> Robert Gilmore

Motivation-01

Motivation-

ontents

.....

Nonlinear-0

Chaos-0

Chaos-0

Chaos-I

haos-05

From Nonlinear Dynamics to BioMedicine The Topology of Chaos

Robert Gilmore

Physics Department Drexel University Philadelphia, PA 19104 robert.gilmore@drexel.edu

August 25, 2009

Rouen: 1-4 September 2009

Basic Question

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-01

Motivation (

ontonto

Nonlinear-0

NI. . . P.

Nonlinear-0

Nonlinear-0

Chaos-

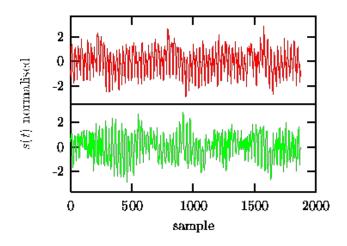
Chann 0'

Chaos-0

Chaos-0

Chaos-05

Is This Predictable or Not?



Science or Not?

From Nonlinear Dynamics to **BioMedicine**

Motivation-02

Fundamental Question

If you believe there is no relation between cause and effect, go home now.

If you believe there is, you must work with dynamical systems (Newtonian) equations.

These have the form:

$$\frac{dx_i}{dt} = f(x_i; c_k)$$

State Variables Chemical Concentrations Potential Differences

 x_i

Control Parameters **Temperature** Magnetic Fields

Table of Contents

From Nonlinear Dynamics to BioMedicine

Gilmore

Motivation-01

Mativation Of

Contents

Nonlinear-01

Monlinear 02

Nonlinear-03

Nonlinear-04

Chaos-0

Chaos-U

Chaos

Chaos-0

C...a05

Outline

- Basics of NLD & Chaos
- Measures: Dynamical, Geometric, Topological
- Operation of the Periodic Orbits and Chaos
- 4 Topological Analysis Program
- Applications to Data
- O Bounding Tori Contain Strange Attractors
- Representations of Strange Attractors

From Nonlinear Dynamics ...

From Nonlinear Dynamics to **BioMedicine**

Nonlinear-01

Nature of Nonlinear Systems

Linear

$$\frac{d}{dt} \left[\begin{array}{c} x \end{array} \right] = \left[\begin{array}{c} M \end{array} \right] \left[\begin{array}{c} x \end{array} \right] \qquad \frac{d}{dt} \left[\begin{array}{c} x \end{array} \right] = \left[\begin{array}{c} M \end{array} \right] \left[\begin{array}{c} x \end{array} \right] +$$

$$\overline{dt} \mid x$$

$$x \mid = \mid M$$

More

Stuff

NonLinear



$$\left[\begin{array}{c} x(t) \end{array}\right] = e^{\left[\begin{array}{c} M \end{array}\right]t} \left[\begin{array}{c} x(0) \end{array}\right]$$

$$x \to 0$$
 if $Re(\lambda_i) < 0$, all i

???

Linear Dynamical Systems

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mart

Nonlinear-0

Nonlinear-02

Nonlinear-U

Chaos-u

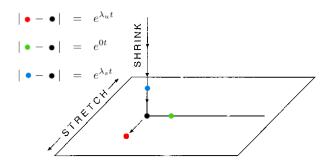
Chaos-02

Chaos-0

Chaos-0

haos-05

Typical Solutions for Linear Equations



Starting Point

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-

ontents

Nonlinear-0

Nonlinear-03

Monlinger (

Chaos 0

Chaos-0

Chaos-u

CI.... 01

Two of Our Heroes

J. B. Fourier



H. Poincare



Starting Point

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-(

-

Monlinear O

. . .

Nonlinear-Us

Nonlinear-04

Chaos-(

Chaos-0

Chaos-0

Chaos-

Chaos-05

Why They Are Our Heroes

- Fourier: taught us how to use periodic orbits to describe linear systems.
- Poincaré: taught us to use periodic orbits to understand nonlinear systems.

Nonlinear Systems

From Nonlinear Dynamics to BioMedicine

Robert Gilmore

Motivation-0

.

ontents

Nonlinear-03

NI . . . l'.

Nonlinear-0

NI . . . C.

Nonlinear-05

Chaos-U2

Chaos-0

Chaos-0

Two Benchmark Nonlinear Systems

Rossler



$$\begin{array}{rcl} \dot{x} & = & -y - z \\ \dot{y} & = & x + ay \\ \dot{z} & = & b - cz & +zx \end{array}$$

Lorenz



$$\begin{array}{lll} \dot{x} & = & -\sigma x + \sigma y \\ \dot{y} & = & Rx - y & -xz \\ \dot{z} & = & -bz & +xy \end{array}$$

Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-02

..

Nonlinear-03

Nonlinear-04

NI - - I'-- - - - OF

Chaos-01

CI 0/

Chaos

Chaos-

Chaos-05

Definition of Chaos

Motion is chaotic if it is

- Oeterministic
- Bounded
- Recurrent
- 4 Sensitive to Initial Conditions

Mechanisms for Generating Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation (

ontonto

Nonlinear-0

Naulinaan O

Nonlinear-05

Chaos-

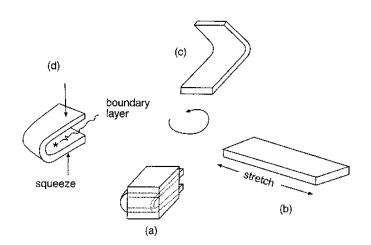
Chaos-02

Chaos-0

Chaos-0

Chaos 05

Stretching and Folding



Mechanisms for Generating Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-C

ontents

Nonlinear-03

Nonlinear-

Nonlinear-0

Noninear-04

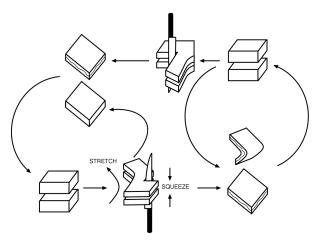
Chaos-0

Chaos-03

Cl.

haos-05

Tearing and Squeezing



Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mativation 0

ontonto

Nonlinear-0

..

Monlinger Of

Tvoillinear 0-

Cl.

Chaos-04

c. .-

Production of Chaos

Chaos (chaotic motion, any strange attractor) is generated by the continuous repetition of two processes:

- Stretching
- Squeezing

Three Measures: Dynamical

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-0

C....

Nonlinear-0

Monlinger O

Nonlinear-05

Chaos-u

Chaos-0

Chaos-(

Consequences of Stretching / Squeezing

- "Stretching" causes nearby points to separate
- Stretching grows exponentially with time (short times)

$$\delta x(t) \simeq e^{\lambda t} \delta x(0) \qquad \lambda > 0$$

- \odot λ is a Lyapunov exponent
- 4 Almost all points move apart
- 6 A measure zero set does not
- O Lyapunov exponents can be tricky to estimate

Mechanisms for Generating Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation (

ontonto

Nonlinear-03

Nonlinear-0

Nonlinear OF

Chann

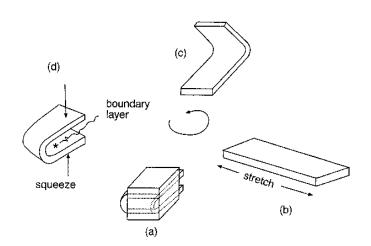
Chaos 0

Chaos-0

Chaos (

CI.... 0F

Stretching and Folding



Studying Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

iviotivation-o.

Motivation-02

Contonto

Monlinger O

Nonlinear-0

Nonlinear-C

Nonlinear-

Nonlinear-C

Chaos-

Chaos-02

Chaos-0

Chaos-

Chaos-05

How to "Measure" Chaos?

Method Measure Data Rqmts.

Dynamical Lyapunov Exponents L, C, -

Geometrical Fractal Dimensions LL, CC, SS

Topological Linking Numbers M, M, S

L = Long, C = Clean, S = Stationary

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

.

. . .

ivoimilear o.

Nonlinear-03

Nonlinear-0

Nonlinear O

Nonlinear-0

. .

CI 0

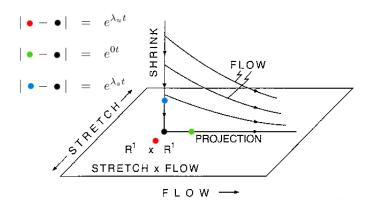
C...a05 0

Chaos-0

Chaos-0

haos-05

Lyapunov exponents measure how points move with respect to each other.



From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

.

ontents

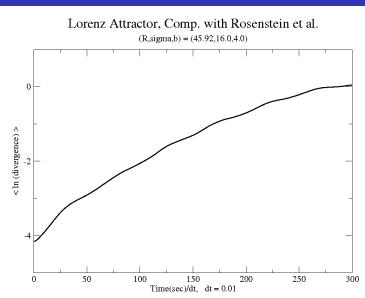
Tvommear o

Nonlinear-0

Monlinear-0

Nonlinear-05

Chaos O





From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

ontonto

Nonlinear-0

. . . .

Nonlinear-C

Nonlinear-05

CHaos-01

Chaos-02

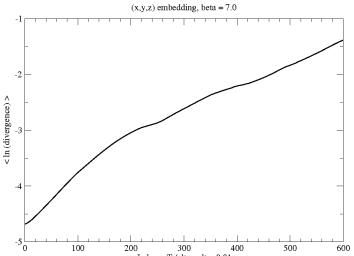
Chaos 03

Chaos 0

Chana OF

Malkus-Robbins Dynamo Model

Find the Right Linear Region



From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Mativation O

Nonlinear-01

Naulinaan 0

..

Chaos-

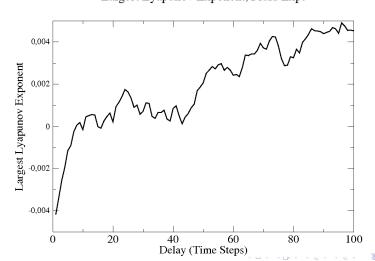
Chaos 02

Chaos-03

Chaos-0

haos-05

On Fluid Data Largest Lyapunov Exponent, Fluid Expt



From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-02

.

Nonlinear-01

N - - 1 - - - - 0

.....

Nonlinear-05

CI 01

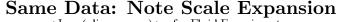
Chaos-02

CHaus-uz

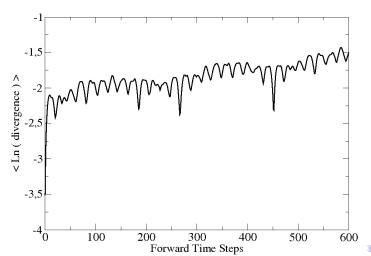
Chaos-03

Chaos-0

naos-05



< Log (divergence) > for Fluid Experiment



From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Martineria C

ontonto

Nonlinear-0

Naulinaan OF

c. ..

Chann 00

C11aU3-U2

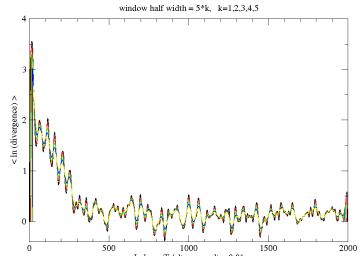
Chaos-03

Chaos-0

Chaos-05

What Value would you Like?

LLE Slope from moving average window



From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

ntente

Nonlinear-0

..

Nonlinear-C

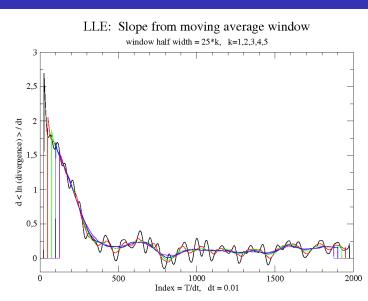
Nonlinear-05

Chaos-01

Chaos-03

Chans-(

c. .-



From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Marinarian C

.

N 11 .

i voiiiiiieai-v

ivoniinear-u

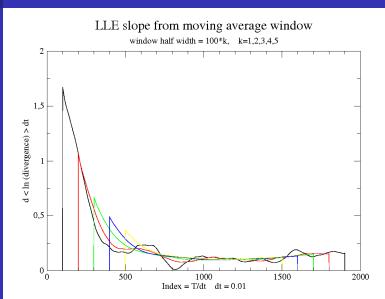
Naulinaan 0

Chaos 01

Chaos-us

Chaos-C





Three Measures: Geometrical

From Nonlinear Dynamics to **BioMedicine**

Consequences of Stretching / Squeezing

- Recurrence causes "Squeezing".
- Squeezing causes nearby points to move closer

$$\delta x(t) \simeq e^{\lambda t} \delta x(0) \qquad \lambda < 0$$

- \bullet λ is another Lyapunov exponent.
- 4 Repetition of Squeezing builds up a flakey, milleffeuil like structure
- This is called a Fractal
- **6** The number of points $|x-y| < \epsilon$ scales like $N(\epsilon) \simeq \epsilon^D$
- Fractal dimensions (e.g., D) are also tricky to estimate.

Stretching and Folding

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

NA-+:...+:--- (

.

Nonlinear-03

.. .. .

Nonlinear O

Nonlinear-05

Chaos-0

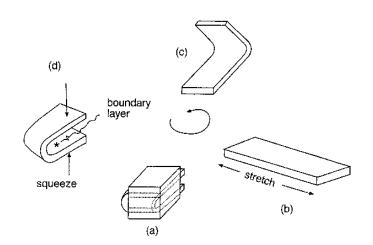
Chaos-02

Chaos-0

Chaos-0

haos-05

One Mechanism for Generating Chaos



Mechanisms for Generating Chaos

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mariantina O

Nonlinear-01

Naulinaan O

Naulinaan Ol

Chaos-0

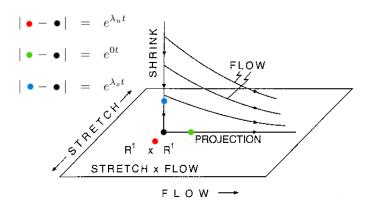
Chaos-0

Chaos-u

CHaos-C

Chaos 05

Stretching and Folding



Problems Estimating Fractal Dimensions

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Martineria C

.

Naulinaan O

Nonlinear-02

Nonlinear-0

N. II. O.

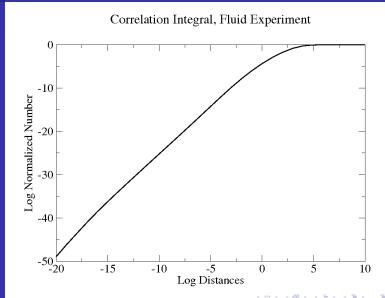
C. ...

Chaos-02

Chaos-0

Chaos-C

haos-05



Robert Gilmore

Motivation-03

Mastinasian 0

_

Nonlinear-01

..

Nonlinear-0

.. .. .

Nonlinear-0!

Chaos (

c. ...

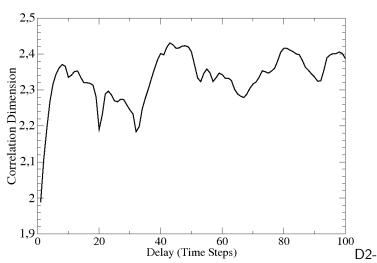
Chaos-u.

Chaos-03

Chaos-0

Chaos 05









> Robert Gilmore

Motivation-01

NA-+:...+:--- (

Nonlinear-01

NI. .. P.

Noninear-C

Nonlinear 05

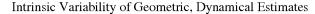
Chaos-

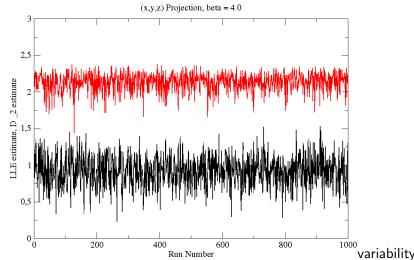
Chaos-0

Chann 03

CI 0

c. .-







> Robert Gilmore

Motivation-0

.

.

Mantinana

..

.....

Nonlinear (

<u>.</u>. . . .

Chaos O

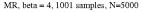
C11aU3-U2

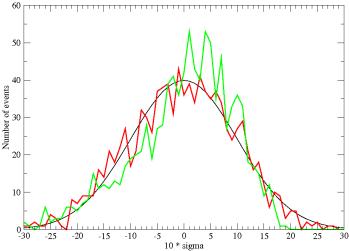
C.........

Cilaos-o-











Strange Attractor

From Nonlinear Dynamics to **BioMedicine**

Periodic Orbits

A strange attractor is the ' Ω ' limit set of the flow.

- There are unstable periodic orbits "in" the strange attractor.
- Many.
- They outline the strange attractor.
- They provide a skeleton for the strange attractor.
- They can be extracted from the attractor.
- Their organization can be determined (in R^3).
- This analysis method applies in R^3 only, for now.

Skeletons

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mativation 0

. . . .

Contents

TVOIIIIICAI O

Nonlinear-0

Nonlinear-

Maulineau

Nonlinear-0

Chaos 01

Chaos-0

. .

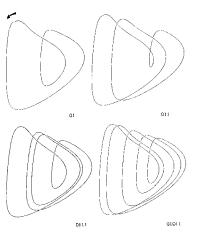
Chaos-0

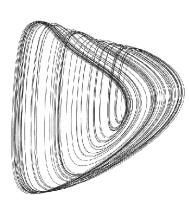
Chaos-0

Chann 05

BZ reaction

UPOs Outline Strange attractors





Skeletons

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

...

Contents

Nonlinear-0

M . . . P O

Nonlinear (

Nonlinear-05

Chaos-

Chaos O

Chaos (

Chass

C...ao5 C

Lefranc - Cargese

UPOs Outline Strange attractors

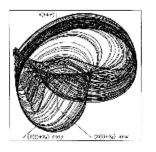




Figure 5. Left: a chaotic attractor reconstructed from a time series from a chaotic laser; Right: Superposition of 12 periodic orbits of periods from 1 to 10.

Dynamics and Topology

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-(

ontents

Mantinana

......

.....

NI - - Post - - - C

Chaos-0

Channe

Chaos-C

Chaos-i

Organization of UPOs in R^3 :

Gauss Linking Number

$$LN(A,B) = \frac{1}{4\pi} \oint \oint \frac{(\mathbf{r}_A - \mathbf{r}_B) \cdot d\mathbf{r}_A \times d\mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|^3}$$

Interpretations of LN $\simeq \#$ Mathematicians in World

Linking Numbers

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

.......

Nonninear-0

Nonlinear-

Nonlinear-0

Nonlinear-0

Nonlinear-0

Chaos-C

Chaos-0

haos-05

Linking Number of Two UPOs

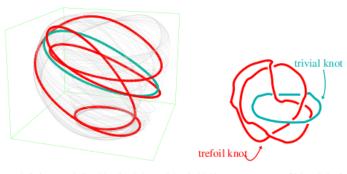


Figure 6. Left: two periodic orbits of periods 1 and 4 embedded in a strange attractor; Right: a link of two knots that is equivalent to the pair of periodic orbits up to continuous deformations without crossings.

Lefranc - Cargese

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

violivation-u.

Conten

Noninear-0.

.....

Naulinaan O

Chann 0'

Chaos 03

Chansel

Chaos-C

Compute Table of Expt'l LN

Table 7.2 Linking numbers for all the surrogate periodic orbits, to period 8, extracted from Belousov-Zhabotinskii data^a

Orbit	Symbolics	1	2	3	4	5	6	7	8a	8Ь
1	1	0	1	1	2	2	2	3	4	3
2	01	1	1	2	3	4	4	5	6	6
3	011	1	2	2	4	5	6	7	8	8
4	0111	2	3	4	5	8	8	11	13	12
5	01 011	2	4	5	8	8	10	13	16	15
6	011 0M1	2	4	6	8	10	9	14	16	16
7	01 01 011	3	5	7	11	13	14	16	21	21
8a	01 01 0111	4	6	8	13	16	16	21	23	24
8Ь	01 011 011	3	6	8	12	15	16	21	24	21

All indices are negative.

Evolution in Phase Space

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

Nonlinear-0

..

Nonlinear-0

Nonlinear-05

Chann 01

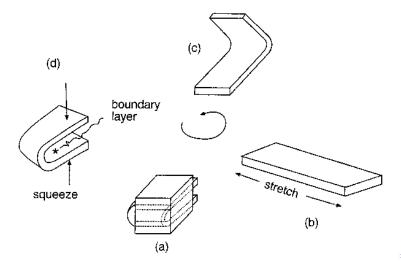
Chaos-02

Chaos-03

Chaos-

Chana 05

One Stretch-&-Squeeze Mechanism



Motion of Blobs in Phase Space (Poincaré)

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-(

ontents

Nonlinear-0

Nonlinear-

Nonlinear-0

Nonlinear-04

Nonlinear-0

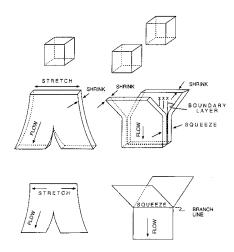
Chaos-0

Chann 03

Channe

CI 05

Stretching — Squeezing



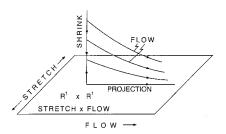
Collapse Along the Stable Manifold

From Nonlinear Dynamics to BioMedicine

Birman - Williams Projection

Identify x and y if

$$\lim_{t \to \infty} |x(t) - y(t)| \to 0$$



This projects the flow donw onto the unstable manifold.



Fundamental Theorem

From Nonlinear Dynamics to BioMedicine

lilmor

iviotivation-U1

Notivation-02

. .

Nonlinear-t

ivoniinear-u

near-03

Nonlinear-04

Chaos-0

Chaos-02

Chaos-0

Chans

Cilaus-u

Birman - Williams Theorem

If:

Then:

Fundamental Theorem

From Nonlinear Dynamics to **BioMedicine**

Birman - Williams Theorem

Certain Assumptions Tf:

Then:

Fundamental Theorem

From Nonlinear Dynamics to BioMedicine

> Robert ilmore

violivation-c

Contents

Nonlinear-0

Nonlinear-0

Nonlinear-0

Nonlinear-04

Nonline

Clia03-01

Cnaos-U2

Chaos-

Chaos

Birman - Williams Theorem

If: Certain Assumptions

Then: Specific Conclusions

Birman-Williams Theorem

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

viotivation-u

Content

Nonlinear-01

Nonlinear-02

Nonlinear-03

Nonlinear-0

Nonlinear-0

Chaos-01

C. ...

Chaos-0

Chans

Chaos-

Assumptions, B-W Theorem

A flow $\Phi_t(x)$

- on R^n is dissipative, $\underline{n=3}$, so that $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 < 0$.
- Generates a <u>hyperbolic</u> strange attractor SA

IMPORTANT: The underlined assumptions can be relaxed.

Birman-Williams Theorem

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Conter

Nonlinear-01

Nonlinear-03

Nonlinear-04

Chaos-0

Charac

Chaos-0

Chaos-I

Conclusions, B-W Theorem

- The projection maps the strange attractor \mathcal{SA} onto a 2-dimensional branched manifold \mathcal{BM} and the flow $\Phi_t(x)$ on \mathcal{SA} to a semiflow $\overline{\Phi}(x)_t$ on \mathcal{BM} .
- UPOs of $\Phi_t(x)$ on \mathcal{SA} are in 1-1 correspondence with UPOs of $\overline{\Phi}(x)_t$ on \mathcal{BM} . Moreover, every link of UPOs of $(\Phi_t(x), \mathcal{SA})$ is isotopic to the correspond link of UPOs of $(\overline{\Phi}(x)_t, \mathcal{BM})$.

Remark: "One of the few theorems useful to experimentalists."

A Very Common Mechanism

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-01

Motivation-02

ontente

Nonlinear-0

Nonlinear-

Nonlinear-0

TVOIIIIICAI (

Chaos-0

Chaos-0

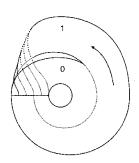
Chaos-

Chaos-05

Rössler:

Attractor Branched Manifold





A Mechanism with Symmetry

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-02

ntents

Nonlinear-0

Nonlinear-0

Nonlinear-0

Chaos

Chaos-0

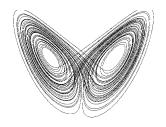
Chaos-0

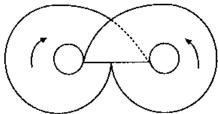
Chaos-I

Chaos-05

Lorenz:

Attractor Branched Manifold





Examples of Branched Manifolds

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

ontonto

Nonlinear-0

Monlinear-

Nonlinear-

Nonlinear-04

Nonlinear-(

Chaos-01

Chaos-0

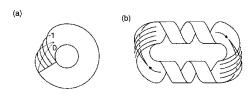
C...a05 0

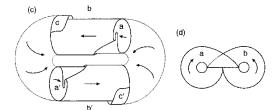
Chaos-0

Chaos-0

hace 05

Inequivalent Branched Manifolds





Aufbau Princip for Branched Manifolds

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

ntente

Nonlinear-0

M . . . P O

Nonlinear-C

N 11 0

Nonlinear-0!

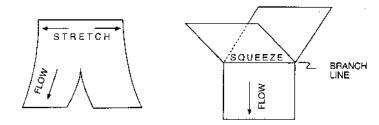
Chaos-0

C...a05 0

Chaos 0

Chaos

Any branched manifold can be built up from stretching and squeezing units



subject to the conditions:

- Outputs to Inputs
- No Free Ends



Dynamics and Topology

From Nonlinear Dynamics to BioMedicine

Rossler System



 $\frac{ds}{dt} = -y$ e

 $\frac{dy}{dz} = x + ay$

 $\frac{dz}{dz}=b+z(z-c)$

(b)



(c)



(f)





(d)



Dynamics and Topology

From Nonlinear Dynamics to BioMedicine

Lorenz System

$$\frac{dx}{dt} = -ax + ay$$

$$\frac{dy}{dt} = Rx \cdot y \cdot xz$$

$$\frac{dz}{dt} = -bz + xy$$

$$\left(+i-1\right)$$

(b)

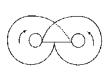












Dynamics and Topology

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

Contents

Nonlinear-U

INOTHINEAT-02

Nonlinear-U3

Noninear-04

Chans-C

Cilaus-u

Chaos-02

Chaos-0

Chaos-

Chaos 05

Poincaré Smiles at Us in R³

- Determine organization of UPOs \Rightarrow
- Determine branched manifold ⇒
- Determine equivalence class of \mathcal{SA}

Determinism

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

antonto

Nonlinear-0

.....

N 11 /

ivoilillear-c

Naulinaan OF

Chaos-0

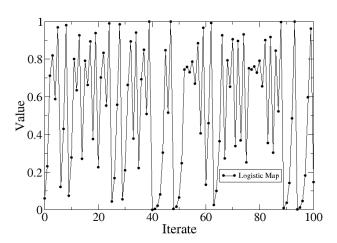
<u>.</u>. ..

Chaos 0

Chaos 0

c. .-

How to predict the future from the past



Some Prediction Results

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-(

ontents

Nonlinear-0

Nonlinear-0

Nonlinear-0

Nonlinear-04

Chann 01

Chaos-u.

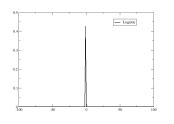
Chann Of

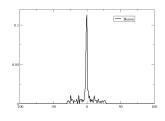
C.........

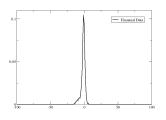
Chaos-(

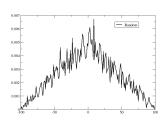
Chaos-05

Tightly binned predictions suggest determinism











Topological Analysis Program

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation (

ontents

Nonlinear-01

linear-03

Vonlinear-0

Nonlinear-0

Chaos-01

Chaos-02 Chaos-03

Chaos-0

Cl. . . . 0F

Topological Analysis Program

Locate Periodic Orbits

Create an Embedding

Determine Topological Invariants (LN)

Identify a Branched Manifold

Verify the Branched Manifold

Model the Dynamics

Validate the Model



Locate UPOs

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Contents

Nonlinear-C

NI. .. P.

Nonlinear (

Nonlinear-0

Chaos

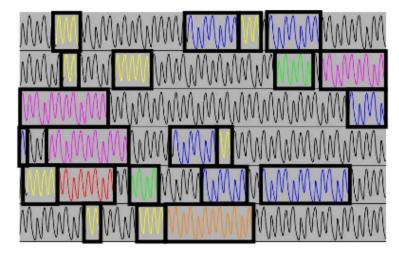
Chaos-0

Chaos-0

Chans-I

haos-05

Method of Close Returns



Locate UPOs

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-(

ontents

Nonlinear-0

Monlinear 0

Chaos-0

Chann

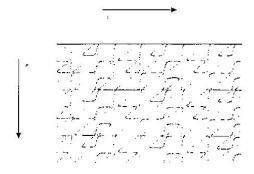
Chaos 0

Chaos-0

Chaos-05

Method of Close Returns

$$|x_i - x_{i+p}| < \epsilon$$
, pixel \to black



Embeddings

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-01

Motivation-02

Conter

Nonlinear-01

..

Nonlinear-0

Nonlinear-05

Chaos-u

Chaos-02

Chaos-0

Chaos-

Chaos-05

Embeddings

This is a tricky business. There are many problems ...

Many Methods: Time Delay, Differential, Hilbert Transforms, SVD, Mixtures, ...

Tests for Embeddings: Geometric, Dynamic, Topological[†]

None Good

We Demand a 3 Dimensional Embedding

Locate UPOs

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation 0

......

Nonlinear-C

.....

N ... 0

Nonlinear-U

Nonlinear-05

Chaos-I

Chaos-0

Chaos-0

Chaos-

Chaos-0

Periodic Orbits Outline the Attractor

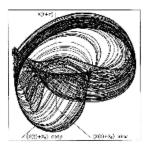




Figure 5. Left: a chaotic attractor reconstructed from a time series from a chaotic laser; Right: Superposition of 12 periodic orbits of periods from 1 to 10.

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-C

ontente

Nonlinear-0

Mantinana

Nonlinear-0

Chaos-C

Chaos-0

haos-05

Linking Number of Orbit Pairs

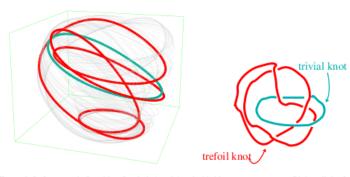


Figure 6. Left: two periodic orbits of periods 1 and 4 embedded in a strange attractor; Right: a link of two knots that is equivalent to the pair of periodic orbits up to continuous deformations without crossings.

Lefranc - Cargese

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Nonlinear-0

Nonlinear-0

Chann 01

Chaos-02

Chaos-03

Chansel

Compute Table of Expt'l LN

Table 7.2 Linking numbers for all the surrogate periodic orbits, to period 8, extracted from Belousov-Zhabotinskii data^a

Orbit	Symbolics	1	2	3	4	5	6	7	8a	8Ь
1	1	0	1	1	2	2	2	3	4	3
2	01	1	1	2	3	4	4	5	6	6
3	011	1	2	2	4	5	6	7	8	8
4	0111	2	3	4	5	8	8	11	13	12
5	01 011	2	4	5	8	8	10	13	16	15
6	011 0M1	2	4	6	8	10	9	14	16	16
7	01 01 011	3	5	7	11	13	14	16	21	21
8a	01 01 0111	4	6	8	13	16	16	21	23	24
8Ь	01 011 011	3	6	8	12	15	16	21	24	21

All indices are negative.

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

.......

Contents

Nonlinear-u

Naulinaan O

Monlinear-C

Monlinear O

. .

c. .

Chaos-U

Chaos-0

Compare w. LN From Various βM

Table 2.1 Linking numbers for orbits to period five in Smale horseshoe dynamics.

	19	1 f	21	3 <i>f</i>	39	41	4_2f	$4_{2}9$	5 ₃ f	539	5_2f	529	$5_1 f$	518
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	2	1	1	1	1	2	2	2	2
01	0	1	1	2	2	3	2	2	2	2	3	3	4	4
001	0	1	2	2	3	4	3	3	3	3	4	4	5	5
011	0	1	2	3	2	4	3	3	3	3	5	5	5	5
0111	0	2	3	4	4	5	4	4	4	4	7	7	8	8
0001	0	1	2	3	3	4	3	4	4	4	5	5	5	5
0011	0	1	2	3	3	4	4	3	4	4	5	5	5	5
00001	0	1	2	3	3	4	4	4	4	5	5	5	5	5
00011	0	1	2	3	3	4	4	4	5	4	5	5	5	5
00111	0	2	3	4	5	7	5	5	5	5	6	7	8	9
00101	0	2	3	4	5	7	5	5	5	5	7	6	8	9
01101	0	2	4	5	5	8	5	5	5	5	8	8	8	10
01111	0	2	4	5	5	8	5	5	5	5	9	9	10	8

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

ontont

Nonlinear-0

Nonlinear

Nonlinear-

Nonlinear-0

Chaos-0

Chaos-0

c. .

a. .

Guess Branched Manifold

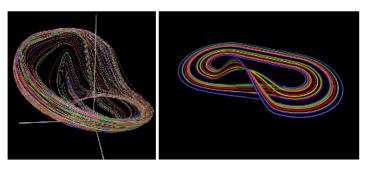


Figure 7. "Combing" the intertwined periodic orbits (left) reveals their systematic organization (right) created by the stretching and squeezing mechanisms.

Lefranc - Cargese

From Nonlinear Dynamics to BioMedicine

Gilmore

Motivation-0

Motivation-02

onten

ivoniniear-01

Nonlinear-us

ivoniinear-u

Chaos

CI 00

CI 0

Chaos

Identification & 'Confirmation'

- ullet \mathcal{BM} Identified by LN of small number of orbits
- Table of LN GROSSLY overdetermined
- Predict LN of additional orbits
- Rejection criterion

From Nonlinear Dynamics to BioMedicine

Gilmore

Motivation-01

Motivation-02

Conter

Nonlinear-0

Nonlinear-0

Nonlinear-C

Chaos-

Chaos-0

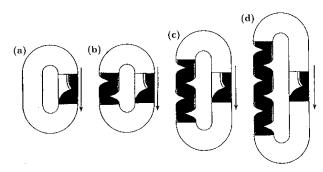
Chaos-0

Chaos-0

Chaos 05

What Do We Learn?

- BM Depends on Embedding
- Some things depend on embedding, some don't
- Depends on Embedding: Global Torsion, Parity, ..
- Independent of Embedding: Mechanism





Perestroikas of Strange Attractors

From Nonlinear Dynamics to BioMedicine

Robert Gilmore

Motivation-03

Motivation (

ontente

Nonlinear-0

Nonlinear-

Nonlinear-

. . . .

Nonlinear-u

C...405 0.

Chaos-0

Chaos-(

Chaos-05

Evolution Under Parameter Change

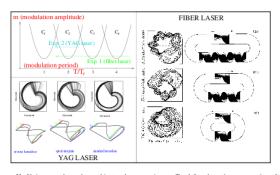


Figure 11. Various templates observed in two laser experiments. Top left: schematic representation of the parameter space of forced nonlinear oscillators showing resonance tongues. Right: templates observed in the fiber laser experiment: global torsion increases systematically from one tongue to the next [40]. Bottom left: templates observed in the YAG laser experiment (only the branches are shown): there is a variation in the topological organization across one chaotic tongue [39, 41].

Perestroikas of Strange Attractors

From Nonlinear Dynamics to BioMedicine

Robert Gilmore

Motivation-0

_

Nonlinear-0

N. 11 0

Nonlinear-0

Nonlinear (

Nonlinear-0

Chana 01

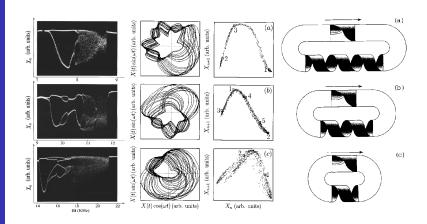
Chaos-02

Chaos-03

Chaos-0

Chaos 05

Evolution Under Parameter Change



Lefranc - Cargese



An Unexpected Benefit

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-C

Motivation-0

ontente

Nonlinear-0

N1 11 /

Nonlinear-05

Chaos-02

Chaos-0

Chaos-(

Chaos-05

Analysis of Nonstationary Data

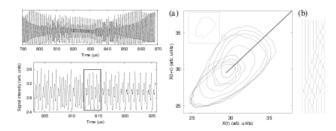


Figure 16. Top left: time series from an optical parametric oscillator showing a burst of irregular behavior. Bottom left: segment of the time series containing a periodic orbit of period 9. Right: embedding of the periodic orbit in a reconstructed phase space and representation of the braid realized by the orbit. The braid entropy is $h_T = 0.377$, showing that the underlying dynamics is chaotic. Reprinted from [61].

Lefranc - Cargese

Last Steps

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-02

Conten

Nonlinear-01

Tvommear o

ivonimear-u

ivoniinear-04

Chann 01

Cilaus-u.

Chaos-0

Chaos-0

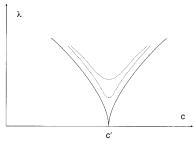
Cnaos-u

Model the Dynamics

A hodgepodge of methods exist: # Methods $\cong \#$ Physicists

Validate the Model

Needed: Nonlinear analog of χ^2 test. OPPORTUNITY: Tests that depend on entrainment/synchronization.



Our Hope \rightarrow Now a Result

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-02

Conte

ivoniinear-u

Nonlinear-03

Nonlinear-04

Chaos-0

CI 00

Chaor

Chaos-

Compare with Original Objectives

Construct a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Experimental Schematic

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

.

ontonto

Nonlinear-03

..

Nonlinear-0

N. II ...

Nonlinear-0

Chaos-

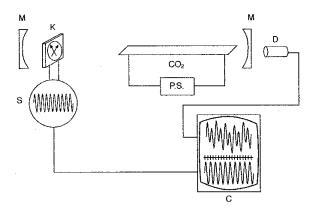
Chaos-0

Chans-N

Chaos-I

Cl. . . . 05

Laser Experimental Arrangement



Experimental Motivation

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation (

ontonto

Nonlinear-0

Naulinaan 0

Nonlinear-C

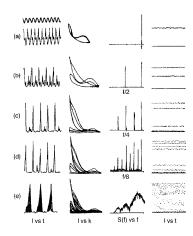
Chaos-01

Cildos 0

CI 0

Chaos-0

Oscilloscope Traces



Some Attractors

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-(

ontonto

Nonlinear-01

Nonlinear-

Nonlinear-0

Nonlinear-0

Chaos-01

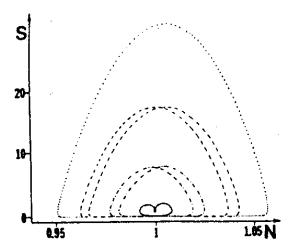
Cildos 0.

Chaos-03

Chaos-(

haos-05

Coexisting Basins of Attraction



Time Series Data

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-C

ontents

Nonlinear-0

Nonlinear-

Nonlinear-0

Nonlinear-05

Chaos-u.

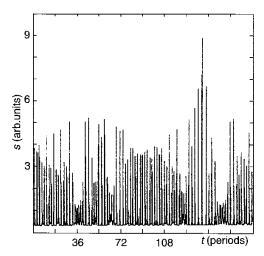
. .

Chaos-0

Chaos C

Chaos 05

Note the Spikiness



Time Series Data

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

iviotivation-

Contents

Nonlinear-0

Nonlinear-0

Chaos-0.

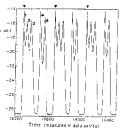
Chaos-u2

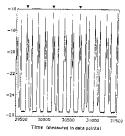
Chaos-0

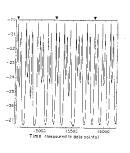
Chaos-I

haos-05

A Short Part of the Time Series







Real Data

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

ontents

Nonlinear-0

Nonlinear-C

Nonlinear-(

Mantinana

Chaos-0

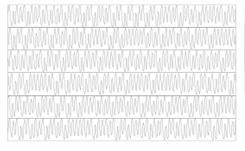
c. .

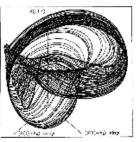
Chaos-0

Chaos-0

Chaos-u

Experimental Data: LSA





Lefranc - Cargese

Real Data

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

Nonlinear-0

NI. . . Para

.....

Nonlinear-C

CI 01

CI 00

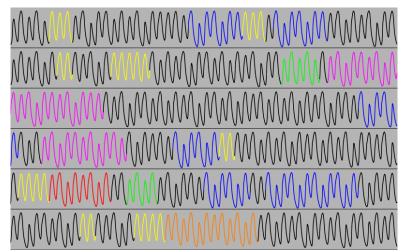
Chaos-02

Chaos-0

Chaos-0

haos-05

Experimental Data: LSA



Mechanism

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation I

ntente

Nonlinear-0

Naulinaa.

Manlinson (

.

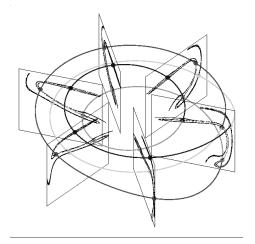
Chaos-u.

Chaos-0

Chaos-0

haos-05

Stretching & Squeezing in a Torus



Time Evolution

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-

ontents

Nonlinear-0

Monlinger (

Nonlinear-0

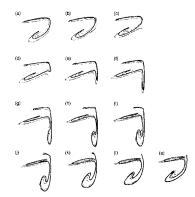
N . . . P 0

CI....

Chaos 0

CHaos-C

Rotating the Poincaré Section around the axis of the torus



Time Evolution

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation (

ontonto

Nonlinear-0

KI 11 .

Nonlinear-0

Chann 0

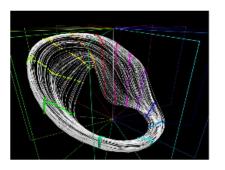
Chaos-0

Chaos-0

Chansel

Chaos-U

Rotating the Poincaré Section around the axis of the torus



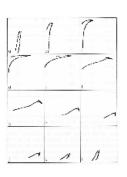


Figure 2. Left: Intersections of a chaotic attractor with a series of section planes are computed. Right: Their evolution from plane to plane shows the intemplay of the stretching and squeezing mechanisms.

Another Visualization

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation (

ontont

Nonlinear-0

Nonlinear.

Nonlinear-0

N OF

Chaos-01

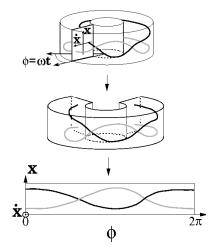
Chaos-u2

Chaos-0

Chaos-0

haos-05

Cutting Open a Torus



Belousov-Zhabotinskii Experimental Configuration

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation

ontonto

Nonlinear-0

Naulinaan O

......

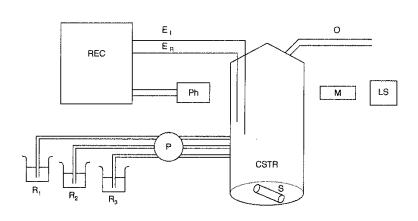
Nonlinear-0

Chaos-u.

Chaos-0

Chaos-C

Chaos 05



Close Returns Plot

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation I

ontent

Nonlinear-0

Monlinear O

Nonlinear-(

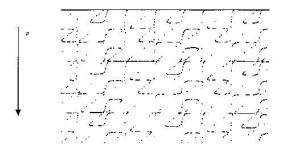
Chaos-U.

Cl. . . . 0:

Chaos (







Embeddings

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-(

ontents

Nonlinear-0

Mantinaan

Nonlinear-0

Nonlinear-0

Monlinear (

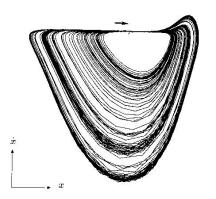
CI....

Chaos O

Chaos-0

Chaos-0

First Embedding Attempt: x, \dot{x}, \ddot{x}



Embeddings

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-(

ontents

Nonlinear-0

KI 11 .

Naulinaan O

Nonlinear-0

Chaos-0

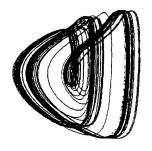
Chann 01

Chann

Chaos-0

Chaos-05

Second Embedding Attempt: $\int x, x, \dot{x}$



Nonstationary!

Embeddings

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-03

Motivation-(

ontents

Nonlinear-0

N. .

Nonlinear-C

Monlinear O

<u>.</u>. ..

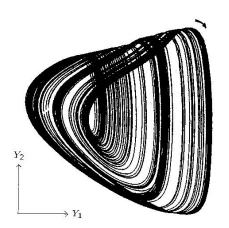
Chaos 0

Chaos-0

Chaos-0

Chaos-05

Third embedding attempt: $\int xe^{-t'/\tau}, x, \dot{x}$



Orbits to Organization

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-0

ontents

Nonlinear-0

Monlinear O'

Nonlinear-03

Nauliaaa 07

Nonlinear-0!

CHaos-0

Chaos-0

Chaos-0

Chaos-

Chaos-05

Once you have an embedding:

- Find a Poincaré Section
- Construct a First Return Map on the Section
- Introduce a Symbolic Encoding
- Encode all Unstable Periodic Orbits
- Find their Linking Numbers

Return Maps

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

NA-+:..-+:--- (

ontents

Nonlinear-0

N 11 0

Naulinaan (

Nonlinear-0

Chaor

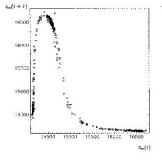
Chaos 0

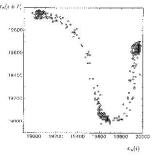
Chaos-0

Chaos-I

haos-05

Two Symbols Suffice! 0 and 1





Embedded Periodic Orbits

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

Nonlinear-0

Monlinear (

.....

Nonlinear-0

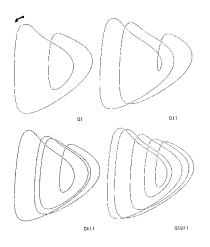
CI 01

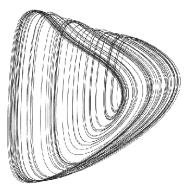
Chaos-02

Chaos-03

CI (

Some Named Low-Period Orbits





Some Extracted and Reconstructed Periodic Orbits

From Nonlinear

Dynamics to BioMedicine

Orbit

3

5

6

8a

8b

9

10a

10b

11

13a

13h

 1_1 2_1

 3_1

 4_1

 5_1

 6_2

 7_2

 8_1

 8_3

 9_3

 10_{6}

 10_{6}

 11_{9}

Name

0111 01 011 011~0M1 $(01)^2011$ $(01)^20111$ $01(011)^2$

 $(01)^3011$

 $(011)^20101$

 $(011)^20111$

 $(01)^3011 0111$

 $01(011)^3$

01

011

Symbolics

Local Torsion

28 33

33 40

Self-Linking

5

8

9

16

23

21

62

Table of Experimental Linking Numbers

From Nonlinear Dynamics to BioMedicine

Orbit	Symbolics	1	2	3	4	5	6	7	8a	8b
1	1	0	1	1	2	2	2	3	4	3
2	01	1	1	2	3	4	4	5	6	6
3	011	1	2	2	4	5	6	7	8	8
4	0111	2	3	4	5	8	8	11	13	12
5	01 011	2	4	5	8	8	10	13	16	15
6	011~0M1	2	4	6	8	10	9	14	16	16
7	$01\ 01\ 011$	3	5	7	11	13	14	16	21	21
8a	01 01 0111	4	6	8	13	16	16	21	23	24
8b	01 011 011	3	6	8	12	15	16	21	24	21
		a A II	::		40 100					

^aAll indices are negative.

Testing the Result

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

ontents

Nonlinear-0

Nonlinear-

Nonlinear-0

Nonlinear-0

Nonlinear-05

Chaos-u.

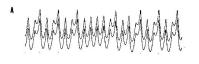
Chaos-0

Chaos-0

Chaos-0

Chaos-05

(a), (c) y_1^m compared with y_1^d . (b), (d) y_3^m compared with y_3^d .



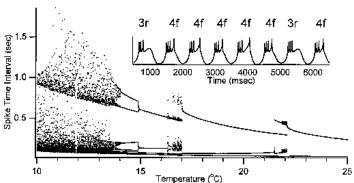




Bifurcation Diagram

From Nonlinear Dynamics to BioMedicine

Is This Chaotic or Not?



Reducing Dimension

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

NA-+:...+:--- 0

. . .

.

Nonlinear-U

.....

Chaos 0

Chaos-C

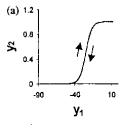
Chaos-u.

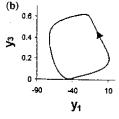
Chaos-0

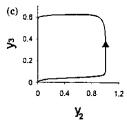
Chaos-0

haos-05

Correlations Imply Fewer Independent Variables







Phase Space Projections

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

NA-+:...+:--- (

_

Contents

Nonlinear-C

Nonlinear-0

Nonlinear-0

.. .. .

TVOIIIIICAI O

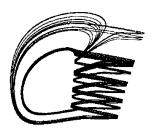
Chaos-0

Chaos-I

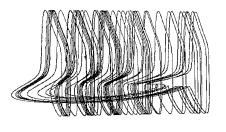
Chaos-

haos-05

Two Different Planar Projections



 y_4 - y_5 Plane



 \dot{y}_4 - y_5 Plane

First Return Map

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation_03

. . .

Contents

Nonlinear-01

. . .

Monlinear-0

Nonlinear-04

Chaos-01

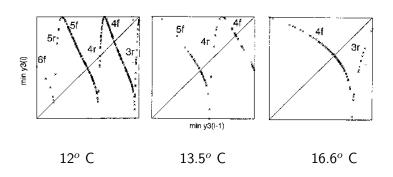
Chaos 0

Chaos-C

Chaos-I

haos-05

Return Map Migrates with Temperature



Scroll Templates

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

ontonto

Nonlinear-0

Monlinear-

Nonlinear-0

Nonlinear-04

.. .. .

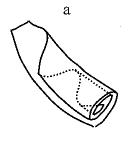
. .

Chaos-0

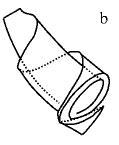
Chaos-0

Chaos-0

hace 05



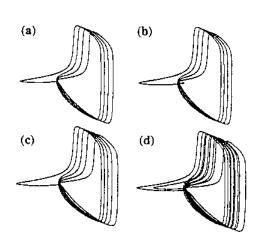
Granch	Array	c	1	2	3	4	5	G	7	8	9
0	IN-U	e	0	0	0	0	IJ	Ü	0	0	0
1	-N+0	0	1	2	2	2	2	2	2	2	2
2	1541	0	2	2	2	2	2	2	2	2	2
3	-N÷3	0	2	2	3	4	4	4	4	4	4
4	(N-2	0	2	2	4	4	4	4	4	4	4
5	-N-2	0	2	2	4	4	5	6	6	5	6
6	-N-3	0	2	2	4	4	6	6	6	6	6
7	-N-3	0	2	2	4	4	6	6	7	8	ß
В	-N-4	0	2	2	4	4	6	6	8	2	8
9	-N+4	0	2	2	4	4	6	6	8	8	ç



Arra y	0	ī	2	3	4	5	6	7	3	-
0	0	0	2	2	4	4	6	б	8	-
-1	0	1	2	2	4	4	5	6	8	1
+1	2	2	2	2	4	4	6	6	S	1
-2	2	2	2	3	4	4	6	6	8	1
+2	4	4	4	4	4	4	6	6	8	٤
-3	4	4	4	ż	4	5	6	6	8	ŧ
+3	6	6	6	6	6	6	6	6	8	8
-4	6	6	6	fi	6	6	6	7	8	4
14	8	8	8	8	5	5	9	E	8	5
-5	8	8	8	8	8	8	8	8	8	4
	0 -1 +1 -2 +2 -3 +3 -4	0 0 1 0 +1 2 -2 2 +2 4 -3 4 +3 6 -4 6 14 8	0 0 0 1 +1 2 2 -2 2 2 +2 4 4 +3 6 6 -4 6 6 14 8 8	0 0 0 0 2 -1 0 1 2 +1 2 2 2 2 -2 2 2 2 +2 4 4 4 -3 4 4 4 +3 6 6 6 -4 6 6 6	0 0 0 0 2 2 -1 0 1 2 2 +1 2 2 2 2 2 -2 2 2 2 3 +2 4 4 4 4 +3 6 6 6 6 -4 6 6 6 6 14 8 8 8 8	0 0 0 2 2 4 -1 0 1 2 2 2 4 +1 2 2 2 2 2 4 +2 2 2 2 3 4 +2 4 4 4 4 4 -3 4 4 4 4 4 +3 6 6 6 6 6 -4 6 6 6 6 6 14 8 8 8 8 8	0 0 0 2 2 4 4 4 4 1 2 2 2 2 2 4 4 4 4 4 4 4 4	0 0 0 2 2 4 4 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	0 0 0 2 2 4 4 5 6 6 1 1 0 1 2 2 2 4 4 5 6 6 6 6 6 6 6 6 6 7 1 4 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	0 0 0 2 2 1 4 6 6 8 8 -1 0 1 2 2 2 4 4 6 6 8 8 -1 0 1 0 1 2 2 2 4 4 6 6 8 8 -1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

Periodic Orbits

From Nonlinear Dynamics to BioMedicine



(a)
$$4f$$

Creating the Branched Manifold

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-(

ontents

Nonlinear-0

Nonlinear-0

Nonlinear-0

Nonlinear-04

Nonlinear-(

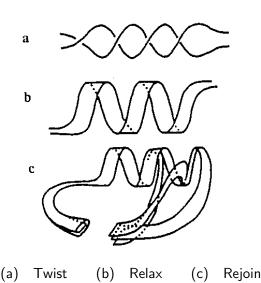
Chaos-

Chaos-0

Chaos-0

Chaos-0

haos-05



Simple Two-Parameter Model

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation (

ontont

Nonlinear-0

. . .

Naulinaan 0

Nonlinear-0

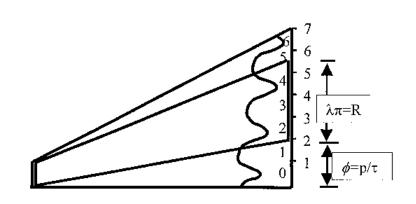
Cildos 0

Cl. . . . 0

Chaos-0

Chaos

CI OF



$$\phi = \mathsf{Drift} \qquad \lambda = \mathsf{Stretch} \; (\mathsf{Lyapunov} \; \mathsf{Exp})$$

The Setup

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

ontent

Nonlinear-(

Nonlinear-

Nonlinear-0

N . . . P O

Chaos-0

Cildos 0

Chaos-u.

Chaos-0

Chaos-I

haos-0!

- Suppose you want real-time data from a certain subject.
- And the subject is:

The Setup

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-

ontento

Nonlinear-0

Monlingar

Nonlinear-0

Nonlinear-0

Nonlinear-

Cilaus-u

Chaos-02

Chaos-0

Chaos-C

Ch.... 05

• Suppose you want real-time data from a certain subject.

• And the subject is:

CALVIN





Brain Rampant

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Nonlinear-0

Nonlinear-03

Nonlinear-0

Naulinaan (

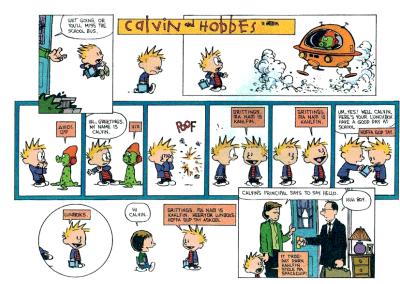
Nonlinear-0

Chaos-

Chaos-0

Chaos-







Dad's of the World – Watch Out!

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-0

Contonts

Nonlinear-0

..

Nonlinear-C

Nonlinear-0

Chaos-I

Chaos-0

Cildos (

CHaus-

Chaos-05

















Advance in Technology

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Contents

Nonlinear-01

Monlinger O

Nonlinear-0

Nonlinear 04

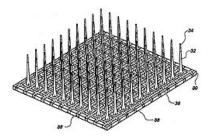
Chann

Chaos-0

Chaos-0

Chann OF

A normal implant would provide one time series. This guy has a behavioral time scale approx. 10^{-1} sec. Use an electrode array implant to record lots of time series simultaneously.



Record from Lots of Spots

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mativation

.

Nonlinear-0

.. .. .

Monlinger (

Nonlinear-0

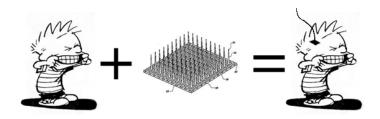
Chaos-0

a. .

Character (

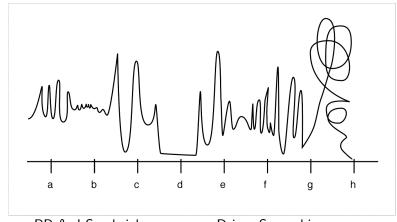
CI 0F

Now Calvin is Wired



Highly Effervescent Time Series

From Nonlinear Dynamics to **BioMedicine**



- a: PB & J Sandwich
- b: Skool
- c: Toboggan on Hobbes
- d: Dad Explains Something

- e: Drives Spaceship
- f: Speaks with Martian
- g: A GIRL !!!



Underlying Assumption

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-02

Conter

Nonlinear-01

Nonlinear-02

onlinear-0

Nonlinear-04

Chaos-0

Chaos-0

Chaos-

Ch . . . 0F

Biological Algorithm

Use the Ergodic Theorem (Hypothesis, Guess, Hope, Desperation Wish) to assume lots of short snippets from the $10^{2.5\pm0.5}$ electrodes can be reconstructed into a single long times series, one for each behavioral mode.

The reconstruction can be carried out via a "biological algorithm".

Biological Algorithm for Data Annealing

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

.............

Contents

Nonlinear-0

..

Nonlinear-0

Nonlinear-0

Nonlinear-0

Chaos-0

Chaos-0

Chaos-0

Chaos-I

Chann 05

- Time-tag time series from each electrode
- Cut out short snippets w. same time-tag from each electrode record
- Use DNA type comparison to join them

$$...G \ A \ C \ T \ C \quad T \ A \ G \ C \\ \qquad \qquad A \ T \ C \ G \quad T \ A \ T \ T...$$

Study each longer time series to determine behavior fingerprint

Biological Algorithm for Data Annealing

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-(

ontents

Nonlinear-01

Manlinson O

Chaos-0

Cildos 0

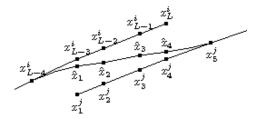
Chaos-02

Chaos-0

Chaos-C

hans-05

Interpolating Connection Between Snippets



Reconstruction

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-

ontents

Nonlinear-0

Monlinear

Nonlinear-C

Nonlinear-04

Chaos-u

Chaos-02

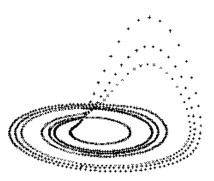
Chaos-0

Chaos-0

Chaos 05

Rossler Attractor

This period-7 orbit was reconstructed from 4 short snippets.



Reconstruction

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-⊦

ontents

Nonlinear-0

Monlingar

Nonlinear-0

Nonlinear 0

Chaos-0

Cl. . . . 00

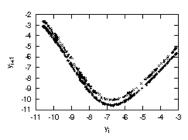
Chann

Chaos-0

Chaos-05

Rossler Attractor

Return maps for the original attractor and the attractor reconstructed from many short snippets.



Reconstruction

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-(

ontents

Nonlinear-0

NI II

Nonlinear-0

Nonlinear-0

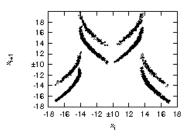
Chaos-0

Chaos-u

Chaos-c

Lorenz Attractor

Return maps for the original attractor and the attractor reconstructed from many short snippets.



Orbits Can be "Pruned"

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-02

Nonlinear-0

..

Nonlinear-C

Nonlinear-0

Chaos-0

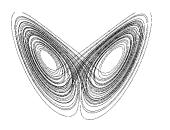
C.1.005 0

Character C

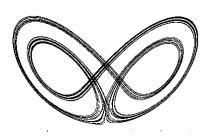
Chaos-I

Chara OF

There Are Some Missing Orbits







Shimizu-Morioka



Linking Numbers, Relative Rotation Rates, Braids

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation (

. . .

Nonlinear-01

Naulinaan O

Nonlinear-0

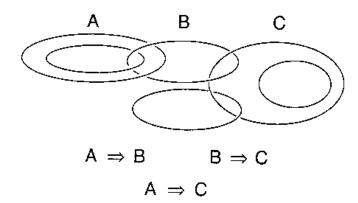
Chaos-0

Chaos-0

Chaos-C

CI 05

Orbit Forcing



An Ongoing Problem

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation (

......

Nonlinear-0

KI 12 .

.....

Nonlinear-0

Chaos (

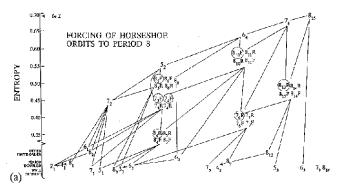
Chaos-u.

Chaos-03

Chaos-0

CI.... 0F

Forcing Diagram - Horseshoe



u - SEQUENCE ORDER



An Ongoing Problem

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-02

~ . .

Nonlinear-01

NI II OI

......

Noninear-04

Chaos O

Cilaus-u

Chaos-u

Chaos-0

Chaos-0

Chaos OF

Status of Problem

- Horseshoe organization active
- More folding barely begun
- Circle forcing even less known
- Higher genus new ideas required

Perestroikas of Branched Manifolds

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-02

Contei

Nonlinear-01

Nonlinear-02

Nonlinear-03

Nonlinear-04

Chaor

CHaos o

Chaos-02

Chaos-0

Chaos-

Chaos-05

Constraints on Branched Manifolds

"Inflate" a strange attractor

Union of ϵ ball around each point

Boundary is surface of bounded 3D manifold

Torus that bounds strange attractor

Torus and Genus

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-01

Motivation-02

Nonlinear-0

..

.....

Nonlinear-04

Cl.

Chaos-C

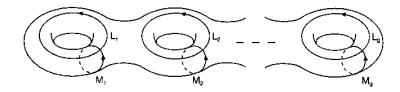
Chaos-u

Chaos-0

Chaos-0

haos-05

Torus, Longitudes, Meridians



Flows on Surfaces

From Nonlinear Dynamics to **BioMedicine**

Surface Singularities

Flow field: three eigenvalues: +, 0, -

Vector field "perpendicular" to surface

Eigenvalues on surface at fixed point: +, -

All singularities are regular saddles

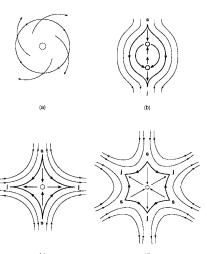
$$\sum_{s.p.} (-1)^{\text{index}} = \chi(S) = 2 - 2g$$

fixed points on surface = index = 2g - 2

Flows in Vector Fields

From Nonlinear Dynamics to BioMedicine

Flow Near a Singularity



Some Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-

ontents

Nonlinear-0

Namiliana O

.....

Chaos 0

Chaos-u

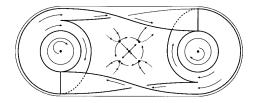
Chaos-U

Chaos-0

Chaos-I

haos-05

Torus Bounding Lorenz-like Flows



Canonical Forms

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-

ontents

Nonlinear-0

Monlinear-C

Nonlinear-0

Chaos

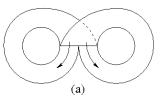
Chaos-0

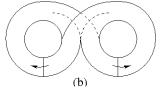
Chaos-0

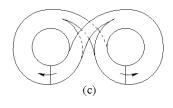
Chaos-I

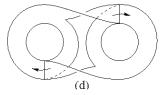
haos-0!

Twisting the Lorenz Attractor









Constraints Provided by Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-(

ontents

Nonlinear-0

N 11 0

Monlinear-0

Nonlinear-04

..

. .

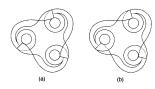
Chaos-u

Chaos-0

Chaos-(

haos-05

Two possible branched manifolds in the torus with g=4.





Use in Physics

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

viotivation-i

C.....

Nonlinear-01

.....

..

Chaos-u

Chaos-0

Chaos-(

Chaos-05

Bounding Tori contain all known Strange Attractors

Tab.1. All known strange attractors of dimension $d_L < 3$ are bounded by one of the standard dressed tori.

Strange Attractor	Dressed Torus	Period $g-1$ Orbit
Rossler, Duffing, Burke and Shaw	A ₁	1
Various Lasers, Gateau Roule	A_1	1
Neuron with Subthreshold Oscillations	A_1	1
Shaw-van der Pol	$A_1 \cup A_1^{(1)}$	1 U 1
Lorenz, Shimizu-Morioka, Rikitake	A_2	$(12)^2$
Multispiral attractors	A_n	$(12^{n-1})^2$
C_n Covers of Rossler	C_n	1 ⁿ
C ₂ Cover of Lorenz ^(a)	C_4	14
C ₂ Cover of Lorenz ^(b)	A_3	$(122)^2$
C_n Cover of Lorenz ^(a)	C_{2n}	1^{2n}
C_n Cover of Lorenz ^(b)	P_{n+1}	$(1n)^n$
$2 \rightarrow 1$ Image of Fig. 8 Branched Manifold	A_3	$(122)^2$
Fig. 8 Branched Manifold	Ps	(14)4
(a) Rotation axis through origin.		
(b) Rotation axis through one focus.		

(6) Rotation axis through one focus

Labeling Bounding Tori

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-0

Motivation-02

Contents

Nonlinear-01

Monlinear 03

Nonlinear-03

Noninear-04

Chaos-0

Chaos-U

CHaus-02

Chaos-0

Chaos-

Chaos-05

Labeling Bounding Tori

Poincaré section is disjoint union of g-1 disks

Transition matrix sum of two g-1 \times g-1 matrices

One is cyclic g-1 \times g-1 matrix

Other represents union of cycles

Labeling via (permutation) group theory

Some Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-

ontent

Nonlinear-0

Naulinaan (

Nonlinear-0

CI 0

<u>.</u>. .

Chaos (

CI.... 0F

Bounding Tori of Low Genus

TABLE I Bnumeration of canonical forms up to genus 9

В	m	ım		TOTH CAT NOT THE
	g	m) n1n2ng-1
	1	1	(0)	1
	3	2	(2)	11
	4 5	3	(3)	111
	5	4	(4)	1111
	5	3	(2,2)	1212
	6	5	(5)	11111
	6	4	(3,2)	12112
	7	6	(6)	111111
	7	5	(4,2)	112121
	7	5	(3,3)	112112
	7	4	(2,2,2)	122122
	7	4	(2,2,2)	131313
	8	?	(?)	1111111
	8	6	(5,2)	1211112
	8	ō	(4,3)	1211121
	8	5	(3,2,2)	1212212
	8	5	(3,2,2)	1 221 221
	8	5	(3,2,2)	1313131
	9	8	(8)	111111111
	9	7	(6,2)	11111212
	9	7	(5,3)	11112112
	9	7	(4,4)	11121112
	9	6	(4,2,2)	11122122
	9	ð	(4,2,2)	11131313
	9	ð	(4,2,2)	11212212
	9	ō	(4,2,2)	12121212
	9	6	(3,3,2)	11212122
	9	ð	(3,3,2)	11221122
	9	ō	(3,3,2)	11221212
	9	ō	(3,3,2)	11311313
	9	5	(2,2,2,2)	12221222
	9	5	(2,2,2,2)	12313132
	9	5	(2,2,2,2)	14141414

Motivation

From
Nonlinear
Dynamics to
BioMedicine

Robert Gilmore

Motivation-03

.

ontont

Nonlinear-0

.. .. .

Monlinear O

.....

Naulinaan O

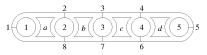
Chaos-0

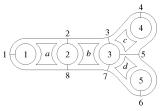
Chaos-0

Chaos-0

Chaos 05









Labels for Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Maritan C

C

Nonlinear-0

.. .. .

Monlinear (

Nonlinear-0

Chaos-0

Chaos-0

Chaos-0

Chaos-0

Chaos-05

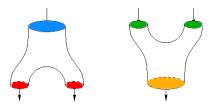
Each is described by a Transition Matrix. This is the sum of a general cyclic rotation and another Permutation Group Element. For second case, previous:

$$= (12345678) + (1)(28)(357)(4)(6)$$

Aufbau Princip for Bounding Tori

From Nonlinear Dynamics to **BioMedicine**

Any bounding torus can be built up from equal numbers of stretching and squeezing units



- Outputs to Inputs
- No Free Ends
- Colorless

Aufbau Princip for Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Motivation-I

ontents

Nonlinear-0

Naulinaan O

Nonlinear-0

..

Nonlinear-0

Chaos-

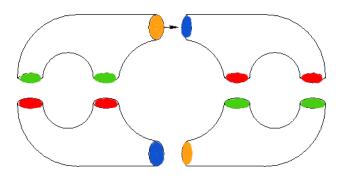
Chaos-0

Chaos-0

Chaos-0

haos-05

Application: Lorenz Dynamics, g=3



Poincaré Section

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Mativation

ontont

Monlinear 0

Naulinaan O

Nonlinear-0

Chaos-0

Cl.

Chaos 05

Construction of Poincaré Section

P. S. = Union



Components = g-1

Exponential Growth

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-0

Contents

Nonlinear-01

Monlinger O

Nonlinear-0

Chaos-0

Chaos-01

Chaos-0

Chaos-0

The Growth is Exponential

TABLE I: Number of canonical bounding tori as a function of genus, g.

g	N(g)	g	N(g)	g	N(g)
3	1	9	15	15	2211
4	1	10	28	16	5549
5	2	11	67	17	14290
ð	2	12	145	18	3 6 824
7	5	13	3 6 8	19	96347
8	6	14	870	20	252927

Exponential Growth

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-01

Motivation-(

ontents

Nonlinear-0

N. 11 /

Nonlinear-0

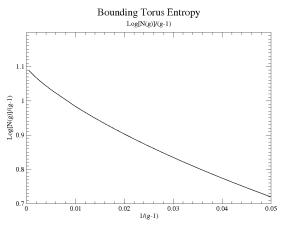
Chaos-01

Chaos-0

Chaos-0

CI 0F

The Growth is Exponential The Entropy is log 3



Extrinsic Embedding of Bounding Tori

From Nonlinear Dynamics to BioMedicine

> Robert Gilmore

Motivation-03

Motivation-0

ontente

Nonlinear-03

Nonlinear 0

Nonlinear-0

Nonlinear-04

Nonlinear-

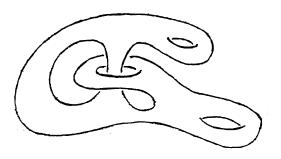
CHaos-0

Chaos-0

Chaos-0

Chaos-05

Extrinsic Embedding of Intrinsic Tori



Partial classification by links of homotopy group generators. Nightmare Numbers are Expected.