Why do Casinos make money?

Casinos are a tremendous money-making venture. Everybody knows this. And, despite other delusions that people may have about a particular "system" of betting, it is nevertheless true that (with the exception of games like poker where you play against other civilians) you are more likely to lose than to win. So why are people still willing to play?

The easy answers are that people are dumb or uninformed or that they are simply paying for "entertainment" and certainly all of these are true to some degree, but consider a game where you lost *every* time. Would you still play?

Let's consider a fairly simple game – Roulette. Now, in Roulette, there is a wheel, which has slots labeled from 1-36 (half in red, and half in black), as well as a slot with a green "0" and another green "00". A ball is released and falls randomly in one of the slots. This game, unlike many others (card games, especially) has no "memory". The outcome of a particular spin is completely independent of what happened previously.

One of the most common bets consists of selecting red or black. Now, it is clear to see that since there are a total of 38 slots, and 18 of them are, say, red, then the probability of red coming up is:

$$p = \frac{18}{38} = 0.473$$

Expectation Value

This means that if you bet, say, a dollar on a given spin then the "expectation value" (simply put, the return for each outcome multiplied times the probability of the outcome, and all added up) of your return is:

$$E(r) = p \times 1 + (1 - p) \times (-1) = -0.054$$

This means, of course, that *on average* for every dollar I put down, I will lose about a nickel each bet. If that loss were steady then ultimately all of your money would be depleted. But wait! This is a random process, and therefore it behaves like a random walk. Consider that if you bet 1 dollar N times in a row, then your expected loss will be:

$$-0.05 \times N \pm \sqrt{N}$$

In other words, if you play 10 spins, and bet on red each time, then, sure, on average you'll lose 50 cents (0.05×10), but your range reasonable range of outcomes will be between -$3.66 to +$2.66. This means that fairly often, you'll come out a winner.

Let's consider the situation generally for a game which pays even odds. In roulette, your expected return is -5%, in Blackjack (with typical rules, and assuming you play optimally), your expected return is about -0.5% per hand. In both cases, though, the standard deviation in your return will be $\sigma\sqrt{N}$ (σ is the standard deviation *per hand*; in this particular case, $\sigma = 1$). That means that if you play for a long time, a reasonable ($\sim 70\%$ of the time) range of outcomes will be:

$$ret = \mu N \pm \sigma\sqrt{N}$$

Now, let's say you're lucky. You're at the upper limit of your expected performance. You're ahead if your return is above zero. The longer you play, the less likely that's to be the case.

Consider the upper limit:

$$ret_+ = \mu N + \sigma\sqrt{N}$$

if you've played for a very long time, the best you can reasonably hope for is to exactly break even. How long?

$$\mu N + \sigma\sqrt{N} = 0$$

or

$$N = \left(\frac{\sigma}{\mu}\right)^2$$

In other words, in Roulette, you can play for about 400 spins before you can be pretty certain of losing. In blackjack, it's more like $40,000$ hands!

And this, in essence, is why casinos make money. Consider that in a typical visit, I'm only likely to play, say, 100 hands of blackjack. On average, I'm only going to lose 1/2 of a hand more than I win. And the scatter is something like 10 hands in either direction. In essence, this means that something like 45% of the time, I'm going home a winner! And that's great for the casino, because it means that I'm more likely to come back next time. Meanwhile, since the casino is averaging over so many players, they consistently make returns equal to the house edge.

Long Odds

There's another type of bet that's available in Vegas (or in your lottery, for example), and that's one with long odds. Besides being able to bet on red vs. black, Roulette also offers you the opportunity to bet that a specific number will come up. If that happens, you get 35 times your bet! Clearly the expected return is:

$$E(r) = 35 \times (1/38) + (-1) \times (37/38) = -0.053$$

A very similar return, but the vast majority of the time you lose, and only occasionally do you win. This means that (for reasons I'm not going to go into), the standard deviation in your return is something like $5.8 \times \sqrt{N}$, and thus, there is a much larger scatter (over many, many spins) of:

$$-0.05 \times N \pm 5.8 \times \sqrt{N}$$

So even though in the long run you do about as well betting a single number or betting red vs. black (and in both cases you lose), the noise is much larger.

Consider that while 400 spins of red vs. black will guarantee you to lose, it'll take something like 13,000 spins of picking a particular number before you're "guaranteed" to lose. (Of course, on the opposite side, when you lose, you're likely to lose much more under this scenario).

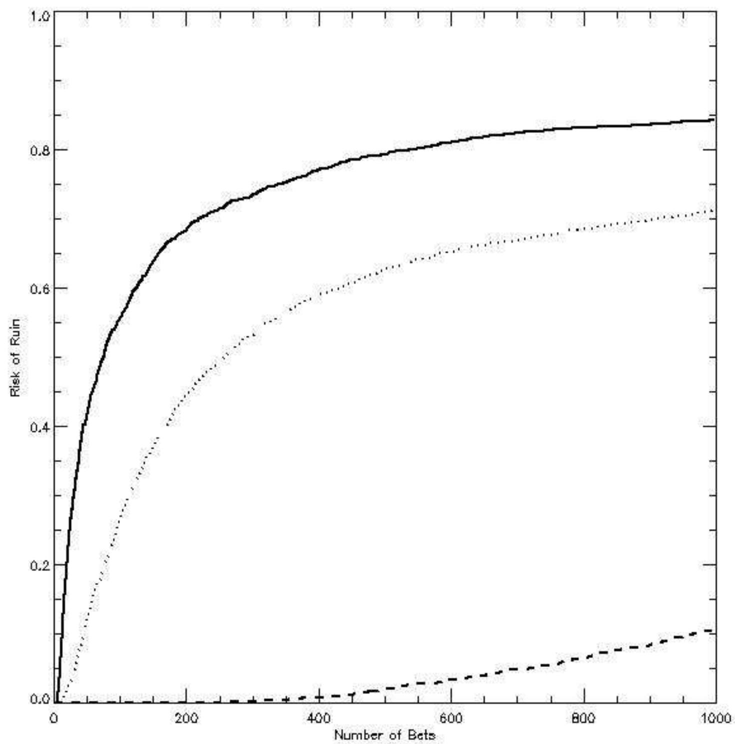
The longer the odds (even if the return is the same), the longer you can play without being guaranteed to lose.

As a particularly salient example consider slot machines. In many casinos, slot machines are set to return -12% on average. If slots returned a simple even odds bet (similar to red vs. black), then after 100 pulls of the slot machine (which you could probably accomplish in just under 5 minutes), the odds of being even (or better), would be something like 18%. After twice as long, the odds would by about 3.4%. How on earth could people keep playing for hours if they were *guaranteed* to lose after such a short time?

The answer is that a typical return from a slot machine is one that pays, say 100:1. However, that happens very, very infrequently (say 1 in 110 pulls). As a result, while a typical player will still lose, the "lucky" ones can end up ahead. The losers, however, can lose everything.

Risk of Ruin

Which brings us to the concept of "risk of ruin." Remember, gambling is a random walk. Imagine that you are betting $10/bet, and you have $100 in your pocket. It doesn't matter if you eventually would have beaten the house if at any point you drop below -10 bets. Once you're out of money, you're out of money. The casino, of course, has a much looser definition of what being "out of money" means, and they thus are kind enough to provide many opportunities for getting more (ATMs, credit card cash advances and the like) at various convenient places. The basic argument still holds, though. At some point you will no longer be able to borrow any more cash, and you're done.



Even in a fair, even-odds game, you have a very high probability of simply running out of money. If you start with, say, 5 bet units, eventually there's something like an 85% probability that you will eventually have a downturn and run out of cash.