

INTERSTELLAR MASERS

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A maser is defined as “a device using the stimulated emission of radiation by excited atoms to amplify or generate coherent monochromatic electromagnetic radiation in the microwave range.”¹ This definition is too constricting because many masers emit electromagnetic radiation outside the microwave regime. For this reason, Charles Townes, who received the Nobel Prize in Physics in 1964 for his research on masers, insisted in his Nobel Lecture that perhaps the *m* in maser should stand for molecular rather than microwave.² This suggestion has yet to catch on, even though *molecular amplification by stimulated emission of radiation* is a more accurate acronym, with almost all astrophysical masers being of molecular origin.

Masers are the product of stimulated emission, which occurs when a photon with energy equal to the energy gap between two quantum states perturbs the excited state. This perturbed excited state then relaxes to a lower energy state releasing a photon with energy equal to that of the original photon. The original photon was never absorbed, so now two photons with the same energy are free to perturb other quantum states and cause the release of more photons with the same energy.

Masers are so interesting because all of the photons emitted by the stimulated emission will have the same energy, phase, polarization, and direction as the original perturbing photon; thus, masers are extremely coherent

and highly amplified types of electromagnetic radiation. The reason that the photons released by stimulated emission have the same characteristics as the stimulating photon is because the external electromagnetic field associated with the moving photon will cause atoms or molecules to oscillate. This oscillation, which is directly related to the properties of the original photon, passes these properties on to the photons produced by the stimulated emission.³ This highly amplified and coherent radiation appears in astronomical spectra as very tall and narrow emission lines (Figure 1) that significantly reduce the error associated with finding redshifts of interstellar gas clouds with maser activity.²

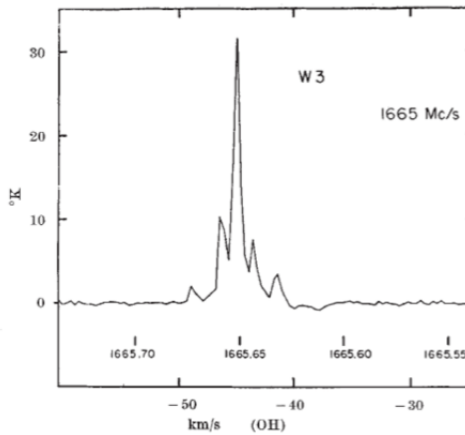


Figure 1. OH maser emission line example¹⁰

But what prevents these photons released by stimulated emission from getting reabsorbed? If we imagine a quantum mechanical system with only two states, we can say there are N_1 particles in the ground state, E_1 , and N_2 particles in the excited state, E_2 . The rate of photon absorption or stimulated

¹“maser, n.” *The Oxford English Dictionary*

²Townes, Charles H., *Production of coherent radiation by atoms and molecules*. Nobel Lecture, (1964).

³Saleh, Bahaa E. A. and Teich, Malvin Carl, *Fundamentals of Photonics*. New York: John Wiley and Sons, (1991).

emission is proportional to N_1 or N_2 , respectively, so if $N_1 > N_2$, then absorption will occur faster than stimulated emission, and no maser can be produced.⁴ As a second case, if $N_1 = N_2$, the rate of photons produced by stimulated emission will be equal to the rate of photons absorbed, so, again, no maser can be produced. The only case in which a maser can be produced is if $N_2 > N_1$; that is, there are more particles in the excited state than the ground state, which is also known as population inversion.⁴ In this case, the rate at which photons are produced by stimulated emission is greater than the rate at which photons are absorbed, so there will be a net number of photons emitted by spontaneous emission that will feed-back into the system causing more stimulated emissions and allowing for the operation of a maser.

In order for population inversion to occur, a quantum system with three or more states is necessary. For clarity's sake, I will call the ground state E_1 , the first excited state E_2 , and the second excited state E_3 with N_1 , N_2 , and N_3 particles in each state, respectively. If all of the particles are initially in E_1 , and a large amount of energy is added to the system, termed *pumping*, most of the particles will now transition to E_3 .⁴ Some E_3 states will now spontaneously emit a photon and drop to the E_2 state, just as some of these new E_2 states will spontaneously emit a photon and drop to E_1 . If the lifetime of a E_3 to E_2 transition ($\tau_{3\rightarrow 2}$) is faster than the E_2 to E_1 transition ($\tau_{2\rightarrow 1}$), then it is evident that more states will get backed up in E_2 rather than E_1 or E_3 . In other words, $N_2 > N_1$, or a population inversion has been created.

Two of the most popular instances of astrophysical masers are HII regions, which are interstellar clouds of ionized hydrogen most commonly referred to as star-forming

regions, and leftover supernova material.⁵ These conditions are highly energetic enough to facilitate pumping, and, as shall be proven, the hydrogen that is present in these gas clouds has appropriate state lifetimes to induce population inversions and create masers.

In order for proton-electron recombination to occur, electrons recombine with a proton to form an excited state of the hydrogen atom. Then, following the selection rules governed by the conservation of angular momentum ($\Delta l = \pm 1$), the quantum states drop to lower energy states while releasing a photon.

Originally, quantum state populations of hydrogen defined by their principal energy level, n , and their orbital angular momentum, l , were thought to be computed statistically; more specifically, physicists believed that these state populations were proportional to their statistical weights, $g_{nl} = 2(2l + 1)$. Comparing this prediction to observed intensities of interstellar gas clouds, things were not agreeing completely, and it was determined that something other than collisional processes have an effect on the occupation of quantum states.⁶

The level populations for a recombining gas are actually given by the Saha equation. The Saha equation employs a statistical distribution to determine the number density of the ionized gas when a system is in local thermal equilibrium.⁷ If one analyzes the simple case of protons and electrons recombining to form neutral hydrogen ($H \leftrightarrow p^+ + e^-$), the Saha equation takes the form

⁴Svelto, Orazio, *Principles of Lasers*, 4th ed. (trans. David Hanna), Springer, (1998)

⁵Fish, V.L., Reid, M.J., Argon, A.L., Zheng, X.W., arXiv:astro-ph/0505148v1. (2005).

⁶Brocklehurst, M. *Mon. Not. R. astr. Soc.*, **153**, 471-490 (1971).

⁷Hansen, C.J., Kawaler, S.D., Trimble, V., *Stellar Interiors*. New York: Springer, (2004).

$$\frac{n_p n_e}{n_H} = \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\frac{13.6eV}{kT}} \frac{g_p g_e}{g_H}$$

where n is the number density of the given species, T is the temperature at which the recombination process is occurring, and g is the degeneracy for the given species ($g_p = 2$, $g_e = 2$, $g_H = 4$).⁸ From the Saha equation, one can see that as $T \rightarrow \infty$, $n_p \rightarrow \infty$ and $n_e \rightarrow \infty$, or, in other words, all of the neutral hydrogen is ionized. In the case of the other extreme, as $T \rightarrow 0$, $n_p \rightarrow 0$ and $n_e \rightarrow 0$, or all the protons and electrons have recombined to form neutral hydrogen.

In Brocklehurst's 1971 paper, in which he calculates level populations for hydrogen-like ions in interstellar gas clouds, he computes the probability that a quantum state will spontaneously transition to another state through electric dipole radiation as

$$A_{n'l, n'l'} = \frac{64\pi^4 \nu^3}{Z^2 3hc^3} \frac{\max(l, l')}{(2l+1)} e^2 a_0^2 \left[\int_0^\infty R(n'l') r R(nl) dr \right]^2$$

where $\nu = \frac{\Delta E}{h}$ is the frequency associated with the energy difference between the initial and final quantum states.⁶

Since the probability of spontaneous emission is proportional to the frequency (or energy) difference cubed ($P_{f \rightarrow i} \propto \nu^3 \propto (\Delta E)^3$), the transitions that undergo the largest change in principle quantum number n while obeying the selection rules will have a higher probability of occurring since $\Delta E \propto \nu \propto \frac{1}{n_f^2} - \frac{1}{n_i^2}$, where n_f and n_i are the final and initial principle quantum numbers, respectively. If one picks any excited state with quantum numbers n, l, m_l , according to the selection rules, it can decay to a state with arbitrary n and $\Delta l = \pm 1$. But the excited state is most likely to transition to a state that maximizes Δn , so a state with saturated quantum numbers (i.e. $l = n - 1, m_l = \pm 1$, which is also known as the "drip line"

of Figure 2) is the most likely final state of a decay. If a state already has saturated quantum numbers, it can only transition to states with $\Delta n = -1$ since $\Delta l = -1$, and the state thusly remains on the "drip line" until it reaches the ground state ($n = 1$) and no longer spontaneously emits energy.

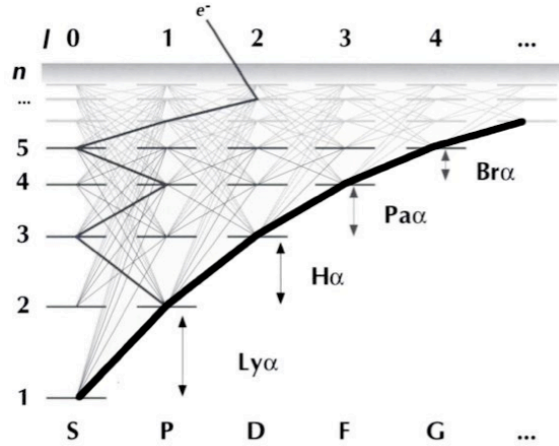


Figure 2. Hydrogen n and l levels. The possible transitions ($\Delta l = \pm 1$) are outlined, with the most probable decay path in bold. This is known as the "drip line," with $l = n - 1, m_l = \pm 1$.⁸

The free space lifetime of the hydrogen state $|n, l = n - 1, m_l = \pm 1\rangle$, as I computed in problem 1 of Professor Gilmore's fifth Quantum III homework assignment, is

$$\tau_{i \rightarrow f} = \frac{16\pi^2 e^3 \hbar^3 (2n-1)^{4n+2} \Gamma(2n-1) l(2l+1)(2l-1)^2}{9 \cdot 2^{4n} e^2 \varepsilon^2 (-1)^{2l} (\Delta E)^3 \Gamma(2n+1) n^{2n+1} (n-1)^{2n+3}}$$

where ε is a polarization vector associated with ΔE , the energy difference between the initial and final states.⁹

Writing this lifetime in terms of the principle quantum number n , one finds that $\tau_{i \rightarrow f} \propto \frac{1}{(\Delta E)^3} \propto \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)^3$. If we imagine a hydrogen atom with infinitely many energy levels, computation shows that $\tau_{p \rightarrow q} > \tau_{q \rightarrow 1}$ for $p > q > 1$, where p and q are integers.

⁸Dopita, M.A., Sutherland, R.S., *Astrophysics of the Diffuse Universe*. Springer, (2003).

⁹Kratzer, Rachael. *Quantum 3, Homework 5, Problem One*.

From the previous discussion of population inversion, it is evident that these are the exact conditions necessary for population inversion to occur, and thus maser production is possible.⁸

The first maser was unknowingly detected by H. Weaver, D.R.W. Williams, N.H. Dieter, and W.T. Lum of the Radio Astronomy Laboratory at the University of California at Berkeley.¹⁰ Spectra were taken at the Hat Creek Radio Observatory of the Orion Nebula, W3 and W49 (HII regions), and NGC 6334 by taking “the difference between the spectr[a] as observed for a specified source . . . and the spectr[a] of a comparison region . . . near the source but sufficiently far away so that the source is out of the antenna beam.”¹⁰ In all of the spectra for these sources, strong OH emission lines at 1665 MHz were observed that could not be identified, so Weaver, et al., attributed them to ‘mysterium’ (See Figure 1 for the strong OH emission line of W3 as observed by Weaver, et al.). This “narrowest line so far observed in the interstellar medium”¹⁰ peaked the interest of many astronomers who continued studying the 1665 MHz OH emission lines of W3 and W49 and finally determined that “maser amplification process[es]” were responsible for these spectral features.¹¹ Maser emission also continues to be observed from other molecules such as water (H₂O), methanol (CH₃OH), formaldehyde (CH₂O), and silicon monoxide (SiO).

I thank Dr. Robert Gilmore at Drexel University for his wisdom and teachings in making this analysis of interstellar masers possible.

¹⁰Weaver, H., Dieter, N.H., Williams, D.R.W., Lum, W.T., *Nature*, **208**, 29-31 (1965).

¹¹Davies, R.D., Rowson, B., Booth, R.S., Cooper, A.J., Gent, H., Adgie, R.L., Crowther, J.H., *Nature*, **213**, 1109-1110 (1967).