Appendix

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#Quantum Paper Work
#Finite Nuclear Size Perturbation Theory
\#   Computes energy shifts due to finite nuclear size.

restart;
with(*orthopoly*);

$$[G, H, L, P, T, U] \tag{1}$$

$L(5, x)$;   *### These are the Laguerres polynomials*

$$1 - 5x + 5x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5 \tag{2}$$

$g(5, 0, x) := L(5, x)$;   *### begins process for computing Associated Laguerres polynomials*

$$1 - 5x + 5x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5 \tag{3}$$

for i **from** 1 **to** 7 **do**
$g(5, i, x) := \mathit{diff}(g(5, i-1, x), x) : \mathit{print}(5, i-1, g(5, i-1, x))$ **:od**:

$$5, 0, 1 - 5x + 5x^2 - \frac{5}{3}x^3 + \frac{5}{24}x^4 - \frac{1}{120}x^5$$

$$5, 1, -5 + 10x - 5x^2 + \frac{5}{6}x^3 - \frac{1}{24}x^4$$

$$5, 2, 10 - 10x + \frac{5}{2}x^2 - \frac{1}{6}x^3$$

$$5, 3, -10 + 5x - \frac{1}{2}x^2$$

$$5, 4, 5 - x$$

$$5, 5, -1$$

$$5, 6, 0 \tag{4}$$

for j **from** 0 **to** 12 **do** $g(j, 0, x) := L(j, x)$; **for** i **from** 1 **to** $j+2$ **do**
$g(j, i, x) := \mathit{diff}(g(j, i-1, x), x)$ **:od:od**:*## Lots of assoc lag.*

for n **from** 1 **to** 5 **do for** l **from** 0 **to** $n-1$ **do**
$R(n, l, x) := g(n+l, 2 \cdot l+1, \mathrm{x}) \cdot \exp\left(-\frac{\mathrm{x}}{2}\right) \cdot (x)^l$; $RR(n, l, r) := \mathit{subs}\left(x = \frac{2 \cdot r}{n}, g(n+l, 2 \cdot l+1,\right.$
    $\left.\mathrm{x}) \cdot \exp\left(-\frac{\mathrm{x}}{2}\right) \cdot (x)^l\right)$;
$N(n, l) := \mathit{int}\left(r^2 \cdot RR(n, l, r)^2, r = 0 .. \infty\right)$;
  $RX(n, l, r) := \frac{RR(n, l, r)}{\mathrm{sqrt}(N(n, l))}$;   *### constructs normalized hydrogenic radial wavefunctions*

  $\mathit{print}(n, l, RX(n, l, r))$
 :od;**od**;

$$1, 0, -2\,\mathrm{e}^{-r}$$

$$2, 0, \frac{1}{4}\,(-2+r)\,e^{-\frac{1}{2}r}\sqrt{2}$$

$$2, 1, -\frac{1}{12}\,e^{-\frac{1}{2}r}\,r\sqrt{6}$$

$$3, 0, \frac{2}{27}\left(-3+2\,r-\frac{2}{9}\,r^2\right)e^{-\frac{1}{3}r}\sqrt{3}$$

$$3, 1, \frac{1}{81}\left(-4+\frac{2}{3}\,r\right)e^{-\frac{1}{3}r}\,r\sqrt{6}$$

$$3, 2, -\frac{2}{1215}\,e^{-\frac{1}{3}r}\,r^2\sqrt{30}$$

$$4, 0, \frac{1}{16}\left(-4+3\,r-\frac{1}{2}\,r^2+\frac{1}{48}\,r^3\right)e^{-\frac{1}{4}r}$$

$$4, 1, \frac{1}{480}\left(-10+\frac{5}{2}\,r-\frac{1}{8}\,r^2\right)e^{-\frac{1}{4}r}\,r\sqrt{15}$$

$$4, 2, \frac{1}{1920}\left(-6+\frac{1}{2}\,r\right)e^{-\frac{1}{4}r}\,r^2\sqrt{5}$$

$$4, 3, -\frac{1}{26880}\,e^{-\frac{1}{4}r}\,r^3\sqrt{35}$$

$$5, 0, \frac{2}{125}\left(-5+4\,r-\frac{4}{5}\,r^2+\frac{4}{75}\,r^3-\frac{2}{1875}\,r^4\right)e^{-\frac{1}{5}r}\sqrt{5}$$

$$5, 1, \frac{1}{1875}\left(-20+6\,r-\frac{12}{25}\,r^2+\frac{4}{375}\,r^3\right)e^{-\frac{1}{5}r}\,r\sqrt{30}$$

$$5, 2, \frac{2}{65625}\left(-21+\frac{14}{5}\,r-\frac{2}{25}\,r^2\right)e^{-\frac{1}{5}r}\,r^2\sqrt{70}$$

$$5, 3, \frac{1}{328125}\left(-8+\frac{2}{5}\,r\right)e^{-\frac{1}{5}r}\,r^3\sqrt{70}$$

$$5, 4, -\frac{2}{4921875}\,e^{-\frac{1}{5}r}\,r^4\sqrt{70} \tag{5}$$

$int\left(RX(1, 0, r)^2 \cdot r^2, r = 0 .. \infty\right);$ *### Normalized??*

$$1 \tag{6}$$

for n **from** 1 **to** 5 **do for** l **from** 1 **to** $n-1$ **do**
$over(n, l) := int\left(RX(n, l, r)\cdot RX(n, l-1, r)\cdot r\cdot r^2, r = 0 .. \infty\right) : print(n, l, over(n, l));$
: **od**:**od**:

$$2, 1, -3\sqrt{3}$$
$$3, 1, -9\sqrt{2}$$
$$3, 2, -\frac{9}{2}\sqrt{5}$$
$$4, 1, -6\sqrt{15}$$
$$4, 2, -12\sqrt{3}$$
$$4, 3, -6\sqrt{7}$$
$$5, 1, -15\sqrt{6}$$
$$5, 2, -\frac{15}{2}\sqrt{7}\sqrt{3}$$
$$5, 3, -30$$
$$5, 4, -\frac{45}{2} \quad \textbf{(7)}$$

for n **from** 1 **to** 5 **do for** l **from** 0 **to** $n-1$ **do**
$over(n, l) := int\left(RX(n, l, r)\cdot RX(n, l, r)\cdot r^2\cdot r^2, r = 0 ..\ \infty\right) :\ print(n, l, over(n, l));$
:od:od:

$$1, 0, 3$$
$$2, 0, 42$$
$$2, 1, 30$$
$$3, 0, 207$$
$$3, 1, 180$$
$$3, 2, 126$$
$$4, 0, 648$$
$$4, 1, 600$$
$$4, 2, 504$$
$$4, 3, 360$$
$$5, 0, 1575$$
$$5, 1, 1500$$
$$5, 2, 1350$$
$$5, 3, 1125$$
$$5, 4, 825 \quad \textbf{(8)}$$

for n **from** 1 **to** 5 **do for** l **from** 2 **to** $n-1$ **do**
$over(n, l) := int\left(RX(n, l, r)\cdot RX(n, l-2, r)\cdot r^2\cdot r^2, r = 0 .. \infty\right); print(n, l, over(n, l));$
:od:od:

$$3, 2, 45\sqrt{10}$$
$$4, 2, 240\sqrt{5}$$

$$4,\ 3,\ 80\,\sqrt{7}\,\sqrt{3}$$

$$5,\ 2,\ 375\,\sqrt{14}$$

$$5,\ 3,\ 250\,\sqrt{7}\,\sqrt{3}$$

$$5,\ 4,\ 750 \tag{9}$$

$pert1 := -\frac{3}{2\cdot K} + \frac{r^2}{2\cdot K^3} + \frac{1}{r};$

$$-\frac{3}{2\,K} + \frac{1}{2}\,\frac{r^2}{K^3} + \frac{1}{r} \tag{10}$$

$pert2 := -\frac{1}{K} + \frac{1}{r};$

$$-\frac{1}{K} + \frac{1}{r} \tag{11}$$

$int\left(RX(1, 0, r)^2\cdot r^2\cdot pert1,\ r = 0..K\right);\ taylor(\%,\ K = 0,\ 10);$

$$\frac{1}{2}\,\frac{2\,K^3 + 3 - 3\,K^2 - 3\,\mathrm{e}^{-2\,K} - 3\,\mathrm{e}^{-2\,K}\,K^2 - 6\,\mathrm{e}^{-2\,K}\,K}{K^3}$$

$$\frac{2}{5}\,K^2 - \frac{1}{3}\,K^3 + \frac{6}{35}\,K^4 - \frac{1}{15}\,K^5 + \frac{4}{189}\,K^6 + \mathrm{O}\left(K^7\right) \tag{12}$$

1

$$1 \tag{13}$$

$int\left(RX(1, 0, r)^2\cdot r^2\cdot pert2,\ r = 0..K\right);\ taylor(\%,\ K = 0,\ 10);$

$$\frac{K - 1 + \mathrm{e}^{-2\,K}\,K + \mathrm{e}^{-2\,K}}{K}$$

$$\frac{2}{3}\,K^2 - \frac{2}{3}\,K^3 + \frac{2}{5}\,K^4 - \frac{8}{45}\,K^5 + \frac{4}{63}\,K^6 - \frac{2}{105}\,K^7 + \frac{2}{405}\,K^8 + \mathrm{O}\left(K^9\right) \tag{14}$$

for n **from** 1 **to** 5 **do for** l **from** 0 **to** $n-1$ **do**
$z := int\left(RX(n, l, r)^2\cdot r^2\cdot pert1,\ r = 0..K\right);\ \ corr1(n, l) := taylor\left(z,\ K = 0,\ 8 + 2^l\right):\ \ print(n, l, corr1(n, l))$
:od:od:

$$1,\ 0,\ \frac{2}{5}\,K^2 - \frac{1}{3}\,K^3 + \frac{6}{35}\,K^4 - \frac{1}{15}\,K^5 + \mathrm{O}\left(K^6\right)$$

$$2,\ 0,\ \frac{1}{20}\,K^2 - \frac{1}{24}\,K^3 + \frac{3}{160}\,K^4 - \frac{11}{1920}\,K^5 + \mathrm{O}\left(K^6\right)$$

$$2,\ 1,\ \frac{1}{1120}\,K^4 - \frac{1}{1920}\,K^5 + \frac{1}{6048}\,K^6 + \mathrm{O}\left(K^7\right)$$

$$3,\ 0,\ \frac{2}{135}\,K^2 - \frac{1}{81}\,K^3 + \frac{46}{8505}\,K^4 - \frac{17}{10935}\,K^5 + \mathrm{O}\left(K^6\right)$$

$$3,\ 1,\ \frac{8}{25515}\,K^4 - \frac{2}{10935}\,K^5 + \frac{68}{1240029}\,K^6 + \mathrm{O}\left(K^7\right)$$

$$3,\ 2,\ \frac{4}{6200145}\,K^6 - \frac{1}{3444525}\,K^7 + \frac{2}{29229255}\,K^8 + \mathrm{O}\left(K^9\right)$$

$$4,\ 0,\ \frac{1}{160} K^2 - \frac{1}{192} K^3 + \frac{81}{35840} K^4 - \frac{13}{20480} K^5 + \mathrm{O}(K^6)$$

$$4,\ 1,\ \frac{1}{7168} K^4 - \frac{1}{12288} K^5 + \frac{37}{1548288} K^6 + \mathrm{O}(K^7)$$

$$4,\ 2,\ \frac{1}{2580480} K^6 - \frac{1}{5734400} K^7 + \frac{31}{778567680} K^8 + \mathrm{O}(K^9)$$

$$4,\ 3,\ \frac{1}{5449973760} K^8 - \frac{1}{14863564800} K^9 + \frac{1}{78721843200} K^{10} - \frac{1}{610397061120} K^{11} + \frac{1}{6183242956800} K^{12} + \mathrm{O}(K^{13})$$

$$5,\ 0,\ \frac{2}{625} K^2 - \frac{1}{375} K^3 + \frac{18}{15625} K^4 - \frac{1}{3125} K^5 + \mathrm{O}(K^6)$$

$$5,\ 1,\ \frac{8}{109375} K^4 - \frac{2}{46875} K^5 + \frac{916}{73828125} K^6 + \mathrm{O}(K^7)$$

$$5,\ 2,\ \frac{4}{17578125} K^6 - \frac{1}{9765625} K^7 + \frac{334}{14501953125} K^8 + \mathrm{O}(K^9)$$

$$5,\ 3,\ \frac{16}{101513671875} K^8 - \frac{8}{138427734375} K^9 + \frac{28}{2618408203125} K^{10} - \frac{118}{88824462890625} K^{11} + \frac{4}{32135009765625} K^{12} + \mathrm{O}(K^{13})$$

$$\begin{aligned}5,\ 4,\ &\frac{4}{164959716796875} K^{10} - \frac{2}{266473388671875} K^{11} + \frac{4}{3374176025390625} K^{12} \\ &- \frac{1}{7873077392578125} K^{13} + \frac{8}{772167205810546875} K^{14} - \frac{1}{1459980010986328125} K^{15} \\ &+ \frac{2}{52397060394287109375} K^{16} - \frac{8}{4343440532684326171875} K^{17} \\ &+ \frac{8}{101943104267120361328125} K^{18} - \frac{4}{1334969222545623779296875} K^{19} \\ &+ \frac{8}{7712800651788711547851 5625} K^{20} + \mathrm{O}(K^{21})\end{aligned} \tag{15}$$

for n **from** 1 **to** 5 **do for** l **from** 0 **to** $n-1$ **do**
$z := int\left(RX(n, l, r)^2 \cdot r^2 \cdot pert2, r = 0\,..K\right);\ \ corr1(n, l) := taylor(z, K = 0, (6 + 2 \cdot l)):\ \ print(n, l, corr1(n, l))$ **:od:od**:

$$1,\ 0,\ \frac{2}{3} K^2 - \frac{2}{3} K^3 + \frac{2}{5} K^4 + \mathrm{O}(K^5)$$

$$2,\ 0,\ \frac{1}{12} K^2 - \frac{1}{12} K^3 + \frac{7}{160} K^4 + \mathrm{O}(K^5)$$

$$2,\ 1,\ \frac{1}{480} K^4 - \frac{1}{720} K^5 + \frac{1}{2016} K^6 + \mathrm{O}(K^7)$$

$$3,\ 0,\ \frac{2}{81} K^2 - \frac{2}{81} K^3 + \frac{46}{3645} K^4 + \mathrm{O}(K^5)$$

$$3,\ 1,\ \frac{8}{10935}K^4 - \frac{16}{32805}K^5 + \frac{68}{413343}K^6 + \mathrm{O}(K^7)$$

$$3,\ 2,\ \frac{4}{2066715}K^6 - \frac{2}{2066715}K^7 + \frac{2}{7971615}K^8 + \mathrm{O}(K^9)$$

$$4,\ 0,\ \frac{1}{96}K^2 - \frac{1}{96}K^3 + \frac{27}{5120}K^4 + \mathrm{O}(K^5)$$

$$4,\ 1,\ \frac{1}{3072}K^4 - \frac{1}{4608}K^5 + \frac{37}{516096}K^6 + \mathrm{O}(K^7)$$

$$4,\ 2,\ \frac{1}{860160}K^6 - \frac{1}{1720320}K^7 + \frac{31}{212336640}K^8 + \mathrm{O}(K^9)$$

$$4,\ 3,\ \frac{1}{1486356480}K^8 - \frac{1}{3715891200}K^9 + \frac{1}{18166579200}K^{10} + \mathrm{O}(K^{11})$$

$$5,\ 0,\ \frac{2}{375}K^2 - \frac{2}{375}K^3 + \frac{42}{15625}K^4 + \mathrm{O}(K^5)$$

$$5,\ 1,\ \frac{8}{46875}K^4 - \frac{16}{140625}K^5 + \frac{916}{24609375}K^6 + \mathrm{O}(K^7)$$

$$5,\ 2,\ \frac{4}{5859375}K^6 - \frac{2}{5859375}K^7 + \frac{334}{3955078125}K^8 + \mathrm{O}(K^9)$$

$$5,\ 3,\ \frac{16}{27685546875}K^8 - \frac{32}{138427734375}K^9 + \frac{28}{604248046875}K^{10} + \mathrm{O}(K^{11})$$

$$5,\ 4,\ \frac{4}{38067626953125}K^{10} - \frac{4}{114202880859375}K^{11} + \frac{4}{674835205078125}K^{12} + \mathrm{O}(K^{13}) \qquad \textbf{(16)}$$

#graph for analytic potential
$with(plots):$
$R := 1.4:$

$Vanalytic :=$ **proc**(r)
$-\frac{1}{r}$
end proc:

$Vr :=$ **proc**(r)

if $r \geq R$ **then**
$-\frac{1}{r}$
elif $r < R$ **then**
$-\frac{3}{2}\frac{1}{R} + \frac{1}{2}\frac{1}{R}\cdot\left(\frac{r}{R}\right)^2$

end if
end proc:

$liney := PLOT(CURVES([[0, -1.8], [0, .1]],)) :$
$linex := PLOT(CURVES([[-.2, 0], [4.5, 0]],)) :$
$Text := textplot([1.5, -.3, \text{"Rn"}, font = [TIMES, ROMAN, 13]], align = right) :$
$t2 := textplot([3, -.42, \text{"V(r)/e"}, font = [TIMES, ROMAN, 13]], align = right) :$
$t3 := textplot([-.03, -.8, \text{"E/e"}, font = [TIMES, ROMAN, 13]], align = left) :$
$t4 := textplot([2, .03, \text{"r"}, font = [TIMES, ROMAN, 13]], align = above) :$
$line := \mathrm{PLOT}(\mathrm{CURVES}([[\mathrm{R}, -1.5], [R, 0]], linestyle = dots)) :$
$Vees := plot\left([\mathrm{Vanalytic}, \mathrm{Vr}], .6..5, labels = \left[r, \frac{V(r)}{e}\right], linestyle = [solid, dash], title = \text{"Potential}\right.$

Curves for a point particle and a finite size nucleus

$\left.\text{"}, font = [TIMES, ROMAN, 13], legend = [V(analytic), V(real)]\right) :$
$display(Vees, line, Text, t2, t3, t4, liney, linex)$