PHYS 201 Winter 2010

Recitation Problems (Week 2)

All problems taken from *University Physics,* Young and Freedman, 12th Ed.

13.62. **IDENTIFY:** Calculate the resonant frequency and compare to 35 Hz.
SET UP: ω in rad/s is related to f in Hz by $\omega = 2\pi f$.

EXECUTE: The resonant frequency is $\sqrt{k/m} = \sqrt{(2.1\times 10^6\ \text{N/m})/108\ \text{kg}} = 139\ \text{rad/s} = 22.2\ \text{Hz}$, and this package does not meet the criterion.
EVALUATE: To make the package meet the requirement, increase the resonant frequency by increasing the force constant k.

13.63. **IDENTIFY:** $ma_x = -kx$ so $a_{max} = \dfrac{k}{m}A = \omega^2 A$ is the magnitude of the acceleration when $x = \pm A$. $v_{max} = \sqrt{\dfrac{k}{m}}A = \omega A$.

$P = \dfrac{W}{t} = \dfrac{\Delta K}{t}$.

SET UP: $A = 0.0500\ \text{m}$. $\omega = 3500\ \text{rpm} = 366.5\ \text{rad/s}$.

EXECUTE: **(a)** $a_{max} = \omega^2 A = (366.5\ \text{rad/s})^2(0.0500\ \text{m}) = 6.72\times 10^3\ \text{m/s}^2$

(b) $F_{max} = ma_{max} = (0.450\ \text{kg})(6.72\times 10^3\ \text{m/s}^2) = 3.02\times 10^3\ \text{N}$

(c) $v_{max} = \omega A = (366.5\ \text{rad/s})(0.0500\ \text{m}) = 18.3\ \text{m/s}$. $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.450\ \text{kg})(18.3\ \text{m/s})^2 = 75.4\ \text{J}$

(d) $P = \dfrac{\frac{1}{2}mv^2}{t}$. $t = \dfrac{T}{4} = \dfrac{2\pi}{4\omega} = 4.286\times 10^{-3}\ \text{s}$. $P = \dfrac{(0.450\ \text{kg})(18.3\ \text{m/s})^2}{2(4.286\times 10^{-3}\ \text{s})} = 1.76\times 10^4\ \text{W}$.

(e) a_{max} is proportional to ω^2, so F_{max} increases by a factor of 4, to $1.21\times 10^4\ \text{N}$. v_{max} is proportional to ω, so v_{max} doubles, to 36.6 m/s, and K_{max} increases by a factor of 4, to 302 J. In part (d). t is halved and K is quadrupled, so P_{max} increases by a factor of 8 and becomes $1.41\times 10^5\ \text{W}$.

15.9. **IDENTIFY:** Evaluate the partial derivatives and see if Eq.(15.12) is satisfied.

SET UP: $\dfrac{\partial}{\partial x}\cos(kx+\omega t) = -k\sin(kx+\omega t)$. $\dfrac{\partial}{\partial t}\cos(kx+\omega t) = -\omega\sin(kx+\omega t)$. $\dfrac{\partial}{\partial x}\sin(kx+\omega t) = k\cos(kx+\omega t)$.

$\dfrac{\partial}{\partial t}\sin(kx+\omega t) = \omega\sin(kx+\omega t)$.

EXECUTE: **(a)** $\dfrac{\partial^2 y}{\partial x^2} = -Ak^2\cos(kx+\omega t)$. $\dfrac{\partial^2 y}{\partial t^2} = -A\omega^2\cos(kx+\omega t)$. Eq.(15.12) is satisfied, if $v = \omega/k$.

(b) $\dfrac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx+\omega t)$. $\dfrac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$. Eq.(15.12) is satisfied, if $v = \omega/k$.

(c) $\dfrac{\partial y}{\partial x} = -kA\sin(kx)$. $\dfrac{\partial^2 y}{\partial x^2} = -k^2A\cos(kx)$. $\dfrac{\partial y}{\partial t} = -\omega A\sin(\omega t)$. $\dfrac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\omega t)$. Eq.(15.12) is not satisfied.

(d) $v_y = \dfrac{\partial y}{\partial t} = \omega A\cos(kx+\omega t)$. $a_y = \dfrac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$

EVALUATE: The functions $\cos(kx+\omega t)$ and $\sin(kx+\omega t)$ differ only in phase.

15.15. **IDENTIFY and SET UP:** Use Eq.(15.13) to calculate the wave speed. Then use Eq.(15.1) to calculate the wavelength.

EXECUTE: **(a)** The tension F in the rope is the weight of the hanging mass:

$F = mg = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$

$v = \sqrt{F/\mu} = \sqrt{14.7\text{ N}/(0.0550\text{ kg/m})} = 16.3\text{ m/s}$

(b) $v = f\lambda$ so $\lambda = v/f = (16.3\text{ m/s})/120\text{ Hz} = 0.136\text{ m}$.

(c) EVALUATE: $v = \sqrt{F/\mu}$, where $F = mg$. Doubling m increases v by a factor of $\sqrt{2}$. $\lambda = v/f$. f remains 120 Hz and v increases by a factor of $\sqrt{2}$, so λ increases by a factor of $\sqrt{2}$.

15.21. **IDENTIFY:** For a point source, $I = \dfrac{P}{4\pi r^2}$ and $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$.

SET UP: $1\,\mu\text{W} = 10^{-6}\text{ W}$

EXECUTE: **(a)** $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (30.0\text{ m})\sqrt{\dfrac{10.0\text{ W/m}^2}{1\times 10^{-6}\text{ W/m}^2}} = 95\text{ km}$

(b) $\dfrac{I_2}{I_3} = \dfrac{r_3^2}{r_2^2}$, with $I_2 = 1.0\,\mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\dfrac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\,\mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\text{ W/m}^2)(4\pi)(30.0\text{ m})^2 = 1.1\times 10^5\text{ W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.