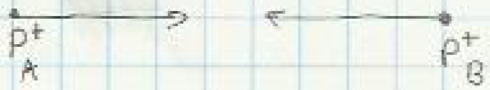
PHYS 201 HW6 solutions

$\boxed{37.59}$



a) Choose S as the lab frame and S' as proton A's frame:

$$v' = \frac{v-u}{1-uv/c^2}$$

$u = 0.5c$ = speed of S' frame relative to S frame

$v = -0.5c$ = speed of other proton in lab frame

v' = speed of other proton in S'

$$v' = \frac{-.5c - .5c}{1-(.5)(-.5)} = \frac{-c}{1+\frac{1}{4}} = -\frac{4c}{5}$$

Speed of each proton relative to the other is $\boxed{0.8c}$

b) Using nonrelativistic mechanics: $.5c - (-.5c) = \boxed{c}$

c) $KE = \gamma mc^2 - mc^2 = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$

(i) Lab frame: $v = c/2$

$$K = 938\text{ MeV}\left(\frac{1}{\sqrt{1-(1/2)^2}} - 1\right) = \boxed{145\text{ MeV}}$$ per proton

(ii) Proton frame: the proton whose rest frame you're in has zero velocity and thus zero KE.

The other proton has $v = 4c/5$

$$K = 938\text{ MeV}\left(\frac{1}{\sqrt{1-(4/5)^2}} - 1\right) = \boxed{625\text{ MeV}}$$

d) Using nonrelativistic mechanics: $KE = \frac{1}{2}mv^2$

(i) Each proton has $v = c/2$ so

$$K = \frac{m}{2}\cdot\frac{c^2}{4} = \frac{938\text{ MeV}}{8} = \boxed{117\text{ MeV}}$$

(ii) Again, one proton has $v=0$ and thus $K=0$. For the other, $v=c$ so

$$K = \frac{mc^2}{2} = \frac{938\text{ MeV}}{2} = 469\text{ MeV}.$$

Note that proton mass was given in terms of $m_p \times c^2$.

$\boxed{37.66}$

a) 80 m/s is not large enough to need relativity:

$K = \frac{1}{2}mv^2 = \frac{1}{2}(.058)(80)^2 = \boxed{185.6\text{ J}}$

b) 1.8×10^8 m/s is a relativistic speed: $K = mc^2(\gamma - 1)$

$K = .058(3\times10^8)^2\left(\frac{1}{\sqrt{1-(1.8/3)^2}} - 1\right) = \boxed{1.31\times10^{15}\text{ J}}$

c) $v' = \frac{v-u}{1-uv/c^2}$

v = rabbit speed in players' frame $= 2.2\times10^8$ m/s
u = speed of ball frame relative to players $= 1.8\times10^8$ m/s
v' = rabbit speed relative to ball

$v' = \frac{(2.2\times10^8)-(1.8\times10^8)}{1-(2.2)(1.8)/9} = \boxed{7.14\times10^7\text{ m/s}}$

d) $l = \frac{l_0}{\gamma} = 20\text{ m}\sqrt{1-(2.2/3)^2} = \boxed{13.6\text{ m}}$

e) According to the players, $l = l_0 = 20$ m and $v = 2.2\times10^8$ m/s, so $t = \frac{l}{v} = \frac{20}{2.2\times10^8} = \boxed{9.09\times10^{-8}\text{ s}}$

f) $t' = \frac{t}{\gamma} = 9.09\times10^{-8}\sqrt{1-(2.2/3)^2} = \boxed{6.18\times10^{-8}\text{ s}}$

Check: to the rabbit, $l = 13.6$ m

$\frac{13.6}{2.2\times10^8} = 6.18\times10^{-8}$ s

Note: $\left(\frac{2.2}{3}\right)^2 = \frac{v^2}{c^2} = \left(\frac{2.2\times10^8}{3\times10^8}\right)^2$

$\boxed{39.12}$

a) Need $\lambda = .15 \times 10^{-9}$ m

deBroglie $\lambda = \frac{h}{p} = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda}$ $\qquad$ e = charge of electron

Use energy conservation: $e\Delta V = \frac{1}{2}mv^2$ $\qquad$ ΔV = potential difference

$$\Delta V = \frac{mv^2}{2e} = \frac{h^2}{2m\lambda^2 e} = \frac{(6.626 \times 10^{-34}\ J \cdot s)^2}{2(9.109 \times 10^{-31}\ kg)(1.5 \times 10^{-10}\ m)^2(1.6 \times 10^{-19}\ C)}$$

$\boxed{\Delta V = 66.9\ V}$

b) photon energy $= hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{1.5 \times 10^{-10}}$

$E = 1.33 \times 10^{-15}\ J$

to get that same energy in electrons, use

$K = e\Delta V = E \qquad \Delta V = \frac{E}{e} = \frac{1.33 \times 10^{-15}}{1.6 \times 10^{-19}} = \boxed{8310\ V}$

$\boxed{39.38}$

a) $\lambda = \frac{h}{p}$ $K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$ given $K = 40\,eV$

Note: mass in kilograms → convert eV to J

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\ J\cdot s}{\sqrt{2(9.109\times10^{-31}\ kg)(40\ eV)(1.6\times10^{-19}\ J/eV)}}$$

$$\boxed{\begin{aligned}\lambda &= 1.94\times10^{-10}\ m\\ &= .194\ nm\\ &= 1.94\ Å\end{aligned}}$$

b) $t = \frac{\text{distance}}{\text{speed}}$ $K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}}$

given $R = 2.5\ m$

$$t = 2.5 \times \sqrt{\frac{9.109\times10^{-31}}{2(40)(1.6\times10^{-19}\ J/eV)}} = \boxed{6.67\times10^{-7}\ s}$$

c) width of the central diffraction pattern: $w = \frac{2R\lambda}{a}$

and using Δv_y = uncertainty in y-direction velocity,

$w = \Delta v_y t = \frac{\Delta p_y t}{m}$, given $a = 5\times10^{-6}$

so $\frac{\Delta p_y t}{m} = \frac{2R\lambda}{a} \rightarrow \Delta p_y = \frac{2mR\lambda}{ta} = \boxed{2.65\times10^{-28}\ kg\cdot m/s}$

d) Use $\Delta p \Delta x \geq \hbar = \frac{h}{2\pi}$ $\Delta x_{min} = \frac{h}{\Delta p(2\pi)} = \boxed{4\times10^{-7}\ m}$

$4\times10^{-7}\ m = .4\ \mu m$, which is within an order of magnitude of the slit size.

$\boxed{40.47}$

a) Given: $T = \left[1 + \dfrac{(U_0 \sinh[KL])^2}{4E(U_0 - E)}\right]^{-1}$

use $\sinh(KL) = \dfrac{e^{KL} - e^{-KL}}{2}$

if $KL \gg 1$ → say KL is large.

$\left.\begin{array}{l} e^{KL} \text{ becomes very large} \\ e^{-KL} \text{ approaches zero} \end{array}\right\}$ so $\dfrac{e^{KL} - e^{-KL}}{2} \rightarrow \dfrac{e^{KL}}{2}$

so $(U_0 \sinh[KL])^2 \rightarrow \dfrac{U_0^2 e^{2KL}}{4}$ and

$T \rightarrow \left[1 + \dfrac{U_0^2 e^{2KL}}{16E(U_0 - E)}\right]^{-1} = \dfrac{16E(U_0 - E)}{16E(U_0 - E) + U_0^2 e^{2KL}}$

Now e^{2KL} grows much faster than $16E(U_0 - E)$ for large KL.

So $16E(U_0 - E) + U_0^2 e^{2KL} \rightarrow U_0^2 e^{2KL}$

Putting it back in:

$T \rightarrow \dfrac{16E(U_0 - E)}{U_0^2 e^{2KL}} = \boxed{16\left(\dfrac{E}{U_0}\right)\left[1 - \dfrac{E}{U_0}\right] e^{-2KL}}$

This is Eq. 40.21

b) If $KL \gg 1$, and $K = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$, L is the barrier width

To make KL large at least one must be large - if L is large, the barrier is wide. If K is large, $U_0 - E$ is large, so E must be much smaller than U_0.

c) $E \to U_0$ means $(U_0 - E) \to 0$

so $k = \frac{\sqrt{2m(U_0-E)}}{\hbar}$ becomes very small

for small kL, $\sinh(kL) \approx kL$

so

$$T \to \left[1 + \frac{U_0^2 k^2 L^2}{4E(U_0-E)}\right]^{-1} = \left[1 + \frac{U_0^2\, 2m(U_0-E)L^2}{\hbar^2\, 4E(U_0-E)}\right]^{-1}$$ using the definition of k

so $T \to \left(1 + \frac{2U_0^2 L^2 m}{4E\hbar^2}\right)^{-1}$ since $U_0 \to E$, $\frac{U_0^2}{E} \to \frac{E^2}{E} = E$

$$\to \left[1 + \frac{2mEL^2}{4\hbar^2}\right]^{-1}$$

Remember that wavenumber $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{p}$

so $k = \frac{2\pi p}{h} = \frac{p}{\hbar}$; $p = \sqrt{2mE}$

so $k = \frac{\sqrt{2mE}}{\hbar}$ and $\frac{2mEL^2}{4\hbar^2} = \frac{L^2}{4}k^2 = \left(\frac{kL}{2}\right)^2$

Put back into T:

$$T \to \left[1 + \left(\frac{kL}{2}\right)^2\right]^{-1}$$