

## Recitation Problems (Week 9)

All problems taken from *University Physics*, Young and Freedman, 12th Ed.

- 37.3. IDENTIFY and SET UP:** The problem asks for  $u$  such that  $\Delta t_0 / \Delta t = \frac{1}{2}$ .

**EXECUTE:**  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$  gives  $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = (3.00 \times 10^8 \text{ m/s})\sqrt{1-\left(\frac{1}{2}\right)^2} = 2.60 \times 10^8 \text{ m/s}$ ;  $\frac{u}{c} = 0.867$

Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for  $u$ .

- 37.9. IDENTIFY and SET UP:**  $l = l_0\sqrt{1-u^2/c^2}$ . The length measured when the spacecraft is moving is  $l = 74.0 \text{ m}$ ;  $l_0$  is the length measured in a frame at rest relative to the spacecraft.

**EXECUTE:**  $l_0 = \frac{l}{\sqrt{1-u^2/c^2}} = \frac{74.0 \text{ m}}{\sqrt{1-(0.600c/c)^2}} = 92.5 \text{ m}$ .

**EVALUATE:**  $l_0 > l$ . The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

- 37.11. IDENTIFY and SET UP:** The  $2.2 \mu\text{s}$  lifetime is  $\Delta t_0$  and the observer on earth measures  $\Delta t$ . The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is  $l$  and  $l_0$  is 10 km.

**EXECUTE:** (a) The greatest speed the muon can have is  $c$ , so the greatest distance it can travel in  $2.2 \times 10^{-6} \text{ s}$  is  $d = vt = (3.00 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}$ .

(b)  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1-(0.999)^2}} = 4.9 \times 10^{-5} \text{ s}$

$d = vt = (0.999)(3.00 \times 10^8 \text{ m/s})(4.9 \times 10^{-5} \text{ s}) = 15 \text{ km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c)  $l = l_0\sqrt{1-u^2/c^2} = (10 \text{ km})\sqrt{1-(0.999)^2} = 0.45 \text{ km}$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

- 37.19. IDENTIFY and SET UP:** Reference frames  $S$  and  $S'$  are shown in Figure 37.19.



Frame  $S$  is at rest in the laboratory. Frame  $S'$  is attached to particle 1.

**Figure 37.19**

$u$  is the speed of  $S'$  relative to  $S$ ; this is the speed of particle 1 as measured in the laboratory. Thus  $u = +0.650c$ . The speed of particle 2 in  $S'$  is  $0.950c$ . Also, since the two particles move in opposite directions, 2 moves in the  $-x'$  direction and  $v'_x = -0.950c$ . We want to calculate  $v_x$ , the speed of particle 2 in frame  $S$ ; use Eq.(37.23).

**EXECUTE:**  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.950c + 0.650c}{1 + (0.950c)(-0.650c)/c^2} = \frac{-0.300c}{1 - 0.6175} = -0.784c$ . The speed of the second particle, as measured in the laboratory, is  $0.784c$ .

**EVALUATE:** The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is  $0.300c$ . The correct relativistic calculation gives a result more than twice this.

**37.30.** The force is found from Eq.(37.32) or Eq.(37.33).

(a) Indistinguishable from  $F = ma = 0.145 \text{ N}$ .

(b)  $\gamma^3 ma = 1.75 \text{ N}$ .

(c)  $\gamma^3 ma = 51.7 \text{ N}$ .

(d)  $\gamma ma = 0.145 \text{ N}, 0.333 \text{ N}, 1.03 \text{ N}$ .

**37.53.** (a)  $E = \gamma mc^2$  and  $\gamma = 10 = \frac{1}{\sqrt{1 - (v/c)^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \Rightarrow \frac{v}{c} = \sqrt{\frac{99}{100}} = 0.995$ .

(b)  $(pc)^2 = m^2 v^2 \gamma^2 c^2, E^2 = m^2 c^4 \left( \left( \frac{v}{c} \right)^2 \gamma^2 + 1 \right)$   
 $\Rightarrow \frac{E^2 - (pc)^2}{E^2} = \frac{1}{1 + \gamma^2 \left( \frac{v}{c} \right)^2} = \frac{1}{1 + (10/(0.995))^2} = 0.01 = 1\%$ .