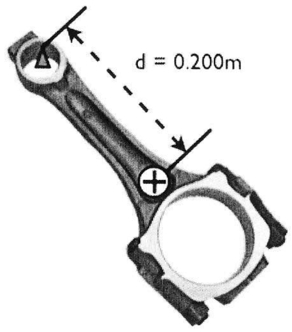
PHYS 201 Winter 2010

Recitation Problems (Week 1)

All problems taken from *University Physics,* Young and Freedman, 12th Ed.

13.49 A 1.80-kg connecting rod from a car engine is pivoted about a horizontal knife edge as shown in the picture. The center of gravity of the rod was located by balancing and is 0.200 m from the pivot. When the rod is set into small-amplitude oscillation, it makes 100 complete swings in 120 s. Calculate the moment of inertia of the rod about the rotation axis through the pivot.



$f = \frac{100 \text{ swings}}{120 \text{ s}} = \frac{5}{6} \text{ Hz}$ $\boxed{\omega = 2\pi f = \frac{5\pi}{3} \text{ rad/s}}$

For a pendulum, $\tau = I\ddot{\theta} \Rightarrow -dmg(\sin\theta) = I\ddot{\theta} \Rightarrow -dmg\theta = I\ddot{\theta}$ for small angles (using $\sin\theta \simeq \theta$). Solution is $\theta(t) = A\cos(\omega t + \phi)$ where $\boxed{\omega = \sqrt{\frac{mgd}{I}}}$ We know m, g, and d, so solve for I.

$$I = \frac{mgd}{\omega^2} = \frac{(1.80)(9.81)(0.200)}{\left(\frac{5\pi}{3}\right)^2} \approx \boxed{0.129 \text{ kg}\cdot\text{m}^2}$$

13.58 A 50.0g hard-boiled egg moves on the end of a spring with force constant k = 25.0 N/m. Its initial displacement is 0.300 m. A damping force $F_x = -bv_x$ acts on the egg, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b.

Solution for damped harmonic oscillators is $\boxed{x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)}$

Since the egg starts with no initial velocity, $\phi = 0$ (or $\pm\pi$, $\pm 2\pi$, etc.)

The initial displacement is $x(0) = Ae^0\cos(0) = A$, so $A = 0.300$ m.

The amplitude at time t is $(0.3)e^{-\frac{b}{2m}t}$, so $(0.3)e^{-\frac{b}{2m}t} = 0.1$ when $t = 5.00$ s.

Solve $(0.3)e^{-\frac{b}{2m}5} = 0.1 \Rightarrow e^{-\frac{b}{2m}5} = \frac{1}{3}$ by finding the natural log of both sides: $-\frac{5}{2}\frac{b}{m} = \ln\left(\frac{1}{3}\right) = -\ln(3) \Rightarrow b = \frac{2m}{5}\ln(3)$

$m = 0.050 \text{ kg} \Rightarrow b = \frac{0.1}{5}\ln(3) = 0.02\ln(3) \simeq \boxed{0.0220 \text{ kg/s}}$

13.61 A sinusoidally varying driving force is applied to a damped harmonic oscillator. (a) What are the units of the damping constant b ? (b) Show that the quantity $\sqrt{(km)}$ has the same units as b. (c) In terms of F_{max} and k, what is the amplitude for $\omega_d = \sqrt{(k / m)}$ when (i) $b = 0.2 \sqrt{(km)}$ and (ii) $b = 0.4 \sqrt{(km)}$?

The equation of motion for a driven, damped harmonic oscillator is

$m\ddot{x} + b\dot{x} + kx = f(t)$. In this case, $f(t)$ is sinusoidal with angular frequency ω_d, so $f(t) = C\sin(\omega_d t)$ for some constant C.

a) $m\ddot{x}$, kx, and $f(t)$ are in Newtons, so $b\dot{x}$ is too. $\dot{x}$ is in m/s, so

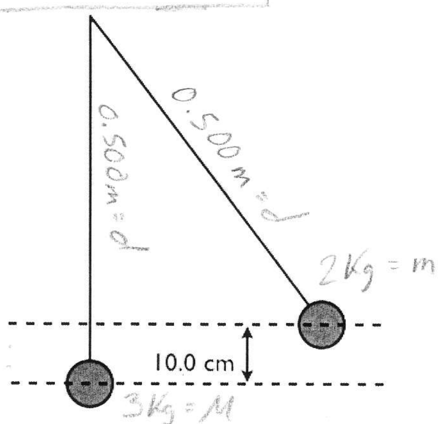
$$b\frac{m}{s} \sim N \Rightarrow b \sim \frac{N\cdot s}{m} = Kg\frac{m}{s^2}\cdot\frac{s}{m} = \boxed{Kg/s}$$

b) k is N/m, so $\sqrt{km} \sim \sqrt{\frac{N}{m}\cdot Kg} = \sqrt{\frac{Kg\, m/s^2}{m}\cdot Kg} = \sqrt{Kg^2/s^2} = \boxed{Kg/s}$

c) Look up driven, damped oscillator amplitude formula, then plug in $b = 0.2\sqrt{km}$ and $0.4\sqrt{km}$.

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}} \quad \text{i)}\ \boxed{A = 5\,\frac{F_{max}\cdot m}{k}} \quad \text{ii)}\ \boxed{A = 2.5\,\frac{F_{max}\cdot m}{k}}$$

13.89 In the picture, the upper ball is released from rest, collides with the stationary lower ball, and sticks to it. The strings are both 50.0 cm long. The upper ball has mass 2.00 kg, and it is initially 10.0 cm higher than the lower ball, which has mass 3.00 kg. Find the frequency and maximum angular displacement of the motion after the collision.



I) Use conservation of E to find velocity V_0 of upper ball just before collision: $\frac{1}{2}mv_0^2 = mgh \Rightarrow V_0 = \sqrt{2gh} \approx \boxed{1.40\ m/s}$

II) Use conservation of p to find velocity of stuck-together balls after collision (E is not conserved in sticky collisions!)

$$mV_0 = (M+m)V_1 \Rightarrow V_1 = \frac{m}{M+m}V_0 \approx \frac{2}{5}(1.40) = \boxed{0.56\ m/s}$$

III) Pendulum equation of motion is $\theta(t) = A\cos(\omega t + \phi)$ $\omega = \sqrt{\frac{K}{I}} = \sqrt{\frac{g}{d}} \approx \boxed{4.43\ \frac{rad}{s}}$

(this is true for small angles... for large angles, it's wrong!) $\omega = 2\pi f \Rightarrow \boxed{f \approx 0.705\ Hz}$

Since $\theta(0) = 0$ but $\dot{\theta}(0) \neq 0$, we know $\theta(t) = A\cos(\omega t \pm \frac{\pi}{2}) = \pm A\sin(\omega t)$.

$\dot{\theta}(0) = \frac{V_1}{d}$ by definition of angular velocity. $\dot{\theta}(t) = \pm\omega A\cos(\omega t)$ by chain rule.

$$\text{Therefore } \frac{V_1}{d} = \pm\omega A \Rightarrow A = \pm\frac{V_1}{d\omega} \approx \frac{0.56}{(0.5)(4.43)} = \boxed{0.253\ rad} = \boxed{14.5°}$$

(Is small-angle still valid? $\sin(0.253) \approx 0.250$, so it's accurate to $\pm 1.2\%$.)