The Topology of Chaos Chapter 7: Covers and Images

> Robert Gilmore

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The Topology of Chaos Chapter 7: Covers and Images

Robert Gilmore

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Physics and Topology Workshop Drexel University, Philadelphia, PA 19104

September 3, 2008

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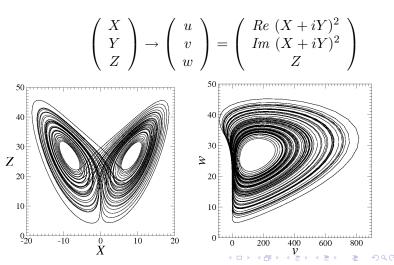
Modding Out a Rotation Symmetry

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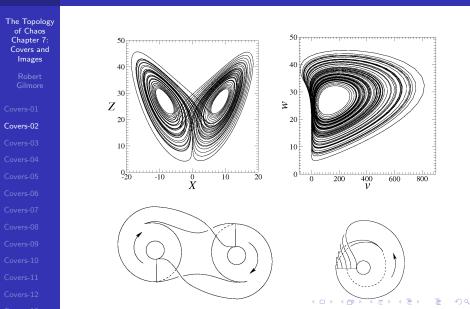
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Covers-01

Modding Out a Rotation Symmetry



Lorenz Attractor and Its Image



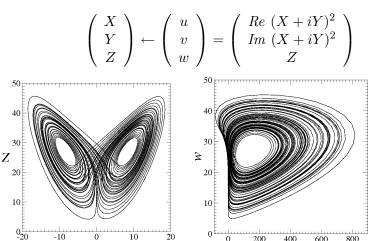
Lifting an Attractor: Cover-Image Relations

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Covers-02 Covers-03 Covers-04 Covers-04 Covers-05 Covers-06 Covers-07 Covers-08 Covers-09 Covers-10 Covers-11

Creating a Cover with Symmetry



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Cover-Image Related Branched Manifolds

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Covers-02 Covers-03 Covers-04 Covers-05

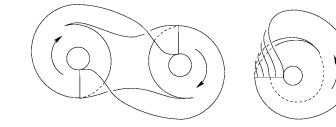
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Cover-Image Branched Manifolds



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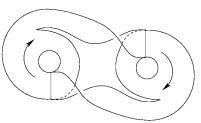
Covering Branched Manifolds

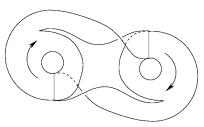
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Covers-05

Two Two-fold Lifts Different Symmetry





Rotation Symmetry Inversion Symmetry

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Topological Indices

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Covers-06 Covers-07

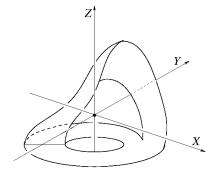
Covers-08

Covers 10

Covers-11

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Topological Index: Choose Group Choose Rotation Axis (Singular Set)



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Locate the Singular Set wrt Image

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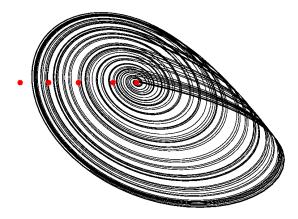
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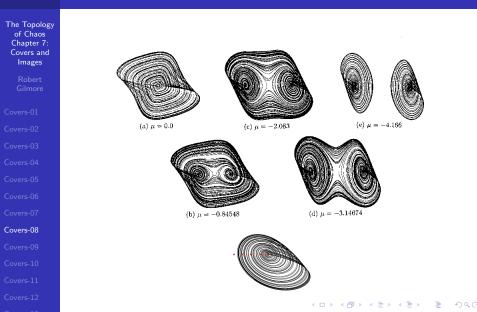
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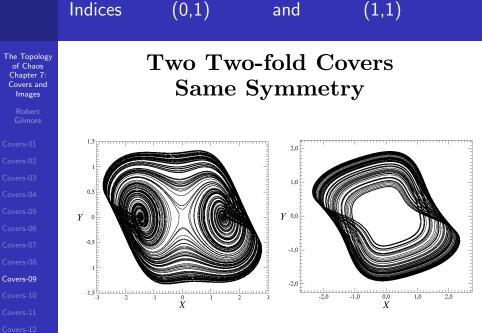
Different Rotation Axes Produce Different (Nonisotopic) Lifts



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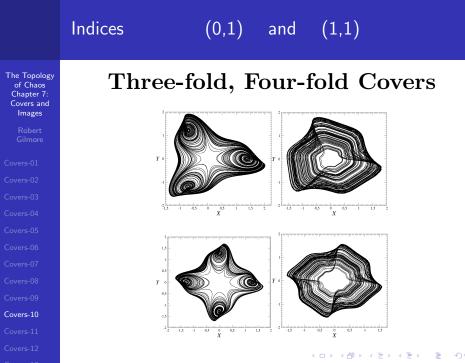
Nonisotopic Locally Diffeomorphic Lifts



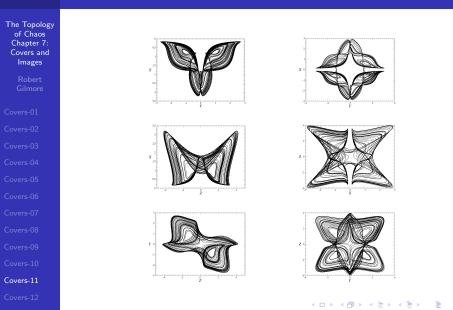


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Two Inequivalent Lifts with V_4 Symmetry



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How to Construct Covers/Images

Algorithm

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- Construct Invariant Polynomials, Syzygies, Radicals
- Construct Singular Sets

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Covers-12

- Determine Topological Indices
- Construct Spectrum of Structurally Stable Covers
- Structurally Unstable Covers Interpolate

Surprising New Findings

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Covers-01 Covers-02 Covers-03 Covers-04 Covers-05 Covers-06 Covers-07 Covers-08 Covers-09 Covers-10

Covers-1

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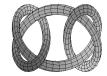
Symmetries Due to Symmetry

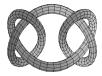
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- Schur's Lemmas & Equivariant Dynamics
- Cauchy Riemann Symmetries
- Clebsch-Gordon Symmetries
- Continuations
 - Analytic Continuation
 - Topological Continuation
 - Group Continuation

Covers of a Trefoil Torus







Granny Knot

Square Knot



Trefoil Knot (日) (雪) (ヨ) (ヨ) (ヨ)

You Can Cover a Cover = Lift a Lift

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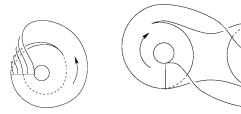
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Covers-10

Covers-11

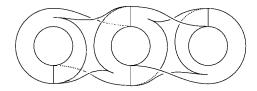
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Covers of Covers of Covers



Rossler

Lorenz

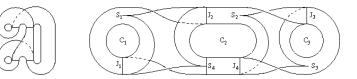


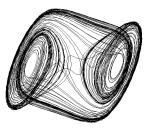
Universal Branched Manifold

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EveryKnot Lives Here







Isomorphisms and Diffeomorphisms

Local Stuff

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Local Isomorphisms Cartan's Theorem Dynamical Systems: Local Diffeomorphisms

Groups:

??? Anything Useful ???

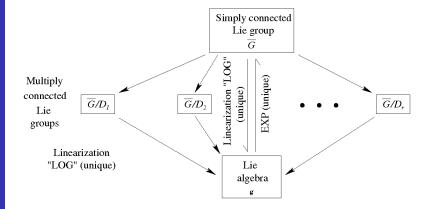
Universal Covering Group

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Cartan's Theorem for Lie Groups



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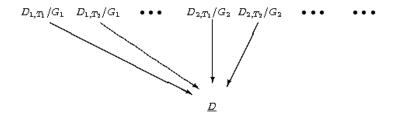
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Universal Image Dynamical System

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Locally Diffeomorphic Covers of \underline{D}



<u>D</u>: Universal Image Dynamical System

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Local Isomorphisms & Diffeomorphisms

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Local Isomorphisms & Diffeomorphisms

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Lie Groups

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Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms

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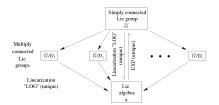
Local Isomorphisms & Diffeomorphisms

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Lie Groups

Local Isomorphisms



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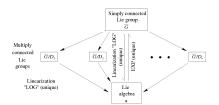
Local Isomorphisms & Diffeomorphisms

Lie Groups Dynamical Systems

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Local Isomorphisms



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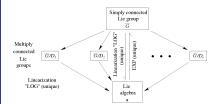
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Local Isomorphisms Local Diffeos



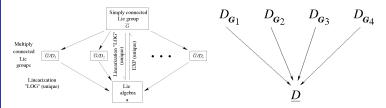
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Local Isomorphisms & Diffeomorphisms

Lie Groups Dynamical Systems

Local Isomorphisms Local Diffeos



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