TDEC 115 - Week 6 Recitation Problems

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Formulas and Concepts

At this stage we're ready to start analyzing some simple circuits. So far all we are dealing with are sources of emf (batteries) and resisters, placed in series and parallel arrangements. The two rules which all you to solve any combination of this sort are Kirchoff's Rules:

1. Junction: The total change in potential around any closed loop in a circuit must be zero. This says that the potential always has one value at every point.

2. Loop: The total current entering a junction is equal to the total current leaving. This says that no charge is building up at a junction.

The procedure to solve a circuit is sort of like drawing a free body diagram. In each branch you put an arrow labeling the current in that branch. The direction of the arrow is *arbitrary*! But, when you solve for the currents, a negative sign indicates that you chose the wrong direction: i.e. the current is flowing opposite the direction you chose. The is just like when you draw a force arrow in one direction and its value comes out negative.

Now, when writing down the Junction Rule, we start at one point, and write down the voltage change each element contributes in order until we've returned to the starting point. Note that the direction we traverse the loop *need not* be in the direction of the current! To do this, you need to know what voltage change each element introduces. We start with resisters.

R$-$ If you go through a resister *in the direction of the current* then the voltage *decreases* by IR.

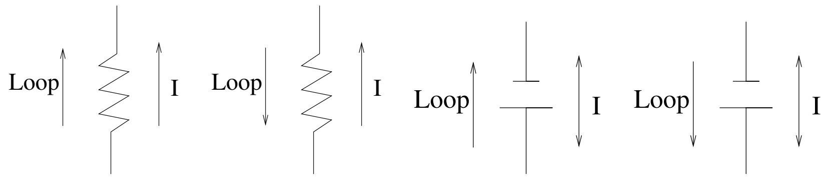
R$+$ If you go through a resister *opposite the direction of the current* then the voltage *increases* by IR

And next we have the batteries.

$\mathscr{E}-$ If you go through an EMF *from the + plate to the − plate, regardless of the current direction*, then the voltage *decreases* by $\mathscr{E}$.

$\mathscr{E}+$ If you go through an EMF *from the − plate to the + plate, regardless of the current direction*, then the voltage *increases* by $\mathscr{E}$

Note that resisters depend on the current direction, but batteries do not!



$V = -IR$ $\quad$ $V = +IR$ $\quad$ $V = -\mathscr{E}$ $\quad$ $V = +\mathscr{E}$

Next, one subtlety concerns the power in a circuit, specifically the battery. Generally, a real battery has two parts: an source of EMF and an internal resistance. Both parts are internal to the battery and thus *not* part of the external circuit! So we have the following quantities:

1. Power generated by the EMF: This is the product $I\mathscr{E}$ of the current in the battery and the EMF in the battery.

2. Power consumed by the battery: This is the product I^2R_{int} of the current in the battery squared and the internal resistance of the battery.

3. Power delivered by the battery to the circuit: This is the difference $I\mathscr{E} - I^2R_{int}$ of the total power generated by the EMF and power eaten by the resistance.

Make careful note of the distinction between these three, and also that they can be asked in different ways, not just the way mentioned here. Finally, this means that the total power consumed by the external circuit must be equal to the total power delivered to it, which is the third item on this list.

Solution

26-55. I am solving this one here since we were asked not to cover it in class.

Comment: 1eV is the energy gained by a particle with charge e, going through a potential difference of 1V, or

$$1\text{eV} = (\text{e})(1\text{V}) = (1.602 \times 10^{-19}\text{C})(1\text{V}) = 1.602 \times 10^{-19}\text{J}$$

The first problem takes a little bit of work. We need to relate the bulk properties of current and such, to the microscopic ones - the fact that we have individual particles flying about. The book has a nice derivation of the current density of a stream of particles $J = nqv$, n the number density, q the charge,and v speed. This simply says that current is moving charge. The faster and more numerous the charge, the more current density you get. This is related to the current by $i = JA$. Thus we have

$$i = nqAv = \frac{N}{\mathrm{Vol}} qAv$$

Where N is the total number in the volume Vol of the beam. The reason for making this substitution is that we want the total number (N), and we want to eliminate the beam dimensions (as these are unknown). Note that volume is area times length - so if we can find a length somewhere, the dimensions can be eliminated. Thus we note that the speed of the particles is related to the length $v = L/t$. Thus finally

$$i = \frac{Nq}{Vol} \frac{AL}{v} = \frac{Nq}{t} \rightarrow N = \frac{it}{q}$$

Don't forget that q=2e, twice the electric charge.

In the previous part we found the number of alphas for a given amount of time. Now we want the number in a given length. We can do this simply if we can convert between length and time. This is done conveniently by the velocity, thus we have

$$N = \frac{it}{q} = \frac{iL}{qv}$$

But we also need the speed, which can be found from the kinetic energy

$$v = \sqrt{\frac{2KE}{m}}$$

Where m is the alpha mass, which is 4 times the proton mass (since an alpha is 2 protons and 2 neutrons). Just don't forget to convert eV to Jules. Note - the velocity can by calculated quicker if the mass is given in units of MeV/c^2, as is common in particle physics (it always helps to use consistent units throughout your calculations!)

The energy the alphas gain from the potential difference becomes kinetic energy (energy conservation). Thus

$$KE = qV \rightarrow V = \frac{KE}{q}$$

Notice that the electric charge e can cancel the e in the eV in the numerator, simplifying this calculation.