Energy I: Week 5 Recitation Problems

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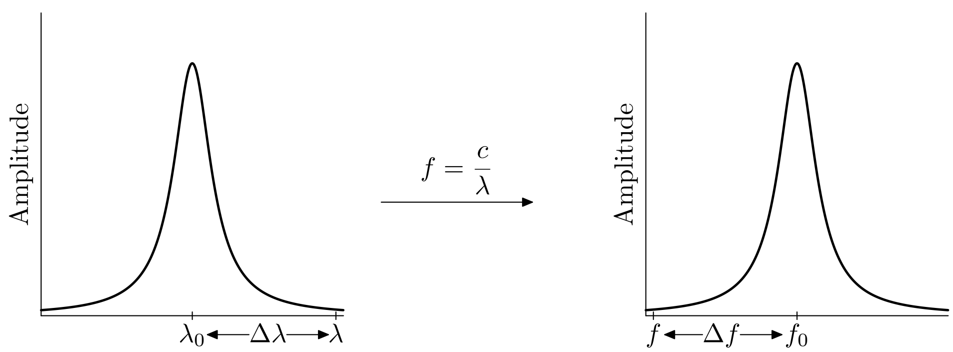
33-4

In this problem we're considering a sharply peaked distribution of wavelengths of a certain width and we wish to computer the corresponding frequency distribution width. Since frequency and wavelength are two complementary ways of describing a wave, we should be able to calculate the frequency width.

The problem doesn't exactly specify what this width is, but we'll take it to be the difference in wavelength between the peak and where the amplitude drops to nearly zero (as viewed on the graph). Thus, our width will be $\Delta\lambda$, so the wavelength where the amplitude drops sufficiently will be $\lambda = \lambda_0 + \Delta\lambda$:



Now, the peak wavelength corresponds to the peak frequency:

$$f_0 = \frac{c}{\lambda_0},$$

and next we have to figure out how the widths correspond. If we take the wavelength $\lambda = \lambda_0 + \Delta\lambda$, this will correspond to the frequency $f = f_0 - \Delta f$. The minus is important - a larger lambda corresponds to a smaller frequency since they are inversely related!

So we have, putting the above together,

$$f = \frac{c}{\lambda} = \frac{c}{\lambda_0 + \Delta\lambda} = f_0 - \Delta f = \frac{c}{\lambda_0} - \Delta f.$$

But we want Δf, so we have

$$\begin{aligned}\Delta f &= \frac{c}{\lambda_0} - \frac{c}{\lambda_0 + \Delta\lambda} \\ &= \frac{c}{\lambda_0} - \frac{c}{\lambda_0}\left(\frac{1}{1+\Delta\lambda/\lambda_0}\right) \\ &= \frac{c}{\lambda_0}\left(1 - \frac{1}{1+\Delta\lambda/\lambda_0}\right),\end{aligned}$$

where the last steps of algebra are necessary to remove numerical error. Δf is going to be much smaller than f, so you would need to keep a large number of significant digits around to not gero zero out of the calculation otherwise! Plugging in the numbers we get

$$\Delta f = 4.741 \times 10^{14}\text{Hz} \cdot \left(1 - \frac{1}{1 + (.01)/(632.8)}\right) = 7.491 \times 10^{9}\text{Hz}$$

33-5

An LC-circuit has a natural angular frequency oscillation given by

$$\omega = \frac{1}{\sqrt{LC}},$$

or in terms of regular frequency, we have

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

But, for an EM-wave we have $c = \lambda f$, or

$$c = \frac{\lambda}{2\pi\sqrt{LC}}.$$

So, solving for L gives

$$L = \frac{1}{C}\left(\frac{\lambda}{2\pi c}\right)^2.$$

Putting in the numbers gives $L = 5 \times 10^{-21}$H, which is a very small value?

33-6

This is the same as the last problem, but we want the wavelength this time. So

$$\lambda = 2\pi c\sqrt{LC} = 4.74\text{m}.$$

33-9

Since we have the relation $P = dE/dt$, if we talk about average values we get

$$P_{avg} = \frac{\Delta E}{\Delta t},$$

which when solved for ΔE gives an energy of 100kJ.

33-15

In strengths of the electric and magnetic fields in an EM-wave are always related by the ration

$$E_m = cB_m,$$

so we get the magnetic field value of 6.67×10^{-9}T.

The intensity is given by the average of the Poynting vector, which is given by

$$\begin{aligned} I &= S_{avg} \\ &= \left(\frac{EB}{\mu_0}\right)_{avg} \\ &= \frac{E_{rms}B_{rms}}{\mu_0} \\ &= \frac{1}{2}\frac{E_mB_m}{\mu_0}, \end{aligned}$$

which gives the value 5.31×10^{-3}W.

Finally, since the wave spreads out *isotropically*, its intensity is everywhere the same on the surface of a sphere. Thus we have

$$I = \frac{P}{A} \rightarrow P = 4\pi r^2 I,$$

which gives 6.67W.