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Modding Out a Rotation Symmetry

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} \rightarrow
\begin{pmatrix}
u \\
v \\
w
\end{pmatrix} =
\begin{pmatrix}
\text{Re} (X + iY)^2 \\
\text{Im} (X + iY)^2 \\
Z
\end{pmatrix}
\]
Lorenz Attractor and Its Image
Lifting an Attractor: Cover-Image Relations

Creating a Cover with Symmetry

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\leftarrow
\begin{pmatrix}
u \\
v \\
w
\end{pmatrix}
=
\begin{pmatrix}
\text{Re} \ (X + iY)^2 \\
\text{Im} \ (X + iY)^2 \\
Z
\end{pmatrix}
\]
Cover-Image Related Branched Manifolds

Cover-Image Branched Manifolds
Covering Branched Manifolds

Two Two-fold Lifts
Different Symmetry

Rotation Symmetry
Inversion Symmetry
Topological Index: Choose Group
Choose Rotation Axis (Singular Set)
Different Rotation Axes Produce Different (Nonisotopic) Lifts
Nonisotopic Locally Diffeomorphic Lifts

(a) $\mu = 0.0$

(c) $\mu = -2.083$

(e) $\mu = -4.166$

(b) $\mu = -0.84548$

(d) $\mu = -3.14674$
Two Two-fold Covers
Same Symmetry

Indices \((0,1)\) and \((1,1)\)
Indices  (0,1)  and  (1,1)

Three-fold, Four-fold Covers
Two Inequivalent Lifts with $V_4$ Symmetry
How to Construct Covers/Images

Algorithm

- Construct Invariant Polynomials, Syzygies, Radicals
- Construct Singular Sets
- Determine Topological Indices
- Construct Spectrum of Structurally Stable Covers
- Structurally Unstable Covers Interpolate
Surprising New Findings

Symmetries Due to Symmetry

- Schur’s Lemmas & Equivariant Dynamics
- Cauchy Riemann Symmetries
- Clebsch-Gordon Symmetries
- Continuations
  - Analytic Continuation
  - Topological Continuation
  - Group Continuation
Covers of a Trefoil Torus

Granny Knot

Square Knot

Trefoil Knot
You Can Cover a Cover = Lift a Lift

Covers of Covers of Covers of Covers

Rossler

Lorenz

Ghrist
Universal Branched Manifold

EveryKnot Lives Here
Local Stuff

Groups:
Local Isomorphisms
Cartan’s Theorem

Dynamical Systems:
Local Diffeomorphisms

??? Anything Useful ???
Cartan’s Theorem for Lie Groups

Simply connected Lie group $\bar{G}$

Multiply connected Lie groups

$\bar{G}/D_i$

Linearization "LOG" (unique)

$\bar{G}/D_2$

EXP (unique)

$\bar{G}/D_r$

Lie algebra $g$
Locally Diffeomorphic Covers of $D$

$D_1, T_1 / G_1$  $D_1, T_2 / G_1$  $\cdots$  $D_2, T_1 / G_2$  $D_2, T_2 / G_2$  $\cdots$  $\cdots$

$D$: Universal Image Dynamical System
Useful Analogs

Local Isomorphisms & Diffeomorphisms
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups

Dynamical Systems

Local Isomorphisms
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups

Dynamical Systems

Local Isomorphisms  Local Diffeos
Useful Analogs

Local Isomorphisms & Diffeomorphisms

Lie Groups

Dynamical Systems

Local Isomorphisms

Local Diffeos

Multiply connected Lie groups

Simply connected Lie group $\tilde{G}$

Linearization "LOG" (unique)

Lie algebra $\mathfrak{g}$

$D_{G_1}$ $D_{G_2}$ $D_{G_3}$ $D_{G_4}$

$\tilde{G}/D_1$ $\tilde{G}/D_2$ $\tilde{G}/D_3$ $\tilde{G}/D_4$

$D$