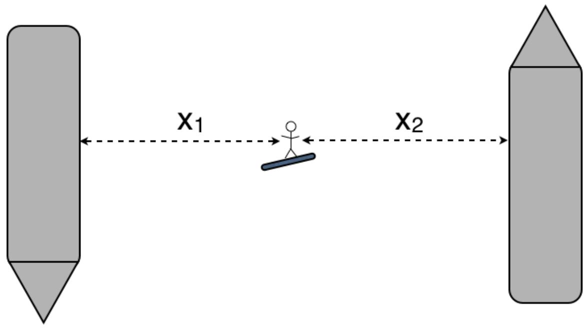
PHYS 201 WINTER 2010

HOMEWORK 3

Solutions

1. Surfing on Sine Waves



1.1. What is the amplitude of her vertical motion as a function of x_2 ?

$$z_{surfer}(t) = 1.6\Big[\sin(kx_1 - \omega t) + \sin(kx_2 + \omega t)\Big]$$
$$= 1.6\Big[2\cos\Big(\tfrac{1}{2}k[x_1 - x_2] - \omega t\Big)\sin\Big(\tfrac{1}{2}k[x_1 + x_2]\Big)\Big]$$

The amplitude (the part that doesn't depend on time) is $3.2\sin\big(k\frac{1}{2}[x_1 + x_2]\big)$. We can figure out k and ω by using $k = \frac{2\pi}{\lambda} \approx 0.785\ m^{-1}$ and $\omega = 2\pi f = \frac{2\pi}{T} \approx 2.51\ s^{-1}$.

Either x_1 or x_2 has to be negative since the ships are on opposite sides of the surfer. Let's choose x_1 to be negative to be consistent with the picture. If so, then $|x_1| + |x_2| = 130$ is equivalent to $-x_1 + x_2 = 130$, so we know $x_1 = x_2 - 130$. Substitution tells us $\frac{1}{2}[x_1 + x_2] = x_2 - 65$ and the amplitude in meters is:

$$A = 3.2\sin\big(0.785[x_2 - 65]\big)$$

The amplitude is zero when $x_2 = x_1 = 65\ m$. (This happens because the ships are exactly alike, moving at exactly the same speed, and pointed in opposite directions, so their wakes interfere destructively at the midpoint.)

2. In Tune and On Time

2.1. **Find the mass of the string in grams.** The mass of the string is its density times its volume. Because the density is in g/cm^3 and we want an answer in grams, we'll use centimeters for length units. For radius r and length L,

$$m = \rho(\pi r^2 L) = (8.908)\pi(0.0127)^2(64.77) = 0.292\ g$$

Note that 0.010 is the *diameter* of the string, not the radius, and that I have converted $r = 0.005$" and $L = 25.5$" into cm. (A useful rule is 1" $\approx 2.54\ cm$.)

2.2. **Find the tension in the string in newtons.**
The speed-of-sound formulas must equal each other, so we can write

$$\sqrt{\frac{T}{\mu}} = \lambda f \qquad \Rightarrow \qquad T = \mu(\lambda f)^2$$

Remember that the fundamental wavelength of a string of length L is $2L$. The mass per unit length is just the total mass of the string divided by its length:

$$\mu = \frac{0.000292\ Kg}{0.6477\ m} = 4.508 \cdot 10^{-4}\ Kg/m$$

Plugging in the numbers gives $T = (4.508 \cdot 10^{-4})(1.295 \cdot 330)^2 = 82.3\ N$. This result is close to a manufacturer's estimate of 15 lb for a 0.010-size high-E string.

3. Don't Touch That Dial

3.1. **How long should the antenna be?**

$$\lambda f = c \quad \Rightarrow \quad \lambda(91.7 \cdot 10^6) = 3 \cdot 10^8 \quad \Rightarrow \quad \lambda = 3.27\ m$$

The minimum practical antenna length is then $\frac{1}{4}(3.27) = 0.818\ m = 81.8\ cm$.

3.2. **If $L = 30.0\ \mu H$ and $R = 0.200\ m\Omega$, what should the capacitance be?**
Instead of m, b, and k, the terms in the circuit equation are L, R, and $\frac{1}{C}$. Substitute these into the formula for ω' and set it equal to $2\pi(91.7 \cdot 10^6)\ Hz$:

$$2\pi(91.7 \cdot 10^6) = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \Rightarrow \quad \frac{1}{(30.0 \cdot 10^{-6})C} - \frac{(0.200)^2}{4(30.0 \cdot 10^{-6})^2} = 3.32 \cdot 10^{17}$$

The $\frac{R^2}{4L^2}$ term is much smaller than the $\frac{1}{LC}$ term; apparently this circuit is very weakly damped. Solving for C gives $C = 1 \cdot 10^{-13}\ F = 0.100$ picofarads.

4. Bonus Question

Let v be the speed of the Porsche relative to the police car. If the radar gun transmits at frequency f_0, the Porsche sees a Doppler-shifted frequency

$$f_{Porsche} = \sqrt{\frac{c-v}{c+v}} f_0$$

Notice that the shift factor is less than 1 when $v > 0$. This is a *redshift*, so positive v means "the observers are moving away from each other." (Also note that this is the relativistic Doppler formula for electromagnetic radiation, not sound!) The radar signals reflecting off the Porsche have frequency $f_{Porsche}$. The police car is moving away relative to the Porsche, so the returning signals are redshifted again:

$$f_{detected} = \sqrt{\frac{c-v}{c+v}} f_{Porsche} = \frac{c-v}{c+v} f_0$$

Since we know the ratio of $f_{detected}$ and f_0, this expression can be used to find v.

$$\frac{f_{detected}}{f_0} = \frac{c-v}{c+v} \quad \Rightarrow \quad c - v = \left(\frac{f_{detected}}{f_0}\right)(c+v)$$

$$c\left(1 - \frac{f_{detected}}{f_0}\right) = v\left(1 + \frac{f_{detected}}{f_0}\right)$$

Plugging in the numbers given in the problem gives $v = 11.18\ m/s$. This is the speed of the Porsche *relative to the police car*. In order to write a speeding ticket, we convert this into $v = 25.0\ mile/hr$ and add it to the speed of the police car. The Porsche is traveling about 90 mph, which is way too fast.