Physics 201 - Homework 1

Ben Coy

January 24, 2011

13.11

(a)

We are given the speed at the equilibrium position as $12\,\text{ms}^{-1}$. This is the maximum speed and is related to the amplitude by

$$v_{max} = A\omega$$

So we need to find the angular frequency. This only depends on two quantities: the mass and spring constant.

$$\omega = \sqrt{k/m} = \sqrt{300/2} = 12.25\,\text{s}^{-1}$$

Rearranging the equation for the amplitude gives

$$A = \frac{v_{max}}{\omega} = \frac{12}{12.25} = 0.98\,\text{m}$$

(b)

The initial displacement is zero and the initial velocity is negative. Our displacement is described by a cosine function

$$x = A\cos(\omega t + \phi)$$

We require the initial displacement to be zero

$$x_i = A\cos(\phi) = 0$$

The first two (positive) solutions are

$$\phi = \pi/2$$

$$\phi = 3\pi/2$$

For $\phi = \pi/2$ the cosine function is decreasing and at $\phi = 3\pi/2$ the cosine function is increasing. The $\phi = \pi/2$ solution corresponds to a negative initial velocity and $\phi = 3\pi/2$ corresponds to a positive initial velocity. Since the initial velocity is in the negative direction the solution is

$$\phi = \pi/2$$

(c)

Here we just need to substitute our values for the amplitude, angular frequency and phase into the equation for the displacement shown above

$$x = A\cos(\omega t + \phi) = 0.98\,\mathrm{m}\cos(12.25\,\mathrm{s}^{-1} + \pi/2)$$

13.33

(a)

There are two forces acting on the 1.5kg ball. These are its weight and the force provided by the glue. At the lowest and highest points the acceleration is maximum and therefore we have the maximum net force. At the lowest point the net force is in the opposite direction to the weight and the force provided by the glue must be greatest.

(b)

The angular frequency only depends on the mass and spring constant so we can calculate it using the new reduced mass

$$\omega = \sqrt{k/m} = \sqrt{165/2} = 9.08\,\mathrm{s}^{-1}$$

The frequency is related to the angular frequency by

$$f = \frac{\omega}{2\pi} = 1.45\,\mathrm{Hz}$$

Since the mass is removed at the lowest point there is no change in the mechanical energy (see example 13.5 from the textbook). At this point all the energy is stored as spring potential energy. Before the smaller mass is removed this energy is given by

$$E = \frac{1}{2}k\left(A + \frac{(M + m)g}{k}\right)^2$$

where A is the original amplitude and M and m are 2kg and 1.5kg respectively. After the lower ball comes loose the energy is

$$E = \frac{1}{2}k\left(A' + \frac{Mg}{k}\right)^2$$

where A' is the new amplitude. These energies must be equal which gives

$$A' = A + \frac{mg}{k}$$

An alternative way to think about the problem is to consider the change in the equilibrium position. The original equilibrium position is given by

$$y_{eq} = \frac{(M + m)g}{k}$$

and the new equilibrium position by

$$y'_{eq} = \frac{Mg}{k}$$

So the shift in the equilibrium position is

$$\Delta y_{eq} = y_{eq} - y'_{eq} = \frac{mg}{k}$$

and the new amplitude is given by

$$A' = A + \Delta y_{eq} = A + \frac{mg}{k}$$

Calculating this value gives

$$A' = 0.15 + \frac{1.5 \times 9.8}{165} = 0.239 \, \text{m}$$

13.58

The displacement of a damped oscillator is given by

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$

where

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

A is the initial amplitude and the coefficient in front of the cosine term is the amplitude for any time

$$x_{max} = Ae^{-(b/2m)t}$$

Rearranging this equation for the damping constant, b, gives

$$b = \frac{2m}{t}\ln\left(\frac{A}{x_{max}}\right)$$

Substituting the values given in the problem

$$b = \frac{2 \times 0.05}{5}\ln\left(\frac{0.3}{0.1}\right) = 0.022 \, \text{kgs}^{-1}$$

13.81

First we will find how long it takes for the boat to move down 10cm from the dock. Since the boat is starting at the highest point the displacement is given by

$$y = A\cos(\omega t)$$

We wish to find the time when $y = 0.1\,\mathrm{m}$ so

$$\cos(\omega t) = y/A = 0.1/0.2$$

$$\omega t = \pi/3$$

Using $\omega = 2\pi/T$ we have

$$t = \frac{\pi}{3\omega} = \frac{T}{6}$$

We know that it will take the same time for the boat to rise from $y = 10\,\mathrm{cm}$ to $y = 20\,\mathrm{cm}$ so the amount of time available to board is $2t$

$$2t = T/3 = 3.5/3 = 1.17\,\mathrm{s}$$

15.9

The wave equation is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

For this problem we evaluate the left hand side and then the right hand side of the equation for each function and see if they are the same. We do not start by assuming they are the same, i.e. setting them equal.

(a)

The function is

$$y = A\cos(kx + \omega t)$$

The l.h.s. of the wave equation gives

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y$$

and the r.h.s. is

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}(-\omega^2 y)$$

Using $v = \omega/k$ we have

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{k^2}{\omega^2}(-\omega^2 y) = -k^2 y$$

So the l.h.s. and r.h.s. are the same and the function is a solution of the wave equation.

(b)

Exactly the same as for part (a).

4

(c)

The l.h.s. is

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \cos(kx)$$

and the r.h.s.

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}(-A\omega^2 \cos(\omega t)) = -Ak^2 \cos(\omega t)$$

which is clearly not the same as the l.h.s. so this function is not a solution of the wave equation.

(d)

To find the transverse velocity we differentiate the function with respect to t

$$v_y = \frac{\partial y}{\partial t} = A\omega \cos(kx + \omega t)$$

and to find the transverse acceleration we take the second derivative

$$a_y = \frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(kx + \omega t) = -\omega^2 y$$