Fermi Surfaces and their Geometries

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1. Introduction

The Pauli exclusion principle is the source of electron degeneracy. Additional electrons to a certain volume cause the already existing electrons to increase their energy. This extra energy in turn translates into pressure, brought about by the compression of the initial volume. Electron degeneracy is responsible for the existence of white dwarfs.

A sea of electrons exists in metals. The electrons feel the force of the ions in the metal. The electron-electron interactions are negligible. But the electrons themselves behave as if they were a liquid. The quasi-particles are described by the Fermi-Dirac statistics. A system of conduction electrons can be thought of as a Fermi liquid.

If we assume that the potential of the metal lattice is the same everywhere, the Fermi liquid of conduction electrons is isotropic. The distribution function of the quasi particles is the Fermi function:

$$f(E) = \frac{1}{(e^{\frac{E - \gamma T}{t}} + 1)}$$

E is the quasi-particle energy. γ is the chemical potential and T is the temperature expressed in energy units.

For T=0, f = 1 if $E < \gamma$

      f =0 if $E > \gamma$

$\gamma(0)$ is called the Fermi energy

The radius of the Fermi sphere is:

$$N = \frac{2V}{(2\pi\hbar)^3} \frac{4\pi}{3} p_F{}^3$$

N: number of quasi particles,

V: occupied volume

p_F: radius of the Fermi sphere

At absolute zero, all orbitals are occupied by the quasi particles. Their momenta are less than the Fermi momentum p_F. The states outside of the Fermi sphere do not contain any quasi particles. At higher temperatures, the states are built by moving the quasi particles from inside the sphere to outside. The radius of the Fermi sphere remains constant when the we take the interactions between the quasi particles into account. But the Fermi energy shifts.

If we were to introduce a quasi particle that has quasi momentum **p** into the electron liquid at absolute temperature, the probability of finding the particle would go as:

$$e^{-\frac{t}{\tau_p}}$$

where,

$$\tau_p = \frac{a}{\hbar\gamma}(E(\boldsymbol{p}) - \gamma)^2$$

with a being of order of unity.

The decay comes from an imaginary part of the quasi particle energy.

The quasi particles are more or less stable when their energies approach the Fermi energy. Band structure and experiments allow us to learn about the different properties of the Fermi surface of metals. Certain values of the quasi momentum correspond to energy minima and maxima.

$$E(\boldsymbol{p}) = E(\boldsymbol{p}_0) + (\frac{1}{m})^{\alpha\beta}_{\boldsymbol{p}=\boldsymbol{p}_0} (p_\alpha - p_{0\alpha})(p_\beta - p_{0\beta})$$

Close to the points of extrema, the constant energy surfaces are ellipsoids. For these energy surfaces, close to a minimum energy point , the surface surrounds a region in which the energy has values that are less than the surface.

2. Local geometry

The observables are impacted by the shape of the Fermi surfaces. As far as high frequency of the electromagnetic or ultrasonic perturbation is concerned, the relationship between observables and Fermi surface shape becomes critical. In these conditions of high-frequency, the electrons whose motion is in synchronism with the perturbation take part in the absorption of the energy. The quasi particles are located on the effective parts of the Fermi surface for which

ω = qv

with,

ω : perturbation frequency

q: perturbation wave vector

v: electron velocity

if the electrons are moving in a transverse way relative to direction of motion of the wave, the above equation is relevant in the case of sound waves and electromagnetic waves.

The local geometry of the Fermi surface has a Gaussian curvature. The points of a surface that have a Gaussian curvature which is finite fall into three different categories. They may either be elliptic, hyperbolic or parabolic. These categories are the positive, negative, or zero values of the second differential form at the point of interest. For the points that are parabolic, the Gaussian curvature of the surface approaches zero; and so does the differential form mentioned above.

When the curvature is equal to zero, the points can be found on lines that separate the hyperbolic parts and the elliptic parts of the surface. These lines are present in most metals. The reason why the lines of zero curvature affect the observables is that the electron density of states in their neighborhood is enhanced. Below is the expression for the electron density of state on the Fermi surface:

$$N_\gamma = \frac{V}{(2\pi\hbar)^3} \int \frac{dS}{v}$$

With *dS* being the element of the surface area. The integration is over the entire Fermi surface. Instead of carrying this integration as it presently is, it is possible to turn the integral into one over the angles that define the normal to the Fermi surface. Calculations lead to:

$$dS = R_1 R_2 sin\theta d\theta d\varphi$$

If we define the quantity on the right hand side of the expression above as $d\Omega/K(\theta,\varphi)$,

then the integral becomes,

$$N_\gamma = \frac{V}{(2\pi\hbar)^3} \int \frac{d\Omega}{K(\theta,\varphi)}$$

The above equation shows that the lines of zero curvature contribute more to the electron density of states than all other lines. This means that, when the curvature of the Fermi surface approaches zero at a point, this gives an enhancement of the electrons corresponding to the vicinity of the point on the Fermi surface.

Very close to a point where the curvature is zero, the shape of the Fermi surface is that of a cylinder or a plane. It becomes a cylinder when one principal radius tends to infinity. It becomes a plane when the two radii of curvature approach infinity.

3. Attenuation of ultrasounds

An ultrasound distorts the lattice structure of metals. This phenomenon is the origin of electric fields that will have an effect on the electrons. The electronic spectrum also changes as a result. The electron energy consequently changes by the amount

$$\Delta E = \Lambda_{ij}(\boldsymbol{r},\boldsymbol{p})\, u_{ij}(\boldsymbol{r},t).$$

Where, u_{ij} is the strain tensor

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

u is the vector of the displacement of the lattice. Λ_{ij} is the deformation potential.

An electron and an ultrasound wave interact by means of the following equation:

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\partial}{\partial \boldsymbol{r}} E + e\boldsymbol{E}$$

The first term represents the deformation of the electron spectra. It is proportional to the deformation potential Λ_{ij}. The second term represents the effect of the electric field on the electron.

The quasi-particles interact with other quasi-particles. We can think of the Fermi-liquid interaction as a self-consistent field. This field originates from the fact that the electrons have an effect on an individual electron. The energy of the quasi particles changes when the state of other quasi particles changes.

4. Magnetic Field and Cyclotron Resonance and Concluding Remarks

Even though no conclusive explanation has yet been found, it is thought that the propagation of the cyclotron wave of the Fermi liquid could be responsible for the resonance peaks in the case of potassium. As far as the specular reflection of electrons is concerned, we notice a very weak singular resonance feature. We have also come to know that the presence of lines of anomalous curvature on the Fermi surface has an effect on the anomalous skin effect characteristics. As a result, the cyclotron resonance can be increased.

References

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