TDEC 115 - Week 1 Recitation Problems

Daniel J. Cross

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New Formulas and Concepts

We begin our discussion of Electromagnetism by considering electric fields. Electric fields behave a lot like gravitational fields. Every particle has mass. This mass creates a gravitational 'field' which pulls other things with mass toward it. The more mass, the greater the field, and the stronger the pull. Finally, the the further away you get from a mass, the weaker the field.

In addition to possessing mass, most matter also possesses something called charge. Charge works almost the same as mass - it creates a field, called the electric field. This field exerts a force on other things that have charge, and the further away the weaker the effect. The major difference is that, while all mass attracts all other mass, charge comes in two flavors - called positive and negative. The rule is that likes repel and opposites attract.

It actually turns out that the electric force is much, much stronger than gravitation (see problem 65). But, because of these two flavors, opposites tend to attract each other, leading to an object that, overall, has no net charge. Most things tend to end up neutralized, so we don't tend to notice this force as much as gravity, since nothing can diminish it (we think).

Now, charge is measured in units called coulombs [C]. Charge, like mass, is a conserved quantity - it is neither created nor destroyed. Finally, the basic equation of force between two charges is Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are the two charges, r is the distance between them, and k is a constant, with value 9×10^9 in MKS units (can you figure out the units from Coulomb's Law?). This formula gives the force magnitude. The direction is given by the above likes/opposites rule.

Finally, this constant is often given in the form

$$k = \frac{1}{4\pi\epsilon_0}$$

where ϵ_0 is called the permittivity of free space (something which means nothing to you now, but will make more sense later) and has the value 8.85×10^{-12} (again, you give the units).

Problems

HRW 21-13

In this problem we are given two charges (q_1, q_2) placed on the x-axis and are asked to place a third charge, q_3 in a place where it will experience no force. If q_3 is placed anywhere off the x-axis the two forces on it due to the other charges will be in different directions and therefore have no hope of canceling (See Fig. 1). Thus, q_3 must be placed on the axis.

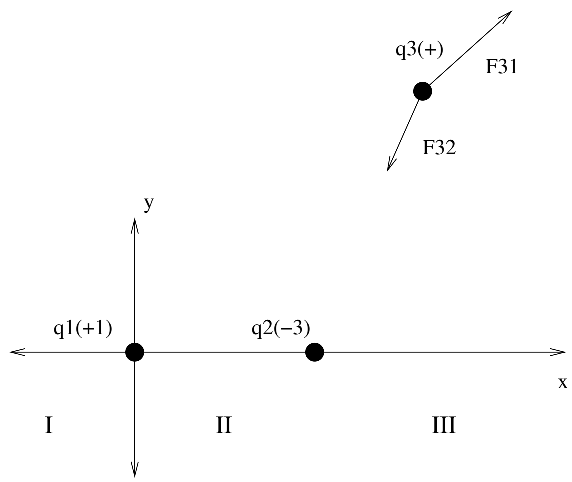

Figure 1: Forces on q_3 are not collinear when off axis.

Now, q_3 must be placed on the axis, but there are now 3 possible regions to consider. These regions (I, II, and III) are separated by q_1 and q_2. Assume that q_3 is positively charged. If it is placed in region II, then both forces on it would be to the right. Thus, they can never cancel in this region.

This leaves I and II. Now, in both regions, the forces will be in different directions, so we have the possibility of cancellation. Now, if were' in III, we're always closer to q_2, which is the larger charge. Since the electric force is proportional to charge and inversely proportional to separation squared (see the equation above), the force due to q_2 will always dominate in III, so there will be no place where the net force goes to zero.

This leaves only region I. In this region we're always closest to q_1, which is the weaker of the two charges. Thus we may expect a cancellation in this region. If we place the third charge a distance x from the axis, then the total force on it from the two charges will be

$$\begin{aligned} F &= F_{31} + F_{32} \\ &= k\frac{q_1 q_2}{x^2} + k\frac{q_2 q_3}{(x+L)^2} = 0. \end{aligned}$$

Thus we can cancel k and q_2 to get

$$\frac{q_1}{x^2} + \frac{q_3}{(x+L)^2} = 0.$$

Cross-multiplying and collecting terms in x gives the quadratic equation

$$x^2(q_1 + q_3) + x(2Lq_1) + q_1 L^2 = 0$$

and the (complicated looking) solutions are

$$x = L\left(\frac{-q_1 \pm \sqrt{q_1^2 - q_1(q_1 + q_2)}}{q_1 + q_2}\right).$$

What is interesting about this solution is that it depends linearly on L. This means that as long as $|q_2| > |q_1|$ we will have a solution somewhere in region I for any given value of L. This was not obvious from the setup of the problem, but is nevertheless true. It can also be seen from the following graph of the magnitudes of the forces from charges 1 and 2 (Fig. 2). In region I the two curves intersect, and this point is where the net force goes to zero. Convince yourself that by sliding q_2 around, as long as it stays to the left of q_1, the two graphs will always intersect somewhere in I. (How does this solution depend on q_1? On q_3?)

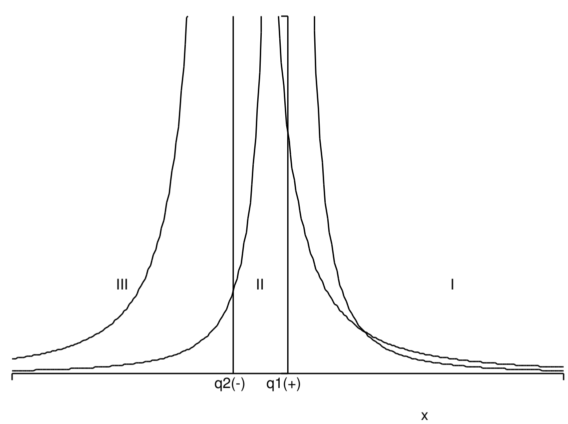

Figure 2: Force magnitudes due to q_1 and q_2 and their cancellation in I.

Now that you've seen this solution in detail, you should be able to check what happens in the following cases:

1. $|q_1| = |q_2|$.
2. q_1 negative and q_2 positive.

3. q_1 and q_2 with the same sign.

Finally, here is a graph of the net force in each region along the x-axis (Fig. 3). If the force is positive then it is toward the right, and if it is negative it is to the left (indicated by the arrows). The forces does cross the axis at $x = .137$, but it's invisible because of the scale of the plot.

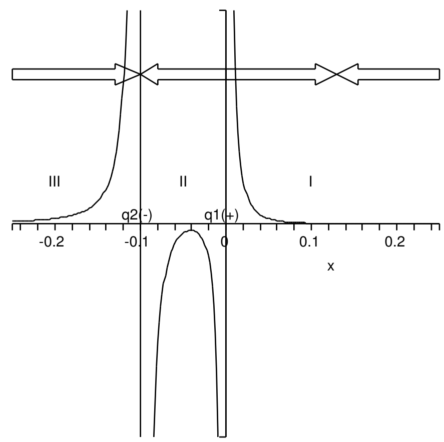

Figure 3: Net force on q_3 as a function of x.