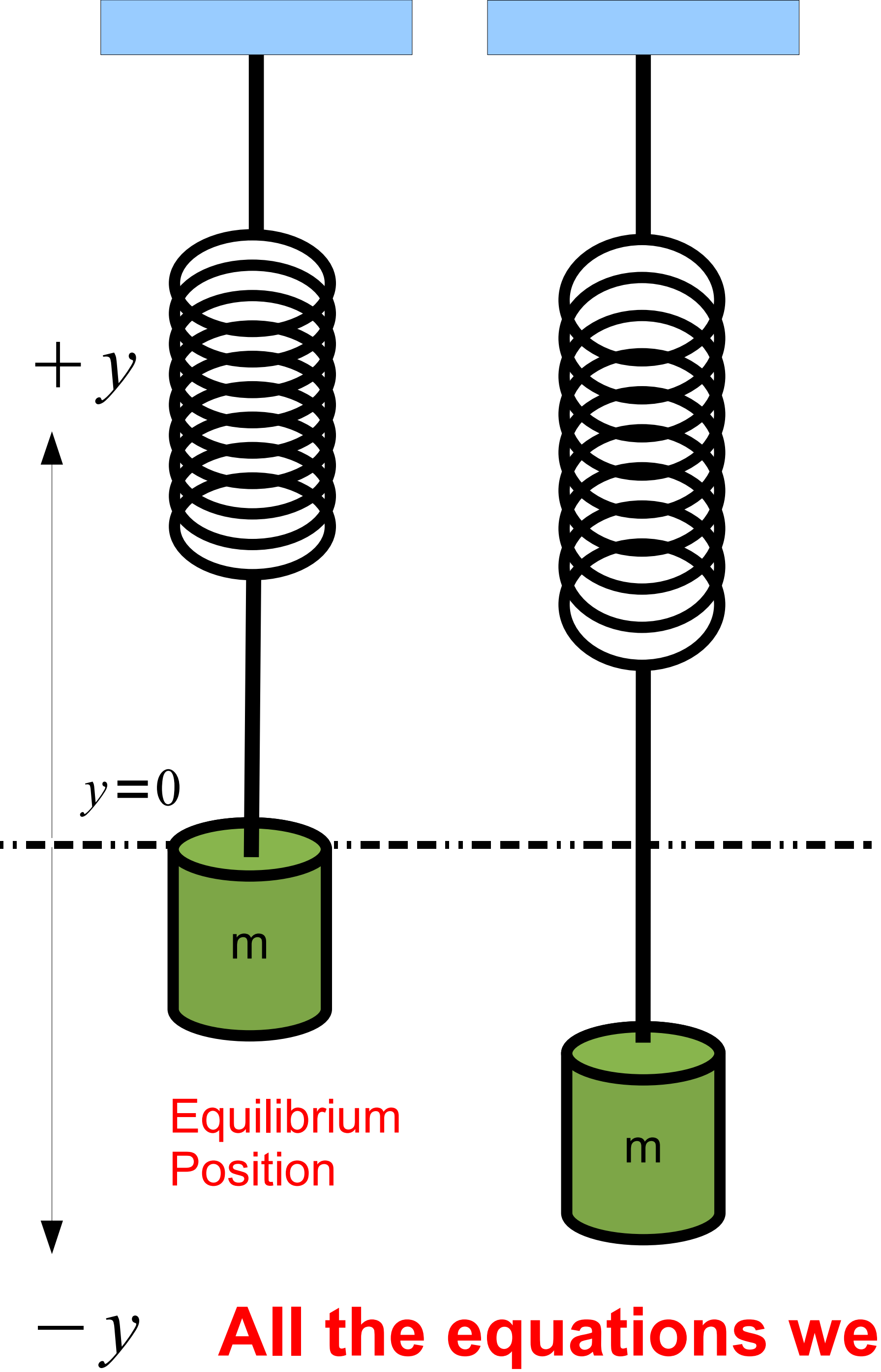
$$F_{net}=F_s-w=k\,\Delta y-mg=0$$
$$k\,\Delta y=mg$$
$$k\left(y_o-y_1\right)=mg$$

$$F_{net}=-k\left(y-y_o\right)-mg$$
$$ma=-k\left(y-y_1\right)$$

Set y_1 as new equilibrium position $y_1=0$

$$ma=-ky$$

All the equations we used for the horizontal spring on a frictionless table can be used here, as long as we take the equilibrium position to be y = 0.



$$F_{net}=F_s-w=k\Delta y-mg=0$$

$$k\Delta y=mg$$

$$k(y_o-y_1)=mg$$

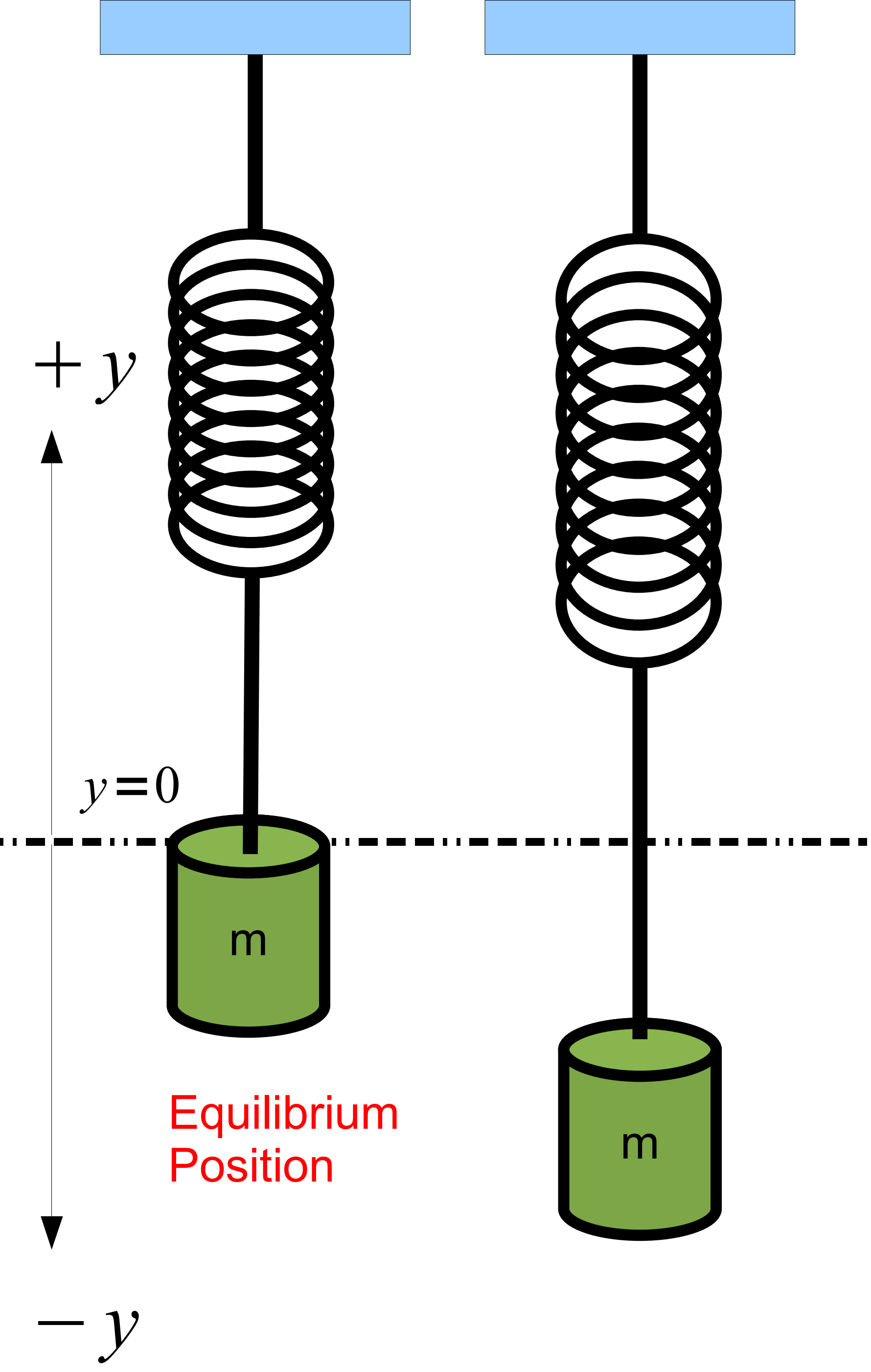
$$F_{net}=-k(y-y_o)-mg$$

$$ma=-k(y-y_1)$$

Set y_1 as new equilibrium position $y_1=0$

$$ma=-ky$$

All the equations we used for the horizontal spring on a frictionless table can be used here, as long as we take the equilibrium position to be y = 0.



A

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$v_{max} = A\omega$$

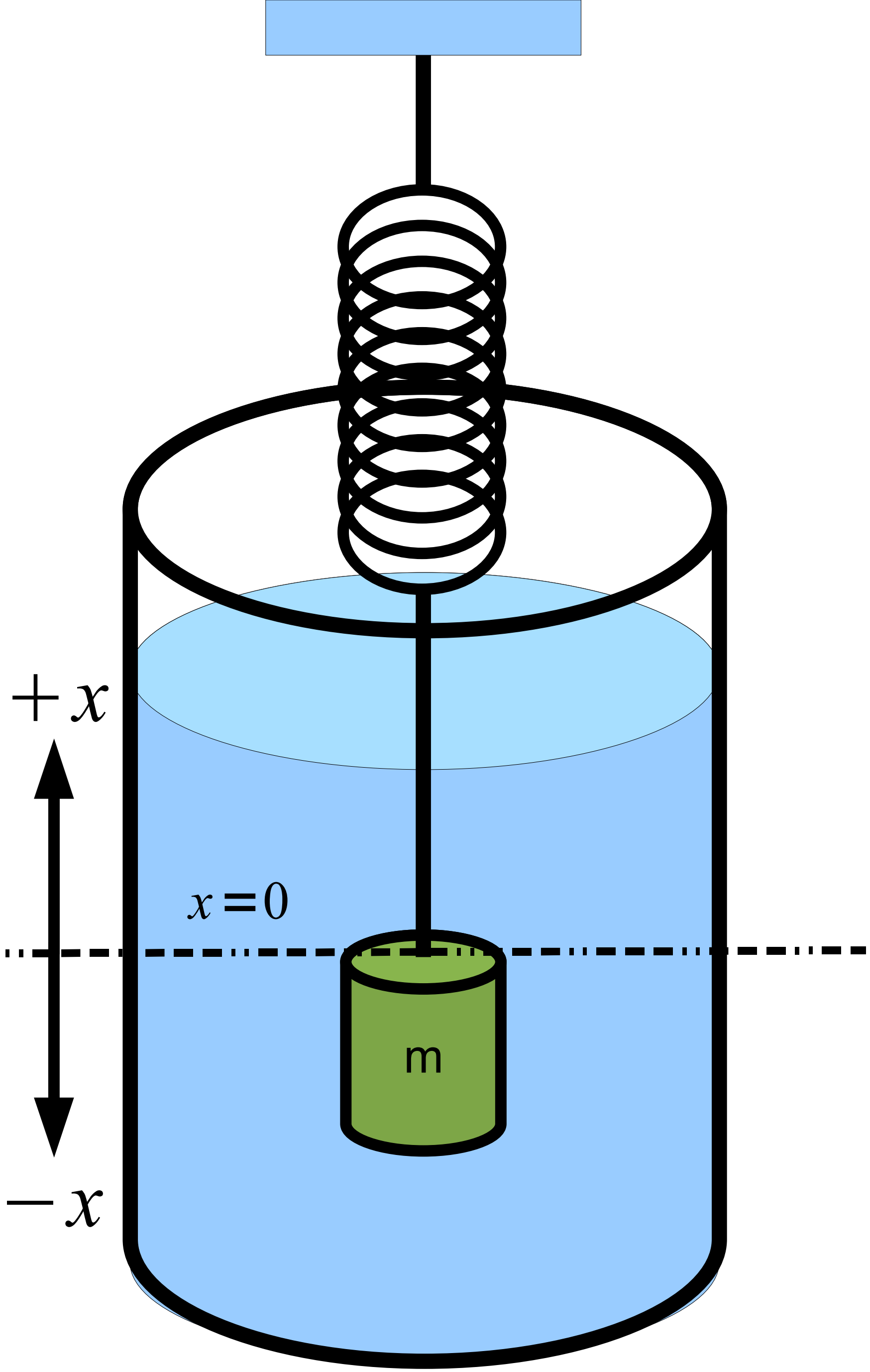
$$a_{max} = A\omega^2$$

$$y(t) = A\cos(\omega t)$$

$$v_y(t) = -A\omega\sin(\omega t)$$

$$a_y(t) = -A\omega^2\cos(\omega t)$$

Damped Harmonic Motion



$$-kx - bv = ma$$

$$x = e^{-\alpha t}\sin(\omega t)$$

$$\alpha = \frac{b}{2\mathrm{m}}$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2\mathrm{m}}\right)^2}$$

$$\omega = \sqrt{\omega_o^2 - \left(\frac{b}{2\mathrm{m}}\right)^2}$$

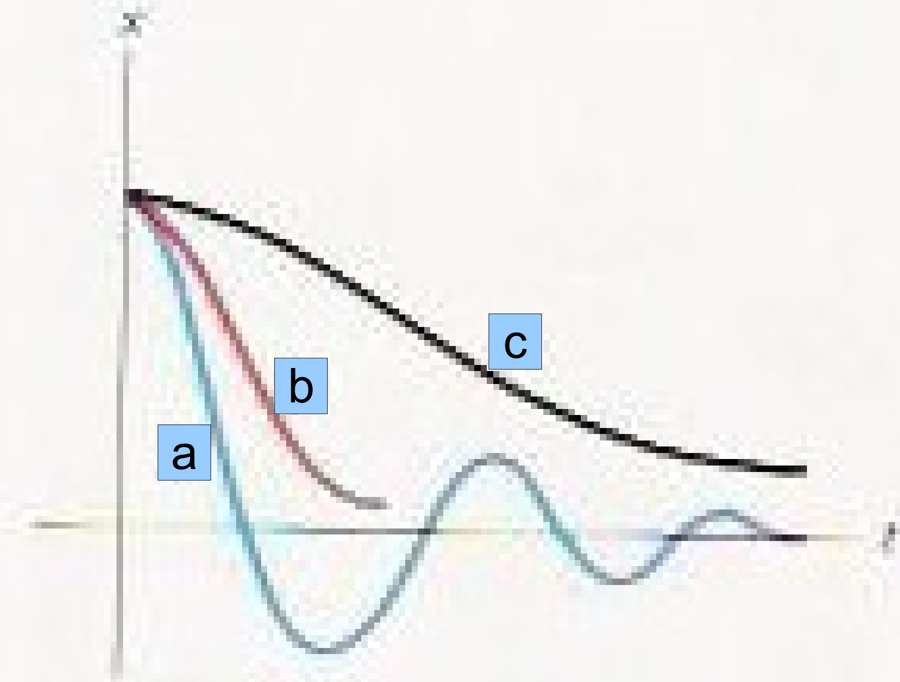


Figure 13.20 Plots of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$x = e^{-\alpha t}\sin(\omega t)$$

$$\alpha = \frac{b}{2\text{m}}$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2\text{m}}\right)^2}$$

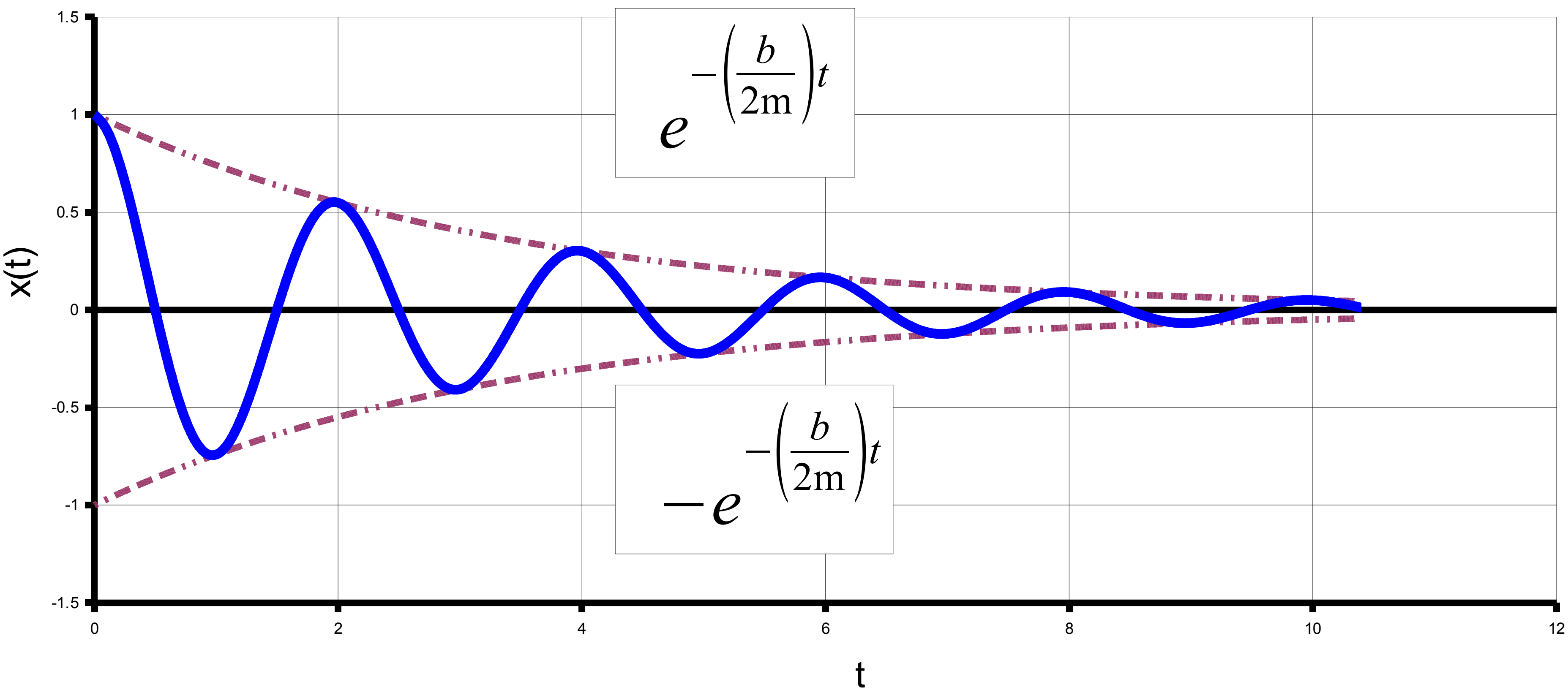
$$b_c = \sqrt{4mk}$$

a.) Underdamped $b < b_c$

b.) Critically Damped $b = b_c$

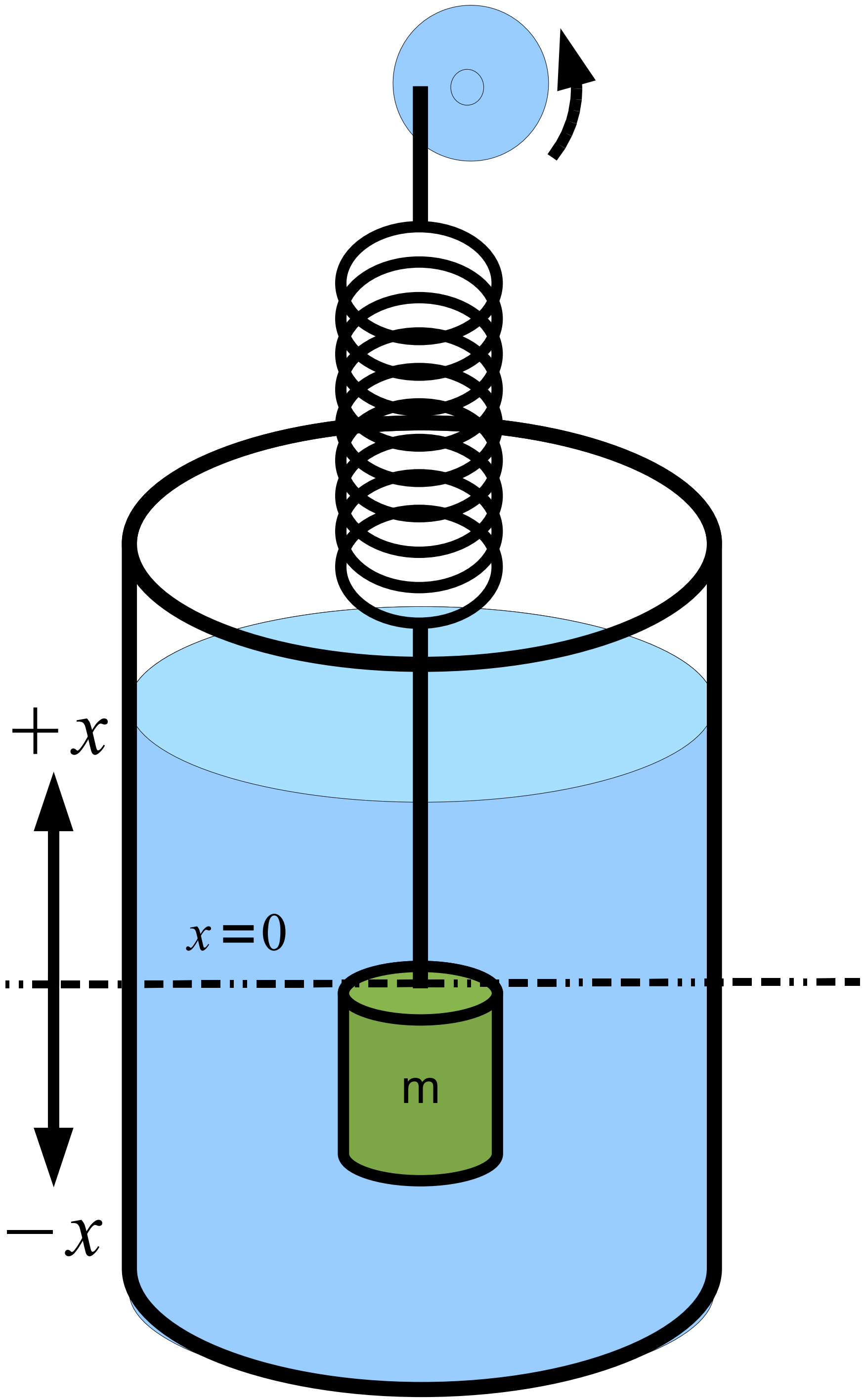
c.) Overdamped $b > b_c$

Under Damped Harmonic Motion



m	10
k	100
b	6
wo	3.16
w	3.15

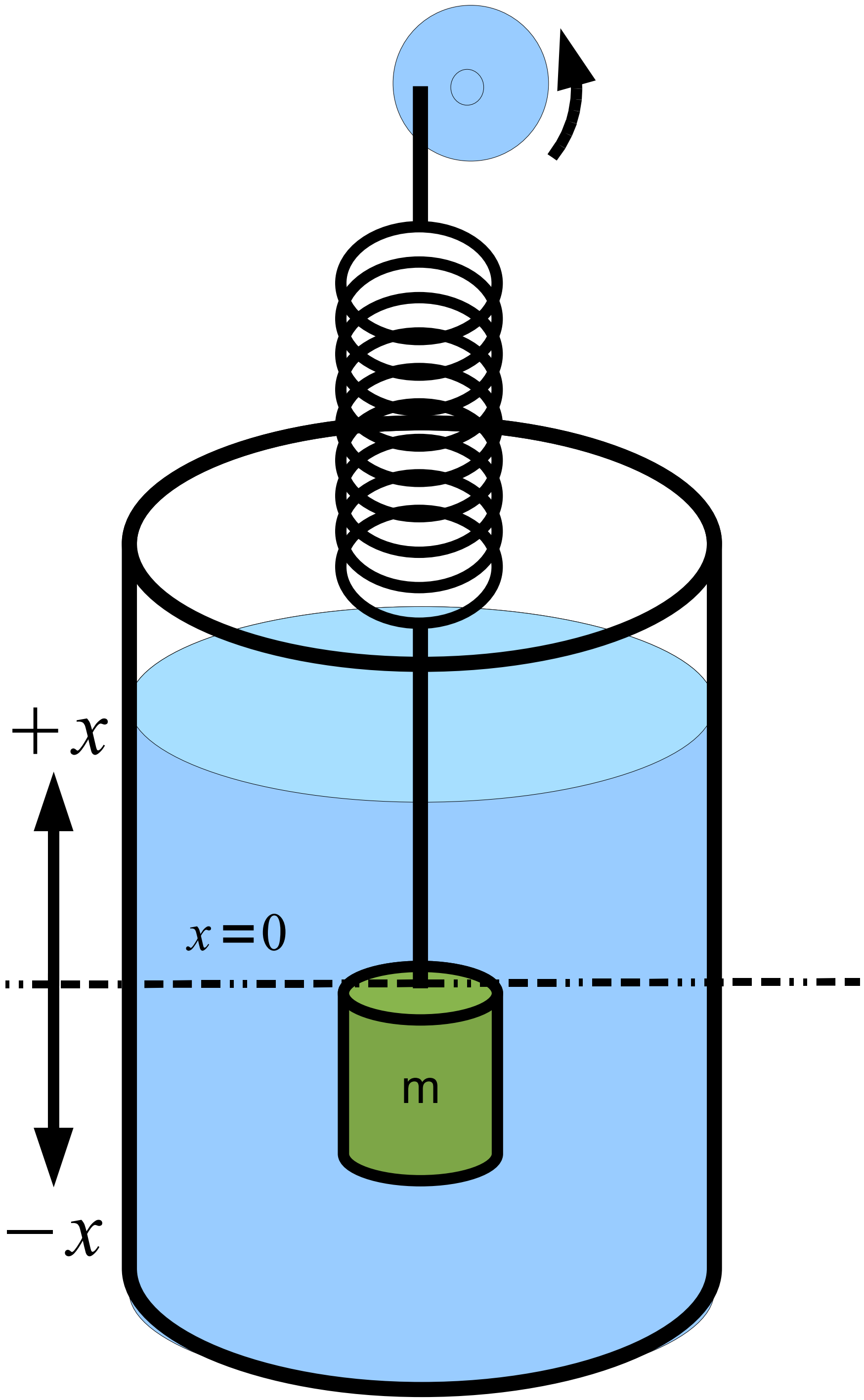
Forced Damped Harmonic Motion



$$-kx - b\frac{dx}{dt} + F_o \sin(\omega t) = m\frac{d^2 x}{dt^2}$$

$$x(t) = A\sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$



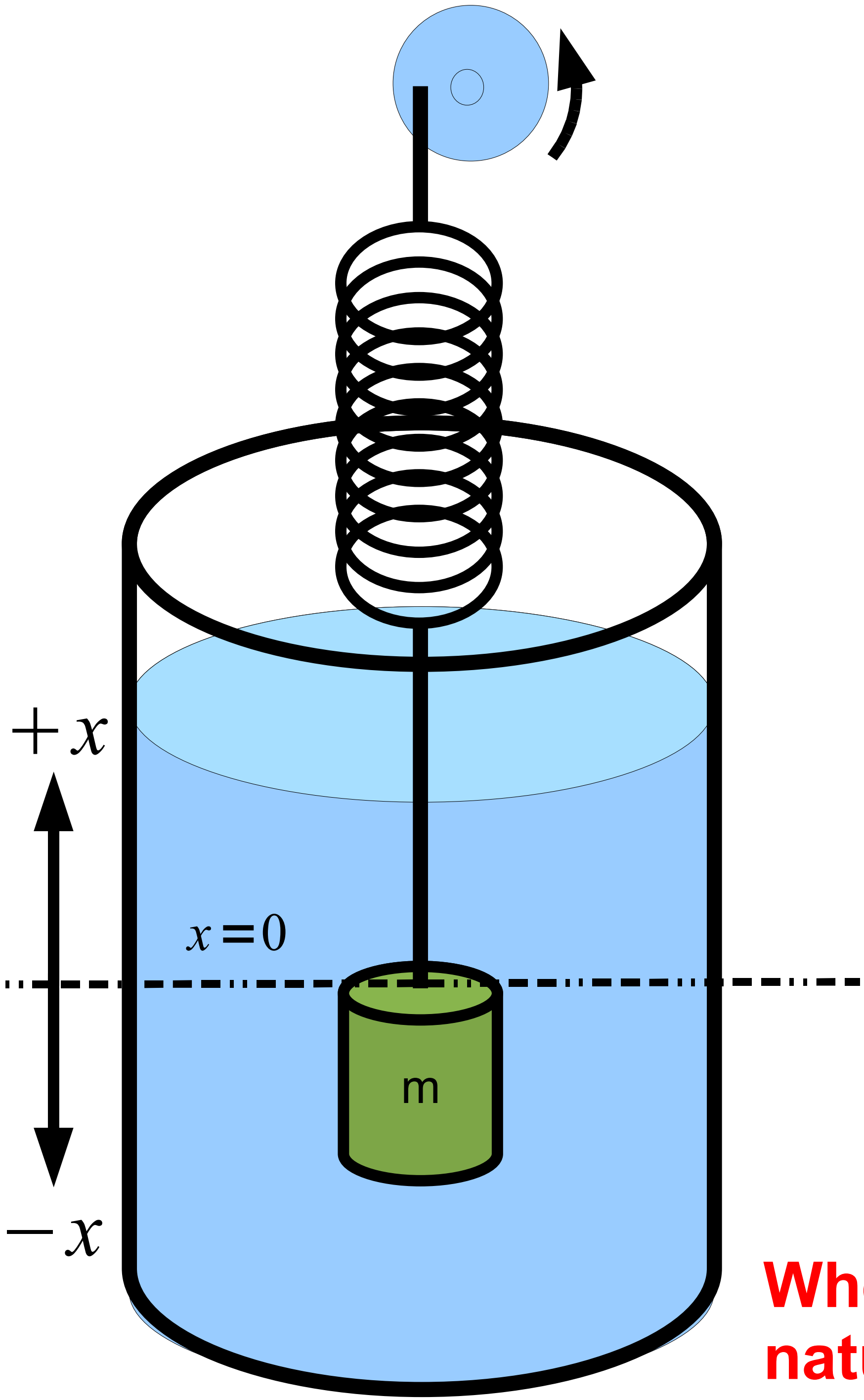
$$-kx - bv + F_o \sin(\omega t) = ma$$

$$x(t) = A \sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$

When $\omega = \omega_o$

$$A_{max} = \frac{F_o}{\sqrt{b^2\omega^2}}$$



$$-kx - bv + F_o \sin(\omega t) = ma$$

$$x(t) = A\sin(\omega t + \phi)$$

$$A = \frac{F_o}{\sqrt{m^2(\omega^2 - \omega_o^2) + b^2\omega^2}}$$

When $\omega = \omega_o$

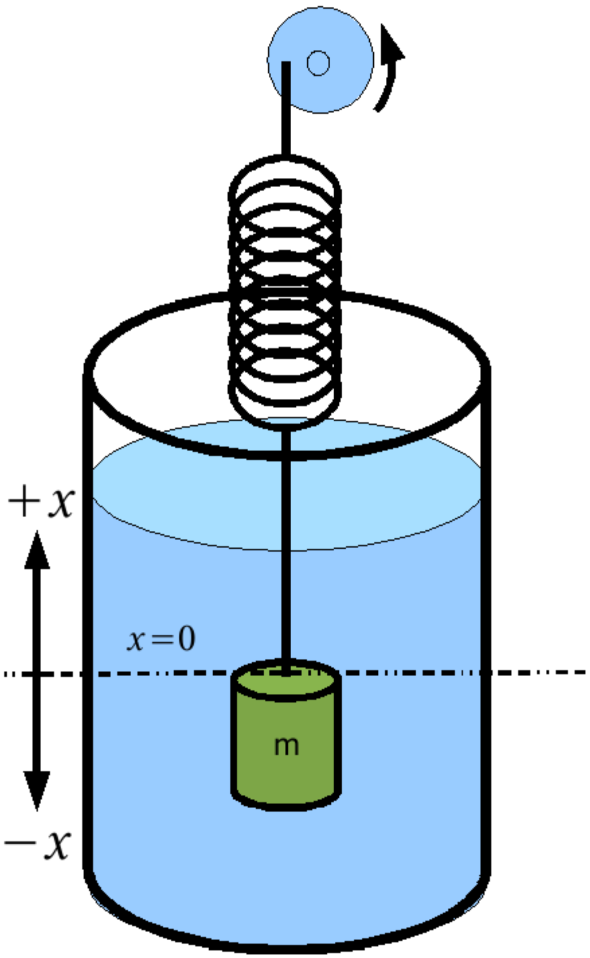
$$A_{max} = \frac{F_o}{\sqrt{b^2\omega^2}}$$

When the driving frequency equals the natural frequency, the amplitude reaches a maximum.

$b = 2\,kg/s$

$b = 3\,kg/s$

$b = 10\,kg/s$



When $\omega = \omega_o = 3.16\,s^{-1}$

$$A_{max} = \frac{F_o}{\sqrt{b^2\omega^2}}$$

Under Damped Harmonic Motion