PHYS 201 Winter 2010

Recitation Problems (Week 10)

All problems taken from *University Physics,* Young and Freedman, 12th Ed.

38.40. The change in wavelength of the scattered photon is given by Eq. 38.23

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1-\cos\phi) \Rightarrow \lambda = \frac{h}{mc\left(\frac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

Thus, $\lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.67\times10^{-27}\text{ kg})(3.00\times10^{8}\text{ m/s})(0.100)}(1+1) = 2.65\times10^{-14}\text{ m}.$

39.3. **(a)** $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(2.80\times10^{-10}\text{ m})} = 2.37\times10^{-24}\text{ kg}\cdot\text{m/s}.$

(b) $K = \frac{p^2}{2m} = \frac{(2.37\times10^{-24}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})} = 3.08\times10^{-18}\text{ J} = 19.3\text{ eV}.$

39.5. **IDENTIFY and SET UP:** The de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then

$\lambda = h\left(\frac{2\pi r_n}{nh}\right) = \frac{2\pi r_n}{n}.$

EXECUTE: **(a)** For $n=1$, $\lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529\times10^{-10}$ m, so $\lambda = 2\pi(0.529\times10^{-10}\text{ m}) = 3.32\times10^{-10}$ m

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n=4$, $\lambda = 2\pi r_4/4$.

$r_n = n^2 a_0$ so $r_4 = 16a_0$.

$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32\times10^{-10}\text{ m}) = 1.33\times10^{-9}$ m

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\frac{1}{n} = \frac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.29. **IDENTIFY and SET UP:** $\psi(x) = A\sin kx$. The position probability density is given by $|\psi(x)|^2 = A^2\sin^2 kx$.

EXECUTE: **(a)** The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2$, $n = 1, 3, 5, \ldots$

$x = n\lambda/4$, $n = 1, 3, 5, \ldots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0$, which occurs where $\sin kx = 0$ and

$kx = 2\pi x/\lambda = n\pi$, $n = 0, 1, 2, \ldots$

$x = n\lambda/2$, $n = 0, 1, 2, \ldots$ so $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$

EVALUATE: The situation is analogous to a standing wave, with the probability analogous to the square of the amplitude of the standing wave.

39.48. **IDENTIFY and SET UP:** The minimum uncertainty product is $\Delta x \Delta p_x = \dfrac{h}{2\pi}$. $\Delta x = r_1$, where r_1 is the radius of the $n = 1$ Bohr orbit. In the $n = 1$ Bohr orbit, $mv_1 r_1 = \dfrac{h}{2\pi}$ and $p_1 = mv_1 = \dfrac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \dfrac{h}{2\pi \Delta x} = \dfrac{h}{2\pi r_1} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.529\times10^{-10}\ \text{m})} = 2.0\times10^{-24}\ \text{kg}\cdot\text{m/s}$. This is the same as the magnitude of the momentum of the electron in the $n = 1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

40.21. $T = 16\dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)e^{-2L\sqrt{2m(U_0 - E)}/\hbar}$. $\dfrac{E}{U_0} = \dfrac{6.0\ \text{eV}}{11.0\ \text{eV}}$ and $E - U_0 = 5\ \text{eV} = 8.0\times10^{-19}\ \text{J}$.

(a) $L = 0.80\times10^{-9}\ \text{m}$: $T = 16\left(\dfrac{6.0\ \text{eV}}{11.0\ \text{eV}}\right)\left(1 - \dfrac{6.0\ \text{ev}}{11.0\ \text{eV}}\right)e^{-2(0.80\times10^{-9}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(8.0\times10^{-19}\ \text{J})}/1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 4.4\times10^{-8}$

(b) $L = 0.40\times10^{-9}\ \text{m}$: $T = 4.2\times10^{-4}$.