

The Topology of Chaos

Chapter 10: Summary

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Summary

**1 Question Answered \Rightarrow
2 Questions Raised**

We must be on the right track !

Original Objectives Achieved

There is now a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Result

**There is now a classification theory
for low-dimensional strange attractors.**

- ① It is topological
- ② It has a hierarchy of 4 levels
- ③ Each is discrete
- ④ There is rigidity and degrees of freedom
- ⑤ It is applicable to R^3 only — for now

The Classification Theory has 4 Levels of Structure

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① Basis Sets of Orbits

The Classification Theory has 4 Levels of Structure

- 1 Basis Sets of Orbits
- 2 Branched Manifolds

The Classification Theory has 4 Levels of Structure

- 1 Basis Sets of Orbits
- 2 Branched Manifolds
- 3 Bounding Tori

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- 1 Basis Sets of Orbits
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- 3 Bounding Tori
- 4 Extrinsic Embeddings

Four Levels of Structure

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Summary-01

Summary-02

Summary-03

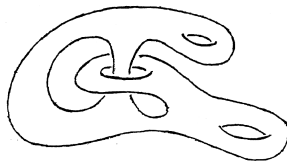
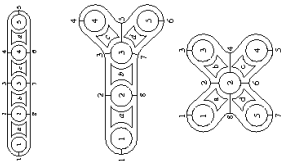
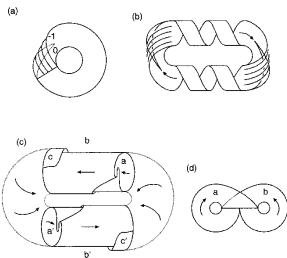
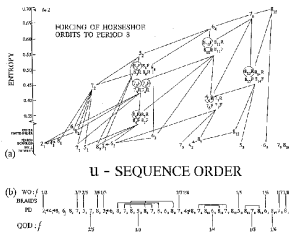
Summary-04

Summary-05

Summary-06

Summary-07

Summary-08



Poetic Organization

LINKS OF PERIODIC ORBITS

organize

BOUNDING TORI

organize

BRANCHED MANIFOLDS

organize

LINKS OF PERIODIC ORBITS

Some Unexpected Results

- Perestroikas of orbits constrained by branched manifolds
- Routes to Chaos = Paths through orbit forcing diagram
- Perestroikas of branched manifolds constrained by bounding tori
- Global Poincaré section = union of $g - 1$ disks
- Systematic methods for cover - image relations
- Existence of topological indices (cover/image)
- Universal image dynamical systems
- NLD version of Cartan's Theorem for Lie Groups
- Topological Continuation – Group Continuation
- Cauchy-Riemann symmetries
- Quantizing Chaos
- Representation labels for inequivalent embeddings
- Representation Theory for Strange Attractors

We hope to find:

- Robust topological invariants for R^N , $N > 3$
- A Birman-Williams type theorem for higher dimensions
- An algorithm for irreducible embeddings
- Embeddings: better methods and tests
- Analog of χ^2 test for NLD
- Better forcing results: Smale horseshoe, $D^2 \rightarrow D^2$,
 $n \times D^2 \rightarrow n \times D^2$ (e.g., Lorenz), $D^N \rightarrow D^N$, $N > 2$
- Representation theory: complete
- Singularity Theory: Branched manifolds, splitting points
(0 dim.), branch lines (1 dim).
- Singularities as obstructions to isotopy