Recitation Problems (Week 9)

All problems taken from *University Physics*, Young and Freedman, 12th Ed.

37.3. IDENTIFY and SET UP: The problem asks for u such that $\Delta t_0 / \Delta t = \frac{1}{2}$.

EXECUTE:
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$
 gives $u = c\sqrt{1 - (\Delta t_0/\Delta t)^2} = (3.00 \times 10^8 \text{ m/s})\sqrt{1 - (\frac{1}{2})^2} = 2.60 \times 10^8 \text{ m/s}$; $\frac{u}{c} = 0.867$

Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for u.

37.9. IDENTIFY and SET UP: $l = l_0 \sqrt{1 - u^2/c^2}$. The length measured when the spacecraft is moving is l = 74.0 m; l_0 is the length measured in a frame at rest relative to the spacecraft.

EXECUTE:
$$l_0 = \frac{l}{\sqrt{1 - u^2/c^2}} = \frac{74.0 \text{ m}}{\sqrt{1 - (0.600c/c)^2}} = 92.5 \text{ m}.$$

EVALUATE: $l_0 > l$. The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

37.11. IDENTIFY and SET UP: The 2.2 μs lifetime is Δt₀ and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is t and t₀ is 10 km.

EXECUTE: (a) The greatest speed the muon can have is c, so the greatest distance it can travel in 2.2×10^{-6} s is $d = vt = (3.00 \times 10^{8} \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}$.

(b)
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - (0.999)^2}} = 4.9 \times 10^{-5} \text{ s}$$

 $d = vt = (0.999)(3.00 \times 10^5 \text{ m/s})(4.9 \times 10^{-5} \text{ s}) = 15 \text{ km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c)
$$l = l_0 \sqrt{1 - u^2/c^2} = (10 \text{ km}) \sqrt{1 - (0.999)^2} = 0.45 \text{ km}$$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.19. IDENTIFY and SET UP: Reference frames S and S' are shown in Figure 37.19.

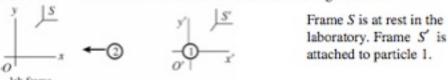


Figure 37.19

u is the speed of S' relative to S; this is the speed of particle 1 as measured in the laboratory. Thus u = +0.650c. The speed of particle 2 in S' is 0.950c. Also, since the two particles move in opposite directions, 2 moves in the -x' direction and $v'_x = -0.950c$. We want to calculate v_x , the speed of particle 2 in frame S; use Eq.(37.23).

EXECUTE:
$$v_z = \frac{v_x' + u}{1 + uv_x'/c^2} = \frac{-0.950c + 0.650c}{1 + (0.950c)(-0.650c)/c^2} = \frac{-0.300c}{1 - 0.6175} = -0.784c$$
. The speed of the second particle,

as measured in the laboratory, is 0.784c

EVALUATE: The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is 0.300c. The correct relativistic calculation gives a result more than twice this.

- 37.30. The force is found from Eq.(37.32) or Eq.(37.33).
 - (a) Indistinguishable from F = ma = 0.145 N.
 - (b) $\gamma^3 ma = 1.75 \text{ N}$.
 - (c) $y^3 ma = 51.7 \text{ N}.$
 - (d) $\gamma ma = 0.145 \text{ N}, 0.333 \text{ N}, 1.03 \text{ N}.$

37.53. (a)
$$E = \gamma mc^2$$
 and $\gamma = 10 = \frac{1}{\sqrt{1 - (v/c)^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{99}{100}} = 0.995.$

(b)
$$(pc)^2 = m^2 v^2 \gamma^2 c^2$$
, $E^2 = m^2 c^4 \left(\left(\frac{v}{c} \right)^2 \gamma^2 + 1 \right)$

$$\Rightarrow \frac{E^2 - (pc)^2}{E^2} = \frac{1}{1 + \gamma^2 \left(\frac{v}{c} \right)^2} = \frac{1}{1 + (10/(0.995))^2} = 0.01 = 1\%.$$