# HW 4 Solutions

1. I drop a 0.1 kg ball from a height of 10m (assuming no air resistance).

   (a) How much work does gravity do on the ball on its way to the ground?
   $W = \Delta E = mg(h_i - h_f) = 10J$

   (b) What is the speed of the ball when it hits the ground?
   $KE = \frac{mv^2}{2} \Rightarrow v = \sqrt{\frac{2KE}{m}} = 10\sqrt{2}m/s$

   (c) Using your projectile motion equations (not using energy), how long did the ball take before hitting the ground?
   $y = y_0 + v_0 t - \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{2y_0}{g}} = \sqrt{2}s$

   (d) At constant acceleration, and only using your result from part c), what will the speed of the ball be when it hits the ground? Compare this answer to that found in part b.
   $v = v_0 - gt = -10\sqrt{2}m/s$
   This is the same as part b.

2. Consider a 6 kg block moving at 0.5c.

   (a) What is the momentum of the block?
   $p = mv\gamma = 1.04 \times 10^9 kgm/s$

   (b) What is the rest energy of the block?
   $E_0 = mc^2 = 5.4 \times 10^{17} J$

   (c) What is the Kinetic energy of the block? (**Hint:** do *not* use the relation you learned in high school.)
   $KE = mc^2(\gamma - 1) = 8.35 \times 10^{16} J$

   (d) If I then do $5 \times 10^{17} J$ of work on the block (admittedly, quite a lot – it's about the same amount of radiant energy the earth gets from the sun every 3 seconds), what will its speed be afterwards?

$$E_i = E_0 + KE = mc^2\gamma_i \quad (1)$$
$$E_f = E_i + W_{ext} = mc^2\gamma_f \quad (2)$$
$$\Rightarrow \gamma_f = \gamma_i + \frac{W_{ext}}{mc^2} = 2.08 \quad (3)$$
$$\Rightarrow \frac{v_f}{c} = \sqrt{1 - \frac{1}{\gamma_f^2}} = 0.877 \quad (4)$$

3. The following nuclear reaction is known to occur:

$$n \rightarrow p + e^- + \nu_e$$

   The neutrino has such low mass that you may assume (for now) that it is massless.

(a) The neutron starts at rest. What is its energy?

Since the masses of all these particles are small it is convenient to work in energy units of MeV ($1MeV = 1.602 \times 10^{-13}J$). Using this:

$$E_{0,n} = m_n c^2 = 939.566MeV = 1.505 \times 10^{-10}J \tag{5}$$

(b) After the decay, what is the mass-energy of the proton and electron (and their total)?

$$\begin{aligned} E_{0,p} &= m_p c^2 = 938.272MeV = 1.503 \times 10^{-10}J && (6)\\ E_{0,e} &= m_e c^2 = 0.511MeV = 8.187 \times 10^{-14}J && (7)\\ \Rightarrow E_{0,tot} &= 938.783MeV = 1.504 \times 10^{-19}J && (8) \end{aligned}$$

(c) What is the total kinetic energy of the proton, electron, and neutrino?

The total kinetic energy will be the neutron's rest mass minus the total rest mass or the proton and electron: $KE = E_{0,n} - E_{0,tot} = 0.783MeV = 1.255 \times 10^{-13}J$

(d) SUPPOSE (remember, this isn't true, but just pretend) that the neutron decays into just the proton and the electron. How fast will the electron fly out? This is a toughie, and you may want to use a computer, or do trial and error. But as a hint, realize that you know that the total momentum of proton and electron has to add up to zero, and the total energy has to add up to your answer in part c. This uniquely gives the energy for both.

We know that energy is conserved:

$$\begin{aligned} E_i &= m_n c^2 = E_f = m_p c^2 \gamma_p + m_e c^2 \gamma_e && (9)\\ \Rightarrow \gamma_p &= \frac{m_n}{m_p} - \frac{m_e}{m_p}\gamma_e && (10) \end{aligned}$$

Momentum is also conserved:

$$\begin{aligned} 0 &= m_e v_e \gamma_e + m_p v_p \gamma_p = m_e c\sqrt{\gamma_e^2 - 1} + m_p c\sqrt{\gamma_p^2 - 1} && (11)\\ \Rightarrow m_e\sqrt{\gamma_e^2 - 1} &= -m_p\sqrt{\gamma_p^2 - 1} && (12)\\ \Rightarrow m_e^2(\gamma_e^2 - 1) &= m_p^2(\gamma_p^2 - 1) && (13) \end{aligned}$$

We can now use (10) to eliminate $\gamma_p$ from (13):

$$\begin{aligned} m_e^2(\gamma_e^2 - 1) &= m_p^2[(\frac{m_n}{m_p} - \frac{m_e}{m_p}\gamma_e)^2 - 1] = (m_n - m_e\gamma_e)^2 - m_p^2 && (14)\\ \Rightarrow \gamma_e &= \frac{m_n^2 - m_p^2 + m_e^2}{2m_n m_e} = 2.53 && (15)\\ \Rightarrow \frac{v_e}{c} &= \sqrt{1 - \frac{1}{\gamma_e^2}} = 0.919 && (16) \end{aligned}$$

(e) Adding a 3rd particle (the neutrino), how would you expect that to affect the speed of the electron?

If you add in the nutrino then the speed of the electron will decrease since some of the kinetic energy will go into moving the nutrino.

4. 5.P.57

   a) $W = \vec{F} \cdot \Delta\vec{r}$, $\Delta\vec{r} = (2, 0, -1)m$, $\Rightarrow W_{Jack} = -1000J$

   b) $W_{Jill} = 0J$

   c) Since the work is zero the angle must be $90^o$.

   d) $\Delta E = \Delta KE = W_{ext} \Rightarrow \frac{m(v_f^2 - v_i^2)}{2} = -1000J \Rightarrow v_f = 1.01m/s$

5. 5.P.79

   a) $W_{ext} = \vec{F} \cdot \Delta\vec{r} = (2 \times 10^{-12}N)(3200m) = 6.4 \times 10^{-9}J$

   $E_f = E_i + W_{ext} = m_e c^2 + W_{ext} \simeq 6.4 \times 10^{-9}J$

   $E_f^2 = (m_e c^2)^2 + (pc)^2 \Rightarrow p = \frac{1}{c}\sqrt{E_f^2 - (m_e c^2)^2} = \frac{1}{c}\sqrt{W_{ext}^2 + 2m_e c^2 W_{ext}} = 2.13 \times 10^{-17}kgm/s$

   $p = mc\sqrt{\gamma^2 - 1} \Rightarrow \gamma = 78171 \Rightarrow \frac{v_e}{c} = 0.9999999999$

   b) Since the electron is traveling close to c we can estimate the time as: $t \simeq \frac{d}{c} = 1.07 \times 10^{-5}s$