Neutrinos And Their Oscillations

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Neutrinos are neutral leptons in the standard model of particle physics. State mixing between the three neutrino species can occur because the flavor eigenstates by which they interact with the weak nuclear force are a superposition of the three mass eigenstates by which they propagate, and vice versa. This neutrino oscillation depends on several things, including the properties of the neutrinos themselves, how many neutrino species are considered, and the medium they are propagating through. These topics are discussed here, along with the current state of neutrino research.

1 Introduction

When beta decay was discovered in the early twentieth century, it put physicists in a jam. This process, which was interpreted at the time as being a neutron converting to a proton and emitting an electron, appeared to violate conservation of energy. The electrons in this decay have a continuous spectrum of energies for a given decay nucleus, so it appeared that energy was disappearing. Wolfang Pauli suggested that a light neutral particle was carrying away the energy undetected.

Pauli's idea was put to the test, and in the early 1950's, Clyde Cowan and Fred Reines set up a detector near a nuclear power plant and were successful in seeing antineutrinos. These neutrinos were soon determined to be associated with the electron. Soon after, neutrinos were discovered for the other charged leptons, the muon and tauon.

The acquisition of an interaction cross section meant that a value for the expected number of neutrinos for an experiment could be calculated. The Homestake experiment was looking at neutrinos from solar fusion and reported a large deficit in the expected number of neutrinos. Some speculated that the three neutrino species were mixing together and that the probability of detecting the electron flavor that was created in the sun as opposed to the other flavors was changing. [1]

The concept of this neutrino oscillation is based on the premise that the flavor eigenstates are not bound to the mass eigenstates, and that they are superpositions of each other. This paper discusses the general idea of neutrino oscillations, and then a two neutrino case both in a vacuum, and through a medium. Then a three neutrino case is considered. Finally, the current state of neutrino research is discussed.

2 A Problem of Solar Proportions

The sun, like all stars on the main sequence, fuses hydrogen into helium in its core. The process by which 98% of this fusion occurs is known as the proton-proton chain reaction. The first step of this reaction involves two hydrogen atoms fusing into deuterium plus a positron and an electron neutrino; the remaining mass difference is released as photons, or is added onto the neutrino as kinetic energy. The deuterium then fuses with another hydrogen atom to form ^3He. From there, several different paths exist for further fusion, the most common of which for the sun terminates at ^4_2He. [2]

The luminosity of the sun has been measured to be $4 \times 10^{26} W$. Knowing that each fusion chain reaction releases 26.7MeV, one can calculate the neutrino flux from the sun, which turns out to be 2×10^{38} neutrinos per second. Asuming spherical symmetry, this translates to a flux of about $6 \times 10^{10} cm^{-2} s^{-1}$ on the surface of the earth. [1]

In the late 1960s, Homestake mine in South Dakota hosted the first experiment dedicated to solar neutrino detection. The detector was filled with perchloroethlyne; upon interacting with a neutrino, the chlorine in this liquid turns into a radioactive isotope of argon. This argon is then collected and counted, each atom corresponding to a neutrino event. Taking what was known about neutrino flux from the sun, and neutrino interaction cross section, physicists at Homestake expected about 7.6 captures per second per 10^{36} atoms. The actual number observed, though, was about 1/3 the expected value. This meant that either the solar model was wrong, the Homestake detector was inaccurate, or something happens to neutrinos as they travel from their source. [2]

3 It's All About The Eigenstates

State mixing occurs in neutrinos because the weak eigenstates which interact via W and Z bosons (ν_e, ν_μ, ν_τ) do not have a one-to-one relationship with the mass eigenstates by which they propagate with a definite mass

(ν_1, ν_2, ν_3). This is only possible if the mass eigenstates all have different magnitudes. In essence, the weak eigenstates ν_α are linear combinations of the mass eigenstates ν_i, as follows:

$$\mid \nu_\alpha(t)\rangle = \sum_{i=1}^{n} U^*_{\alpha i} \mid \nu_i(t)\rangle \tag{1}$$

Where the mixing matrix U is a unitary matrix, ensuring that the number of neutrinos is conserved. The index n is the number of neutrino flavors participating in the model. Note the time-dependence. [3]

Ignoring matter effects, the probability of an oscillation from ν_α to ν_β is

$$\begin{aligned} P_{\alpha\beta} &= \mid \langle \nu_\beta \mid \nu_\alpha(t)\rangle \mid^2 \\ &= \mid \sum_{i=1}^{n}\sum_{j=1}^{n} U^*_{\alpha i} U_{\beta j} \langle \nu_j(0) \mid \nu_i(t)\rangle \mid^2 \end{aligned} \tag{2}$$

If we assume that we are dealing with a plane wave, the time dependent part can be simplified. [3]

$$\mid \nu_i(t)\rangle = \mid \nu_i(0)\rangle e^{-iE_i t} \tag{3}$$

And finally, after multiplying everything out:

$$\begin{aligned} P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} Re\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right] sin^2\left(x_{ij}\right) \\ +2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} Im\left[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}\right] sin\left(x_{ij}\right) \end{aligned} \tag{4}$$

Where

$$x_{ij} = 1.27\frac{\Delta m^2_{ij} L}{E}\frac{MeV}{m\ eV^2} \tag{5}$$

Here, $\Delta m^2_{ij} = m^2_i - m^2_j$ is the mass squared difference of the mass eigenstates i and j, L is the distance (in meters) the oscillating neutrino traveled, and E is the energy of the neutrino. It is easy to see now that no oscillations occur if either the mass squared difference, or a matrix element U is zero. [3] Experimental evidence of neutrino oscillations suggests that the masses of the neutrinos are indeed non-zero.

The $\delta_{\alpha\beta}$ term that arises represents the charge-parity violating phase. This term and the imaginary matrix elements are zero for our purposes here, as CP violation in leptons has yet to be confirmed.

4 Two Neutrinos In A Vacuum

For a two neutrino case, the unitary matrix U is defined simply as

$$U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{6}$$

Because there is only one mass squared difference and one mixing angle to consider for the two neutrino case, the calculation of the oscillation probability becomes much simpler:

$$P_{\alpha\beta} = sin^2(2\theta)\ sin^2\left(1.267\frac{\Delta m^2 L}{E}\frac{MeV}{eV^2 m}\right) \tag{7}$$

[1]

Plugging in 0.59 radians for the mixing angle and $8\times10^{-5}eV^2$ for the mass difference yields the plot seen in Figure 1.

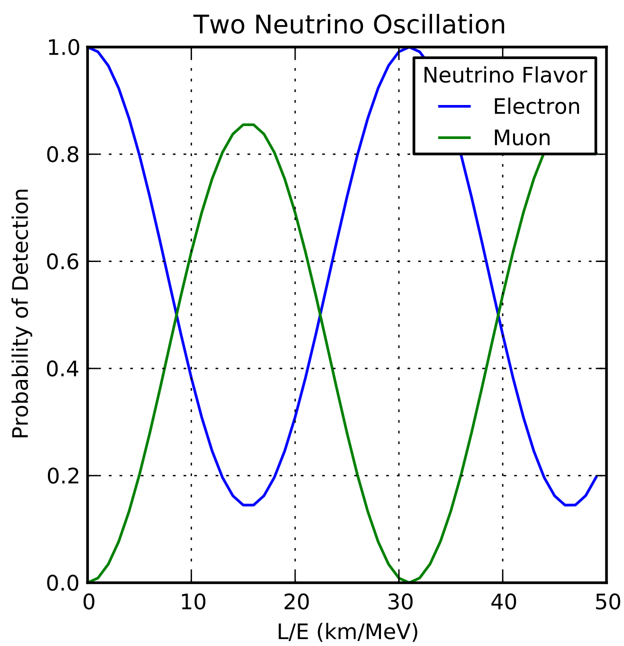

Fig. 1. Graphical example of two-neutrino oscillation

5 Two Neutrinos In Matter

When a neutrino is propagating through a medium, its properties are affected by that medium. Although neutrinos have an astronomically small interaction cross section at around $10^{-43}cm^2\left(\frac{E}{1\ MeV}\right)^2$, elastic scattering must be taken into account. In one approximation for the matter effects on neutrinos, an effective potential is used to describe how the medium changes the way neutrinos propagate. [3]

All three neutrino species are subject to an effective potential that arises from neutral current interactions due to matter. In addition to this, electron neutrinos see a charge current interaction. The standard model says that these two describe all of the interactions that neutrinos engage in. Because the neutral current interactions affects all flavors of neutrinos equally, they do not affect the neutrino oscillation probabilities. The charge current interactions exclusive to the electron flavor, however, do affect oscillations. [3]

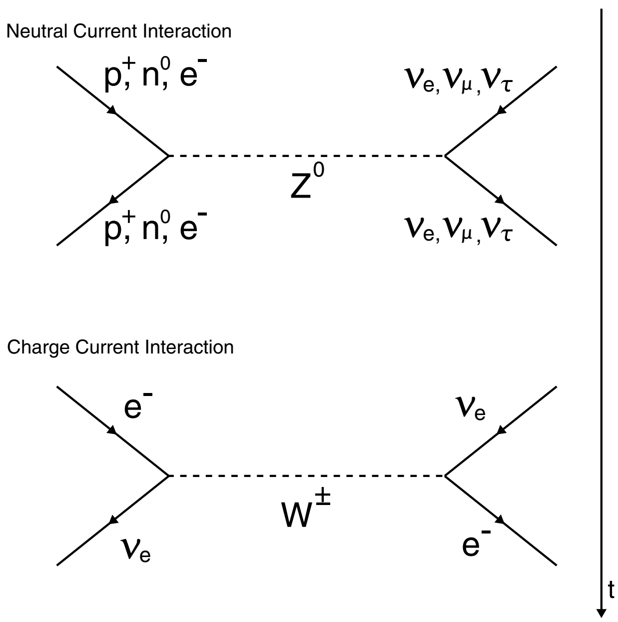

Fig. 2. Feynman Diagrams of the two possible neutrino interactions: neutral current and charge current

The charge current interactions between electron neutrinos and the medium leads to an effective potential of

$$V_{CC} = \sqrt{2} G_F N_e \approx 7.6 Y_e \frac{\rho}{10^{14}} \frac{cm^3}{g} eV \quad (8)$$

Here, G_F is the Fermi coupling constant, N_e is the electron number density of the medium, and Ye is the relative number density of the medium, equal to $\frac{N_e}{N_p+N_n}$. [3]

This effective potential leads to differences in the oscillation parameters for the mass squared difference and the mixing angle.

$$(\Delta m^2)^m = \sqrt{(\Delta m^2 cos2\theta - A)^2 + (\Delta m^2 sin2\theta)^2} \quad (9)$$

$$sin2\theta^m = \frac{sin2\theta \Delta m^2}{(\Delta m^2)^m} \quad (10)$$

Here, $A = \pm 2\sqrt{2} G_F N_e E$, where E is the energy of the neutrino. The plus or minus is used for neutrinos and antineutrinos respectively. [1]

6 Three Neutrinos In A Vacuum

For a three neutrino case, things get more complicated. Now there are two mass squared differences, Δm_{12} and Δm_{23}, and three mixing angles, θ_{12}, θ_{23}, and θ_{13}, that must be considered to accomodate the three weak and mass eigenstates. The unitary matrix U used to describe oscillations involving three neutrinos is called the PMNS matrix:

$$U = U_{\theta_{12}} U_{\theta_{13}} U_{\theta_{23}} \quad (11)$$

$$U_{\theta_{12}} = \begin{pmatrix} cos\theta_{12} & sin\theta_{12} & 0 \\ -sin\theta_{12} & cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (12)$$

$$U_{\theta_{13}} = \begin{pmatrix} cos\theta_{13} & 0 & sin\theta_{13} \\ 0 & 1 & 0 \\ -sin\theta_{13} & 0 & cos\theta_{13} \end{pmatrix} \quad (13)$$

$$U_{\theta_{23}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_{23} & sin\theta_{23} \\ 0 & -sin\theta_{23} & cos\theta_{23} \end{pmatrix} \quad (14)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \quad (15)$$

Where $c_{ij} = cos(\theta_{ij})$ and $s_{ij} = sin(\theta_{ij})$. [1] Plugging accepted values for the mixing angles and mass squared differences into equation 4 gives the following plot for the behavior of three neutrinos in a vacuum, given an initial electron neutrino.

7 What We Know

Ever since the concept of neutrino oscillations was developed as an explanation for the solar neutrino

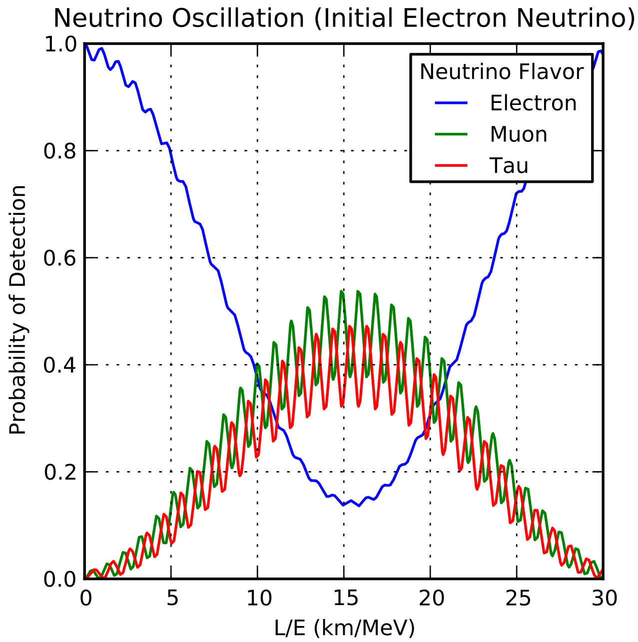

Fig. 3. Graphical example of three-neutrino oscillation beginning with an electron neutrino

deficit, many neutrino detection experiments have been carried out not only with the goal of confirming oscillations, but also to pin down the oscillation parameters. Sources of neutrinos for these experiments come from solar fusion, cosmic ray decay in the atmosphere, nuclear reactor, and particle accelerators. Each experiment either looks for an absence in expected detections from a single neutrino flavor source (disappearance experiment), or looks for a flavor of neutrino not created from the source (appearance experiment). [3] Data from various sources in both appearance and disappearance contexts has helped to piece together our understanding of how neutrinos function.

Experiments such as KamLAND in Japan have looked at electron neutrinos created during the solar fusion process. Solar neutrinos have relatively low energy and a very long path to travel between the Sun and Earth. Taking into account matter effects from the core of the sun, as well as day-night effects on Earth [1], experiments looking at solar neutrino oscillations have been able to put constraints on Δm^2_{21} and $sin^2\theta_{12}$. [3]

Cosmic rays are high energy particles from space which decay upon entering Earth's atmosphere. The subsequent decay chains include mesons (namely pions) which in turn results in atmospheric neutrinos:

$$\pi \rightarrow \mu + \nu_\mu \qquad \mu \rightarrow e + \nu_\mu + \nu_e \tag{16}$$

This results in a 2:1 ratio of muon neutrinos to electron neutrinos as a result of cosmic ray decays. [3] Like the solar neutrino deficit discussed earlier, experiments such as Super-Kamiokande also saw a variation in the expected number of muon neutrinos which changed as a funcion of the zenith angle between the detector and the neutrino source. Analysis of this data led to constraints on Δm^2_{31} and $sin^2\theta_{23}$. [1]

Constraints on the remaining mixing angle, θ_{13}, are far from precise, however, information from the acquisition of the other parameters has offered a loose upper bound for this third mixing angle. This angle may very well be zero, but if it isn't, then the possibility of a non-zero CP-violating phase δ_{CP} comes into play as well. This would be the first instance of CP violation in the lepton sector. [1]

Another unknown in the realm of neutrinos is the ordering of the mass eigenstates. Evidence from solar neutrino experiments suggests with a high degree of certainty that $\Delta m^2_{21} = m^2_2 - m^2_1$ is positive. The same cannot be said for the atmospheric data, thus the sign of $\Delta m^2_{31} = m^2_3 - m^2_1$ is unknown. As a result, there are two possible neutrino mass hierarchies; they are the so called 'normal hierarchy' where $m^2_3 > m^2_2 > m^2_1$, and the 'inverted hierarchy' where $m^2_2 > m^2_1 > m^2_3$. [1]

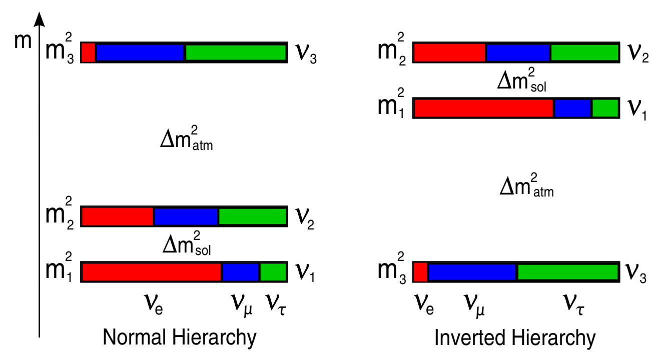

Fig. 4. Normal mass hierarchy vs. inverted mass hierarchy

The three unknown parameters discussed here, θ_{13}, δ_{CP}, and $sgn(\Delta m^2_{31})$ can all be probed by looking at oscillations between electron neutrinos and muon neutrinos: $\nu_e \rightarrow \nu_\mu$ or $\nu_\mu \rightarrow \nu_e$. Several upcoming experiments including T2K in Japan and NOνA in the United States hope to do just that. [1]

While finding these unknowns are all very important to our understanding of how neutrinos behave, the CP-violating phase in particular, if found to be non-zero, has some exciting implications. If the effect is large enough, CP-violating neutrinos may be able to explain the matter anti-matter asymmetry in the universe. Other properties yet to be figured out include, but are not limited to, the absolute scale of neutrino masses (i.e., the difference between zero and the lightest mass eigenstate), whether neutrinos are dirac or majorana, and an explanation as to why neutrinos seem to be so light in the first place. [1]

Oscillation Parameters	
$\Delta m^2_{21}[eV^2]$	7.6×10^{-5}
$\|\Delta m^2_{31}\|[eV^2]$	2.4×10^{-3}
$sin^2\theta_{12}$	0.32
$sin^2\theta_{23}$	0.50
$sin^2\theta_{13}$	0.007

Table 1. Current best-fit values for the oscillation parameters [1]

Clearly we have a long way to go before the nature of neutrinos is fully explained. The analysis of oscillation parameters has revealed much, and they will surely continue to do so in the future.

8 Conclusion

Here we have seen the basic progression that neutrino physics has taken since it started in the early twentieth century. Physicists saw that something was wrong when the neutrino count from the sun didn't add up, so oscillations were offered as a solution to this problem. Oscillations are possible because the neutrino flavor states are mixed among the mass eigenstates, which themselves have different magnitudes.

Oscillations depend first and foremost on the number of neutrino species, and the medium of propagation. They also depend on the mass squared differences of the mass eigenstates, and the mixing angles. Currently, constraints have been put on the two mass squared differences, and two of the three mixing angles. The big three that have yet to be measured are the mixing angle θ_{13}, the sign of Δm^2_{31}, and the CP-violating phase δ_{CP}. Each has its own implications in the grand scheme of things, but all are important in fully understanding neutrinos.

References

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[3] M.C. Gonzalez-Garcia, Y. N. "Neutrino masses and mixing: Evidence and implications". *Rev.Mod.Phys.,* **75**.