PHYS 201 WINTER 2010

HOMEWORK 2

Solutions

1. What's the Frequency, ω?

1.1. Find ω, λ, and k for sound waves with $f = 20\ Hz$ and $f = 20\ KHz$.

Let's start with the 20 Hz sound wave. Use $\lambda f = c$ to find $\lambda = \frac{343}{20} = 17.15\ m$. Then $k = \frac{2\pi}{\lambda} = 0.366\ rad/m$ and $\omega = 2\pi f = 40\pi \approx 126\ rad/s$.

For $f = 20,000\ Hz$, the numbers are $\lambda = 17.15\ mm$, $k = 366\ rad/m$, $\omega = 40,000\pi\ rad/s \approx 126,000\ rad/s$. The numbers 20 Hz and 20 KHz are generally accepted as the lowest- and highest-frequencies that can be heard by humans.

2. The Light At The End of the Tunnel

2.1. How much time passes before the distortions reach Monster 1?

The speed of transverse waves in an ideal string is $c = \sqrt{T/\mu}$ where T is tension in the string and μ is mass per meter. The tension in the string is 75% of Monster 0's weight: $(.75)(30,000\ Kg)(9.81\ m/s^2) = 221,000\ N$. The mass density is:

$$\frac{155\ lb}{3\ ft} \cdot \frac{1\ Kg}{2.205\ lb} \cdot \frac{1\ ft}{12\ in} \cdot \frac{1\ in}{2.54\ cm} \cdot \frac{100\ cm}{1\ m} = 76.9\ Kg/m$$

Wave speed is then $\sqrt{221,000/76.9} = 53.6\ m/s$. The distance to Monster 1 is 80 meters, so $t = \frac{80}{53.6} = 1.49$ seconds.

2.2. How rapidly is Monster 0 transferring energy to Monster 1?

From the book, the formula for average power transfer on an ideal string is

$$P_{av} = \frac{1}{2}\sqrt{\mu T}\omega^2 A^2$$

We've already found μ and T, ω is just $2\pi f = 4\pi$, and the amplitude A is 1.5 meters. $P_{av} = \frac{1}{2}\sqrt{(76.9)(221000)}(4\pi)^2(1.5)^2 = 732,000$ watts ≈ 1000 horsepower.

3. Shaking the Tree

3.1. Estimate the resonance frequency of the tree.

When you lean against a spot 1.2 meters from the base with a force of 500 newtons, you put a torque of $1.2(500) = 600\ Nm$ on the tree and the tree fights back with a torque of $600\ Nm$ in the other direction. Since $\tau = -\kappa\theta$, you now know $-600 = -\kappa\theta$ when $\theta = 0.157$, so $\kappa = 3,822\ Nm$.

You estimate the moment of inertia to be $I = \frac{1}{3}mL^2 = \frac{1}{3}(300)(3.5)^2 = 1,225\ Kgm^2$. The equation of motion $-\kappa\theta = I\ddot{\theta}$ has solutions with angular frequency

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{3822}{1225}} = 1.766\ rad/s$$

Remember, $\omega = 2\pi f$, so the tree naturally oscillates at $\frac{1}{2\pi}(1.766) = 0.28\ Hz$. If the tree is shaken at (or near) this frequency, it will resonate.

3.2. Experimentally, you find that the tree oscillates most violently when you shake it once every 4.0 seconds. Using this new information, improve your estimate for the mass of the tree.

The experimental value for f is now $\frac{1}{4.0} = 0.25\ Hz$. Using the same expression as before, $\omega = \sqrt{\kappa/I}$, we can solve for I and use it to find the tree's mass.

$$\frac{\kappa}{I} = \omega^2 = (2\pi f)^2 = \frac{1}{4}\pi^2$$

Using the previous value $\kappa = 3822$, we find $I = 1549\ Kg \cdot m^2$. Since $I = \frac{1}{3}m(3.5)^2$, we can find $m = 380\ Kg$. The tree is slightly heavier than you estimated.

4. Bonus Problem

We don't know what f is except that it is a function of one variable and its first and second derivatives exist. We don't know what those are, so let's call them f' and f''. Using the chain rule, the partial derivatives of $f(kx \pm \omega t)$ are

$$\partial_t f(kx - \omega t) = -\omega f'(kx - \omega t) \qquad \partial_t^2 f(kx - \omega t) = \omega^2 f''(kx - \omega t)$$
$$\partial_x f(kx - \omega t) = k f'(kx - \omega t) \qquad \partial_x^2 f(kx - \omega t) = k^2 f''(kx - \omega t)$$

Notice that $\omega^2 f''(kx - \omega t) = k^2 f''(kx - \omega t)$ no matter what f or its derivatives are! As long as $\frac{\omega^2}{k^2} = c^2$, we can be sure that $f(kx - \omega t)$ is a solution of the wave equation. Exactly the same method shows that $f(kx + \omega t)$ is also a solution.