The Brown-Twiss Experiment

By Kaleb Politis for Quantum II

In 1956, Robert Brown and Richard Twiss devised an interferometer that would measure the intensity correlations of light. Their purpose was to determine the apparent angular diameter of any given star. Though this experiment can be explained entirely through classical physics, it has played a part in furthering the field of quantum optics.

I. Introduction

Since stars are essentially point-like when viewed from Earth, directly measuring the apparent angular diameter of a star is not possible. With their published work, "*A test of a new type of stellar interferometer on Sirius"* Brown and Twiss described in detail their method and its results. Two Photomultiplier tubes were aimed at the Star. Light was collected into the tubes with the use of two SiO-coated mirrors of diameter 6.5m. The interference effect observed between the two intensities revealed a positive correlation between the two signals, even though no phase information had been collected.

This is not the first time coherence (meaning the quantization of the degree of correlation between two separate beams of light) had been used experimentally. In 1920 Michelson and Pease measured the angular diameter of Betelgeuse. Their apparatus had been devised to measure the correlation of the electric field; the end of a telescope tube is closed with a mask with two apertures, and thus fringes are produced at the focus. But attempts to enlarge the phase interferometer and make the method applicable to dimmer stars failed.

Unlike this former experiment though, the Brown and Twiss method examines the correlation between the intensities of two beams of light (intensity interferometry). In the words of quantum mechanics, one measures the rate of photons arriving at each sensor. This method proves to be much less sensitive to outer influences, (such as scintillation), and thus gives far better results than the Michelson interferometer in determining the apparent radius of stars.

II. Classical Perspective

We treat the incident electromagnetic radiation as a classical wave. Light has an amplitude that moves on propagating wavefronts from its source. If E is the amplitude, then the intensity is:

$$I = \left\langle E \cdot E^{*} \right\rangle$$

The equations of the electric field for two light waves from the same source, passing through two pinholes and reaching a point P after covering different distances are:

$$E_1 = E_{01}\sin(kx - \omega t)$$
$$E_2 = E_{02}\sin(k(x+\Delta x) - \omega t) = E_{02}\sin(kx - \omega t + \delta)$$

where: $\delta = k\Delta x = (2\pi\Delta x)/\lambda$

The effects caused by the addition of amplitudes are known as *interference* (constructive, destructive). The energy received is equal to the average value of the square of the resultant amplitude. Thus for two interfering waves, the resultant intensity at a certain point is:

$$I = E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\delta$$

Thus for two interfering waves of equal amplitude, the intensity at a point can vary from zero to four times the average intensity due to a single wave. This is visually confirmed by the formation of *fringes,* which were first observed in the 17th century, and were not satisfactorily accounted for until 1801 and Thomas Young's experiments.

In the example above, the waves had constant phase and amplitude. Laser light may have such characteristics (to a certain degree), but the light of stars though does not. Produced from a multitude of individual atomic emissions, this light has a wide frequency spectrum and a constantly fluctuating phase. Thus, in order to observe interference fringes, we must arrange that the signals coming from the same atomic sources are constant. By having two limited regions (pinholes) from which we receive the light of a star we can observe *coherent* signals (signals that produce fringes when interfering). The limits of the regions are directly related to the apparent angular extent of the source. It is useful to introduce the term *first degree of coherence:*

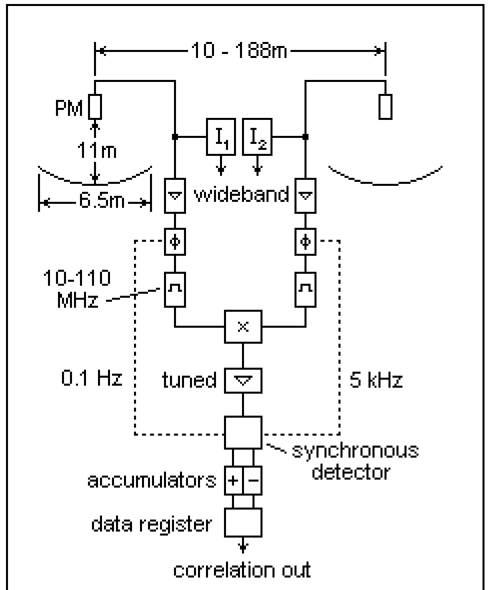
$$g^{(1)}(r_1,t_1;r_2,t_2) = \frac{\left\langle E^*(r_1,t_1)E(r_2,t_2)\right\rangle}{\left[\left\langle |E(r_1,t_1)|^2\right\rangle\left\langle |E(r_2,t_2)|^2\right\rangle\right]^{1/2}}$$

The stellar Michelson interferometer provides a direct method of measuring the function $g^{(1)}(r_1,r_2)$.

Using the Van Cittert-Zernike Theorem we have the dependence of the coherence on the distance for an extended source (such as a star). *"The complex degree of coherence between P_1 and P_2 in a plane illuminated by an extended quasi-monochromatic source is equal to the normalized complex amplitude in the diffraction pattern centered on P_2 that would be obtained by replacing the source by an aperture of the same size and illuminating it by a spherical wave converging on P_2, the amplitude distribution proportional to the intensity distribution across the source."*

In the case of intensity interferometry we examine the fluctuations of the intensity of the incident waves. The fluctuations in phase are present but are unnecessary information. The correlation measured in the intensity interferometer is proportional to $\langle \Delta I_1 \Delta I_2 \rangle$,

where ΔI the average fluctuation of the intensity I at each detector.



Brown and Twiss Interferometer

We use the second order correlation function which measures the coherence of the intensity

$$g^{(2)}(r_1,t_1;r_2,t_2) = \frac{\langle E^*(r_1,t_1)E^*(r_2,t_2)E(r_1,t_1)E(r_2,t_2)\rangle}{\langle |E(r_1,t_1)|^2 \rangle \langle |E(r_2,t_2)|^2 \rangle}$$

which can be written in terms of intensities:

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

The two photocathodes detect the light from two different points. The signals are multiplied and integrated in a correlator from which we get:

$$C = \langle I(r_1)I(r_2)\rangle = I_0^2 g^{(2)}(r_1,r_2)$$

If we examine two monochromatic plane waves:

$$C = \langle E^*(r_1)E^*(r_2)E(r_1)E(r_2)\rangle \propto \left\langle \left(|E_k|^2 + |E_{k'}|^2 \right)^2 \right\rangle + 2\left\langle |E_k|^2 |E_{k'}|^2 \right\rangle \cos(kr_0\varphi)$$

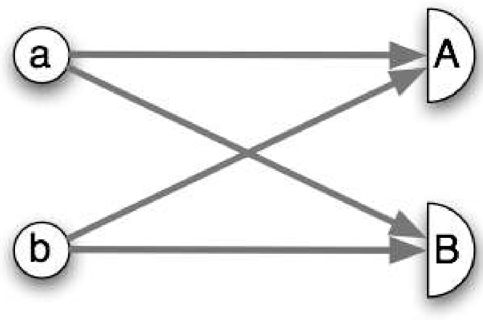
where: $r_0 = r_1 - r_2$

The Brown and Twiss interferometer has a number of advantages. Larger light-gathering capacity allows for determining the apparent angular diameter of much dimmer stars, it relies on electronic rather than visual observation and it is not affected by scintillation.

III. Photons

Using quantum mechanics, the second order correlation function can be regarded as being proportional to the probability of detecting a second photon a time τ after the first one.

Two bosonic particles, a and b, are detected by two detectors, A and B. The amplitude of detecting one of the particles in a detector is $\langle a \mid i \rangle$ and $\langle b \mid i \rangle$ respectively. We assume that the amplitudes of detecting a particle in either detector are equal.



$$\langle a \mid A \rangle = \langle a \mid B \rangle = a$$
$$\langle b \mid A \rangle = \langle b \mid B \rangle = b$$

We examine the probability of detecting exactly one particle in each detector. This depends on whether the particles are *identical* or not. For non-identical, the total probability is equal to the sum of detecting (a in A, b in B) plus (a in B, b in A).

$$P_n = \left|\langle a \mid A \rangle \langle b \mid B \rangle\right|^2 + \left|\langle a \mid B \rangle \langle b \mid A \rangle\right|^2 = 2|a|^2|b|^2$$

When dealing with identical, bosonic particles the probability is:

$$P_i = \left|\langle a \mid A \rangle \langle b \mid B \rangle + \langle a \mid A \rangle \langle b \mid A \rangle\right|^2 = |2ab|^2 = 4|a|^2|b|^2 = 2P_n$$

Thus we see in the case of the identical particles the two amplitudes interfere constructively to give a joint detection probability twice that for two independent events. This results in the effect known as *photon bunching*, whereupon photons have a statistical tendency to arrive simultaneously at a detector.

Sources:
R.H. Brown and R.Q. Twiss (1956) “Correlation between photons in two coherent beams of light”
S. Rath (2004) “The Hanbury Brown-Twiss and related Experiments”
J.B. Calvert (2002) “Sizes of the Stars”