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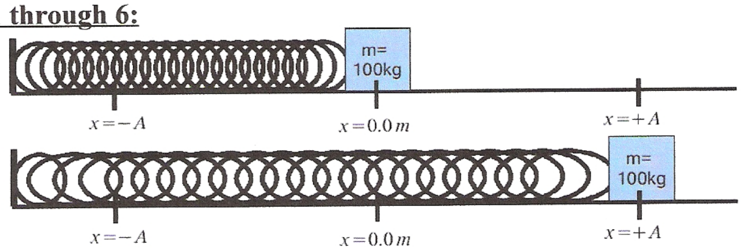
SHOW ALL WORK ON THE TEST PAPER. No other papers will be accepted.
Fundamentals of Physics III – Phys280 – Spring 2009-2010

## Multiple Choice ( 5 Points Each):

**Problems 1 through 6:**




A spring (which can be stretched and compressed) is placed on a frictionless table. One end is anchored to a wall and the other end is attached to a mass of 100 kg. The equilibrium is marked as x=0.0 m. The mass is pulled out to x=+A= +0.2 m and released from rest. The mass then oscillates in Simple Harmonic Motion, between x=+A=+0.2 m and x=-A= -0.2 m. Ignore all drag forces and other frictional forces. The spring constant of the spring is 300 N/m.

1.) What is the angular frequency of the motion of the mass?

a.) 1.41 $s^{-1}$
b.) 31.41 $s^{-1}$
c.) 0.58 $s^{-1}$
d.) 1.73 $s^{-1}$
e.) None of the above.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{300\ N/m}{100\ kg}} = 1.73\ s^{-1}$$

2.) What is the period and frequency of the oscillations of the mass on the spring?

a.) $T=6.0\,s$ , $f=0.17\,Hz$
b.) $T=10.88\,s$ , $f=0.09\,Hz$
c.) $T=3.63\,s$ , $f=0.28\,Hz$
d.) $T=4.44\,s$ , $f=0.23\,Hz$
e.) None of the above.

$$T = \frac{2\pi}{\omega} = 3.63s$$
$$f = \frac{1}{T} = 0.28Hz$$

3.) What is the total energy of the system?

a.) 193.1 J
b.) 6.00 J
c.) 18.89 J
d.) 4.50 J
e.) None of the above.

$$E = \frac{1}{2} k A^2$$
$$= \frac{1}{2}(300\tfrac{N}{m})(0.2m)^2$$
$$= 6J$$

4.) What is the position and velocity of the mass at t = 21.0 seconds after the mass is released?

a.) $x = -0.16\,m$ $v = +0.21\,m/s$
b.) $x = 0.04\,m$ $v = +0.34\,m/s$
c.) $x = 0.16\,m$ $v = -0.21\,m/s$
d.) $x = 0.06\,m$ $v = -0.42\,m/s$
e.) None of the above.

$X = 0.2m \cos(1.73s^{-1}(21s)) = $
$= 0.04m$

$v = -0.2m(1.73s^{-1})\sin(1.73s^{-1}(21s)) =$
$= +0.34\,m/s$

5.) The acceleration of the mass is maximum and points in the negative x direction at:

a.) x = 0.0 m
b.) x = -A
c.) x = +A
d.) Both b.) and c.) are correct.
e.) None of the above.

6.) If I wanted to apply a driving force to the system to reach resonance, what frequency driving force would I use?

a.) 1.73 Hz
b.) 0.28 Hz
c.) 1.41 Hz
d.) 0.56 Hz
e.) None of the above.

Problems 7 through 9:

A standing waves are produced in a medium by driving the medium at one end to produce waves of the form:

$$y(x,t)=0.01\,m\sin(2\,m^{-1}x+12\,s^{-1}t)$$

The waves reflect off a soft, stationary surface and reflect back towards the source of the original wave.

7.) What is the speed of the waves?

a.) 6.00 m/s
b.) 0.17 m/s
c.) 343.0 m/s
d.) 3000 m/s
e.) None of the above.

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{12s^{-1}}{2m^{-1}} = 6\,m/s$$

8.) What is the maximum velocity of the medium perpendicular to the direction of propagation (not the speed of the wave, but the vertical velocity of the medium)?

a.) 6.00 m/s
b.) 0.17 m/s
c.) 0.02 m/s
d.) 0.12 m/s
e.) None of the above.

$$\frac{\partial y(x,t)}{\partial t} = +0.01m\,(12s^{-1})\cos(2m^{-1}x + 12s^{-1}t)$$
$$= .12\,m/s\ \cos(2m^{-1}x + 12s^{-1}t)$$

9.) Which of the following equations would best describe the reflected wave:

a.) $y(x,t)=0.01\,m\sin(2\,m^{-1}x+12\,s^{-1}t)$
b.) $y(x,t)=0.01\,m\cos(2\,m^{-1}x+12\,s^{-1}t)$
c.) $y(x,t)=0.01\,m\sin(2\,m^{-1}x-12\,s^{-1}t)$
d.) $y(x,t)=-0.01\,m\sin(2\,m^{-1}x+12\,s^{-1}t)$
e.) None of the above.

10.) In order to determine the distance to a distant cliff, a hiker named Sam on a winter hike with friends ( T = 12 °C ), shouts and times the time it takes for the echo to return. It takes 5.6 seconds. One of his friends determines the distance to be approximately 1.92 km. The hiker, Sam, having taken a physics course at Drexel, determines the distance to be approximately:

a.) 18.90 km
b.) 1.89 km
c.) 19.20 km , .95km
d.) 1.92 km
e.) None of the above.

$$v = 331\,m/s + .6\,T_C$$
$$= 331\,m/s + .6\,\tfrac{m/s}{°C}(12°C) = 338.2\,m/s$$

$$\Delta x = vt = 338.2\,m/s\left(\frac{5.6s}{2}\right) = 947m$$

Problem One ( 25 Points ):

A standing wave has the form:

$$y(x,t) = (2A\sin kx)\cos\omega t = 0.9\,m\sin(3\,m^{-1}x)\cos(6.28\,s^{-1}t)$$

1. Prove that y( x,t ) is a solution of the linear wave equation: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$
2. What is the speed of the waves?
3. For what values of x will nodes appear?
4. What is the maximum displacement of the anti-nodes?

1) $\frac{\partial y}{\partial x} = 2Ak\cos kx\cos\omega t$ $\qquad$ $\frac{\partial y}{\partial t} = -2A\omega\sin kx\sin\omega t$

$\frac{\partial^2 y}{\partial x^2} = -2Ak^2\sin kx\cos\omega t$ $\qquad$ $\frac{\partial^2 y}{\partial t^2} = -2A\omega^2\sin kx\cos\omega t$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

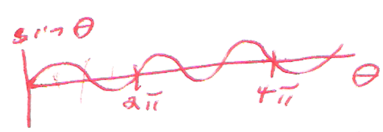
$$(\cancel{-2A}k^2\,\cancel{\sin kx\cos\omega t}) = \left(\frac{1}{v^2}\right)(\cancel{-2A}\omega^2\,\cancel{\sin kx\cos\omega t})$$

$$k^2 = \frac{\omega^2}{v^2}$$

$$v = \frac{\omega}{k}$$

2) $v = \frac{\omega}{k} = \frac{6.28\,s^{-1}}{3\,m^{-1}} = 2.09\,m/s$

3) $\sin kx = 0$




$kx = 0, \pi, 2\pi, 3\pi \ldots$

$\frac{2\pi}{\lambda}x = 0, \pi, 2\pi, 3\pi \ldots$

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3}{2}\lambda$ $\quad n = 0, 1, 2, \ldots$

$= \frac{n}{2}\lambda = 0, \frac{3}{2}m, 3m, 4.5m$

$\sin kx = \pm 1$

$\frac{2\pi}{\lambda} = kx = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi$

$x = \frac{n}{4}\lambda$ $\quad n = 1, 3, 5, \ldots$

$= \frac{3}{4}m, \frac{9}{4}m, \frac{15}{4}m, \ldots$

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

Problem Two ( 25 Points ):

A bat can detect very small insects whose length is approximately equal to one wavelength of the sound the bat emits. The bat emits a sound with a frequency of 60 kHz and the temperature of the air is 25°C.

1. What is the approximate size of the smallest insect the bat can detect?
2. What is the observed frequency, observed by the bat, of the wave reflected off the insect if the bat is moving in the positive x direction at 30 m/s and the insect is moving in the positive x direction at 25 m/s?

$$v = 331\,m/s + 0.6\,\frac{m/s}{°C}(25°C) \approx 346\,m/s$$

1) $v = \frac{\lambda}{T} = \lambda f$

$$\lambda = \frac{v}{f} = \frac{346\,m/s}{60\times10^{3}\,s^{-1}} \simeq 5.8\,mm$$

2) $$f'_{at\ insect} = f\left(\frac{v - v_o}{v - v_s}\right)$$

$$= 60\,kHz\left(\frac{346\,m/s - 25\,m/s}{346\,m/s - 30\,m/s}\right)$$

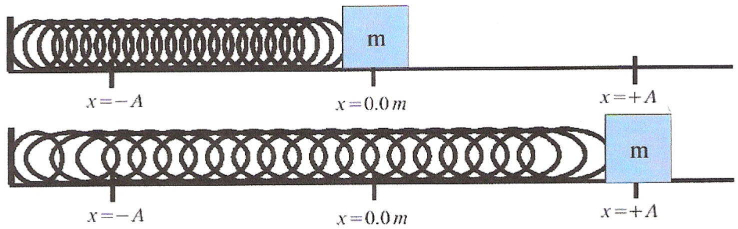
$$= 60.95\,kHz$$

$$f'_{at\ bat} = f\left(\frac{v + v_o}{v + v_s}\right)$$

$$= 60.95\,kHz\left(\frac{346\,m/s + 30\,m/s}{346\,m/s + 25\,m/s}\right)$$

$$\simeq 61.77\,kHz$$

Extra Credit ( 5 Points):




A mass (m) is attached to a spring that has a spring constant (k). The mass and spring is set on a frictionless table and the free spring is attached to the wall. The equilibrium position is marked as x=0.0m. The mass is then pulled to x=+A and released from rest. The mass oscillates back and fourth between x=-A and x=+A, but with a slowly decreasing amplitude due to a damping force, $F_D = -bv$

a.) Write down Newton's Second Law for this experiment.

b.) Prove that $x(t) = A\,e^{-\left(\frac{b}{2m}\right)t}\cos(\omega t + \phi)$ is a solution to part a.) . Under what conditions?

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

$$x=Ae^{-\frac{b}{2\mathrm{m}}t}\cos(\omega t+\phi)$$

$$\frac{dx}{dt}=-A\frac{b}{2\mathrm{m}}e^{-\frac{b}{2\mathrm{m}}t}\cos(\omega t+\phi)-A\omega e^{-\frac{b}{2\mathrm{m}}t}\sin(\omega t+\phi)$$

$$\frac{d^2x}{dt^2}=A\left(\frac{b}{2\mathrm{m}}\right)^2e^{-\frac{b}{2\mathrm{m}}t}\cos(\omega t+\phi)-A\frac{b}{2\mathrm{m}}\omega e^{-\frac{b}{2\mathrm{m}}t}\sin(\omega t+\phi)+A\omega\frac{b}{2\mathrm{m}}e^{-\frac{b}{2\mathrm{m}}t}\sin(\omega t+\phi)-A\omega^2e^{-\frac{b}{2\mathrm{m}}t}\cos(\omega t+\phi)$$