Fourier Transform Interferometer

Xiang Liu

Abstract

The basic mechanism of Fourier transform interferometer has been briefly introduced in this paper. In Fourier transform interferometer, the intensity directly recorded by the detector is a function of the retardation, which is a Fourier transform of the intensity as a function of the wavenumber. The application of the Fourier transform interferometer in the Fourier transform infrared spectroscopy has been briefly introduced. With the Fourier transform interferometer working as the light source, several valuable advantages can be obtained compared to a monochromator light source.

1 Introduction to Interferometer

Interferometry utilizes the principle of superposition to combine waves with same frequency in a way that will cause the resulting wave of their combination to show some meaningful property as diagnostic of the original state of the waves. When two waves with the same frequency (ω) combine, the amplitude of the resulting wave (E_T) is determined by the phase difference between the two waves,

$$E_T = E_1 e^{i\omega t} + E_2 e^{i\omega t+\theta} \tag{1.1}$$

where the E_1 and E_2 are the amplitudes of the two original waves and θ represents the phase difference between the two waves. Therefore, the waves that are in-phase will undergo constructive interference while the waves that are out-of-phase will undergo destructive interference.

A well-known class of interferometers is called Michelson interferometer, which is known as amplitude-splitting interferometer. It was famous as being used by Michelson and Morley through their experiment in a failed attempt to demonstrate the effect of the hypothetical "aether wind".

The Figure 1 shows the structure of a basic Michelson Interferometer. A light source *S* emits light that hits a beam splitter *M* at point *C*. M is partially reflective, so one beam is transmitted through *M* to point *B* while the other is reflected to point *A*. The *M1* and *M2* are reflective mirrors that the two beams will be reflected at point *A* and

B respectively. Then the two reflected beams will recombine at point C′ to produce an interference pattern which can be detected by the detector E. Hence, the optical path difference or retardation between the two beams is

$$\Delta = (AC + AC') - (BC + BC') \quad (1.2).$$

This can be simplified if we assume the point C and point C′ are a same point as

$$\Delta = 2(AC - BC) \quad (1.3).$$

Then the phase difference between the two beams when they recombine is

$$\theta = \frac{2\pi}{\lambda}\Delta = 2\pi\sigma\Delta \quad (1.4),$$

where the λ is the wavelength of the waves and σ is the wavenumber of the waves, which is simply the inverse of the wavelength ($\sigma = 1/\lambda$).

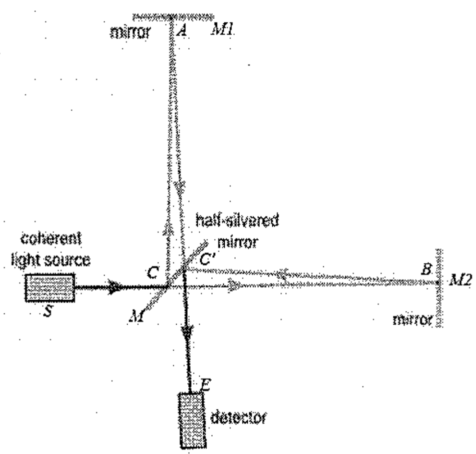

Figure 1 The diagram of a basic Michelson interferometer

Then the amplitude of the interference beam can be written as

$$E_T = E_0 e^{i\omega t} + E_0 e^{i\omega t + \theta} = E_0 e^{i\omega t} + E_0 e^{i\omega t + 2\pi\sigma\Delta} \quad (1.5).$$

And the intensity is

$$I(\delta) = \left| E_T E_T^* \right| = \frac{1}{2} I_0 (1 + \cos(2\pi\sigma\Delta)) \quad (1.6)$$

The I_0 is the intensity at zero path difference (ZPD) that $\Delta = 0$. For simplicity, we assume the original beam from the light source is split into two equal part that the two recombining beams will have the same amplitude ($E_1 = E_2 = E_0$). For a certain light source, the only variable in this equation is the retardation (Δ). Therefore, in Michelson interferometer, the interference is simply determined by the difference between the distances from the two mirrors M1 and M2 to the beam splitter M, which can be easily and precisely controlled.

2 Fourier Transform Interferometer

2.1 Introduction

A Fourier transform interferometer uses the same basic configuration as a Michelson interferometer, but one of the mirrors (*M1* and *M2*) is movable. The recombined beam is detected synchronously with the motion of the mirror. The name, Fourier transform interferometer, comes from the fact that the intensity ($I(\Delta)$) of the recombined beam as a function of the retardation is the Fourier transform of the intensity of the light source, $I(\sigma)$, as a function of the wavenumber.

As discussed before, the intensity recorded at the detector, for a monochromatic source, is given by

$$I(\Delta) = \frac{1}{2} I_0 (1 + \cos(2\pi\sigma\Delta)) \tag{1.7}$$

In practice, the beam from the light source is a spectral radiance with a distribution given by $I(\sigma)$. Since the lights at different wavenumbers are incoherent, the total intensity recorded at the detector can be written as an integral over all wavenumbers

$$I(\Delta) = \frac{1}{2}\int_0^{\infty} I(\sigma)(1 + \cos(2\pi\sigma\Delta))\,d\sigma \tag{1.8}$$

The detector signal, shown in equation(1.8), is composed of an unmodulated , or "DC" part, and a modulated, or "AC" part.

$$\begin{aligned} I_{DC}(\Delta) &= \frac{1}{2}\int_0^{\infty} I(\sigma)d\sigma = \frac{1}{2} I_0 \\ I_{AC}(\Delta) &= \frac{1}{2}\int_0^{\infty} I(\sigma)\cos(2\pi\sigma\Delta)\,d\sigma \end{aligned} \tag{1.9}$$

The DC part is just equal to one half of the signal at ZPD. The AC part is called the interferogram. Now let's just consider the interferogram, the modulated part.

$$I(\Delta) = \frac{1}{2}\int_0^{\infty} I(\sigma)\cos(2\pi\sigma\Delta)\,d\sigma \tag{1.10}$$

If we let the spectral signal is symmetrical about some central σ_0 and then the distribution can be shifted by $-\sigma_0$ to make the distribution as an even function, then the interferogram can be expressed as

$$I(\Delta) = \frac{1}{2}\int_{-\infty}^{\infty} I(\sigma)(\cos(2\pi\sigma\Delta) + i\sin(2\pi\sigma\Delta))\,d\sigma = \frac{1}{2}\int_{-\infty}^{\infty} I(\sigma)e^{i\pi\sigma\Delta}\,d\sigma \tag{1.11}$$

Since the $I(\sigma)$ is a real even function, the imaginary odd part will vanish after integral. Then the resulting function $I(\Delta)$ is still a real function.

Finally, we see the intensity as a function of retardation and the intensity as a function of wavenumber form a Fourier transform pair with a coefficient 1/2. The intensity as a function of wavenumber, which is really what we are interested in, can be recovered from the interferogram by an inverse Fourier transform as

$$I(\sigma) = 2\int_{-\infty}^{\infty} I(\Delta)e^{-i\pi\sigma\Delta}d\Delta \qquad (1.12).$$

In practice, the movable mirror moves at a constant velocity and the detector will record the total intensity as a function of time, which is actually a function of the retardation considering the constant velocity, then the intensity as a function of wavenumber can be obtained in a computer with a simple inverse Fourier transform code.

2.2 Light Source

Now, let's discuss different types of light sources and their interferogram as well as their original signal spectrum. The simplest example is the spectral distribution is a single Gaussian line as shown in Figure 2.

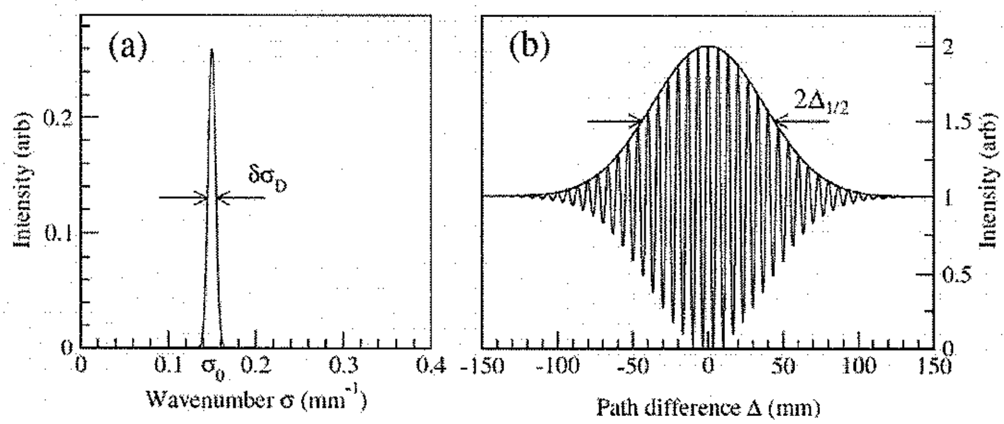

Figure 2 A single Gaussian "line" and its Fourier transform. (a) The line has a central wavenumber $\sigma_0 = 0.15\,mm^{-1}$ and a full-width at half maximum (FWHM) $\delta\sigma_D = 0.01\,mm^{-1}$. (b) The Fourier transform of the line has a Gaussian envelop of FWHM = 88mm.

The Gaussian distribution can be expressed as

$$I(\sigma) = I_0 e^{\frac{4\ln 2(\sigma-\sigma_0)^2}{(\delta\sigma)^2}} \qquad (1.13)$$

The Fourier transform of this Gaussian line can be expressed as

$$I(\Delta) = I_0' e^{-\frac{(\pi\,\delta\sigma\,\Delta)^2}{4\ln 2}} \cos(2\pi\,\sigma_0\,\Delta) \tag{1.14}$$

The cosine term contributes to the rapid oscillation in intensity as the mirror is moved while the exponential term varies much more slowly. The maximum is obtained in ZPD and drops to half its maximum value for a retardation

$$\Delta_{1/2} = \frac{2\ln 2}{\pi\,\delta\sigma} \tag{1.15}$$

Another usual example is a spectrum consisting of two equal-intensity Gaussian lines centered at a wavenumber σ_0 and separated by a wavenumber difference $2s$ which is shown in Figure 3. A common case is a sodium lamp which has two closely spaced yellow lines at 589.0nm and 589.6 nm respectively.

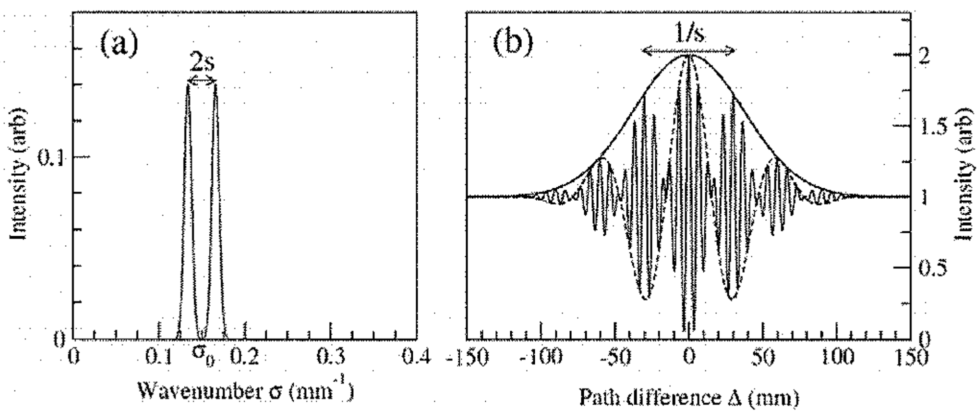

Figure 3 Two Gaussian lines and its Fourier transform. (a) The lines are centered at $\sigma_0 = 0.15\,mm^{-1}$ and separated by $2s = 0.032\,mm^{-1}$. Each line has a FWHM $\delta\sigma_D = 0.01\,mm^{-1}$. (b) The Fourier transform of the lines has Gaussian envelop (solid line) of FWHM= 88 mm, and a cosine envelop (dashed line) 1/s of wavelength 62.5 mm.

In this case, the intensity as a function of retardation can be expressed as

$$I(\Delta) = I_0' e^{-\frac{(\pi\,\delta\sigma\,\Delta)^2}{4\ln 2}} \cos(2\pi\,\sigma_0\,\Delta)\cos(2\pi\,s\,\Delta) \tag{1.16}$$

The term $\cos(2\pi\,\sigma_0\,\Delta)$ describes an extra oscillation where a maximum goes through a minimum to a new maximum when Δ goes from 0 to $1/(2s)$. Thus, in this case, the intensity as a function of retardation (1) oscillates rapidly as the mirror is moved by $\lambda/2$; (2) is modulated periodically when the mirror is moved by additional $1/(4s)$. (Note: the retardation Δ is twice the distance the mirror is moved); (3) finally dies away as the retardation gets much larger than $\Delta_{1/2} = 2\ln 2/(\pi\,\delta\sigma)$, as shown in Figure 3.

In practical cases, the real condition would be much more complicated than these simple Gaussian lines. Nevertheless, the basic underlying principles are the same. The choice of the light source depends on the subjects what we are interested in.

2.3 Resolution

The interferogram, $I(\Delta)$, is what is directly recorded by the detector by detecting the signal as a function of the retardation. According to the equation(1.12), the interferogram should be recorded for Δ from $-\infty$ to ∞. However, this obviously is impractical. Instead, there is a certain maximum retardation, L. In this case, Δ is contained in $[-L, L]$, the actual spectrum is obtained by approximation as

$$I(\sigma)' = 2\int_{-L}^{L} I(\Delta)e^{-i\pi\sigma\Delta}d\Delta \qquad (1.17)$$

This is equivalent to

$$I(\sigma)' = 2\int_{-\infty}^{\infty} \Pi\left(\frac{\Delta}{2L}\right) I(\Delta)e^{-i\pi\sigma\Delta}d\Delta \qquad (1.18),$$

where Π is the symmetric, unit rectangular window function. And the Fourier transform this window function is given by

$$F\left[\Pi\left(\frac{\Delta}{2L}\right)\right] = 2L\mathrm{sinc}(2L\sigma) = \frac{\sin(2\pi L\sigma)}{\pi\sigma} \qquad (1.19).$$

Therefore, the Fourier transform in equation(1.18) can be written as a convolution,

$$I(\sigma)' = 2L\mathrm{sinc}(2L\sigma) * I(\sigma) \qquad (1.20),$$

where the symbol $*$ denotes for convolution. From equations (1.19) and (1.20), the spectral resolution of the Fourier transformed spectrum is limited by this finite interferometer mirror stroke. The true spectrum is convolved with an instrument resolution function, $\mathrm{sinc}(2L\sigma)$, which has its first zero at

$$\delta\sigma = \frac{1}{2L} \qquad (1.21)$$

This term is called the unapodized spectral resolution of the interferometer. From this equation(1.21), it can be seen that the resolution of a Fourier transform interferometer can be easily increased by simply increasing the maximum moving distance of the movable mirror.

3 Applications of Fourier transform interferometer

3.1 Fourier transform infrared spectroscopy

Infrared spectroscopy (IR) describes the absorption, emission or Raman scattering of a solid, liquid or gas in the infrared region. Since the infrared region is the characteristic region for the vibration and rotation of molecules, IR is a powerful technique in studying the structure of molecules. In general, a kind of chemical group has its characteristic absorption in a specific IR region which is relatively independent with their chemical environment. By detecting the IR absorption spectrum of a unknown sample, we can qualitatively identify which chemical groups exist in this sample. A simple IR instrument uses a monochromator as the light source. The absorption spectrum is directly recorded by scanning over all wavenumbers.

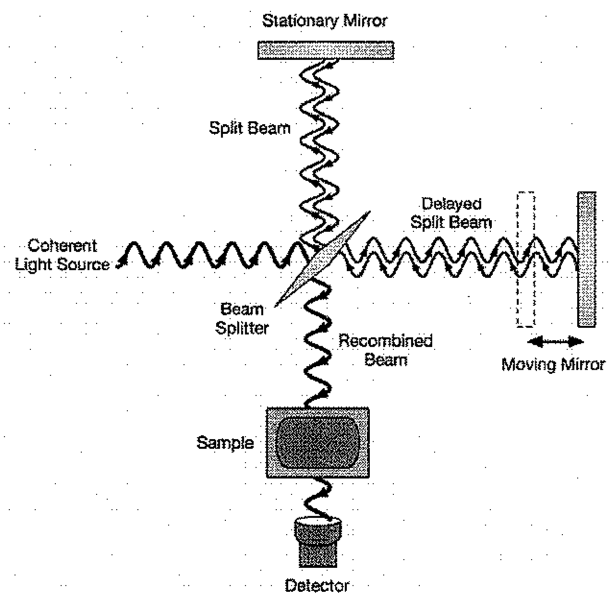

Figure 4 A Schematic diagram of an FTIR instrument

In Fourier transform infrared spectroscopy (FTIR), the monochromator is replaced by a Fourier transform interferometer to work as the light source. A schematic diagram for an FTIR instrument is shown in Figure 4. The interferometer part is the same as a typical Fourier transform interferometer. The only difference is the recombined beam would go through a sample cell before it arrives at the detector. Then the signal recorded by the detector is actually the interferogram subtracting the absorption spectrum of the sample. Then the detector signals are processed by computers to obtain the absorption spectrum of the sample molecules.

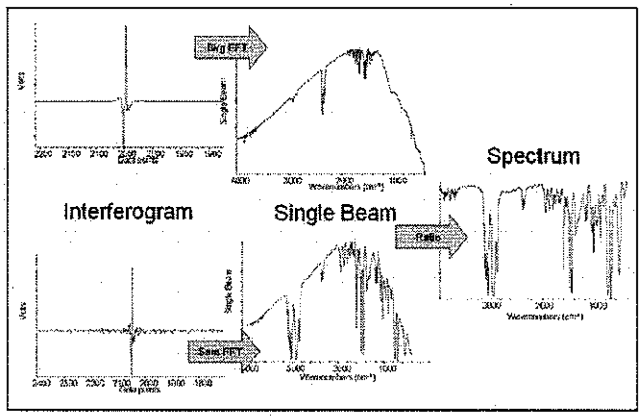

Figure 5 The process of collecting an infrared spectrum in FTIR. Bkg is for background signal. Sam is for signal with sample.

The process of obtaining an IR spectrum is briefly shown in Figure 5. In most cases, the absorbance of the molecule in a specific wavenumber is the property that we are really interested in rather than the absolute intensity of the spectra. The absorbance (A_σ) describes the logarithmic ratio of the radiation falling upon a sample (I_0) to the radiation transmitted through the sample (I_1).

$$A_\sigma = \log_{10}\left(I_0/I_1\right) \tag{1.22}$$

In order to obtain the absorbance at each wavenumber, both the unabsorbed spectrum, or background spectrum, and the absorbed spectrum are required. Since the FTIR is a kind of "single beam" spectrum, that only one spectrum can be recorded at the same time, two measurements should be done, one is to record the background spectrum with no sample and another is to record the absorbed spectrum with exactly the same instrument and settings but with sample. By doing so, some systematic errors come from the instruments can be annihilated. Otherwise, since the measurement is always done in regular condition, the spectrum detected is the sum of the absorption spectrum of the sample and the absorption spectrum of some other molecules like CO_2 and H_2O in the air, which have strong absorption in IR region. Therefore, a background spectrum is necessary in order to rule out the absorption of such non-interested molecules.

3.2 The advantage of FTIR

A modern FTIR spectrometer has three major advantages over a typical non-FT IR spectrometer.

Multiplex advantage

Multiplex advantage or Fellgett's advantage states that a multiplex spectrometer such as Fourier-transform spectrometer which does not separate energy into individual frequencies for measurement will produce a relative improvement in signal-to-noise ratio, compared to scanning monochromator, of the order of the square root of the number of sampling points in the spectrum.

Throughput advantage

The FTIR detect the total intensity of all wavenumbers at a single time and does not limit the amount of light reaching the detector. This means more energy reaches the sample and the detector in FTIR than in a monochromator spectrometer. The higher signal leads to an improved signal-to-noise ratio. Especially in a high-resolution spectrometer where the slit in a monochromator spectrometer becomes very narrow and strongly decrease the intensity of the signals, resulting in poor quality spectra for the samples. This means in monochromator spectrometer the resolution is difficult to increase while in FTIR the resolution can be easily increased by increasing the maximum moving distance of the movable mirror as discussed before.

Precision advantage

An FTIR spectrometer uses a laser to control the velocity of the moving mirror and to time the collection of sampling points throughout the mirror stroke, which is also used as a reference signal within the instrument. The interferogram of the laser is a constant sine-wave, providing the reference for both precision and accuracy of the IR spectrometer. Well-designed FTIR exclusively rely on this reference lase, rather than any other external sample. In this case, the measuring result is of great repeatability compared to a spectrum from a monochromatic spectrometer.

4 Conclusion

The Fourier transform interferometer has similar structure as a Michelson interferometer but with a movable mirror. In Fourier transform interferometer, the signal directly recorded by the detector is actually a function of the moving distance of the movable mirror and is called interferogram, which is a Fourier transform of the real signal which is function of the wavenumbers. The real signal can be easily obtained by an inverse Fourier transform process. Some simple light resources and their characteristic spectrum and interferogram have been introduced. The resolution of an Fourier transform interferometer depends on the maximum moving distance of the movable mirror. With this feature, the resolution of an Fourier transform interferometer can be easily improved by increasing the moving distance.

FTIR has been introduced as a successful application of Fourier transform interferometer. The Fourier transform interferometer works as an light source in the IR spectrometer instead of a monochromator. This offer several valuable advantages in FTIR which can improve the efficiency, accuracy and repeatability of the measurement.

In summary, Fourier transform interferometer provides a great amount of benefits to our scientific research with a relatively simple structure.

Reference

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