Physics of Muons

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Abstract

Similar to the electron, the muon is an elementary particle classified as a lepton with a negative charge and half-spin. Naturally-occurring muons are created due to cosmic rays and travel at relativistic speeds and thus feel the effects of time dilation. Due to this the lifetime of the muon is heavily dependent on the speed at which it is moving. We will discuss the lifetime and decay of the muon as well as the experiment which proves the effects of time dilation on the muon. We will also discuss the implications of muon transition from the $2p$ state to the $1s$ state in mesic atoms.

1 Discovery

In 1934, Hideki Yukawa proposed the first significant theory of the strong force which predicted the existence of a mediating particle that was approximately 300 times more massive than the electron, or about one-sixth the mass of the proton. For this reason, the particle was termed "meson," meaning "middle-weight." Confirmation of these particles came in 1937 from two separate groups involved in studies of cosmic rays; these particles were termed mu-mesons as more middle-weight particles were discovered. However, in 1946, it became increasingly obvious that the mu-meson was not a mediator of the strong force, as experiments showed that these cosmic ray particles did not interact strongly with atomic nuclei. This was explained in 1947, when Cecil Powell's group discovered that cosmic rays were not only comprised of mu-mesons but also pions. Powell's group showed that pions decay into mu-mesons quickly, and as the mu-meson is longer-lived, it was more easily detected. So, although mu-mesons were originally attributed to Yukawa's theory, they actually had nothing to do with strong force interactions.

2 Background

Further observation of the mu-meson showed that it is not comprised of quarks like other mesons. Mesons, for example, were discovered to be comprised of quarks, whereas the mu-meson lacks a quark structure. The mu-meson also decays into both a neutrino and anti-neutrino; this behavior does not match with mesons which decay in either a neutrino or an anti-neutrino, but not both. With this new information, the term mu-meson was dropped and the particle became known as the muon.

The muon shares many of the same characteristics of the electron and thus is similar in behavior. Both particles are part of the lepton family, have a spin of $\frac{1}{2}$ and a negative electric charge. However, the muon is approximately 200 times more massive than the electron with a mass of 105.659 MeV. Like the other leptons, it has an associated antiparticle of equal mass as well as an associated muon neutrino.

All naturally occurring muons are created by collisions of cosmic rays with molecules in the atmosphere. When cosmic ray protons collide with atomic nuclei in the upper atmosphere, pions are created. Shortly after creation, the pions decay into muons and neutrinos. The muons that are created are, at this point, traveling at relativistic velocities. Their lifetime without relativistic effects would only allow for relatively short distances to be traveled; however, due to the effects of time dilation, they are able to survive long enough to reach and even penetrate through the ground. Because of this, approximately 10,000 muons hit every square meter of the Earth's surface per minute. Of course, muons can also be created in the laboratory setting by colliding hadrons to produce pion beams, which will decay into muons.

3 Muon Decay and Lifetime

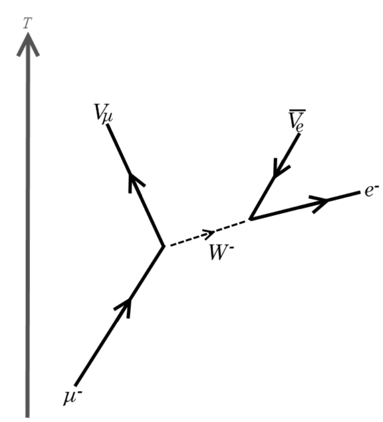

Figure 1: Principal Decay of a Muon

Muon decay, which occurs via the weak interaction, occurs in three modes that do not violate lepton family number; in these modes the muon decays into: an electron, an electron antineutrino, and a muon neutrino; an electron, an electron antineutrino, a muon neutrino, and a photon; or two electrons, an electron antineutrino, a muon neutrino, and a positron. The electron, electron antineutrino, and muon neutrino mode and the electron, electron antineutrino, muon neutrino and photon mode cannot be clearly separated and so the mode that includes the photon is considered to be a subset of the former mode. The modes that violate lepton family number are the modes in which the muon decays into: an electron, an electron neutrino, and a muon antineutrino; an electron and a photon; two electrons and a positron; or an electron and two photons. These modes have been established with a 90% confidence level.

The dominant decay mode is the one in which the products of decay are an electron, an electron antineutrino, and muon neutrino. This is represented formulaically by:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

From this interaction, we can derive the lifetime of the muon. The Feynman diagram for this mode of decay is given in figure 1. The Feynman propagator for this interaction can be written as

$$\frac{ig_w}{(M_W c)^2}$$

where $g_w = \sqrt{4\pi\alpha_W}$ is the weak coupling constant, and $M_W = 82 \pm 2 GeV/c^2$ is the mass of W^-, the mediator particle for weak interactions. This yields scattering amplitude of

$$A = \frac{g_w^2}{8(M_W c)^2}[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)][\bar{u}(4)\gamma_\mu(1-\gamma^5)v(2)]$$

where γ^μ is a set of 4x4 matrices that satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ where $2g^{\mu\nu}$ is the Minkowski metric. In this case we are using the "Bjorken and Drell" convention used in [1] which is as follows:

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}$$

where σ^i $(i = 1, 2, 3)$ is the indicated Pauli spin matrix. u are the particle solutions to the Dirac Equation, v are the antiparticle solutions to the Dirac Equation, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which is a 4x4 matrix given as follows:

$$\gamma^5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We will not derive these quantities but direct the reader to the derivations given in [1]. However, it would behoove us to mention that the u and v matrices contain elements that are dependent on the momentum of the particle, which therefore makes the entire calculation dependent on the velocity at which the particle is moving.

With the scattering amplitude now known, we can calculate the total decay rate which, after some mathematical manipulation which can be found in [1], is given by

$$\Gamma = \left(\frac{g_w}{M_W c}\right)^4 \frac{m_\mu^2}{2\hbar(4\pi)^3} \int_0^{1/2 m_\mu c^2} E^2 \left(1 - \frac{4E}{3m_\mu c^2}\right) dE = \left(\frac{m_\mu g_w}{M_W}\right)^4 \frac{m_\mu c^2}{12\hbar(8\pi)^3}$$

.

Thus the lifetime of the muon follows as

$$\tau = \frac{1}{\Gamma} = (M_W m_\mu g_w)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2}$$

which, when expressed in terms of the Fermi coupling constant, $G_F = \frac{\sqrt{2}}{8}(\frac{g_w}{M_W c^2})^2(\hbar c)^3$, becomes

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

.

A muon at rest has a mean lifetime of 2.197 microseconds. However, because muons are traveling at relativistic speeds when they are created in the atmosphere, their lifetime increases due to time dilation.

4 Time Dilation

We previously stated that the lifetime of the muon is dependent on the velocity at which it is moving. In their experiment measuring time dilation using muons, David Frisch and James Smith postulated that since the decay probability and thus the lifetime of a particle is dependent on forces that are internal to their structure, any dependence of the decay probability on the speed of the particle can be used as an example of a clock in motion relative to an observer. Since we know this to be the case, time dilation should have an observable effect on muons.

One possible way to observe this effect is to measure the number of muon decays per unit time at different speeds and compare the measurements; if decay is affected by time dilation then the number of muon decays per unit time for those traveling at higher speeds should be fewer in number than those traveling more slowly. However, this method requires the use of a particle accelerator, as it is difficult to utilize it with the relatively low intensities of muons present in cosmic rays.

It is possible to use cosmic ray muons to measure the time dilation factor, however, if the number of muons can be measured at different altitudes, which would allow for the measurement of the number of muons decayed as a function of the speed at which they traveled. Frisch and Smith used this idea for their experiment in which they detected muons at the top of Mount Washington and at sea level to determine whether time dilation has an effect on muon decay.

In order to show this, Frisch and Smith prepared a "scintillator" in which, when muons passed through it, excitation energy given to the molecules in the scintillator would be emitted as light. Because cosmic ray muons have a variety of energies, many passed directly through the scintillator. However, those moving at slow enough speeds came to a stop within the scintillator; those were the particles whose decays were observed. "Slow enough," of course, may give a false impression of how fast these particles were moving; in order to test the time dilation factor it was necessary to measure muons that were traveling at relativistic speeds. To accomplish this, the scintillator was covered by a $2\frac{1}{2}$ foot layer of iron, which stopped muons with speeds less than $0.9950c$. Muons moving at speeds higher than $0.9954c$ went through both the layer of iron and scintillator. Therefore, only decay signals for those muons that fell in that range of speeds were counted.

For a one-hour run at the top of Mt. Washington, 568 muons were counted. Taking the average speed of the detected muons to be $0.9952c$, we would expect that it would take 6.4 microseconds for the muons to travel the 6255 feet between the two detectors. Thus, after this amount of time we would expect to see only about 27 muons survive. However, in a one-hour run at 10 feet about sea level, 412 muons were detected. Frisch and Smith concluded that at relativistic speeds relative to a stationary observer, the muons decay much more slowly than if the muons were at rest relative to a stationary observer. Thus, the lifetime of the muon is clearly dependent on the velocity at which it travels, and muons do feel the effects of time dilation.

5 $2p$-$1s$ Muon Transitions in Mesic Atoms

5.1 Influence on Nuclear Fission

In heavy mesic atoms, it has been shown that transitions of the muon from the $2p$ energy state to the $1s$ ground state occur via the transfer of all transition energy to the nucleus of the atom. In this process it is possible that the atom will undergo nuclear fission. The probability that the atom will undergo nuclear fission if there is a muon in the innermost shell is different than that of the atom if there is no muon; this is due to the muon preventing deformation of the nucleus while it in the innermost shell. The closer the threshold fission energy is to the nuclear excitation energy, the larger the influence of the muon on the probability of nuclear fission will be. D. Zaretski and V. Novikov performed calculations to determine whether the fission threshold in the presence of a muon is greater than the excitation energy for a few representative heavy mesic atoms.

We can qualitatively follow the method used by Zaretski and Novikov in [6] to determine the fission threshold. To begin, the potential curve for nuclear fission in the presence of the muon must first be calculated; this is the sum of the potential curve without the presence of a muon and the muon binding energy as a function of nuclear deformation parameters. Finding the muon binding energy requires solving the Schrodinger equation for the muon in the Coulomb field of the deformed nucleus. Zaretski and Novikov use an ellipsoid as a reasonable approximation to the shape of the nucleus; they adjust the potential used in the Schrodinger equation accordingly. Integration of the Schrodinger equation for the ground state of the muon yields the binding energy of the muon. For uranium-238, the fission threshold in the presence of a muon is higher than the excitation energy; for plutonium-239, it is slightly below the excitation energy. From these results we see that the muon keeps the uranium atom from undergoing fission, whereas it does not prevent fission in the plutonium atom.

5.2 Using Transitions as a Nuclear Probe

Another consequence of the $2p$-$1s$ transition in a mesic atom is the emission of a gamma ray; this is known as "Chang radiation."This gamma ray contains separable components which, with measurements of their energies, can provide information about the nuclear radius, nuclear quadrupole moments, and nuclear polarizability. While much of this information is readily available using other particles as nuclear probes, they would be more precisely determined by using muons.

The expected energy of the $2p$-$1s$ transition varies greatly depending on the atom in which the muon is transitioning. For example, for oxygen the expected energy is 0.14 MeV, whereas for lead it is around 5 MeV. Changing the nuclear radius, however, will affect the energy; for example, in lead, changing the nuclear radius by 10% will change the energy by about 0.4 MeV. By using the muon as a probe particle it is possible to determine the effective radius of the nucleus for electric interactions with a precision of 1-2%; this is more precise than what is possible with other probe particles. Using muons to also gather information about nuclear quadrupole moments is more efficient than using electrons, as the quadrupole moments and deviations of the nuclear electric field produce a much greater effect in muonic spectra than in electronic level; this is because the muon spends more time within the nucleus than the electron. The polarizability and compressibility of the nucleus can be determined if swelling of the nucleus due to the addition of a proton can be determined. Some anomalies in isotrope shift of atomic spectral lines have been attributed to this nuclear swelling; this swelling would be much greater in the case of muonic radiation and thus would be allow for a more precise measurement.

6 Summary

We have discussed the primary mode of muon decay and shown the basics of the derivation of the muon lifetime starting from the Feynman propagator. This calculation holds when the muon is moving at relativistic speeds; this is essential as many cosmic ray muons are traveling close to the speed of light. The effects of time dilation on muon decay were first shown by Frisch and Smith in the 1960s.

Muon transitions from the $2p$ to the $1s$ energy states have many implications, including the possible nuclear fission of the mesic atom in which the muon is transitioning. It is also possible to use these transitions as a way to obtain more precise measurements of the nuclear radius, nuclear quadrupole moments, and nuclear polarizability than we are able to achieve with other particles.

7 References

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