Neutrino Oscillations

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Abstract

Neutrinos were first theorized to conserve energy in beta decay. Since the initial postulation of the neutrino, three flavors have been theorized and discovered. Projects designed to detect a specific neutrino flavor have turned up only one third of the expected number of neutrinos. This is due to flavor oscillation, in which one flavor of neutrino transforms into another as a function of distance traveled. There have since been experiments to fully understand neutrino oscillations; these experiments have not been able to get exact values for all of oscillation parameters, but have gotten values for some and also put upper limits on values for others. Using the current parameters we are able to calculate the probability of neutrino oscillation.

1 Background

1.1 History

Neutrinos were first postulated by Wolfgang Pauli in 1930 to conserve energy and momentum in beta decay. Since then, three flavors of neutrinos have been postulated and discovered: electron neutrino, mu neutrino, and tau neutrino. They are like their lepton counterparts in that they are fermionic particles; however, they are different in that they do not carry charge and interact only via weak interactions.

Neutrinos also differ from their lepton counterparts in that they oscillate between flavors. Neutrino oscillations were postulated by Bruno Pontecorvo in the 1950s as he believed this behavior would be analogous to that of neutral kaons; calculations to represent the oscillations were done in the early 1960s. [1] However, experiments verifying these oscillations did not begin until the late 1960s, and oscillations were not concretely confirmed until 2002. [2]

1.2 Solar Neutrino Problem

The Homestake experiment, which took place in the Homestake Gold Mine in South Dakota, began operations in the late 1960s. This experiment was designed to count the number of solar neutrinos emitted by nuclear fusion in the sun. The detector, however, only counted about one-third of the neutrinos predicted. This discrepancy became known as the solar neutrino problem. Initially, the scientific community believed that something was either wrong with the prediction of the number of neutrinos for the experiment, or the experiment itself. However, other experiments designed with the same purpose in mind returned the same result. [2]

A notable experiment that provided the first evidence for mu neutrino oscillations was that of a Japnaese group in 1998 using the SuperKamiokande detector. This experiment studied atmospheric neutrinos, which are produced by cosmic rays. SuperKamiokande is located in the Kamioka mine in Japan; its location is advantageous as the surrounding rocks shield it from background. It is a water Cherenkov detector that uses 50 metric kilotons of pure water, and is lined with photomultiplier tubes to detect Cherenkov radiation. [2]

When cosmic ray protons collide with atoms in the atmosphere, pions are created; pions then quickly decay via the following sequences:

$$\pi^- \to \mu^- + \overline{\nu}_\mu, \qquad \pi^+ \to \mu^+ + \nu_\mu.$$

The mu neutrinos then decay via the following sequences:

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu, \qquad \mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu.$$

From these decay paths one would expect to detect two mu neutrinos for every electron neutrino; however, the experiment detected only about 1.3 mu neutrinos for every electron neutrino; this required that the mu neutrino had to be oscillating into something else. [2]

The Sudbury Neutrino Observatory, or SNO, in Canada was the first to show that neutrinos oscillate into other neutrino flavors. Solar neutrino detectors previous to the SNO experiment were only sensitive to electron neutrinos; for example, the detector for the Homestake experiment studied the reaction:

$$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$$

and so used a tank of C_2Cl_4 which produced an argon-37 atom every few days. These atoms were then extracted from the tank with helium gas and counted by observing their decays; the number of electron neutrinos was counted by this method. [2] The detector for the SNO experiment used heavy water, or D_2O; the deuterium allowed for the detection of all flavors of neutrinos by studying the following reactions: [3]

$$\nu_e + d \to e^- p + p, \quad \nu_x + d \to \nu_x + p + n, \quad \nu_x + e^- \to \nu_x + e^-$$

where the subscript x indicates that this can be any flavor of neutrino. The observed flux of all possible neutrinos was consistent with the original calculations done for the Homestake experiment, confirming that the model used was correct. However, the observed flux of the first reaction was only one-third of the expected value, meaning that two-thirds of the electron neutrinos had oscillated into mu and tau neutrinos; [2] the confirmation of neutrino oscillations finally solved the solar neutrino problem.

2 Neutrino Oscillations

2.1 PMNS Matrix

Each neutrino flavor state does not directly correspond to a mass eigenstate; instead, each flavor state is a linear superposition of all three mass eigenstates. The relationship between the flavor states and mass eigenstates is given by

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

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where $\alpha = e, \mu, \tau$ are the flavor indices and $i = 1, 2, 3$ are the indices for the mass eigenstates. $U_{\alpha i}$ represents the Pontecorvo-Maki-Nakagawa-Sakata matrix, which is

$$\begin{aligned} U &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \end{aligned} \tag{1}$$

where s_{ij} and c_{ij} stands for $\sin\theta_{ij}$ and $\cos\theta_{ij}$, respectively. There is no loss in generality to use the convention $0 \leq \theta_{ij} \leq \pi/2$ and $0 \leq \delta < 2\pi$. [4] The δ terms are CP violating terms; if neutrinos violate CP symmetry, this will be a nonzero value. CP violation has been observed experimentally in neutrinos, but as the findings are not yet statistically sound $\delta = 0$ is the currently used parameter. [5] In addition to the three mixing angles and CP violation term, neutrino oscillations are also sensitive to two of the mass squared differences. [4]

2.2 Oscillations

The propagation of the mass eigenstates can be described by the plane wave solution

$$|\nu_i(t)\rangle = e^{-i(E_i t - \vec{p}_i \cdot \vec{x})}|\nu_i(0)\rangle \ .$$

Quantities in this equation are expressed in natural units; that is, $c = \hbar = 1$. E_i is the energy of the mass eigenstate, t is the time elapsed, $\vec{p}_i$ is the three dimensional momentum, and $\vec{x}$ is the current position of the particle relative to its starting position.

Since we are using natural units, $t \approx L$ where L is the distance traveled. Therefore we can say that the state of a neutrino of flavor α at $x = 0$ is

$$|\nu_\alpha(x=0)\rangle = |\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle$$

and at $x = L$, the same state is

$$|\nu_\alpha(x=L)\rangle = \sum_{i=1}^{3} e^{ip_i L} U^*_{\alpha i}|\nu_i\rangle \ .$$

where p_i is the momentum four-vector. In the ultrarelativistic limit, we can approximate the momentum to be

$$p_i = \sqrt{E^2 - m_i^2} = E - \frac{m_i^2}{2E} + \cdots$$

This limit applies to all currently observed neutrinos; since the neutrino masses are on the order of eV and their energies are at least on the order of MeV, the Lorentz factor is greater than 10^6 in all cases. This approximation gives us

$$|\nu_{\alpha,0}(x=L)\rangle = e^{iEL}\sum_{i=1}^{3} \exp\left(-i\frac{m_i^2}{2E}L\right) U^*_{\alpha i}|\nu_i\rangle \ .$$

Neglecting the overall phase, which is irrelevant, the amplitude of observing the neutrino of flavor β at $x = L$ is

$$\begin{aligned}
\langle\nu_\beta|\nu_{\alpha,0}(x=L)\rangle &= \left[\sum_{j=1}^{3}\langle\nu_j|U_{\beta j}\right]\left[\sum_{i=1}^{3}\exp\left(-i\frac{m_i^2}{2E}L\right) U^*_{\alpha i}|\nu_i\rangle\right] \\
&= \sum_{j=1}^{3} U_{\beta j}\,\exp\left(-i\frac{m_j^2}{2E}L\right) U^*_{\alpha j}
\end{aligned}$$

and the probability of oscillation from $|\nu_\alpha\rangle$ to $|\nu_\beta\rangle$ with energy E and baseline L is

$$\begin{aligned}
P(\nu_\alpha \to \nu_\beta) &= \left|\sum_{j=1}^{3} U_{\beta j}\,\exp\left(-i\frac{m_j^2}{2E}L\right) U_{\alpha j}\right|^2 \\
&= \delta_{\alpha\beta} - 4\sum_{i>j}\Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\,\sin^2\Delta_{ij} + 2\sum_{i>j}\Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\,\sin 2\Delta_{ij}
\end{aligned}$$

where

$$\Delta_{ij} = \frac{\delta m^2_{ij}}{4E} L \ , \qquad \delta m^2_{ij} = m^2_i - m^2_j \ .$$

Since $\delta m_{32} = \delta m_{31} - \delta m_{21}$, $\Delta_{32} = \Delta_{31} - \Delta_{21}$, only two of the three Δ_{ij} terms are independent. [4] It is of some interest to eliminate Δ_{32} from the probability equation and expand it in terms of the other two; however, we will direct the reader's attention to the work of Honda *et al*. for calculations of this nature.

It is useful to write Δ_{ij} with c and $\hbar$ restored as follows:

$$\Delta_{ij} = \frac{\delta m^2_{ij} c^3 L}{4\hbar E} = \frac{\text{GeV fm}}{4\hbar c} \times \frac{\Delta m^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E} \approx 1.267 \times \frac{\Delta m^2}{\text{eV}^2} \frac{L}{\text{km}} \frac{\text{GeV}}{E} \ .$$

This form is convenient for plugging in the oscillation parameters as the mass differences are on the order of eV^2, baselines in experiments are on the order of kilometers, and neutrino energies used in experiments are usually on the order of GeV. [4]

The most current oscillation parameters are $\theta_{13} < 10.3°$ [4], $\theta_{12} = 33.9°^{+2.4°}_{-2.2°}$ [6], $\theta_{23} = 45 \pm 7°$ [7], $\delta m^2_{21} = 7.9^{+0.6}_{-.04} \times 10^{-5}\text{eV}^2$ [6], and $|\delta m^2_{31}| \approx |\delta m^3_{32}| = 2.4^{+0.6}_{-0.5} \times 10^{-3}\text{eV}^2$. [7] As previously stated, δ is currently unknown but is currently taken to be $\delta = 0$ in these calculations. We will see how making this term nonzero changes the oscillation probability between the neutrino and anti-neutrino. Also unknown is the sign of δm^2_{32}.

Figures 1, 2, and 3, which describe the oscillation probability of the electron neutrino, mu neutrino, and tau neutrino respectively, were generated using the current oscillation parameters; since we are using $\delta = 0$, these graphs are valid for both the neutrino and anti-neutrino. As a point of interest, figures 4, 5, and 6 were generated using the same parameters except $\delta = \pi/2$. Note how a nonzero value of δ changes the phase between the neutrino and anti-neutrino case.

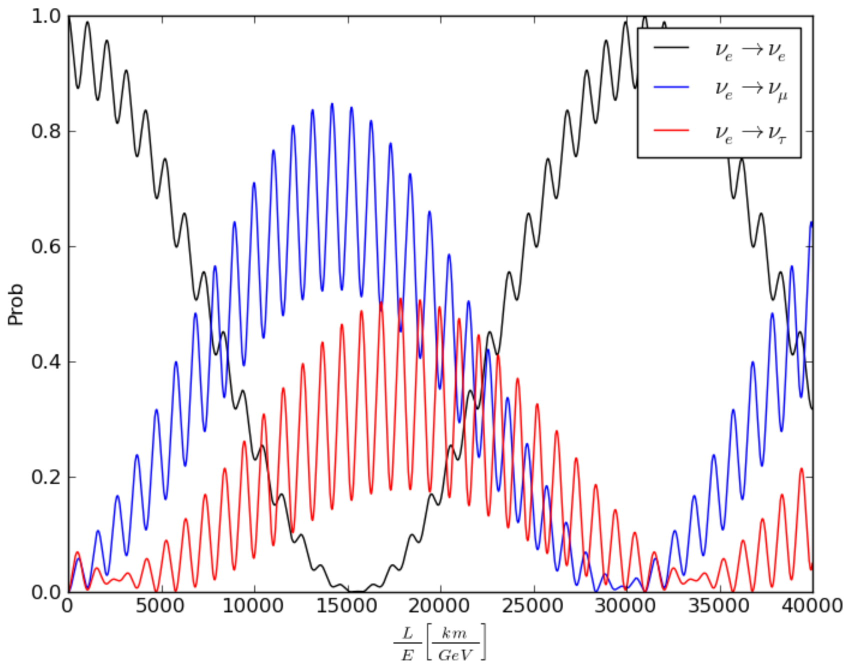

Figure 1: Oscillation probability of the electron neutrino using current oscillation parameters.

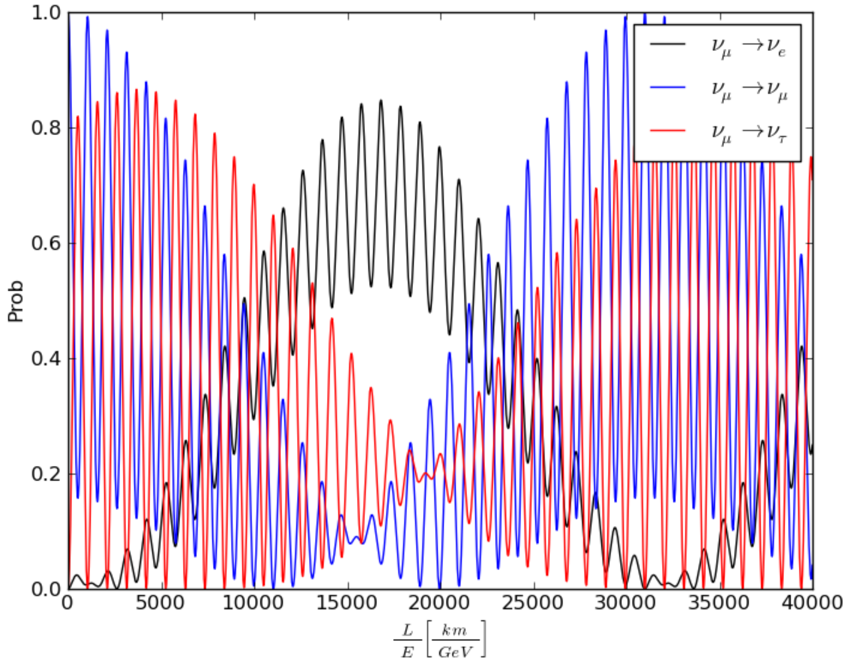

Figure 2: Oscillation probability of the mu neutrino using current oscillation parameters.

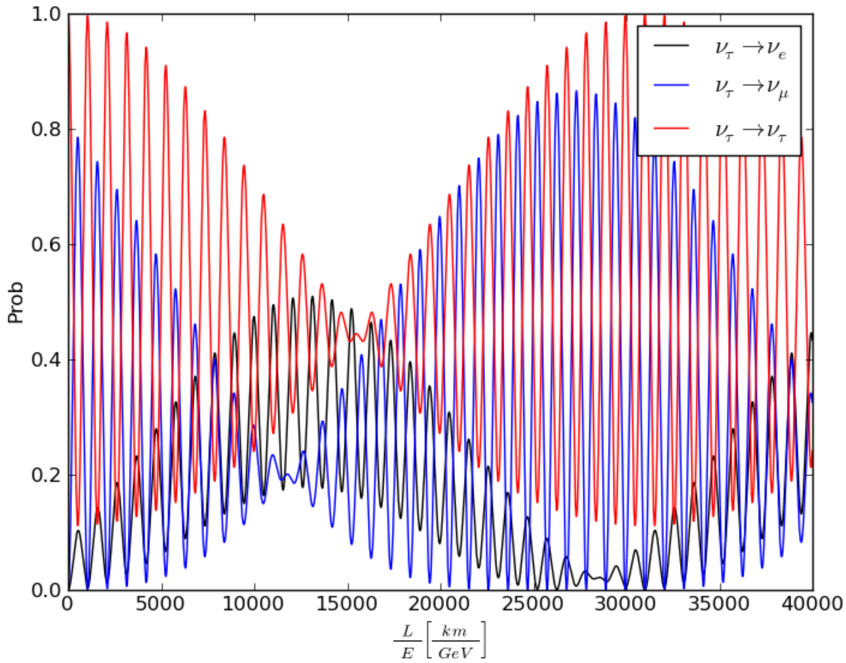

Figure 3: Oscillation probability of the tau neutrino using current oscillation parameters.

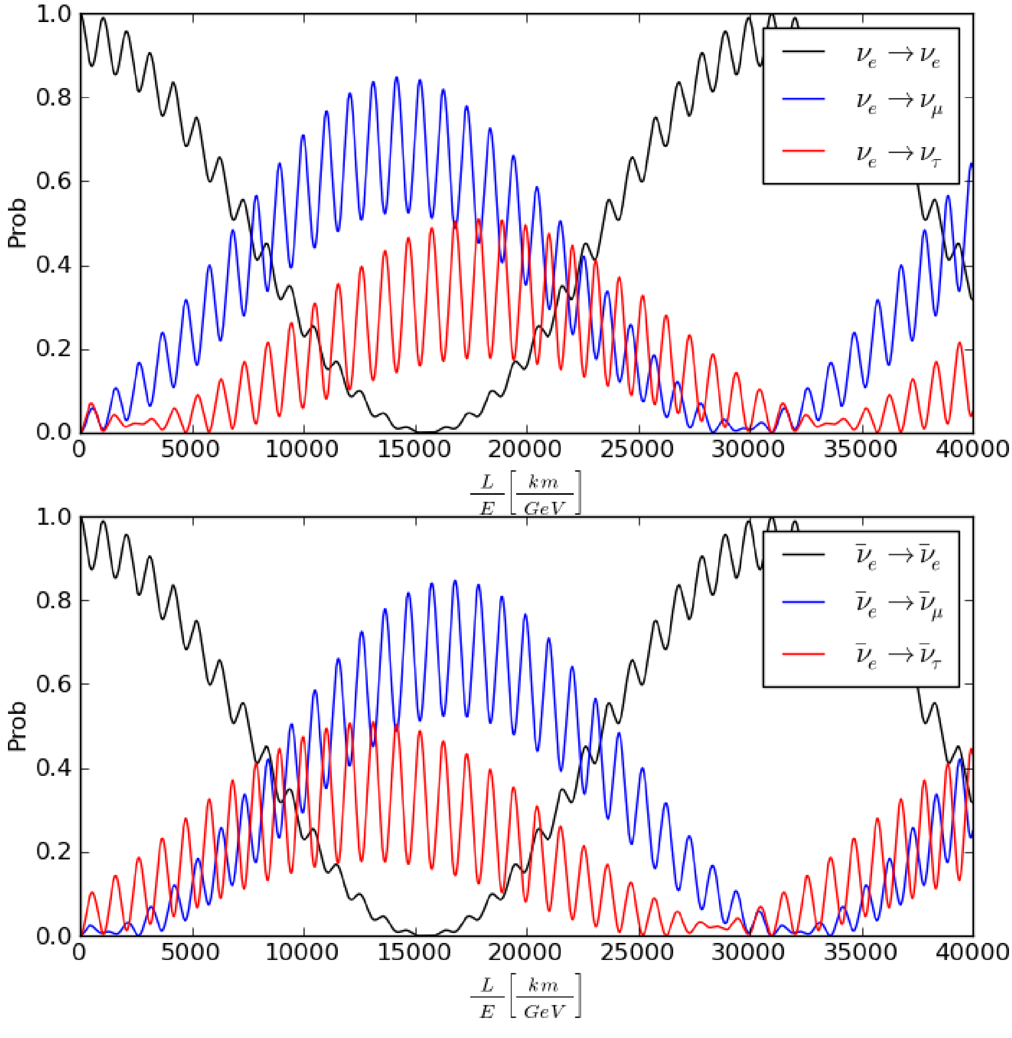

Figure 4: Oscillation probability of the electron neutrino (top) and anti-neutrino (bottom) using $\delta = \pi/2$.

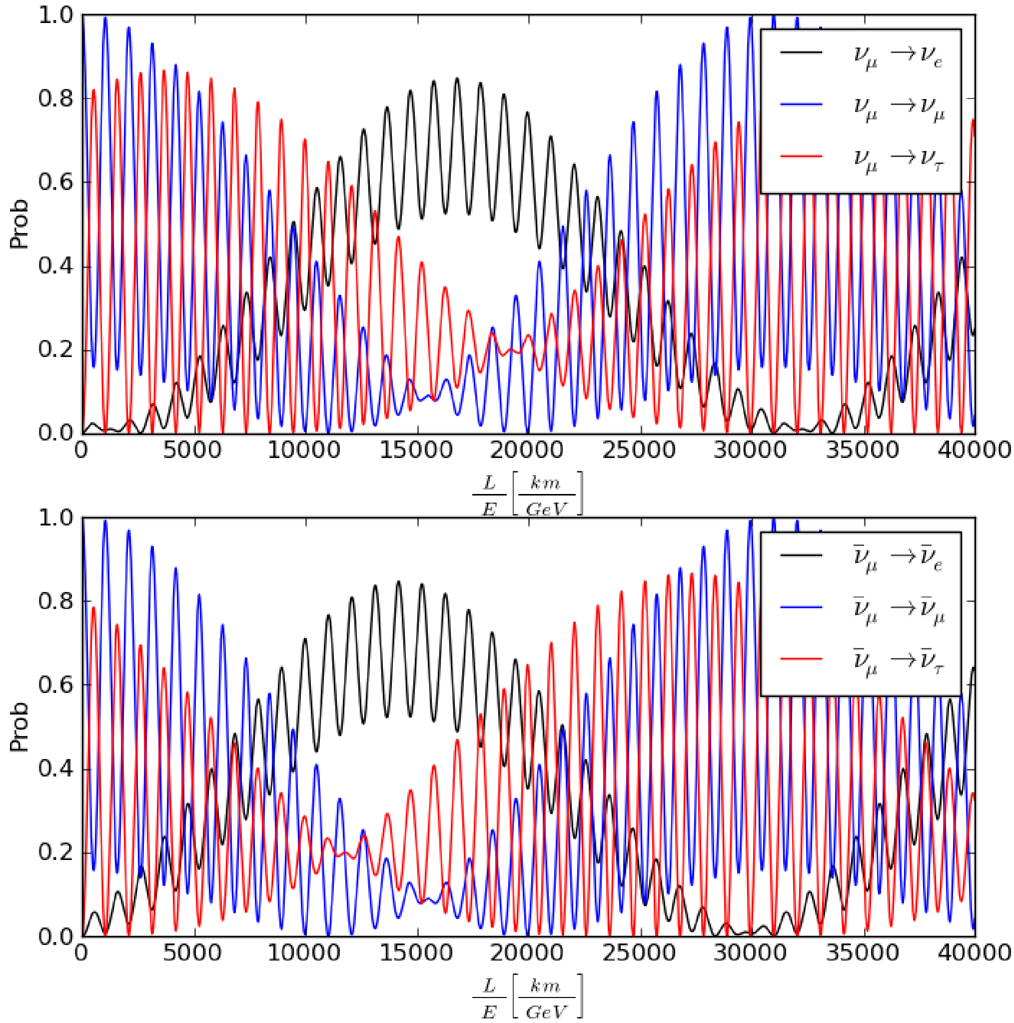

Figure 5: Oscillation probability of the mu neutrino (top) and anti-neutrino (bottom) using $\delta = \pi/2$.

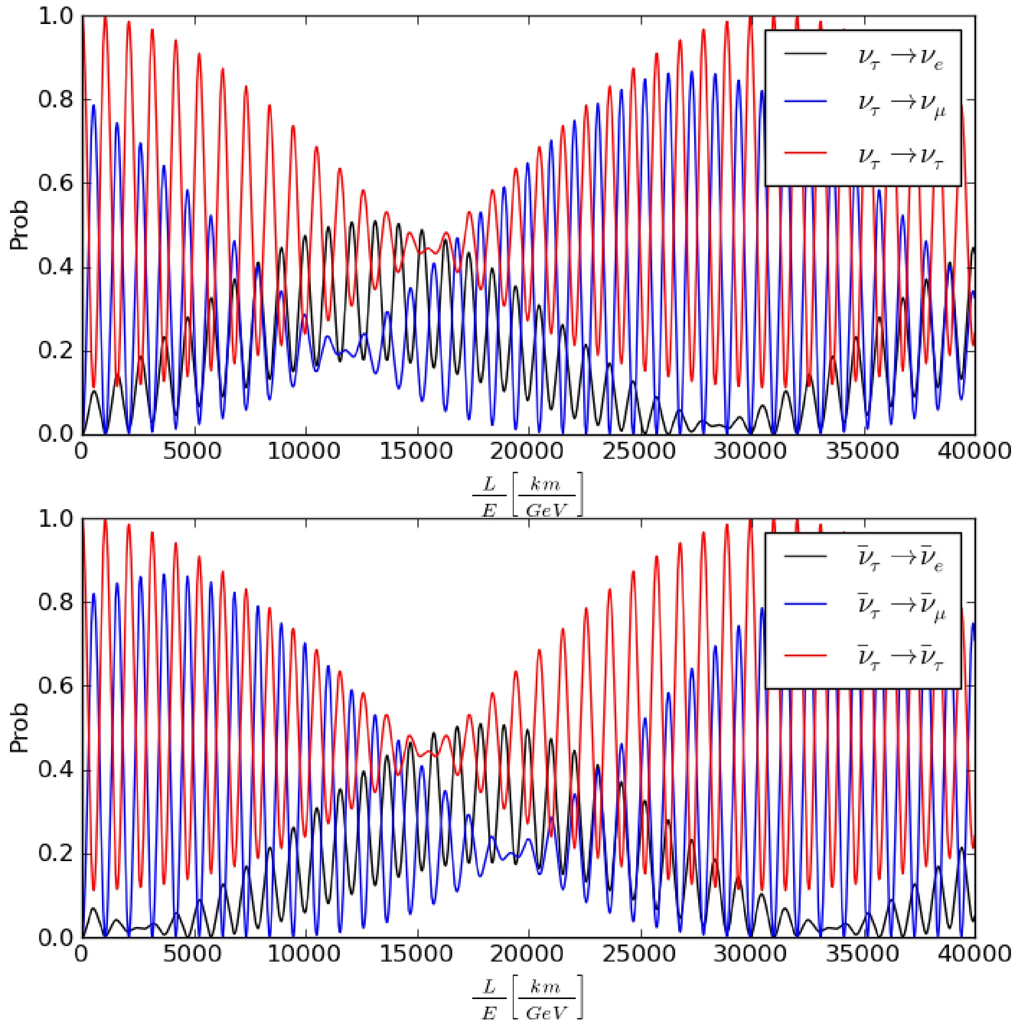

Figure 6: Oscillation probability of the tau neutrino (top) and anti-neutrino (bottom) using $\delta = \pi/2$.

References

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