The Topology
of Chaos
Chapter 7:
Covers and
Images

Robert
Gilmore

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Chapter 7: Covers and Images

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Physics Department
Drexel University
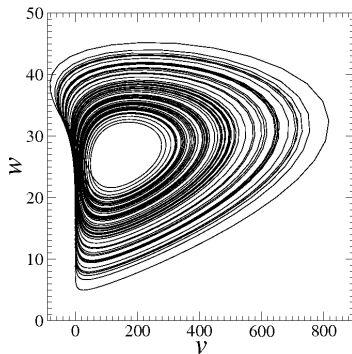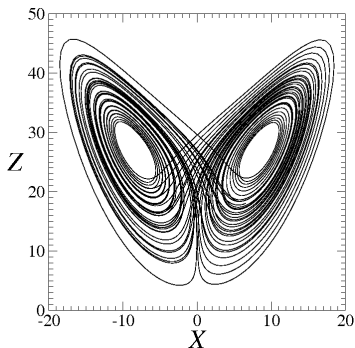Philadelphia, PA 19104
robert.gilmore@drexel.edu

Physics and Topology Workshop
Drexel University, Philadelphia, PA   19104
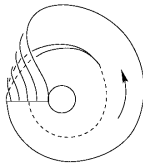
September 3, 2008

Modding Out a Rotation Symmetry
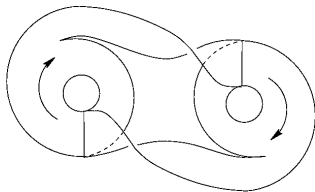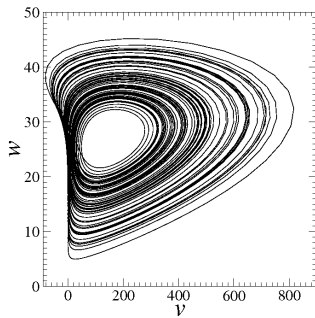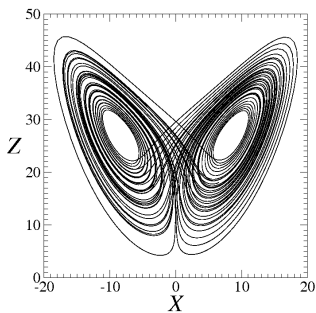
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} Re\ (X + iY)^2 \\ Im\ (X + iY)^2 \\ Z \end{pmatrix}$$

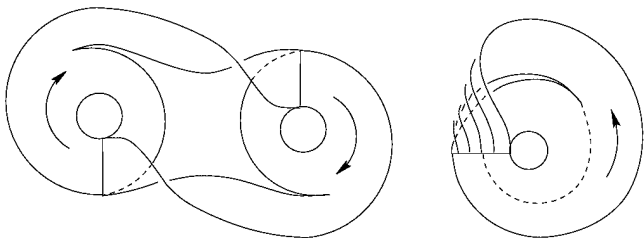# Lorenz Attractor and Its Image

Creating a Cover with Symmetry

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \leftarrow \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} Re\ (X+iY)^2 \\ Im\ (X+iY)^2 \\ Z \end{pmatrix}$$
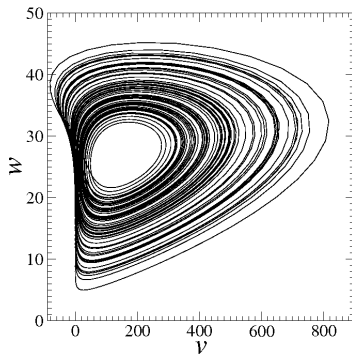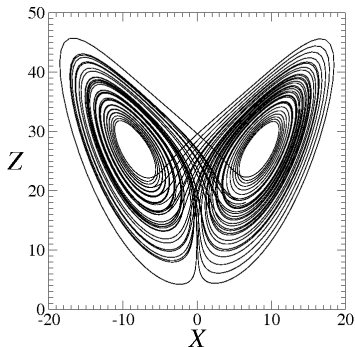
Cover-Image Branched Manifolds
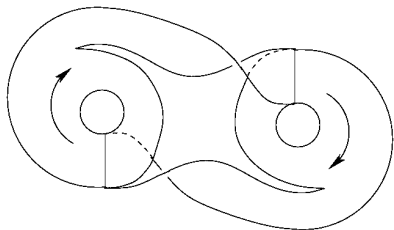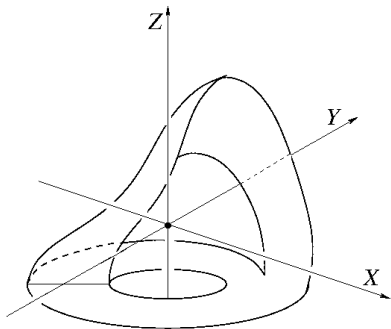
Two Two-fold Lifts
Different Symmetry
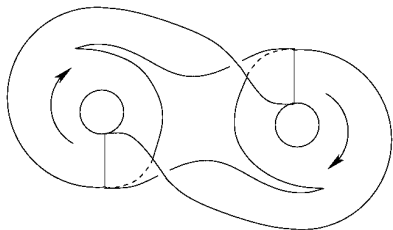
**Rotation
Symmetry**

**Inversion
Symmetry**

Topological Index: Choose Group Choose Rotation Axis (Singular Set)

Different Rotation Axes Produce Different (Nonisotopic) Lifts
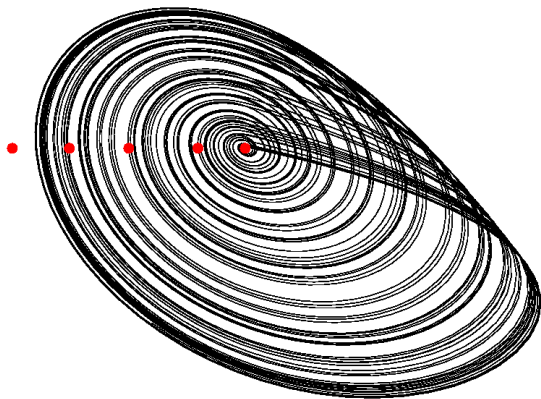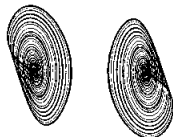
(a) $\mu = 0.0$

(c) $\mu = -2.083$

(e) $\mu = -4.166$

(b) $\mu = -0.84548$

(d) $\mu = -3.14674$
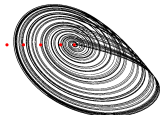
Two Two-fold Covers
Same Symmetry
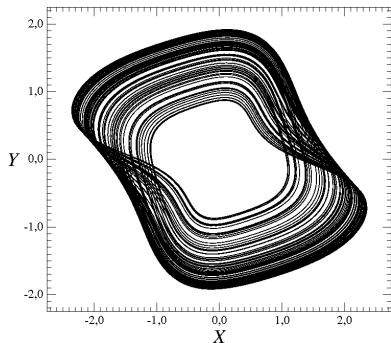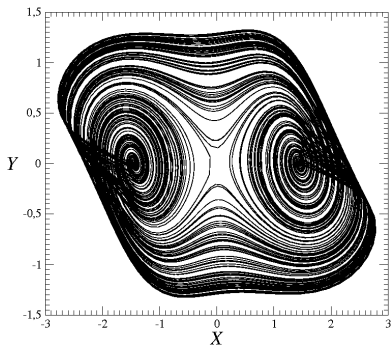
Three-fold, Four-fold Covers
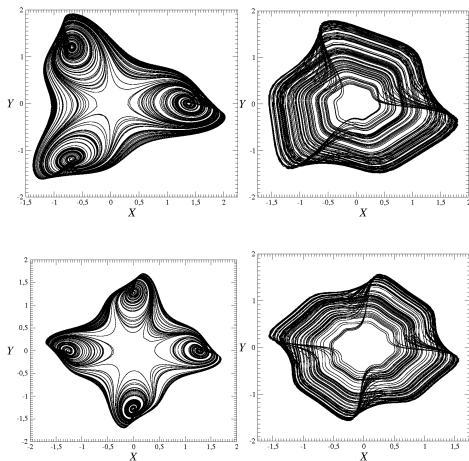
Two Inequivalent Lifts with V_4 Symmetry
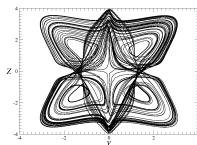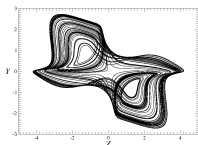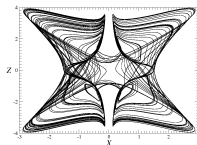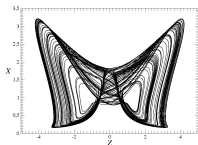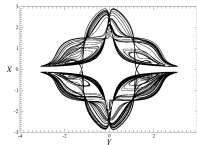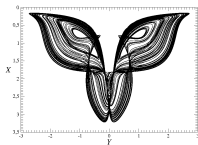
Algorithm

- Construct Invariant Polynomials, Syzygies, Radicals

- Construct Singular Sets

- Determine Topological Indices

- Construct Spectrum of Structurally Stable Covers

- Structurally Unstable Covers Interpolate

Symmetries Due to Symmetry

- Schur's Lemmas & Equivariant Dynamics
- Cauchy Riemann Symmetries
- Clebsch-Gordon Symmetries
- Continuations
  - Analytic Continuation
  - Topological Continuation
  - Group Continuation

Covers of a Trefoil Torus

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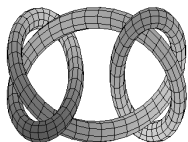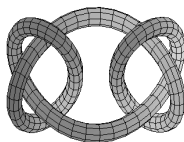
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Granny Knot

Square Knot
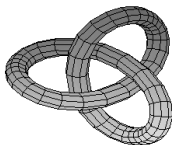
Trefoil Knot

Covers of Covers of Covers

Rossler

Lorenz

Ghrist
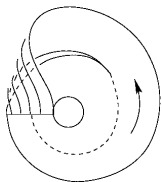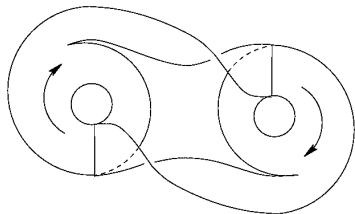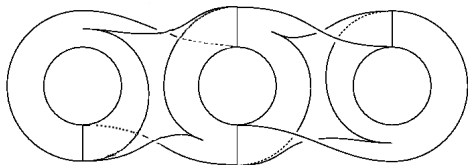
EveryKnot Lives Here
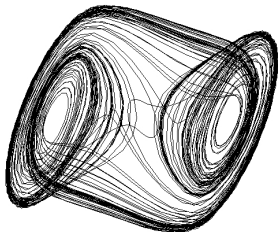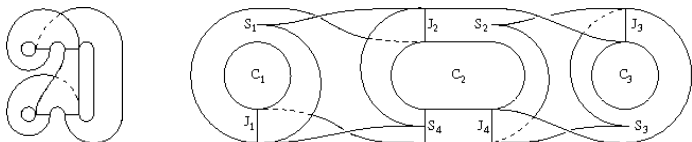
Local Stuff

Groups:

Local Isomorphisms

Cartan's Theorem

Dynamical Systems:

Local Diffeomorphisms

??? Anything Useful ???

Cartan's Theorem for Lie Groups

Locally Diffeomorphic Covers of $\underline{D}$

$D_{1,T_1}/G_1 \quad D_{1,T_2}/G_1 \quad \bullet\bullet\bullet \quad D_{2,T_1}/G_2 \quad D_{2,T_2}/G_2 \quad \bullet\bullet\bullet \quad \bullet\bullet\bullet$

$\underline{D}$
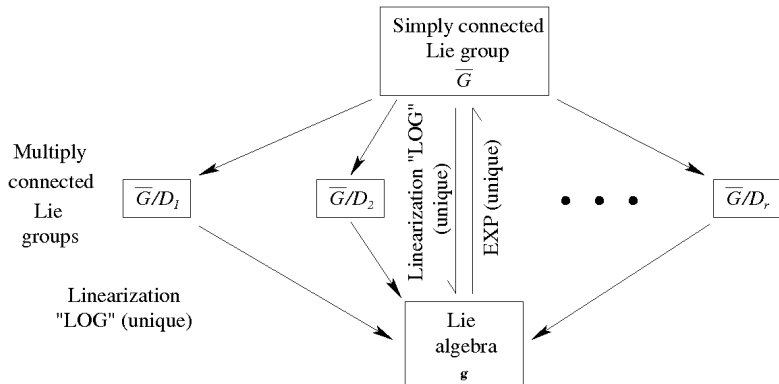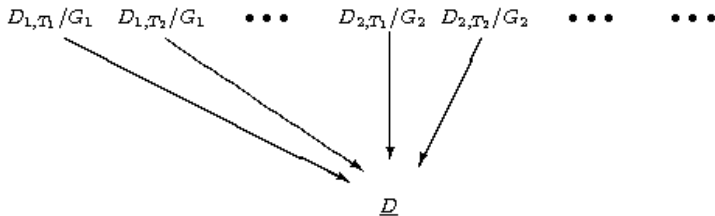
$\underline{D}$: **Universal Image Dynamical System**

Local Isomorphisms & Diffeomorphisms

Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms & Diffeomorphisms

Lie Groups

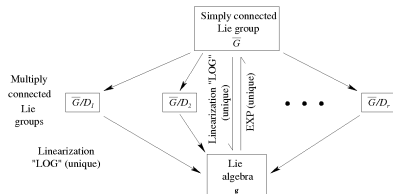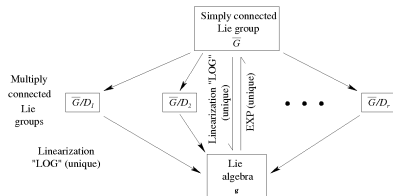**Local Isomorphisms**

Local Isomorphisms & Diffeomorphisms

Lie Groups

Local Isomorphisms

Local Isomorphisms & Diffeomorphisms

Lie Groups   **Dynamical Systems**

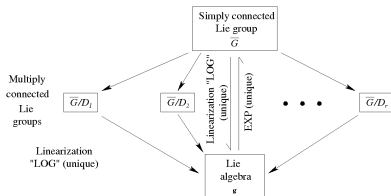## Local Isomorphisms

Local Isomorphisms & Diffeomorphisms

Lie Groups   **Dynamical Systems**

Local Isomorphisms   **Local Diffeos**

Local Isomorphisms & Diffeomorphisms

Lie Groups

Dynamical Systems

Local Isomorphisms

Local Diffeos