Physics 201 - Homework 5

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38.25

(a)

For the hydrogen atom the energy levels are given by

$$E_n = \frac{-1}{\epsilon_0{}^2}\frac{me^4}{8n^2h^2}$$

For a hydrogen-like ion the charge on the nucleus is Ze and the energy levels are

$$E_{n,ion} = \frac{-1}{\epsilon_0{}^2}\frac{mZ^2e^4}{8n^2h^2}$$

The ground state ($n = 1$) energy level is

$$E_{1,ion} = \frac{-1}{\epsilon_0{}^2}\frac{mZ^2e^4}{8h^2} = Z^2E_1$$

For the Be^{3+} ion $Z = 4$ and the ground state energy level is

$$E_{1,ion} = 16E_1 = 16 \times -13.6\,\text{eV} = -217.6\,\text{eV}$$

The ground state energy of the Be^{3+} ion is 16 times greater than that of the hydrogen atom.

(b)

The ionization energy is the amount of energy required completely remove the electron from the atom when it is in the ground state.

$$E_{ionization} = E_\infty - E_1 = 0 - E_1 = -E_1$$

That is, the ionization energy is simply the negative of the ground state energy. For the Be^{3+} ion the ionization energy is $217.6\,\text{eV}$. Again, this is 16 times larger than the ionization energy for the hydrogen atom.

(c)

The energy lost by the ion from the $n = 2$ to $n = 1$ transition is

$$\Delta E = |E_2 - E_1| = 217.6 \left(1 - \frac{1}{4}\right) = 163.2\,\mathrm{eV}$$

The energy lost by the ion is equal to the energy of the photon. The wavelength of the photo is given by

$$\lambda = \frac{hc}{\Delta E} = \frac{1240}{163.2} = 7.598\,\mathrm{nm}$$

(d)

For the hydrogen atom

$$r_H = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$$

and for an ion

$$r_{ion} = \epsilon_0 \frac{n^2 h^2}{\pi m Z e^2}$$
$$r_{ion} = \frac{r_H}{Z}$$

So the radius of the ion is Z times smaller than that for the hydrgoen atom. The radius of the Be^{3+} ion is four times smaller than that of the hydrogen atom, for a given n.

38.56

(a)

The work function is the amount of energy required to remove an electron from the metal. It is the difference between the energy of the photon and the maximum kinetc energy of the released electrons.

$$\Phi = \frac{hc}{\lambda} - K_{max} = \frac{1240}{124} - 4.16 = 5.84\,\mathrm{eV}$$

(b)

The energy of an individual photon is given by

$$E_\gamma = \frac{hc}{\lambda} = 10\,\mathrm{eV} = 1.6 \times 10^{-18}\,\mathrm{J}$$

The number of photons per second is

$$\text{Photons/sec} = \frac{P}{E_\gamma} = \frac{2.5}{1.6 \times 10^{-18}} = 1.56 \times 10^{18}$$

Assuming there is one electron released per photon then 1.56×10^{18} electrons are ejected each second.

(c)

Since the wavelength is unchanged the energy of each photon remains the same. For the power to be half the original value there must be half as many photons. This means that half as many electrons will be ejected. 7.8×10^{17} electrons will be ejected each second.

(d)

If the wavelength is halved then the energy of each photon is doubled. For the power to remain the same there must be half as many photons. If there are half as many photons then half as many electron will be ejected. 7.8×10^{17} electrons will be ejected each second.

38.58

(a)

The threshold wavelength occurs when the energy of the photon is equal to the work function

$$\frac{hc}{\lambda_T} = \Phi$$

$$\Rightarrow \lambda_T = \frac{hc}{\Phi}$$

Cesium

$$\lambda_T = \frac{1240}{2.1} = 590.48\,\mathrm{nm}$$

Copper

$$\lambda_T = \frac{1240}{4.7} = 263.83\,\mathrm{nm}$$

Potassium

$$\lambda_T = \frac{1240}{2.3} = 539.13\,\mathrm{nm}$$

Zinc

$$\lambda_T = \frac{1240}{4.3} = 288.37\,\mathrm{nm}$$

(b)

Copper and Zinc would not emit electrons when irradiated with visible light. Since the threshold wavelength for these metals is shorter than the wavelength of visible light the photons would not have enough energy to release the electrons.

38.75

(a)

The change in energy of the photon is

$$\Delta E = E_f - E_i = \frac{hc}{\lambda'} - \frac{hc}{\lambda} = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$\Delta E = 1240\left(\frac{1}{0.1132} - \frac{1}{0.11}\right) = -318.66\,\text{eV}$$

The kinetic energy gained by the electron is equal to the energy lost by the photon. Since the electron was initally at rest the kinetic energy after the collision is

$$K = 318.66\,\text{eV}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mc^2\left(\frac{v}{c}\right)^2$$

Rearranging the kinetic equation for $\left(\frac{v}{c}\right)^2$ gives

$$\left(\frac{v}{c}\right)^2 = \frac{2K}{mc^2} = \frac{2 \times 318.66}{0.51 \times 10^6} = 1.25 \times 10^{-3}$$

$$\left(\frac{v}{c}\right) = 0.035$$

$$v = 0.035c = 1.06 \times 10^7\,\text{ms}^{-1}$$

(b)

The energy of the created photon is equal to the kinetic energy of the electron

$$K = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{K} = \frac{1240}{318.66} = 3.89\,\text{nm}$$

38.39

(a)

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\phi) = \frac{hc}{m_e c^2}(1 - \cos\phi)$$

$$\Delta\lambda = \frac{1240}{0.51 \times 10^6}(1 - \cos 35^\circ) = 4.3971 \times 10^{-4}\,\text{nm}$$

(b)

$$\Delta\lambda = \lambda' - \lambda$$

$$\lambda' = \lambda + \Delta\lambda = 0.0425 + 4.3971 \times 10^{-4} = 0.0429\,\text{nm}$$

(c)

$$\Delta E = E_f - E_i = \frac{hc}{\lambda'} - \frac{hc}{\lambda} = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$\Delta E = 1240\left(\frac{1}{0.0429} - \frac{1}{0.0425}\right) = -298.77\,\text{eV}$$

The change in energy is negative indicating that energy is lost.

(d)

Due to conservation of energy

$$\Delta E_{e^-} = +298.77\,\text{eV}$$

The electron gains $298.77\,\text{eV}$ of energy.