PHYS 201 WINTER 2010

HOMEWORK 2

Write down all steps towards the solution to obtain maximum credit.
Don't forget to specify units!

1. What's the Frequency, ω?

The symbols f, ω, λ, k, and c have conventional meanings in wave equations that are important but somewhat easy to confuse: f is frequency, ω is angular frequency, λ is wavelength, k is wave number, and c is wave speed. Putting a factor of 2π in the wrong place can seem like a small mistake, but as Richard Feynman said, "If you can't get the 2π's right, you don't know nothing." Here's some practice.

1.1. **Find ω, λ, and k for sound waves with $f = 20\ Hz$ and $f = 20\ KHz$.** Use $c = 343\ m/s$ for the speed of sound at room temperature and atmospheric pressure.

2. The Light At The End of the Tunnel

A railroad track is made of two long steel beams connected by wood blocks. The heaviest rails ever mass-produced were the "Pennsylvania Special" rails which connected Pittsburgh and Philadelphia. Each rail weighed 155 pounds/yard.

Two giant monsters are playing tug-of-war with an 80 meter section of Pennsylvania Special rail. Monster 0, whose mass is 30 megagrams (about as much as an apatosaurus), is pulling the rail in the $-x$ direction with a force equal to 75% of his weight. Monster 1 is pulling with about equal force in the $+x$ direction. Attempting to make Monster 1 drop the rail, Monster 0 shakes it violently. The rail distorts but does not break, and the distortions propagate in the $+x$ direction.

2.1. **How much time passes before the distortions reach Monster 1?** Treat the rail as a string and use the wave speed formula from your book.

2.2. **How rapidly is Monster 0 transferring energy to Monster 1?** Assume the distortions are roughly sinusoidal with amplitude 1.5 meters and frequency 2 Hz, so the displacement of y of the rail at a distance x from Monster 0 is

$$y(x, t) = (1.5)\cos(0.234\ x - 4\pi\ t)$$

Use the average power formula from your book and give your answer in watts.

3. Shaking the Tree

You are stuck on a desert island with only a coconut tree and a volleyball. Since you can't eat the volleyball, you try to shake some coconuts loose. If you shake the tree at its natural resonance frequency, then your small inputs will (hopefully) interfere constructively and cause the tree to oscillate violently.

When you lean against the tree, it bends by an angle θ and exerts a restorative torque τ roughly proportional to θ. When you let go, it oscillates in approximately simple harmonic motion. The equation of motion for θ is then $-\kappa\theta = I\ddot{\theta}$ for some κ.

3.1. **Estimate the resonance frequency of the tree.**

When you press with a force of about 500 newtons on a spot 1.2 meters from the base of the tree, it bends by about $9° \approx 0.157$ rad. Since $\tau = -\kappa\theta$ you can find κ. Modeling the tree as a rod of uniform density, its moment of inertia I about its base is $\frac{1}{3}mL^2$. You estimate that the tree has mass 300 kilograms (neglecting the weight of the coconuts and palm leaves) and height 3.5 meters.

3.2. **Experimentally, you find that the tree oscillates most violently when you shake it once every 4.0 seconds. Using this new information, improve your estimate for the mass of the tree.**

4. Bonus Problem

A wave equation is any solution to a differential equation of the form

$$\frac{\partial^2}{\partial t^2}y(x,t) = c^2\frac{\partial^2}{\partial x^2}y(x,t)$$

where c^2 is a positive real number. This can be written more compactly as

$$\partial_t^2 y(x,t) = c^2\partial_x^2 y(x,t) \qquad \textit{or} \qquad \ddot{y} = c^2 y''$$

Show that the functions $y_1(x,t) = f(kx - \omega t)$ **and** $y_2(x,t) = f(kx + \omega t)$, **are solutions to the wave equation** where f can be *any* function of one variable[1] and ω, k are any two real numbers such that $\frac{\omega^2}{k^2} = c^2$. Thus any smooth function of x and t can be a wave solution as long as it only depends on x and t in the specific combinations $kx + \omega t$ and $kx - \omega t$.

[1] Technically f must be twice-differentiable in x and t or the wave equation would be meaningless. This simply means its first and second derivatives must exist at all points in its domain.