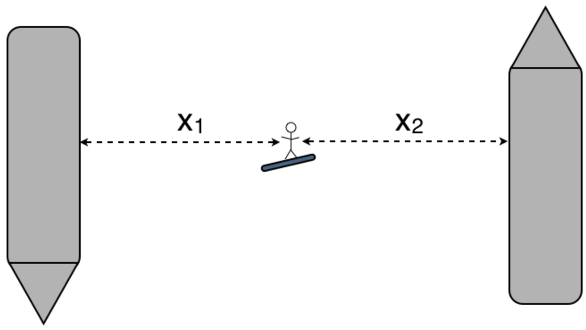
PHYS 201 WINTER 2010

HOMEWORK 3

Write down all steps towards the solution to obtain maximum credit. Don't forget to specify units!

1. Surfing on Sine Waves



A surfer is in deep water with almost no wind or waves until two large, nearly identical ships pass in opposite directions on either side of her. The wakes of the two ships collide and the surfer is caught in the middle. She bobs up and down with the waves but does not move significantly forward, back, right, or left.

Model the ships' wakes as sinusoidal waves moving in the $+x$ and $-x$ directions. At the surfer's location, the heights of the two wakes in meters are approximately

$$z_1(t) = 1.6 \sin \big(kx_1 - \omega t\big) \qquad z_2(t) = 1.6 \sin \big(kx_2 + \omega t\big)$$

with wavelengths $\lambda = 8.00\ m$ and periods $T = 2.50\ s$. The x coordinates of the ships (from her point of view) are are x_1 and x_2 and the ships are 130 m apart.

1.1. **What is the amplitude of her vertical motion as a function of x_2 ?**

The wave height at her location will be the sum of the heights of the component waves: $z_{surfer}(t) = z_1(t) + z_2(t)$. Use the trigonometric identity

$$\sin a + \sin b = 2 \cos \left[\frac{1}{2}\big(a - b\big)\right] \sin \left[\frac{1}{2}\big(a + b\big)\right]$$

to find $z_{surfer}(t)$. Then use $|x_1| = 130 - |x_2|$ to eliminate $|x_1|$ from your answer. (Note that either x_1 or x_2, but not both, should be negative.)

2. In Tune and On Time

The top of a guitar is conventionally tuned to a fundamental frequency of about 330 Hz. The string is a nickel cylinder of length 25.5" and diameter 0.010".

2.1. Find the mass of the string in grams.
The density of nickel at 20° C and 1 atm pressure is about 8.908 g/cm^3.

2.2. Find the tension in the string in newtons.
The speed of sound in a string is $\sqrt{T/\mu}$, where T is the tension on the string and μ is the mass per unit length. The speed of sound in a string is also equal to λf, where λ is its fundamental wavelength and f is its frequency of vibration.

3. Don't Touch That Dial

A (crude) radio receiver consists of an RLC circuit connected to an antenna. A listener tunes the circuit's resonance frequency to 91.7 MHz so she can listen to Drexel's student-run radio station WKDU. The resistance R and inductance L of the circuit are fixed, but the capacitance can be varied by turning a knob.

3.1. How long should the antenna be?
A rule-of-thumb in antenna design says that a straight antenna should be at least $\frac{1}{4}\lambda$ long, where λ is the wavelength the antenna is designed to detect. For electromagnetic waves, $\lambda f = c$ where c is the speed of light.

3.2. If $L = 30.0$ μH and $R = 0.200$ $m\Omega$, what should the capacitance be?
The equation for charge on the capacitor in an RLC circuit is

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = V(t)$$

where V is the driving voltage, which we assume to be sinusoidal with angular frequency $\omega_d = 2\pi(91.7 \cdot 10^6)$ Hz. This is a damped harmonic oscillator equation. The resonance frequency of a damped harmonic oscillator $m\ddot{x} + b\dot{x} + kx = f(t)$ is:

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

The receiver will be in tune with the radio station when $\omega' \approx \omega_d$.

4. Bonus Question

A police officer is driving at exactly the speed limit of 65 mph on a highway when a Porsche speeds past him in the left lane. The police officer's radar gun sends a signal of frequency f_0 directly forward which bounces off the Porsche and heads directly back towards the police car. The returning signal has frequency $(1 - 7.46 \cdot 10^{-8})f_0$. **What is the speed of the Porsche relative to the Earth?** (Use the exact value $c = 299,792,458$ m/s for the speed of light.)