

# The Topology of Chaos

## Chapter 8: Quantizing Chaos

Robert Gilmore

Physics Department  
Drexel University  
Philadelphia, PA 19104  
robert.gilmore@drexel.edu

Physics and Topology Workshop  
Drexel University, Philadelphia, PA 19104

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## Rotating the Attractor

$$\frac{d}{dt} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_1(X, Y) \\ F_2(X, Y) \end{bmatrix} + \begin{bmatrix} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{bmatrix}$$

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix}$$

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = R\mathbf{F}(R^{-1}\mathbf{u}) + R\mathbf{t} + \Omega \begin{bmatrix} -v \\ +u \end{bmatrix}$$

$$\Omega = n \omega_d$$

$$q \Omega = p \omega_d$$

Global Diffeomorphisms

Local Diffeomorphisms

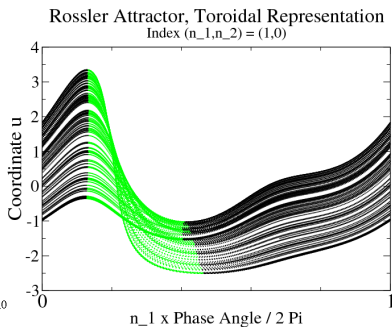
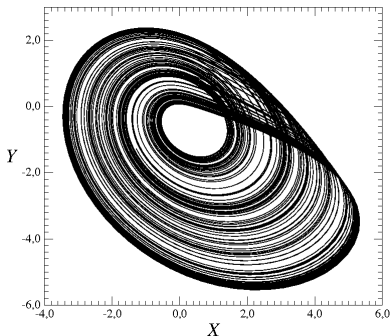
( $p$ -fold covers)

# Two Phase Spaces: $R^3$ and $D^2 \times S^1$

## Rosler Attractor: Two Representations

$R^3$

$D^2 \times S^1$



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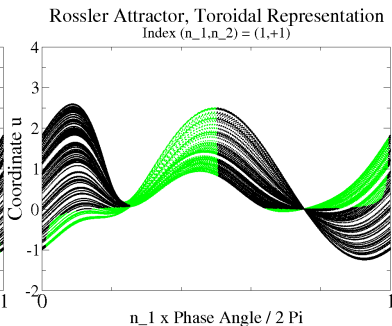
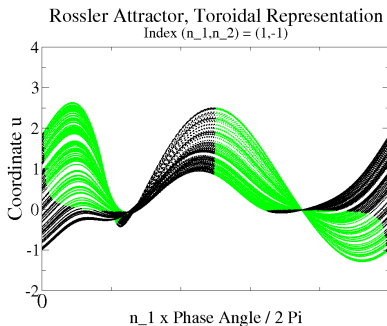
Quantizing  
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Quantizing  
Chaos-08

# Other Diffeomorphic Attractors

## Rossler Attractor:

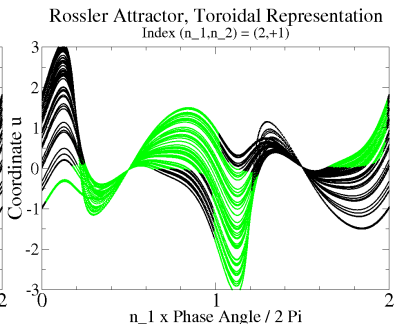
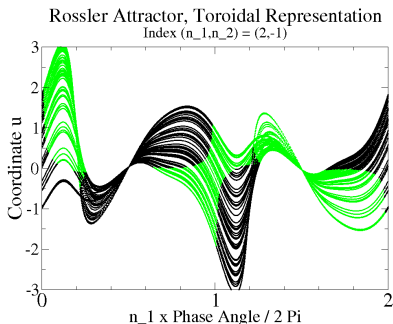
### Two More Representations with $n = \pm 1$



# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

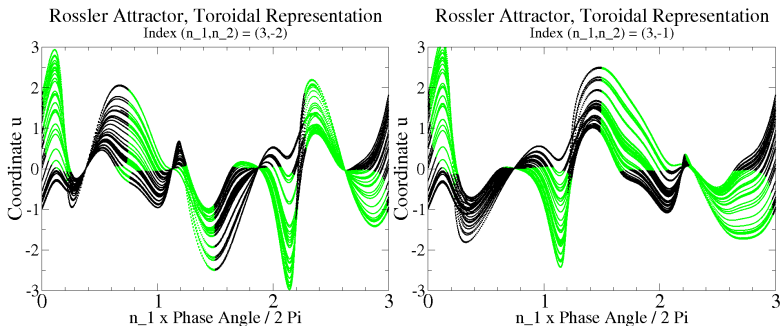
### Two Two-Fold Covers with $p/q = \pm 1/2$



# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

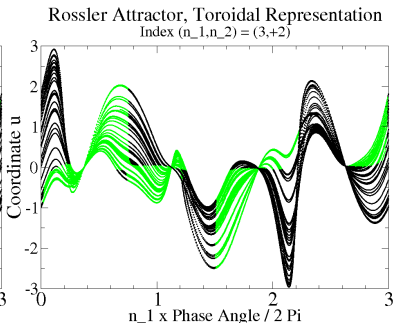
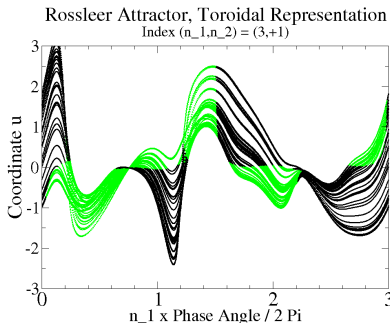
Two Three-Fold Covers with  $p/q = -2/3, -1/3$



# Subharmonic, Locally Diffeomorphic Attractors

## Rossler Attractor:

And Even More Covers (with  $p/q = +1/3, +2/3$ )



## Angular Momentum and Energy

$$L(0) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau X dY - Y dX \quad K(0) = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \frac{1}{2} (\dot{X}^2 + \dot{Y}^2) dt$$

$$L(\Omega) = \langle uv - vu \rangle \quad K(\Omega) = \left\langle \frac{1}{2} (\dot{u}^2 + \dot{v}^2) \right\rangle$$

$$= L(0) + \Omega \langle R^2 \rangle \quad = K(0) + \Omega L(0) + \frac{1}{2} \Omega^2 \langle R^2 \rangle$$

$$\langle R^2 \rangle = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau (X^2 + Y^2) dt = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau (u^2 + v^2) dt$$



## Energy and Angular Momentum

### Diffeomorphic, Quantum Number $n$

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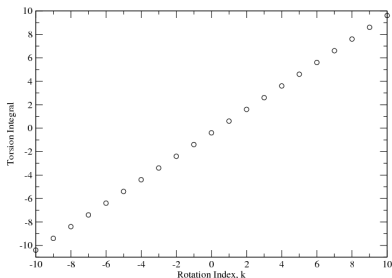
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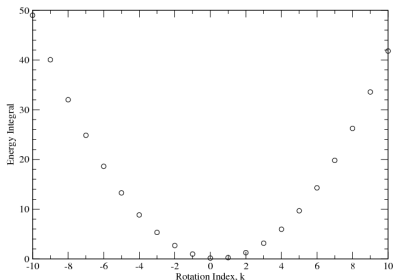
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Torsion Integral



Energy Integral



# New Measures, Subharmonic Covering Attractors

## Energy and Angular Momentum Subharmonics, Quantum Numbers $p/q$

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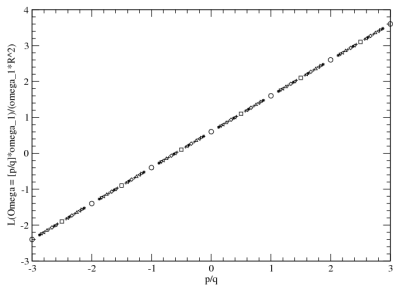
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Torsion Integral



Energy Integral

