The Topology of Chaos
Chapter 8: Quantizing Chaos

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Physics and Topology Workshop
Drexel University, Philadelphia, PA 19104

September 2, 2008

Rotating the Attractor

$$\frac{d}{dt}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_1(X,Y) \\ F_2(X,Y) \end{bmatrix} + \begin{bmatrix} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} \cos\Omega t & -\sin\Omega t \\ \sin\Omega t & \cos\Omega t \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix}$$

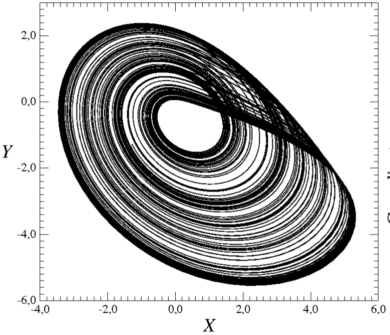
$$\frac{d}{dt}\begin{bmatrix} u \\ v \end{bmatrix} = R\mathbf{F}(R^{-1}\mathbf{u}) + R\mathbf{t} + \Omega \begin{bmatrix} -v \\ +u \end{bmatrix}$$

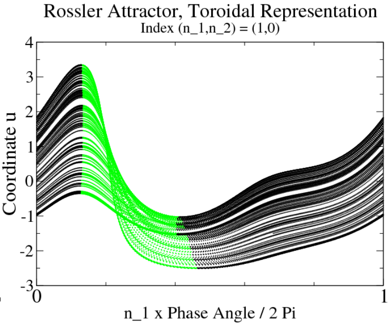
$\Omega = n\ \omega_d$ — Global Diffeomorphisms

$q\ \Omega = p\ \omega_d$ — Local Diffeomorphisms (p-fold covers)

Rossler Attractor: Two Representations

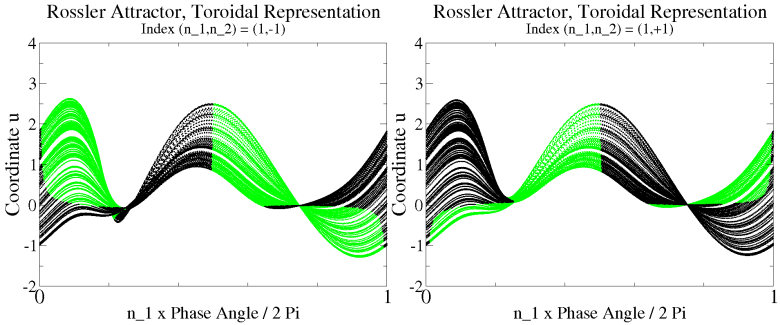
R^3 $\qquad$ $D^2 \times S^1$





Rossler Attractor:

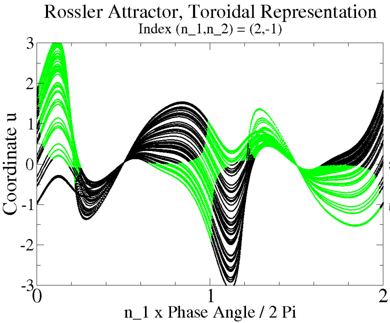
Two More Representations with $n = \pm 1$

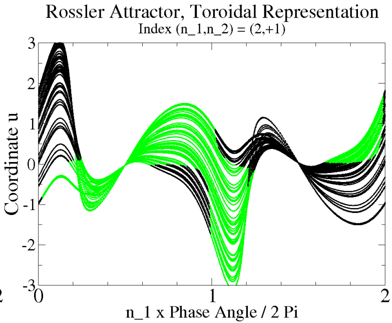


Subharmonic, Locally Diffeomorphic Attractors

Rossler Attractor:

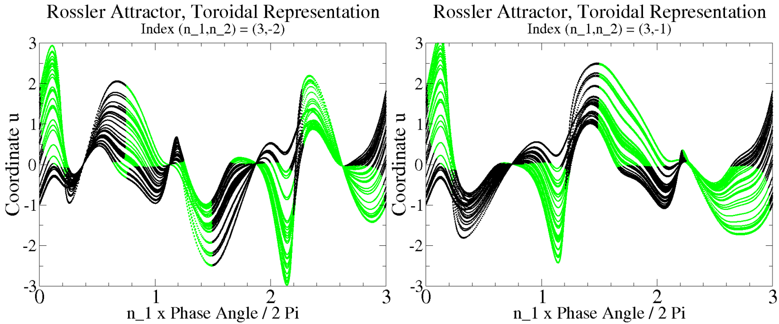
Two Two-Fold Covers with $p/q = \pm 1/2$





Rossler Attractor:

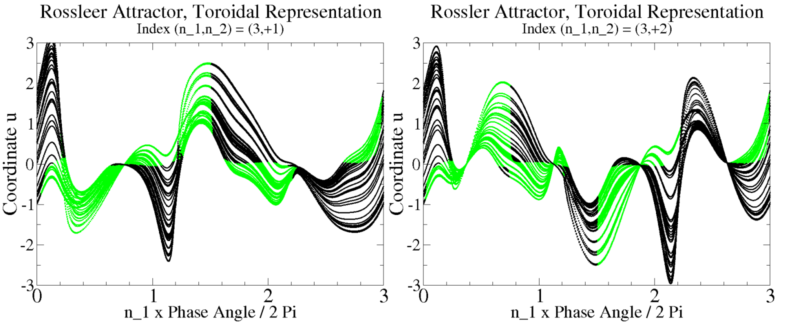
Two Three-Fold Covers with $p/q = -2/3, -1/3$



Subharmonic, Locally Diffeomorphic Attractors

Rossler Attractor:

And Even More Covers (with $p/q = +1/3, +2/3$)



Angular Momentum and Energy

$$L(0) = \lim_{\tau\rightarrow\infty} \frac{1}{\tau} \int_0^\tau XdY - YdX \qquad K(0) = \lim_{\tau\rightarrow\infty} \frac{1}{\tau} \int_0^\tau \frac{1}{2}(\dot{X}^2 + \dot{Y}^2)dt$$

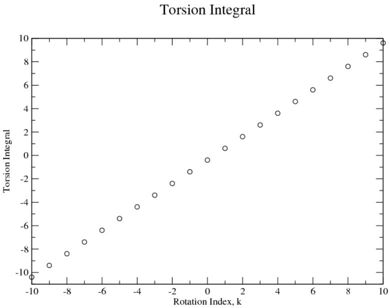
$$L(\Omega) = \langle u\dot{v} - v\dot{u} \rangle \qquad K(\Omega) = \langle \frac{1}{2}(\dot{u}^2 + \dot{v}^2) \rangle$$

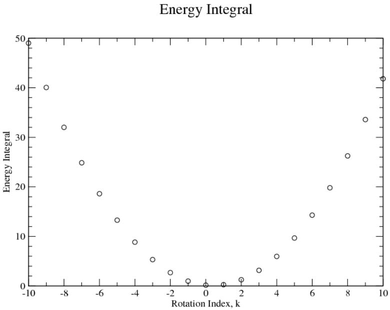
$$= L(0) + \Omega\langle R^2 \rangle \qquad = K(0) + \Omega L(0) + \frac{1}{2}\Omega^2\langle R^2 \rangle$$

$$\langle R^2 \rangle = \lim_{\tau\rightarrow\infty} \frac{1}{\tau} \int_0^\tau (X^2 + Y^2)dt = \lim_{\tau\rightarrow\infty} \frac{1}{\tau} \int_0^\tau (u^2 + v^2)dt$$

Energy and Angular Momentum

Diffeomorphic, Quantum Number n





Energy and Angular Momentum

Subharmonics, Quantum Numbers p/q