

Spin Flip Transition of Hydrogen in Astrophysics

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I. BACKGROUND

The ultimate purpose of this research project is to help identify optically dark galaxies to assist in solving the problem of the void phenomenon. The void phenomenon is the discrepancy between the number of observed dwarf halos in cosmic voids and that expected from CDM simulations.¹ We suspect that there are, in fact, dwarf systems in the voids, but they are optically dark. There exist large amounts of cosmic dust and other particles in the interstellar medium, thus some light particles in the visible portion of the electromagnetic spectrum could get dispersed, making their detection more difficult. From this one could imagine that galaxies that consist of lots of gas and very few stars (low surface brightness galaxies) could easily go undetected by telescopes here on earth. One solution to this problem is to look for spectral lines in other parts of the electromagnetic spectrum. Our research looks into emission lines in the radio portion of the spectrum. One of the main benefits to radio astronomy is that radio waves can penetrate interstellar cosmic dust that light cannot penetrate, therefore allowing us to observe these optically dark galaxies.

The universe is filled with hydrogen atoms in different states, and astronomers can observe these different states to piece together information about the universe. Roughly 97% of hydrogen in the interstellar medium is neutral, and less than half of that is in an atomic form. This cold neutral hydrogen emits radio waves at frequencies of 1420MHz. The emission lines come from a transition between the hyperfine splitting of the ground state of hydrogen. This detected emission line is commonly referred to as the 21 cm line and corresponds to a forbidden spin flip transition of neutral hydrogen. Radio astronomy relies primarily on the detection of this 21 cm line. Our paper presents the detailed quantum mechanical calculations behind the spin flip transition of neutral hydrogen as well as deuterium.

II. THE THEORY

The Arecibo Legacy Fast Arecibo L-band Feed Array (ALFALFA) is a blind extragalactic neutral hydrogen (HI) survey. ALFALFA's goal is to detect low stellar mass, hydrogen rich galaxies within the local universe in order to explore the void phenomenon. Most surveys to date have been either optical or near-infrared surveys, both of which detect galaxies with large stellar masses. ALFALFA offers the ability to detect low stellar mass, high gas mass

galaxies which go undetected by many surveys.²

Again, our interest in ALFALFA lies in the fact that it detects neutral hydrogen. Neutral hydrogen is made up of two spinning particles: one proton and one electron. We know that a spinning particle puts off a small magnetic field; so particle 1 puts off its own magnetic field and also interacts with the field created by particle 2. The spin of each particle is one-half, allowing for only two possible states; one with the particle spins parallel and the other with the spins antiparallel. There is a small energy difference between the parallel and antiparallel states which we calculate in Section III. We will see from this calculation that the configuration of the proton and electron spinning antiparallel to each other is lower in energy and more stable than the alternative state.

The collisions of atoms in the universe excite hydrogen atoms into higher energy levels. Even at the coldest temperatures in our universe, (the CMB temperature is approximately 2.725K) the atomic kinetic energies, $E \approx kT \approx 10^{-4}$, are still significantly larger than the energy required to excite a neutral hydrogen atom into this hyperfine level ($\sim 10^{-6}$).³ Once in this higher energy level, the hydrogen atom will eventually relax into a less energetic state, i.e. it will make a transition from the higher energy level to the lower energy level. This is a forbidden transition; that is, it is highly unlikely to happen-the probability is $2.9 \times 10^{-15} \text{sec}^{-1}$, or one transition every 10 million years. Alternatively, the hydrogen atoms could jostle around bumping into each other, thus decreasing the lifetime of the excited atom causing it to release energy in the form of a radio photon of wavelength 21cm.⁴ Fortunately for radio astronomers, photons of this wavelength are able to pass through our atmosphere with little interference. Because there is so much neutral hydrogen in the interstellar medium, this deexcitation happens often making the emission lines stonger and, therefore, relatively easy to detect.

Both electrons and protons are spin 1/2 particles, and can be arranged in one of the four following ways within a hydrogen atom:

$$\begin{aligned} &|\uparrow\uparrow\rangle, \\ &\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \\ &|\uparrow\uparrow\rangle, \end{aligned}$$

and

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle).$$

These four states make up the hyperfine states of the hydrogen atom in the ground state. The first three correspond to the $F = 1$ state, and the last corresponds to the $F = 0$ state. Here, F represents total angular momentum. That is, F is the sum of the spins of the proton and the electron's orbital angular momentum plus/minus one-half. The triplet levels corresponding to the $F = 1$ state are degenerate when no magnetic field is present. The difference in potential energy between these two states is what puts off the radiation of interest. We can see this energy splitting in Figure 1.

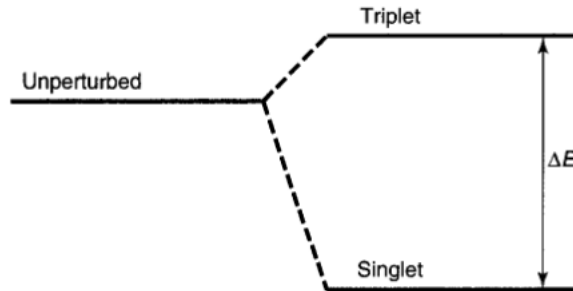


FIG. 1: The hyperfine splitting of neutral hydrogen, showing the triplet and singlet states. The singlet state corresponds to $F = 0$ and the triplet state corresponds to the $F = 1$ state.⁵

The energy difference between the $F = 0$ and $F = 1$ states is due to the magnetic moments of the particles. There is a lower energy when the magnetic moments of two particles are aligned and a higher potential energy when the two are antialigned. We know that

$$E_{pot} = -\boldsymbol{\mu} \cdot \mathbf{B}. \quad (1)$$

Here, the magnetic moment, $\boldsymbol{\mu} \equiv g \frac{q}{2m} \mathbf{S}$, where g is the g-factor and \mathbf{S} is the spin of a particle of mass m and charge q . We see that the magnetic moment is charge dependent; therefore, for an electron and a proton, the lowest energy level occurs when the particle spins are antiparallel to each other. We will show that this is the case in our calculations.

Because the spinning particles are creating magnetic fields and the fields interact with the particle magnetic moments, we must take each case into account when formulating our Hamiltonian. The first interaction that we will look into, is the interaction of the magnetic moment of the proton with the magnetic field due to the electron. The second is the interaction of the magnetic moment of the electron with the magnetic field due to the proton while the electron is outside the radius of the proton. As there is a finite probability

that the electron is located within the radius of the proton, we must take this into account as a separate term; thus we will consider the interaction between the two magnetic moments while the proton and electron radii overlap. (See Figure 2.) The probability of finding the

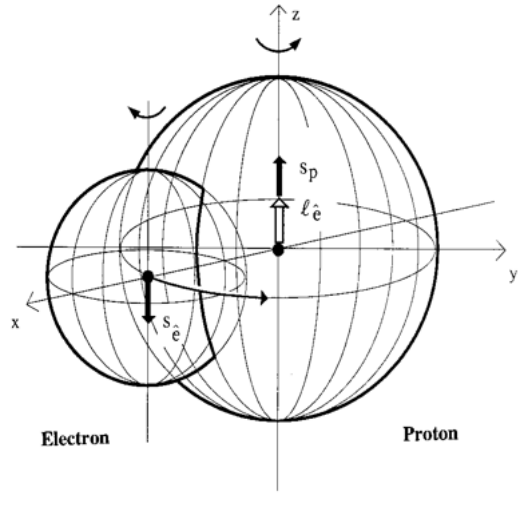


FIG. 2: The electron radius and the proton radius overlap.⁶

electron inside the proton is⁵

$$P = \frac{4R_p^3}{3a_0^3}$$

where R_p is the radius of a proton, and $a_0 \equiv \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$ is the Bohr radius. The first contribution to the Hamiltonian is zero, because in the ground state, the orbital angular momentum of the electron is zero. For the second term, we consider the magnetic field generated by the proton. The magnetic field lines are shown in Figure 3. Integrating over the entire range of the proton's magnetic field to find the second term's contribution to the Hamiltonian yields a net effect of zero. According to Bradt,³ if we assume the proton and electron are both pointlike dipoles and the electron is within the confines of the proton—assumed to be a small sphere with a uniform field inside—then, with the electron's magnetic moment parallel to the proton's magnetic field, “this entire region contributes a negative (low) potential energy.” In the case of antiparallel moments, the potential energy difference is positive. These two potential energy differences correspond to the $F = 0$ and $F = 1$ levels of the ground state hydrogen atom respectively.

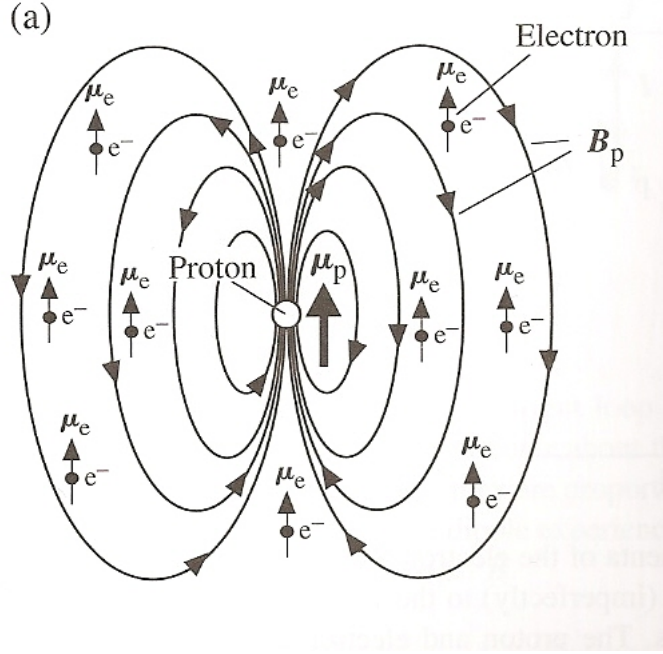


FIG. 3: The magnetic field lines of the proton with the magnetic moment vectors of the electron and proton.³

III. THE CALCULATION

In order to calculate this energy difference, we must use Equation (1) and the probability, P , of finding the electron within the radius of the proton.

$$E_{pot} = -P(\boldsymbol{\mu}_e \cdot \mathbf{B}_p) \quad (2)$$

We know $\boldsymbol{\mu}_e \equiv g_e \frac{e}{2m_e} \mathbf{S}$ where the g -factor for an electron is -2.002319 . Again, we assume the proton is a sphere with a uniform magnetic field passing through, so inside the proton, the magnetic field is classically defined by the following:

$$\mathbf{B}_p = g \frac{\mu_0}{2\pi R_p^3} \boldsymbol{\mu}_p \quad (3)$$

where μ_0 is the permeability of free space. Using Equations (2) and (3) and the definition of the proton's magnetic moment to calculate the energy, we get

$$E_{pot} = \frac{4}{3} \frac{\mu_0}{2\pi} \frac{g_e g_p \mu_B \mu_N}{a_0^3} \frac{\mathbf{S} \cdot \mathbf{I}}{\hbar^2}. \quad (4)$$

Here, $\mu_B = \frac{e\hbar}{2m_e}$ is the Bohr magneton, $\mu_N = \frac{e\hbar}{2m_p}$ is the nuclear magneton, the g -factor for a proton is 5.58569 , and \mathbf{I} is the spin of a proton. We now have that the potential energy

is proportional to the dot product of the spins of the electron and proton; this relationship between spins is called spin-spin coupling. This is what breaks the degeneracy of the ground state into the triplet and singlet states resulting in the ultimate energy gap that radio astronomers observe. In order to obtain the spin-spin coupling term, we will take the dot product of the total angular momentum, \mathbf{F} , with itself and rearrange the equation.

$$\mathbf{F} \cdot \mathbf{F} = \mathbf{S} \cdot \mathbf{S} + 2\mathbf{S} \cdot \mathbf{I} + \mathbf{I} \cdot \mathbf{I} \quad (5)$$

This leads us to

$$\mathbf{S} \cdot \mathbf{I} = \frac{1}{2} (\mathbf{F} \cdot \mathbf{F} - \mathbf{S} \cdot \mathbf{S} - \mathbf{I} \cdot \mathbf{I}). \quad (6)$$

Applying these spin operators, to the wave function, we get the following eigenfunction:

$$\mathbf{S} \cdot \mathbf{I}|\psi\rangle = \frac{\hbar^2}{2} (F(F+1) - S(S+1) - I(I+1))|\psi\rangle \quad (7)$$

Plugging in the values of the total angular momentum $F = 0, 1$, electron spin $S = 1/2$, and proton spin $I = 1/2$, we get

$$\mathbf{S} \cdot \mathbf{I}|\psi\rangle = -\frac{3}{4}\hbar^2|\psi\rangle \quad (8)$$

for the $F = 0$ state, and we get

$$\mathbf{S} \cdot \mathbf{I}|\psi\rangle = \frac{1}{4}\hbar^2|\psi\rangle \quad (9)$$

for the $F = 1$ state. With our eigenvalues for each state, we can now find the energy difference between the $F = 0$ and the $F = 1$ states. Taking the difference in potential energies, we obtain the following:

$$\Delta E_{pot} = \frac{4}{3} \frac{\mu_0}{2\pi} \frac{g_e g_p \mu_B \mu_N}{a_0^3} \left(\frac{1}{4} - \left(-\frac{3}{4} \right) \right). \quad (10)$$

Plugging in the known values of these constants, we see that

$$E_{pot} = 9.427 \times 10^{-25} \text{J} = 5.88 \times 10^{-6} \text{eV}$$

. This corresponds to a frequency of

$$f = \frac{\Delta E}{h} = 1420 \text{MHz} \quad (11)$$

and a wavelength of

$$\lambda = \frac{c}{f} = .21 \text{m} = 21 \text{cm}. \quad (12)$$

Neutral hydrogen emits photons in the form of radio waves with a wavelength of 21 cm, hence the name of the emission line: the 21 cm line.

About 5 years ago, astronomers discovered that deuterium, like hydrogen, emits radiation in the form of photons with wavelengths in the radio portion of the electromagnetic spectrum. We can follow the same procedure as above for deuterium by replacing the proton with deuterium. Deuterium, which is twice as massive as a proton, has a spin of $+1$, a charge of $+e$, and a g-factor of $+0.857$. Plugging these values into the equations above, we obtain the following results:

$$E_{deut} = \frac{8\pi}{3} g_e g_D \mu_B \mu_D (F(F+1) - S(S+1) - I(I+1)). \quad (13)$$

Where $S = 1/2$ is the spin of the electron, $I = 1$ is the spin of deuterium, μ_D represents the contribution from the nuclear magneton—in the case of deuterium, the effect is twice that of neutral hydrogen—and the total angular momentum $F = L + S \pm 1/2 = 1/2, 3/2$ where L , the electron's orbital angular momentum, is zero. Calculating the energy with these constants gives us

$$E_{deut} = (0.2304)E_{pot} = 0.2304(5.88 \times 10^{-6} \text{eV}) = 1.35 \times 10^{-6} \text{eV}.$$

This corresponds to a frequency of

$$f = \frac{\Delta E}{h} = 327.134 \text{MHz} \quad (14)$$

and a wavelength of

$$\lambda = \frac{c}{f} = .92 \text{m} = 92 \text{cm} \quad (15)$$

Like neutral hydrogen, cosmic deuterium also has a spin flip transition which emits photons at the 92cm line.

IV. FUTURE WORK

We would like to determine if optically dark galaxies exist within voids. In order to do so, we need to first detect all HI sources that ALFALFA can measure, then compare these detections with the SDSS optical galaxies to determine whether or not a true match exists; if there is no true match, we consider a galaxy to be optically dark. We, then, need to determine whether these optically dark galaxies are in voids using Void Finder—an algorithm to detect

voids in the universe. By observing emission lines of the spin flip transition of neutral hydrogen, we hope to come up with a solution to the void phenomenon.

¹ Giovanelli, Riccardo. “ The Void Phenomenon Revisited”. *ASP Conference Series* (2009).

² The 21 cm Line: Why Observe It?

<http://alfalfasurvey.wordpress.com/2009/01/17/the-21-cm-line-why-do-we-care/>

³ Bradt, Hale. *Astrophysics Processes*. Cambridge University Press. 2008.

⁴ Swinburne. Centre for Astrophysics and Supercomputing.

<http://astronomy.swin.edu.au/cosmos/S/Spin-flip+Transition>

⁵ UIUC. Physics 214 Lecture. <http://online.physics.uiuc.edu/.../7-2%20Inside%20the%20proton%20Sol.pdf>

⁶ Institute for Basic Research. <http://www.neutronstructure.org/part5.htm>