PHYS 201 WINTER 2010

HOMEWORK 6

Write down all steps towards the solution to obtain maximum credit.
Don't forget to specify units!

1. Space Oddity

In October 2008, a radio telescope in Ukraine sent a message to the planets orbiting the star Gliese 581, which is 20 light-years from Earth in the Libra constellation. (1 light-year $\approx 9.46 \cdot 10^{15}$ meters.) The planets are older than Earth, and astronomers suspect that at least one of them could have liquid water on its surface.

Imagine there is a civilization on the planet Gliese 581d. In the year 2029 (according to Earth clocks), they receive the message and send a reply and a spaceship towards Earth in the year 2030. As approximations, assume that Earth and Gliese 581d do not move relative to each other and neither planet is accelerating.

1.1. Thanks to the Gliesians' advanced technology, the ship quickly accelerates to its cruising speed of $2.80 \cdot 10^8$ m/s relative to Earth. On Earth, we receive the Gliesian radio message on April 1, 2050. **When does the spaceship arrive?**

1.2. To the Gliesian navigators onboard the moving ship, the distance to Earth will seem shorter than 20 light-years due to length contraction. **How far is the journey from their point of view?** Give your answer in light-years.

1.3. From the point of view of the Gliesian navigators, how much time do they spend on the voyage to Earth? Give your answer in Earth years.

1.4. After the spaceship uses enough fuel to accelerate to cruising speed, it has a mass of 250 metric tonnes = 250,000 kg. **What is the kinetic energy of the ship according to an observer on Earth?** Recall that the total energy of a moving, massive object is $E = \gamma mc^2$ where γ is defined to be

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In special relativity, the kinetic energy of a moving, massive object is its total energy minus its rest energy, or $K = (\gamma - 1)mc^2$.

2. X-Ray Specs

An injured emergency room patient is irradiated with x-rays of wavelength 50 picometers. A doctor holds a digital x-ray spectrometer at an angle of 135° from the x-ray beam. The spectrometer shows a plot of x-ray intensity as a function of wavelength, and two peaks are apparent. One is centered at $\lambda = 50$ picometers. **At what wavelength, in picometers, is the center of the other peak?** Hint: Look up "Compton scattering" in your textbook or online.

3. Goalie de Broglie

The de Broglie wavelength of a particle is $\lambda = h/p$ where p is the object's momentum and h is Planck's constant. A hockey puck has mass 165 grams. **Find the de Broglie wavelength, in meters, of a hockey puck moving at 150 km/h.** Is this length comparable to the size of a hockey puck? If not, we can reasonably approximate the behavior of hockey pucks using classical mechanics.

4. What's In The Box?

The wavefunction for a particle in the $n = 7$ state in a "box" of length L is:

$$\psi_7(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{7\pi}{L} x\right)$$

This function is periodic. **Find its wavelength λ in terms of L and show that the energy $E = \frac{p^2}{2m}$ of the $\psi_7(x)$ state is:**

$$E_7 = \frac{49h^2}{8mL^2}$$

Hint: Use λ to find p using the de Broglie relation $\lambda = h/p$.

5. Uncertainty Principle

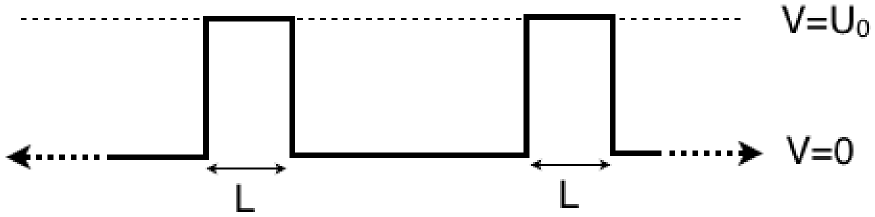
Heisenberg's uncertainty principle says that the product of position uncertainty Δx and momentum uncertainty Δp_x is always at least $h/2\pi$.

$$\Delta x \Delta p_x \geq \frac{h}{2\pi}$$

If Δx for a proton is 1.0 angstrom (10^{-10} meters), **what is its minimum possible momentum uncertainty?** Give your answer in kg·m/s.

6. Bonus Problem

The usual 1-dimensional "particle in a box" problem assumes a potential energy function $V(x)$ that is infinite except in some region $0 < x < a$. A more realistic "box" would not be infinitely high. Consider a potential function $V(x)$ like this:



If a particle initially on one side of a barrier is detected on the other side, it is said to have "tunneled." The probability T of tunneling through a single barrier is[1]

$$T = Ge^{-2\kappa L} \qquad G \equiv 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) \qquad \kappa \equiv \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

An electron (mass $m_e = 9.11 \cdot 10^{-31}$ kg) trapped between the barriers has kinetic energy $E = 2.00$ keV and the height of the barriers is $U_0 = 5.00$ keV. Classically, the electron cannot escape, but quantum mechanics allows it to tunnel through either barrier. **Find the minimum value of L such that the probability of detecting the particle outside the barriers is $\leq 1\%$.**

[1]This result is not easy to find; it comes from the Wentzel-Kramers-Brillouin approximate solution to the Schrödinger equation. For our purposes, just treat it like a magic formula.