PHYS 201 WINTER 2010

HOMEWORK 5

Solutions

1. Blue Laser

The energy of each laser photon is $E = hf = h\frac{c}{\lambda}$. When an electron in the calcium plate absorbs a photon, some of that electron's energy is spent escaping the plate. The (minimum) amount of energy lost is the work function of the material; in this case, it's $\Phi = 2.87$ eV $= 4.598 \cdot 10^{-19}$ J. The remaining energy is:

$$E = h\frac{c}{\lambda} - \Phi = (6.626 \cdot 10^{-34})\frac{3.00 \cdot 10^8}{420 \cdot 10^{-9}} - 4.598 \cdot 10^{-19} = 1.35 \cdot 10^{-20} \text{ J}$$

2. Photons and Electrons

2.1. The frequency of each photon is given by $E = hf$:

$$5 \text{ eV} = (4.136 \cdot 10^{-15}\text{eV} \cdot \text{s})f \quad \Rightarrow \quad f = 1.209 \cdot 10^{15} \text{ Hz}$$

(Looking up Planck's constant in units of eV·s lets us skip converting to Joules and back.) Now convert this to a wavelength using $\lambda f = c$:

$$\lambda = \frac{c}{f} = \frac{3.00 \cdot 10^8 \text{ m/s}}{1.209 \cdot 10^{15} \text{ Hz}} \approx 250 \text{ nm}$$

2.2. Total energy absorbed by the emitter per second is 2 mJ, and each photon has 5 eV of energy. The number of photons per second must therefore be:

$$\frac{.002 \text{ J/s}}{5 \text{ eV / photon}} = \frac{.002}{8.010 \cdot 10^{-19}} \frac{\text{J / s}}{\text{J / photon}} \approx 2.5 \cdot 10^{15} \text{ photons / s}$$

2.3. The maximum kinetic energy is given by $E = hf - \Phi$:

$$E = 5 \text{ eV} - 3 \text{ eV} = 2 \text{ eV} \approx 3.2 \cdot 10^{-19} \text{ J}$$

2.4. Each photoelectron has kinetic energy 2 eV. If the collector voltage is -2 V or less, the photoelectrons will not have enough energy to overcome the collector's electric field. (Remember, electrons are repelled by *negative* voltages!)

2.5. Recall that amperes measure of charge per second: 1 A = 1 C/s. A current of 200 pA means the number of electrons escaping the emitter per second is

$$\frac{200 \cdot 10^{-12} \text{ C/s}}{1.602 \cdot 10^{-19} \text{ C/electron}} \approx 1.248 \cdot 10^{9} \text{ electrons/s}$$

From part 1.2, the total number of photons absorbed per second is $2.5 \cdot 10^{15}$. The fraction of photons that actually contribute to the photocurrent is

$$1.248 \cdot 10^{9} / 2.5 \cdot 10^{15} \approx 5.0 \cdot 10^{-16}$$

3. Color Me Excited

The Rydberg formula for the Balmer series emission wavelengths is:

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

where n is any integer larger than 2 and R is Rydberg's empirical constant $R \approx 1.097 \cdot 10^{7}\ m^{-1}$. The longest-wavelength transitions will occur when $n = 3, 4, 5$, or 6. Plugging those numbers in gives wavelengths of:

$$\lambda = 656, 486, 434, \text{ and } 410 \text{ nanometers}$$

Note that these are all visible: red, blue, purple, and purple. (A hydrogen atom can emit photons of many other wavelengths, but the others are not visible.)

4. A Speedy Electron

Solve the kinetic + potential energy equation for v:

$$\frac{1}{2}mv^2 = \frac{-13.6 \text{ eV}}{n^2} + \frac{ke^2}{a_0 n^2} = \left(-2.179 \cdot 10^{-18} + 4.366 \cdot 10^{-18}\right)\frac{1}{n^2}$$

Here we have converted -13.6 eV into joules, plugged in the numbers for k and e, and factored out the $1/n^2$. Now to finish the job:

$$v^2 = \frac{2}{9.11 \cdot 10^{-31} \text{ kg}}\left(2.187 \cdot 10^{-18} \text{ J}\right)\frac{1}{n^2} \quad \Rightarrow \quad v = \frac{1}{n}\left(2.19 \cdot 10^{6} \text{ m/s}\right)$$

For the case $n = 2$, the velocity is about 1.10 million m/s. (Bohr's model is wrong; protons and electrons are not simply little spheres in circular orbits. Bohr knew this, however, and the model is useful as an introduction to quantum mechanics.)

5. Bonus Problem

The angular momentum of an electron in the $n = 2$ orbit is $L_2 = mv(4a_0)$.

$$L_2 = (9.11 \cdot 10^{-31})(1.10 \cdot 10^{6})(4 \cdot 5.29 \cdot 10^{-11}) = 2.11 \cdot 10^{-34} \text{ J s}$$

The units here are kg$\cdot$(m/s)$\cdot$m = kg$\cdot$m^2/s = J s . For the $n = 1$ orbit, $v = 2.19 \cdot 10^6$ m/s from problem 5. The $n = 1$ orbital radius is just a_0, so $L_1 = mv_1a_0$:

$$L_1 = (9.11 \cdot 10^{-31})(2.19 \cdot 10^{6})(5.29 \cdot 10^{-11}) = 1.06 \cdot 10^{-34} \text{ J s}$$

The difference is $L_p = L_2 - L_1 = 1.05 \cdot 10^{-34}$ J s. This number equals $\hbar$, pronounced "h-bar," which is defined $\hbar = \frac{h}{2\pi}$. In fact, the angular momentum difference between the nth orbit and the $(n+1)$th orbit is $\hbar$ for any choice of n.

In quantum electrodynamics, *all* photons have an angular momentum of $\hbar$. Bohr's model may be wrong, but it still correctly predicts the "spin" of a photon!