Finding an Upper Bound on Neutrinos Mass

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1 Introduction

1.1 Oscillation - Neutrinos have mass!

The electron neutrino is a neutral particle postulated by Wolfgang Pauli in 1930 to explain the missing energy in beta decay. This new particle was determined to be very weakly interacting and assumed to be virtually massless by the Standard Model (SM) of particle physics. In 1933, Enrico Fermi developed the theory of weak interactions based on the properties of neutrinos. Four years later, muon neutrinos were discovered in cosmic rays and were named neutretto because they were thought to be different from electron neutrinos. [1] SM assumes neutrinos are massless and therefore cannot oscillate between flavors. With this assumption, the Standard Solar Model (SSM) predicted the neutrino flux coming from the Sun to be 100% electron neutrinos. In the late 1960s, Ray Davis's Homestake Experiment observed the solar neutrino problem: the observed flux of solar neutrino was only a third of the flux value predicted by the SSM.

In 1998, Super-Kamiokande (Super-K) presented data on the atmospheric muon neutrino deficit, which depended on the distance traveled by the particles. The distance-dependent deficits implied that the neutrino oscillates between the different flavors. [6] In 2001, Sudbury Neutrino Observatory published their first scientific result on the detection of the electron neutrino and total neutrino fluxes and discovered that the electron neutrino flux value was indeed only a third of that of total neutrino flux, providing the strongest evidence of neutrino oscillation yet. [2]

The three flavor states, ν_e, ν_μ and ν_τ, are naturally occurring states and are linear combinations of the mass states of neutrinos, ν_1, ν_2 and ν_3. The phase factors of the three mass states advance at different rates due to the differences between the three masses, resulting in the neutrino oscillations. The periodicity of the phase factors causes the neutrino to return to its original flavor state after a certain distance. If the neutrino is a Majorana particle, which means it is identical to its antiparticle, the antineutrino, the the phase factors become physically meaningful. Setting the phase factors to 1, the unitary transformation matrix becomes:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\ \theta_{23} & \sin\ \theta_{23} \\ 0 & -\sin\ \theta_{23} & \cos\ \theta_{23} \end{bmatrix} \begin{bmatrix} \cos\ \theta_{13} & 0 & \sin\ \theta_{13} \\ 0 & 1 & 0 \\ \sin\ \theta_{13} & 0 & \cos\ \theta_{13} \end{bmatrix} \begin{bmatrix} \cos\ \theta_{12} & \sin\ \theta_{12} & 0 \\ -\sin\ \theta_{12} & \cos\ \theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Using the plane wave solution to describe the propagation of the mass eigenstates and setting c = $\hbar$ = 1,

$$|\nu_i(t)> = e^{-i(E_i t - \vec{p_i}\cdot\vec{x}}|\nu_i(0)>, \tag{2}$$

where E_i is the energy of the mass eigenstate i, t is the interval of propagation, $\vec{p_i}$ is the 3-dimensional momentum and $\vec{x}$ is the position of the neutrino at time t relative to the origin. Since neutrinos are light, relativistic conditions can be assumed:

$$E_i \approx E + \frac{m_i^2}{2E} \tag{3}$$

where E is the total energy of the neutrino. Using this energy in combination with $t \approx L$ in the relativistic limit,

$$|\nu_i(L)> = e^{-im_i^2 L/2E}|\nu_i(0)>, \tag{4}$$

the probability of a neutrino oscillating from flavor α to flavor β is then:

$$\begin{aligned} P_{\alpha\to\beta} &= |<\nu_\beta|\nu_\alpha(t)>|^2 \\ &= \left|\sum_i M^*_{\alpha i} M_{\beta i} e^{-im_i^2 L/2E}\right|^2, \end{aligned} \tag{5}$$

where E_i is the energy of the mass eigenstate i and L is the distance traveled. [9][10]

1.2 Hierarchy

Once it was established that neutrinos have masses, the question then became what are the masses? In 2004, KamLAND published a difference in the squares of the masses of mass eigenstates 1 and 2, Δm^2_{21}, of 0.000079 eV^2. [5] Two years later, FermiLab updated their press room website with Δm^2_{32} = 0.0031 +/- 0.0006 (statistical) +/- 0.0001 (systematic) eV^2. [6] The mass squared differences determine spacing between mass eigenstates on the mass scale but do not provide a absolute mass scale. Since

$$|\Delta m^2_{32}| = |m^2_3 - m^2_2| \tag{6}$$

and if m_3 or m_2 are 0 eV^2, then the other mass eigenstate would have to have at least energy equivalent to the square root of Δm_{32}, or 0.04 eV^2. [7]

As long as the spacings satisfy the findings of KamLAND and FermiLab, the mass eigenstates can be arranged from lowest to highest in eV^2: $\nu_1 \rightarrow \nu_2 \rightarrow \nu_3$ or $\nu_3 \rightarrow \nu_1 \rightarrow \nu_2$; the former is named Normal Hierarchy (NH) and the latter is Inverse Hierarchy (IH).

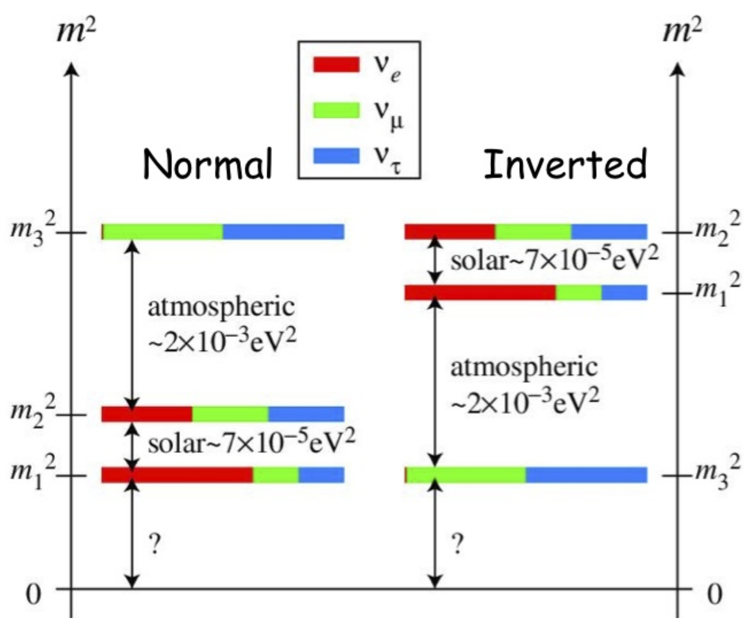

Figure 1: The normal and inverse hierarchy of the three neutrinos mass eigenstates. [10]

In addition to being an important piece of information in development of the absolute mass scale, the two hierarchies also calculate for the effective Majorana neutrino mass differently, assuming neutrino is a Majorana particle. In either case, the lowest mass eigenstate is not at 0 eV^2, so it is important to experimentally find an upper bound in order to determine the precise energy of the lowest mass eigenstate of the absolute scale.

2 Upper-Bound on Neutrino Mass

2.1 Cosmology

The strongest neutrino mass bound is from analyzing cosmology published in 2006. Analysis of the Cosmological Microwave Background (CMB), Large Scale Structure (LSS) and Type Ia Supernovae (SN-Ia) from Supernovae Legacy Survey (SNLS) data in the full 11-dimensional parameter space provided a conservative upper bound on the neutrino masses of $\Sigma m_\nu \leq 1.72$ eV with 95% confidence level (C.L.). Restrictions were placed on the effective number of neutrino species ($N_\nu = 3$) and the running of the primordial spectral index ($\alpha_s = 0$) in the 8-dimensional parameter space; the constraints improved the upper bound to 0.70 eV. The baryon acoustic peak from Baryon Acoustic Oscillations (BAO) and Lyman-α forest (Ly-α) were then incorporated in the analysis separately and brought the upper bound down to 0.48 eV and 0.35 eV respectively. Including both the BAO and Ly-α data resulted in a upper bound of 0.27 eV with 95% C.L., the strongest neutrino mass upper bound. [4] In March 2013, the Planck collaboration published their findings of a effective $N_\nu = 3.30 \pm 0.27$ and $\Sigma m_\nu \leq 0.23$ eV using the BAO and Cosmic Microwave Background (CMB) data. [11]

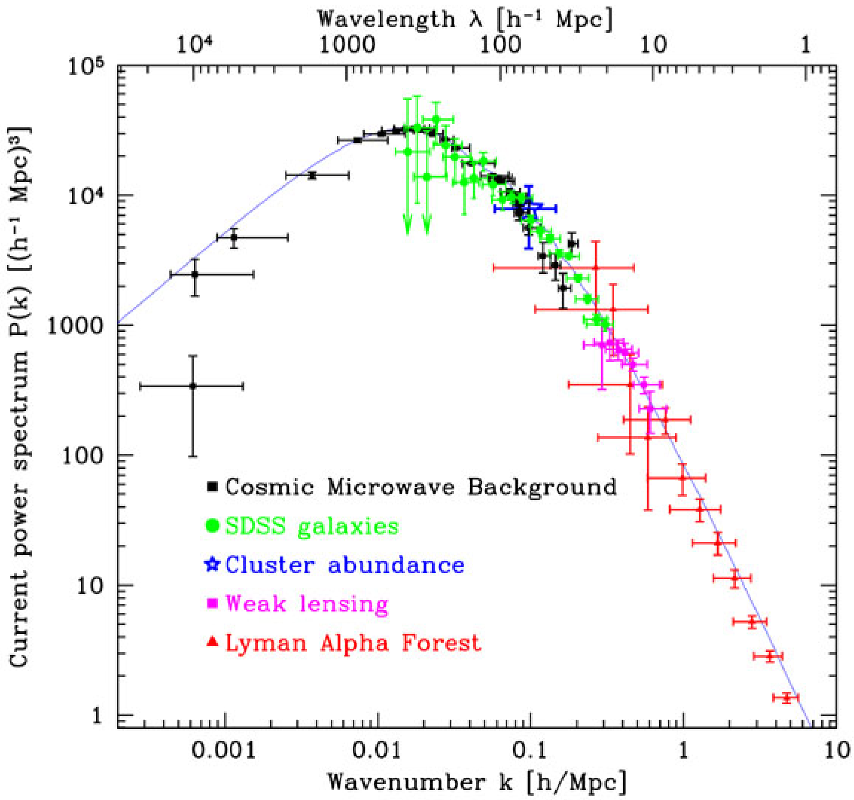

Figure 2: Theoretical prediction for the consensus cosmology model and the power spectrum of density inhomogeneity today from various measurements.[14]

2.2 Neutrinoless Double Beta Decay

An indirect way of finding neutrino masses is by finding the half-life of neutrinoless double beta decay. Most particles derive their masses from the Dirac mechanism, but if neutrino is a Majorana particle, then the different masses can be calculated using the half life of neutrinoless double beta ($0\nu\beta\beta$) decay. The $0\nu\beta\beta$ decay rate is affected by the phase factors of the various neutrinos and is related to the square of the effective Majorana neutrino mass $<\mathrm{m}_\nu>$ by the product of a phase space factor and a nuclear matrix element squared. Using Fermi's Golden Rule, the inverse half-life $0\nu\beta\beta$ decay has the general form of:

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} \times |M^{0\nu}|^2 \times < m_\nu >^2 \tag{7}$$

where $G^{0\nu}$ is the phase space factor, which is proportional to [Q factor]^5, $M^{0\nu}$ is the nuclear matrix element and $< m_\nu >$ is the effective Majorana mass. Substituting in the CKM parameters in Wolfenstein parameterizations ($< \eta > and < \lambda >$) and the nuclear matrix elements of the decaying nuclei dependent coeffecients (C_i), the half-life equation becomes:

$$
\begin{aligned}
[T^{0\nu}_{1/2}(0^+ \to 0^+)]^{-1} = C_1 \left(\frac{< m_\nu >}{m_e} \right)^2 + C_2 < \lambda > \frac{< m_\nu >}{m_e} \cos(\psi_1) \\
+ C_3 < \eta > \frac{< m_\nu >}{m_e} \cos(\psi_2) + C_4 < \lambda >^2 + C_5 < \eta >^2 \\
+ C_6 < \lambda >< \eta > \cos(\psi_1 - \psi_2),
\end{aligned}
\tag{8}
$$

Differences between sets of nuclear matrix elements will therefore yield a narrow, but significant, range of mass limits. The effective Majorana mass is then related to the absolute mass scale of neutrinos by:

$$
< m_\nu >= \sum_{i=1}^{3} |U_{ei}|^2 e^{i\alpha_i} m_i \qquad (for\ all\ m_i \geq 0). \tag{9}
$$

Using standard representation of the PNMS matrix,

$$
\begin{aligned}
< m_\nu > = m_1(1 - \sin^2\theta_{12})(1 - \sin^2\theta_{13}) \\
+ m_2 \sin^2\theta_{12}(1 - \sin^2\theta_{13}) e^{i(\alpha_2 - \alpha_1)} \\
+ m_3 \sin^2\theta_{13} e^{-i\alpha_3},
\end{aligned}
\tag{10}
$$

where α_i are the phase factors. Note that this relation is for a light neutrino exchange mechanism only. [9]

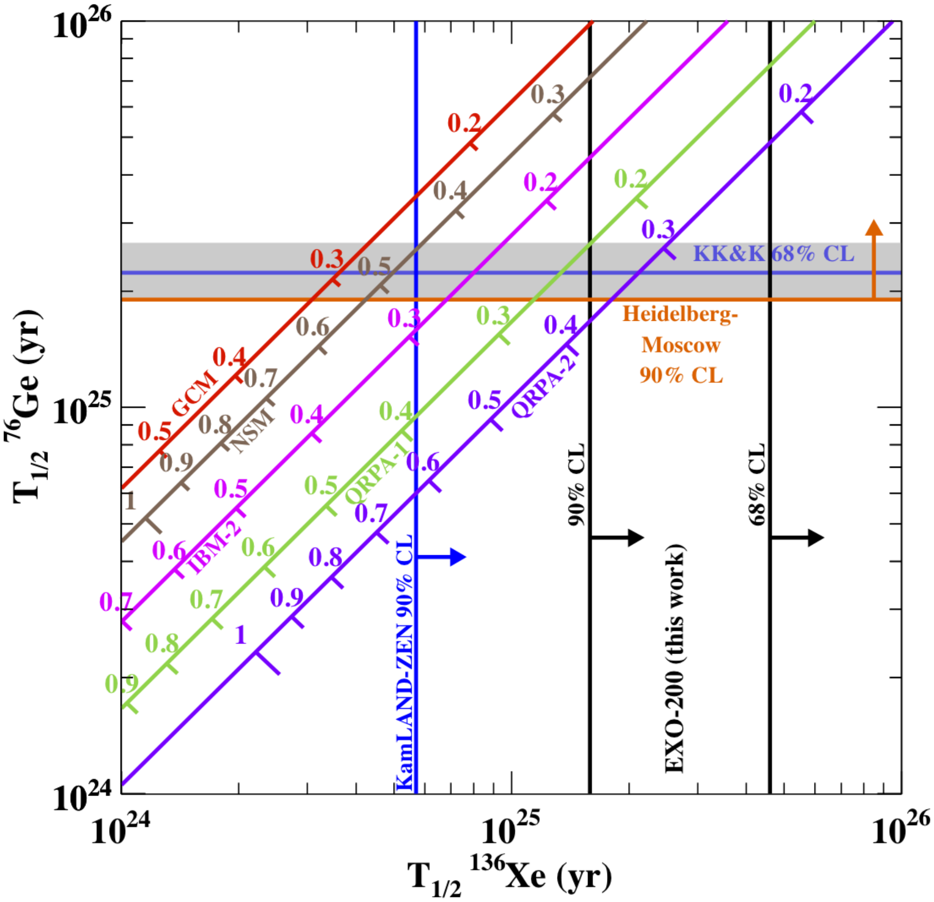

Figure 3: Relation between the $T_{1/2}^{0\nu\beta\beta}$ in ^{76}Ge and ^{136}Xe for different matrix element calculations. [8]

The Enriched Xenon Observatory (EXO) uses 0.1 tonne of liquid xenon-136 as its decay nuclei and in 2012 published a lower limit on the half-life of xenon neutrinoless double beta decay ($T_{1/2}^{0\nu\beta\beta}$) of 1.6×10^{25} years with 90% confidence level and set the upper limit on the Majorana mass of neutrinos to be 140 - 380 meV. [8] EXO is now planning its successor; the next EXO (cleverly named nEXO) will be a single-phase time projection chamber with 4.5 tonne liquid xenon with scintillation readout. nEXO has a higher sensitivity than EXO and will explore the inverted light Majorana neutrino mass hierarchy. In the initial phase, nEXO will have a half-life sensitivity of 2.5×10^{27} years and with barium tagging, the sensitivity will be 2.2×10^{28} years.

2.3 Nuclear Beta Decay

The upper bound of electron antineutrino mass is currently set at 2.3 eV/c^2 by experiments at Mainz and Troitsk. The Karlsruhe Tritium Neutrino Experiment (KATRIN) is a high precision experiment that aims to measure the the actual mass of electron antineutrino or at least improve the existing upper bound by one order of magnitude down to 0.2 eV/c^2 with tritium beta decays. A tritium atom can beta decay into helium-3 with very low energy.

$$^3_1T \rightarrow ^3_2 He^{1+} + e^- + \bar{\nu_e}$$

Usually in tritium beta decays, the total energy is 18.6 keV and the neutrino and electron carry away about 50% of the reaction energy each. KATRIN is looking for the the event in which the electron carries away almost all of the energy, which has a probability of one-in-a-trillion, and using

$$E = mc^2 \tag{11}$$

to calculate the mass of the neutrino. A neutrino mass of 0.35 and 0.30 eV would realistically be discovered with 5 and 4 sigma significance, respectively. KATRIN plans to have full system integration in 2014 and begin testing by 2015. [12]

3 Conclusion

Ever since it was discovered that neutrinos can oscillate between flavors and are therefore not massless, a lot of effort has gone into finding their masses. Cosmology set an upper bound of 0.27 eV with 95% confidence level with CMB, LSS, SNIa, BAO, Ly-alpha data. EXO assumes the neutrino is a Majorana particle and found a 140 - 380 meV mass upper bound. Future more sensitive $0\nu\beta\beta$ (e.g. nEXO) and nuclear beta decay (e.g. KATRIN) experiments will hopefully be able to improve the upper bound of neutrino masses, or even just discover the mass values, and provide a more complete, and correct, SM of particle physics and improve cosmological models.

4 Citations

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