Hydrogen, Positronium, and Quarkonium

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1 Introduction

Most students of physics beyond the very lowest levels are familiar with the spectrum of Hydrogen, however, to approach even slightly differt system seems an insuperable task. However, in truth, the spectrum of species as diverse as Positronium and Charmonium bear a striking resemblence to that of Hydrogen, at least at low energies. The aim of this paper is to showcase the similarites and differences of the specrtra of Hydrogen, Positronium, Charmonium and Bottomonium, and to illustrate the fundemental similarity of these four divergent structures. To this end, we will begin with a brief overview of the Hydrogen atom, which we will then use to form the spectrum of the e^-e^+ bound system, commonly called Positroninum. Afterwords, we will examine the expermental spectrum of the $c\bar{c}$ and $d\bar{d}$ mesons, discuss the discovery of the Charmonium meson, look at the decay modes of both quarkonium systems, and, finally, relate the observations of these states to the predictions of QCD.

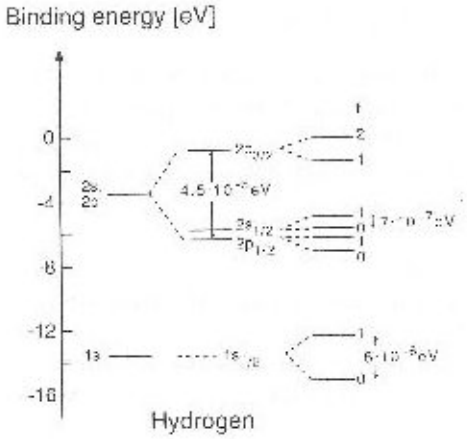
2 Hydrogen

Hydrogen is one of the first systems covered in most undergraduate quantum mechanics courses, and for good reason. Not only is it a simple system to solve (at least to first order) using the 3-d Schrodiner equation in spherical co-ordinates, but, one can apply basic perturbation theory to get the so-called fine and hyperfine structure of the system, due to the relativistic correction to the particle's energies and spin-orbit coupling, and the magnetic interaction between the proton and electron, respectively.

The basic approach to the theoretical determination of the Hydrogen atom spectrum is to begin with a Hamintonian of the form:

$$(-\frac{\hbar^2}{2m}\Delta - \frac{\alpha\hbar c}{r}) \tag{1}$$

which gives energy bound states of the form $E_n = \frac{\alpha^2 mc^2}{2n^2}$ where n is the principle quantum number, which depends on the number of nodes in the radial part of the wavefunction, N, and l, the orbital angular momentum. The m used is the reduced mass of the system, which is approximately the electron mass. On top of this basic potential, we apply fine and hyperfine corrections, which are of order α^2, and $\alpha^2\mu_p/\mu_e$, respectively. For the hyperfine correction, we also need to take into account the total spin of the system, which gives us a set of 4 quantum numbers to completely characterize a state, n, l (typically given by s, p, d, etc.), j (given by the total angular momentum of the electron, $l + s$) and, finally, f (given by $j + s_p$, the proton spin). Below are the first couple of bound states in hydrogen. In this chart, you can clearly see the hierarchy of structure, from the fundemental levels, to the fine structure, and, finally, the hyperfine structure.



3 Positronium

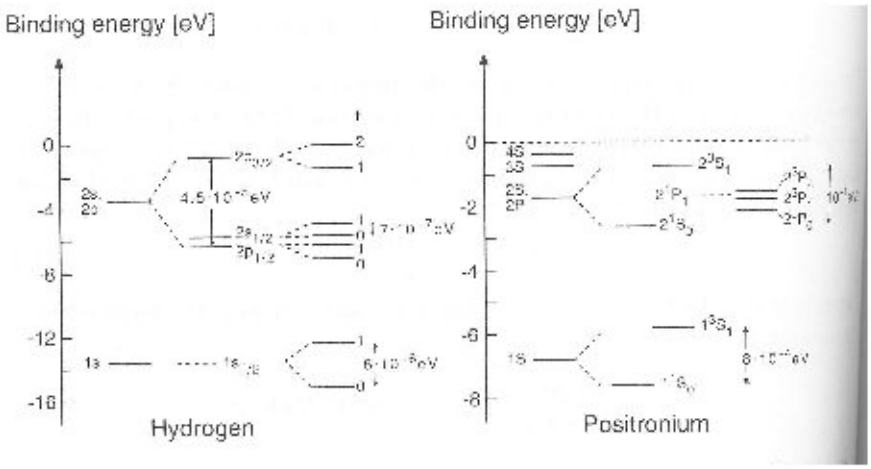
How, then, to proceed to a study of the e^+e^- system? Since the charge on the positron is the same as the charge on the proton, it should be relatively straight forward to move from the Hydrogen system to the positronium system. The Hamiltonian is fundementally the same, except the reduced mass of the system changes to $m_e/2$, while the spin-spin coupling, responsible for the hyperfine interaction term, becomes much larger than before, as the magnetic moment of a positron is on the order of 650 times larger than that of a proton. However, the implications of the second change are profound, as it is no-longer possible to identify, purely from spectral data, the contributions of fine vs. hyperfine structure as they are of similar size.

Moreover, since positronium consists of two fermionic particles, the total spin of the system breaks into a singlet s = 0 and a triplet s= 1 state. Thus, only three quantum numbers are needed to specify a given state, n, s, and l. Positronium states also, unlike Hydrogen atom states, have a finite lifetime, since the constituant particles of the system are a particle/anti-particle pair, and, so, will annihilate to produce either two or three photons, depending on the s quantum number of the system at the time of decay. The width of the decay has been shown to be, in the n = 1, l = 0 s = 0 state.

$$\Gamma = \frac{4\pi\alpha^2\hbar^3}{m_e^2 c}|\psi(0)^2| \tag{2}$$

(Nachmann 1990)

A final difference from hydrogen, is that some states of positronium are invarient under the action of the charge conjagation operator, C. This is due to the fact that the action of C on the system is to effectively exchange the positions of the positron and the electron. Regardless of the slight peculiarities of the Positronium system, the spectrum, at least for low- lying states, is very close to that of Hydrogen. Below are the spectrum of Hydrogen (on the left) and Positronium (on the right). Especially for the n =1 states, the two systems are very close.



4 Charmonium

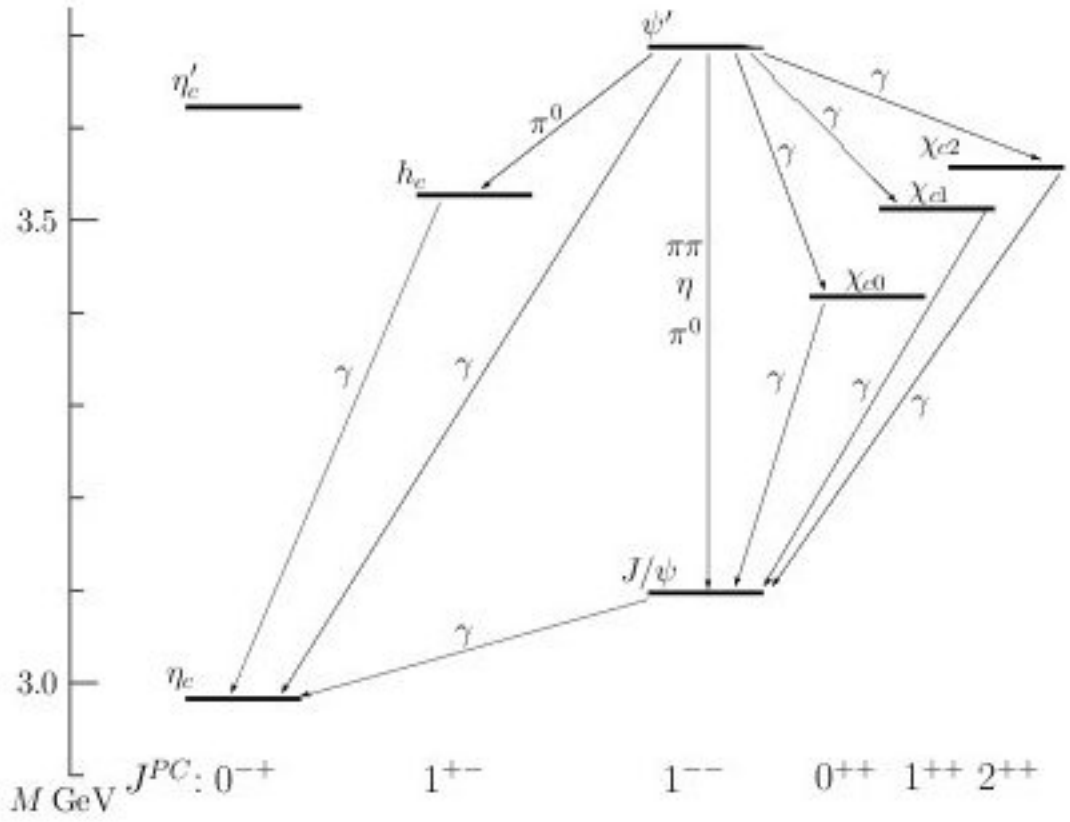
4.1 Discovery

The intial discovery of bound states of the $c\bar{c}$ system is actually a matter of a fair bit of interest to a historian of science. For this reason, we will take a short diversion to discuss the history of the discovery of the J/Ψ state. Back in the 1970's, there were three large particle accelerator experiments searching for new vector mesons, SPEAR, a machine based on electron positron collisions, Brookhaven, which accelerated protons against a fixed target, and ADONE, a European electron positron collider. At that time, there were only three known quarks, s, d, and u, although there were theoretical reasons to believe in the existence of a fourth quark, now known as c. All three of the groups were engaged in a search for new physics in (what were at the time) high energy regimes. Of the two detectors, the e^-1e^+ based systems had a much tighter control over the energies that the collisions occured at, but, that very control meant that scanning over a large energy range was a slow process, whereas proton type detectors could scan a large number of energies simultaniously, but, unless strong precausions were taken, identifying particular resonences was difficult. This problem was compounded by the

extrodinarily small width of the J/Ψ, resonence. Therefore, a great deal of praise should rest with the careful work of Dr. Samuel Ting, who, according to most accounts, was the first to discover the J/Ψ at the Brookhaven lab. However, praise for the care he took in his work should be tempered by some amount of blame, as he did not annouce his discovery for several months, instead choosing to search for additional peaks in the same energy range (3.1 Gev). The end result was that credit for the discovery was split between the group at SPEAR, whose data prompted Ting to finally publish his results and Brookhaven, with ADONE acting only to confirm the results of the first two. A very interesting account of the hunt can be found in Khare 1999, which I would reccommend to those interested in more detail.

4.2 Experimental Results

Below are detailed the currently discovered (and predicted), spectral lines of Charmonium. The reason we begin by looking at the data, and, afterwords, examining the Hamiltonian is that we wish to compare this spectrum to the prior two without any preconceptions. You'll notice that the Charmonium spectrum is very similar to that of Positronium, especially at low energies. The increased horizontal spread of the data is due to a different classification scheme, whereby states are arranged by not only quantum numbers, but also by their behavior under the P parity operator and the C charge conjagation operator. The very fact that several of the Charmonium states are invarient under the action of C to be a sign that the system consists of a particle/anti-particle pair. Moveover, the similarity between the spectra of Charmonium and Positronium suggests that the two system are very similar. As a final point of similarity, note that there is no solid hierarchy of fine vs. hyperfine perturbations: all four of the 1P states are spaced very closly, but even that closeness is about the level of seperation between the 1P and 1S states, another marker of a particle-antiparticle pair.



The quantum numbers used in the graph of the bound states of Charmonium are very similar to those used previously. J^{pc} refers to the total angular momentum of the system, with p and c referring to the behavior of the state under the action of those two operators. L refers to the angular momentum of the system (and can be referenced by the same set of letters as before). In short, though the states of Charmonium have other, stranger names, most of them are capable of being described much like the states of more familiar systems.

4.3 Potential and Transitions

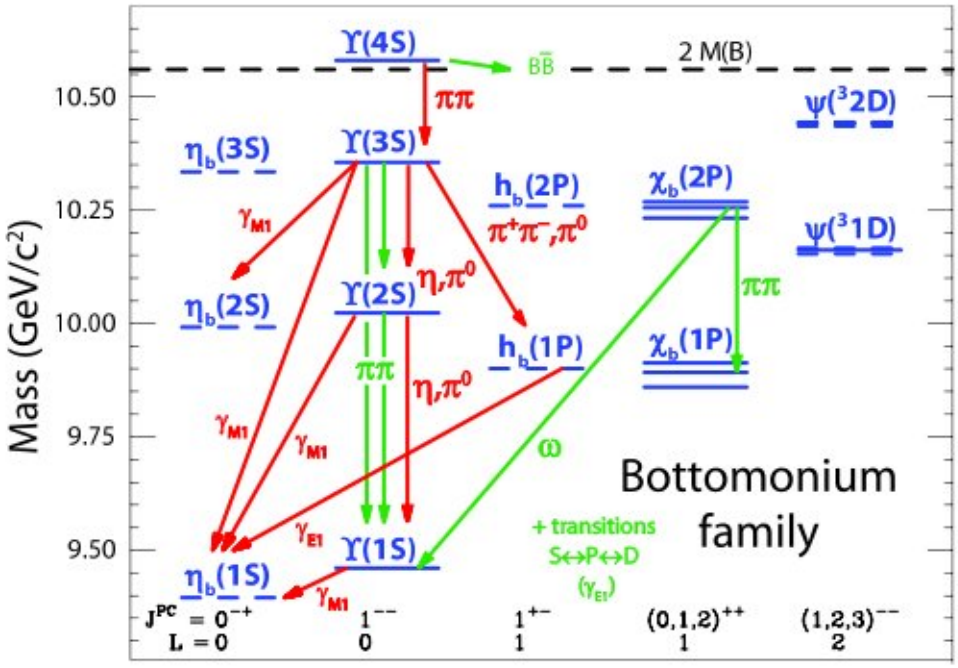
There are, nevertheless, a number of important differences between Charmonium and Positronium, most of which are applicable to a difference in the fundemental forces which hold them together. Positronium is held together solely by the electromagnetic force, as it consists of a pair of colorless leptons, while Charmonium not only feels electromagnetic attraction, but, also, the much stronger strong atomic force, which dominates the potential. The strong force has two primary components, as can be seen from the diagram. It must be approximately coloumbic at close range, to approximate the same low level states as in Positronium, but, for larger seperation, it grows without bound, in order to ensure colorlessness of all observed particles. Thus, an ansatz for the potential of the form $V = \frac{-4\alpha_s(r)\hbar c}{3/r} + kr$ is in order.

Despite the fact that the strong interaction is primarily responsible for the system, most of the decays of of Charmonium are electromagnetic, as can be seen on the previous chart. The reason for this is that any strong interaction below the so-called $D\bar{D}$ threshold (the energy at which a down-antidown pair can be created) requires the anihilation of the two initial quarks. These transitions are supressed by the Zweig rule, which leave only weak and e-m forces to transition between states, and the e-m force is much, much stronger than the weak force. Thus, most transitions between Charmonium states are electromagnetic, and, so, follow the rules of electromagnetic transitions we are familiar with from similar interactions. Nevertheless, some strong transitions do take plase, as indicated on the chart. Below is a summary of the widths of lower lying states on the chart, along with their energies.

State	Quantum # s (n L J pc)	Mass (Mev)	Width (Mev)
η_c	1 0 0 $++$	2980	25.5
J/ψ	1 0 1 $--$	3096	93.4 kev
χ_{c0}	1 1 0 $++$	3414	10.4
χ_{c1}	1 1 1 $++$	3510	.89
χ_{c2}	1 1 2 $++$	3556	2.06
h_{c0}	1 1 1 $+-$	3525	< 1
$\psi(3770)$	1 2 1 $--$	3771	23.0
η_c	2 0 0 $+-$	3638	14

5 Bottomonium

Finally, we arrive at the Bottomonium system. The Bottomonium system is quite similar to the previously discussed Charmonium system, having the same basic potential, except the particles involved are heavier, which, beyond the obvious effects of giving the system more energy, makes some states extremely difficult to observe, as there are now many more states that can decay strongly via meson emission. Thus, only states which decay via slow processes, like magnetic quadropole-type emissions have been experimentally verified. Thus, the table of widths for the $b/barb$ system has a large number of empty spaces. Below are the table of energies and width of predicted Bottomonium states, along with a graph of the lower lying bound states of the particle-antiparticle pair.



State	Quantum # s (n L J pc)	Mass (Mev)	Width (Mev)
$\Upsilon(1S)$	1 0 1 $--$	9460.30	54.02 keV
$\chi_{b0}(1P)$	1 1 0 $++$	9859.44	unknown
$\chi_{b1}(1P)$	1 1 1 $++$	9892.78	unknown
$\chi_{b2}(1P)$	1 1 2 $++$	9912.21	unknown
$\Upsilon(1D)$	1 2 2 $--$	10161.1	unknown
$\Upsilon(2S)$	2 0 1 $--$	10023.26	31.98 keV
$\chi_{b0}(2P)$	2 1 0 $++$	10232.5	unknown
$\chi_{b1}(2P)$	2 1 1 $++$	10255.46	unknown
$\chi_{b2}(2P)$	2 1 2 $++$	10268.65	unknown
$\Upsilon(3S)$	3 0 1 $--$	10355.2	20.32 keV
$\Upsilon(4S)$ 7 4 0 1 $--$	10579.4	20.5 MeV	

6 References

References

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