Physics 201 - Homework 4

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36.6

(a)

$$\lambda = vT = 800 \times 1 = 800\,\text{km}$$

(b)

$$a \sin\theta = m\lambda$$
$$a \sin\theta_1 = \lambda$$

For $a = 4500\,\text{km}$,

$$\sin\theta_1 = \frac{800}{4500} \Rightarrow \theta_1 = 10.24°$$

For $a = 3700\,\text{km}$,

$$\sin\theta_1 = \frac{800}{3700} \Rightarrow \theta_1 = 12.49°$$

36.8

(a)

$$a = \frac{\lambda}{\sin\theta_1} = \frac{580\,\text{nm}}{1} = 580\,\text{nm}$$

(b)

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

where $\beta = \left(\frac{2\pi a}{\lambda}\right)\sin\theta$

$$\beta/2 = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \left(\frac{\pi \times 580}{580}\right)\sin(\pi/4) = 2.22$$

$$\frac{I}{I_0} = \left[\frac{\sin 2.22}{2.22}\right]^2 = 0.128$$

36.32

(a)

$$d \sin\theta = m\lambda$$

For the first order we have $d \sin\theta = \lambda$. We can find d from the number of slits per millimeter

$$d = \left(\frac{1}{350}\right) \text{mm} = 2.85 \times 10^{-3}\,\text{mm} = 2.857 \times 10^{-6}\,\text{m}$$

For a wavelength of 400nm we have

$$\sin\theta = \frac{\lambda}{d} = \frac{400 \times 10^{-9}}{2.857 \times 10^{-6}} = 0.14$$

$$\Rightarrow \theta = 8.05^\circ$$

For a wavelength of 700nm we have

$$\sin\theta = \frac{\lambda}{d} = \frac{700 \times 10^{-9}}{2.857 \times 10^{-6}} = 0.245$$

$$\Rightarrow \theta = 14.18^\circ$$

Therefore the angular width is

$$\Delta\theta = 14.18 - 8.05 = 6.1^\circ$$

(b)

For the third order we have $d \sin\theta = 3\lambda$. For a wavelength of 400nm we have

$$\sin\theta = \frac{3\lambda}{d} = \frac{3 \times 400 \times 10^{-9}}{2.857 \times 10^{-6}} = 0.42$$

$$\Rightarrow \theta = 24.84^\circ$$

For a wavelength of 700nm we have

$$\sin\theta = \frac{3\lambda}{d} = \frac{3 \times 700 \times 10^{-9}}{2.857 \times 10^{-6}} = 0.735$$

$$\Rightarrow \theta = 47.31^\circ$$

Therefore the angular width is

$$\Delta\theta = 47.31 - 24.84 = 22.5^\circ$$

36.40

(a)

The Bragg condition is

$$2d \sin\theta = m\lambda$$

$$m = 1 \Rightarrow \lambda = 2d \sin\theta$$

$$\lambda = 2 \times 3.5 \times 10^{-10} \times \sin(15^\circ)$$

$$\lambda = 1.812 \times 10^{-10}\,\mathrm{m} = 0.181\,\mathrm{nm}$$

Comparing with Figure 32.4 from the textbook we see that this corresponds to the x-ray part of the spectrum.

(b)

$$\sin\theta = \frac{m\lambda}{2d} = m(0.259)$$

Since $\sin\theta \leq 1$, $m_{max} = 3$. This means there are two more angles where strong interference maximia occur. They are

$$\theta_2 = 31.18^\circ$$

$$\theta_3 = 50.95^\circ$$

36.70

$$\sin\theta_1 = 1.22 \left(\frac{\lambda}{d}\right)$$

(a)

First we find the diameter of the aperture from the focal length,

$$D = \frac{f}{2.8} = \frac{35 \times 10^{-3}}{2.8} = 0.0125\,\mathrm{m}$$

Next we need the wavelength of the light. We can find the wavelength in air from the frequency

$$\lambda_{air} = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}\,\mathrm{m}$$

The wavelength in water can be found using the wavelength in air and the refractive index of water $n = 1.33$

$$\lambda_{water} = \frac{\lambda_{air}}{n} = \frac{5 \times 10^{-7}}{1.33} = 3.76 \times 10^{-7}\,\mathrm{m}$$

The angular radius is given by

$$\sin\theta_1 = 1.22 \times \left(\frac{3.76 \times 10^{-7}}{0.0125}\right) = 3.67 \times 10^{-5}$$

$$\theta_1 = 3.67 \times 10^{-5}\,\mathrm{rad}$$

If we denote the distance from the aperture to the object by L then the width is given by

$$w = L\tan\theta_1 = 2.75 \times \tan\theta_1 = 1.01 \times 10^{-4}\,\mathrm{m} = 0.101\,\mathrm{mm}$$

(b)

$$\sin\theta_1 = 1.22 \times \left(\frac{5 \times 10^{-7}}{0.0125}\right) = 4.88 \times 10^{-5}$$

$$\theta_1 = 4.88 \times 10^{-5}\,\mathrm{rad}$$

$$w = L\tan\theta_1 = 1.34 \times 10^{-4}\,\mathrm{m} = 0.134\,\mathrm{mm}$$