TDEC 115 - Week 4 Recitation Problems

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October 19, 2005

1 New Formulas and Concepts

1.1 Moments

By now we're very familiar with Forces. A Force is something that causes an object move. More specifically if I apply a force in a certain direction it will cause the object to move (accelerate) in that direction. But objects can do more than just move in a certain direction, they can rotate as well. Just as the cause of linear motion is a force, the cause of a rotation is called a moment (or torque).

A moment is caused by a force, but it is more than a force. It depends on where and in what direction a force is applied. For example, you know that you will never open a door by pushing at the hinge (Fig. 1a), and the farther away from the hinge you push the easier it is to open (Fig. 1b). Moreover, if you push far away from the hinge, but push straight toward the hinge, the door isn't going to open either (Fig. 1c).

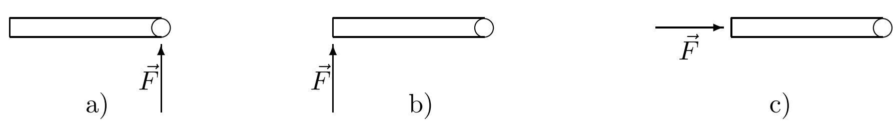

Figure 1. Forces applied in different places on a door. The hinge is on the right.

As cases b and c above illustrate, the moment applied by a force depends on the angle the force makes with the hinge. In fact, we can easily write down precisely what this dependence is by taking our force and writing it into components - one perpendicular to the door and one parallel to it. Then we have exactly are two cases above - one force that has maximal moment and another which has zero. If the force makes angle θ with the axis of the door, then the perpendicular component is

$$F_\perp = F \sin\theta$$

And finally, since the moment is greater the further away we push from the hinge, we have for the moment

$$m = F_\perp \times d = Fd\sin\theta$$

where d is the distance from where the force is applied and the hinge (See Fig. 2).

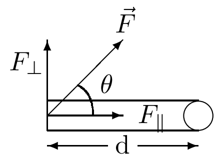

Figure 2. Perpendicular and Parallel components of a force.

Note that that we can regroup the above expression in a different way:

$$m = F(d \sin\theta) = F \times d_\perp$$

where $d_\perp$ is called the *moment arm*. It is the perpendicular (shortest) distance between the force vector and the hinge.

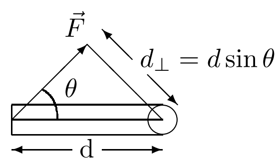

Figure 3. Moment arm demonstrated.

This means that the force has an "effective" distance $d_\perp$ for the exertion of the moment. It is as if the force was applied at a distance of $d_\perp$ away from the hinge and at a right angle.

1.2 Static Equilibrium

The second new concept is Static Equilibrium, that is, the object under study is static – it has no motion. Equilibrium means that everything is balanced – forces in this case. First thing we always do is draw a diagram of the object being acted on and draw all the forces and their directions. We have to know this before we can proceed! Equilibrium means that all the forces balance: the vector sum of all forces adds to zero, but it also means all the moments balance: the moments sum to zero as well. Symbolically:

$$\sum \vec{F} = 0 \tag{1}$$

$$\sum m_O = 0 \tag{2}$$

where O is the point we take our moments about (like the door hinge above).

Now the first equation really stands for two equations. First we pick axis (x and y, or at some angle - whatever is most convenient) and then resolve the forces into their components along these axes. Then we set their sum equal to zero. This gives us 2 equations for finding unknowns.

Next we have to do the moments. Once all our forces are described and draw out, we have to pick a point for our moments. It doesn't matter which point, but since any force acting through the point will give no moment (remember the door!), it's best to pick a spot with the most (unknown) forces passing through it, like an actual hinge. This will makes things easier (i.e. less algebra), but any point will give you the right answer (If the object is in equilibrium somewhere it's in equilibrium everywhere!).

Finally, we need a sign convention for moments. In two dimensional problems we have two senses of rotation: clockwise (cw) and counter-clockwise (ccw). The convention is that if a moment would tend to rotate the object ccw about the axis, then the moment is positive, and if the tendency is to rotate cw the moment is negative. The we add all the moments with the appropriate sign and set equal to zero, and this gives us a third equation for out unknowns.

In Figure 2 above, imagine if we pulled the door in the direction of $\vec{F}$. The door would go up and to the right around the hinge – this is the cw-direction, so in this case the moment is negative.