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Quantum II

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Zero-Point Energy: Huh?

According to the Oxford English Dictionary, a vacuum is defined as "the emptiness of space" [1]. Scientifically speaking, this is not quite the entire story. After all, light from distant celestial objects travels through a vacuum in order for us to view them demonstrating that oscillating electric and magnetic fields exist within a vacuum when light is propagating through it. Even when the electric and magnetic fields within a vacuum are zero, energy still exists. Using Quantum Field Theory, the vacuous universe can be likened to an infinite number of decoupled quantum harmonic oscillators. Applying Schrödinger's equation to each harmonic oscillator in space, their associated energies, in the absence of any outside forces and fields, are found to be $E_n = (\frac{1}{2} + n)\,\hbar\omega$, where n = 0, 1, 2, … is an integer energy state of the quantum harmonic oscillator. For the ground state of the system, or the lowest possible energy that the quantum harmonic oscillator can have, n = 0, or $E_0 = \frac{1}{2}\,\hbar\omega$. Thus, there appears to be a minimum energy of $\frac{1}{2}\,\hbar\omega$ associated with each point in "empty" space known as the zero-point energy.

Representing a vacuum as a set of uncoupled quantum harmonic oscillators is not an approximation but rather results directly from Maxwell's equations in a vacuum. Using the expression $\nabla^2 A = \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}$, which is derived directly from Maxwell's equations, one can determine a solution for the magnetic vector potential, $A = \sum_k \left(b_k \mathbf{e}_k e^{-i\omega t} e^{ik\cdot r} + c.c.\right)$, where $\mathbf{e}_k$ is a polarized unit vector, and the frequency ω is assumed to be constant for all k points in space [2]. Using this solution, the electrodynamic relations $E = -\partial A/\partial t$ and $B = \nabla \times A$, and the Hamiltonian for the electromagnetic field, $H = \frac{1}{2}\int dV(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2)$, the Hamiltonian can be transformed into $H = \frac{1}{2}\sum_k (p_k^2 + \omega^2 x_k^2)$, by defining $b_k = \sqrt{1/4V\epsilon_0}(x_k - ip_k/\omega)$ [2]. This Hamiltonian resembles that for k one-dimensional harmonic oscillators with unit mass so the energy eigenvalues for these Hamiltonians should correspond to one another; thus, each discretized point in space, k, will have an energy $E_n = (\frac{1}{2} + n)\hbar\omega$ corresponding to it. The zero-point energy for each point in space is then $\frac{1}{2}\hbar\omega$ (the lowest energy occurs when n = 0).

Although this energy exists in theory, physically measuring the zero-point energy associated with a vacuum further supports its existence. One such experiment that claims to have proven the reality of a zero-point energy is the measurement of the Casimir Effect. When two uncharged metallic plates are placed in close proximity (~ micrometers) of eachother in a vacuum with no external electromagnetic fields, classically, the two plates should feel no forces. Quantum mechanically, if a zero-point energy exists, the plates should either be attracted to or repelled from one another because of the Casimir effect; "the presence of plates" in a vacuum "restricts … the allowed wave modes between them to discrete values. This discreteness leads to a finite lowering" or raising "of the vacuum energy, resulting in a force of attraction" or repulsion [3]. The experiments that have sought to measure the Casimir effect have successfully proven its existence by not only measuring it but also by matching its dependence on separation predicted by theory [4].

Not everyone is convinced that measuring the Casimir Effect is proof that an omnipresent zero-point energy exists. R.L. Jaffe claims that the Casimir Effect is not actually a consequence of zero-point energy but rather is a result of the relativistic van der Waals force between the two plates [5]. According to Jaffe, "When the plates were idealized as perfect conductors, assumptions were made about the properties of the materials and the strength of the [Quantum Electrodynamic] coupling … that obscure the fact that the Casimir force originates in the forces between charged particles in the metal plates" [5]. In 1978 Schwinger, DeRaad, and Milton derived the Casimir Force equation "in terms of the trace of the Greens function for the fluctuating field in the background of interest (*e.g.* conducting plates)," and Jaffe claims that since the zero-point energy derivation was much more straightforward (but what isn't more straightforward than Greens functions, honestly!) it became the accepted explanation for the Casimir Effect [5].

Regardless of whether or not the zero-point energy exists, this energy associated with the quantum mechanical properties of a vacuum has excited cosmologists because it resembles the so-called dark energy that is used to account for the accelerated expansion of the universe. Since $\varepsilon_{vac} = - p_{vac}$, where ε_{vac} is the zero-point energy density (in a vacuum) and p_{vac} is the pressure in a vacuum, a positive zero-point energy density would result in a negative vacuum pressure and would tend to accelerate the expansion of the universe, as is currently observed [6]. In a nutshell, "the vacuum energy density acts like a cosmological constant." [7].

Unfortunately for cosmologists, the mysterious accelerated expansion of the universe was not resolved by the discovery of the zero-point energy. The total zero-point energy for every point in space is determined by integrating over the energy associated with each point in space ($\hbar\omega /2$) over all space (note that $p = \hbar\omega/c$): $(\hbar / 2\pi^2 c^3)\int \omega^3 d\omega$ [4]. This integral diverges when one integrates over all frequencies, so in order to obtain a finite value for the total zero-point energy of the vacuous universe one must integrate up to a maximum frequency. Traditionally, the maximum frequency has been taken to be on the Planck scale because many argue that "general relativity is valid up to the Planck scale" [8]. Using atomic units (where $\hbar = e = m_e = 1$), O'Connell computed the above integral with a $\omega_{max} = 4.49 \times 10^{26}$, and evaluated the total zero-point energy density as 1.2×10^{96} g/cm^3 [8]. Comparing this with the energy density necessary to expand the universe at its current rate ($\approx 10^{-29}$ g/cm^3), it is apparent that the estimated value of the total zero-point energy density is much too big (~ 125 orders of magnitude too big) to be the dark matter that cosmologists have been looking for [8]. Accordingly, in order to have the total zero-point energy density match the required "dark energy" density, only the frequencies up to $\omega_{max} = 2.4 \times 10^{-5}$ should be accounted for.

A few papers have sought to resolve this extremely large discrepancy in various ways. O'Connell asserts that "one should not consider vacuum fluctuations in isolation but rather in interaction with matter fields and, as a consequence, their contribution to the energy of the vacuum is much less" [8]. Since many of the Quantum Electrodynamic effects that we observe (such as the Casimir Effect and the Lamb Shift) occur because of the vacuum's interaction with matter, this dependence on matter should be taken into account when computing the total zero-point energy [8]. O'Connell proposed that ω_{max} should be taken to be $\Omega = \dfrac{M - m}{M\tau_e}$, where M is the physical mass of the electron, $m \approx (1 - \alpha)$ M, α is the fine structure constant, and $\tau_e = \dfrac{2e^2}{3Mc^3}$ [8]. Plugging in for these values, $\omega_{max} = 2.82 \times 10^4$ resulting in a zero-point energy density equal to 3.1×10^7 g/cm^3. The fact that accounting for the interaction between the vacuum fluctuations and matter reduced the difference between the two energy densities by 89 orders of magnitude leads many to "expect that the energy contributed to the vacuum by the zero-point fluctuations occurs because of the interaction between these fluctuations and matter and not by consideration of these fluctuations in isolation" [8]. On the other hand, the vacuum energy density is still 36 orders of magnitude too big, which means there is a lot to learn in order to reconcile the theory and the data [8].

Dragoman also agrees that these fluctuations need to be taken into account when interacting with matter because the "quantum vacuum is not considered as existing independent of the material particle[s], but as generated by it, and so as localized around the material particle[s]" [4]. He claims that the necessary cosmological energy density and that of the zero-point energy should match since the zero-point energy "should not exist far from matter" [4].

So what exactly do we completely know about the zero-point energy? Theoretically it should exist, but the scientific community is not 100% sure that the physical evidence of the zero-point energy, the Casimir Effect, actually derives from the zero-point energy but rather may be a result of the relativistic van der Waals force. The universe is, in fact, expanding, but what is the maximum frequency that the total zero-point energy is integrated up to in order to match the necessary cosmological energy density to have the universe expand at its current rate. Maybe the initial assumption in modeling the vacuous universe as uncoupled unit masses on springs is incorrect. Apparently, there are "many paradoxes associated to quantum vacuum," but once they are reconciled, many powerful properties of the universe will be unveiled [4].

References

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