Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Alice in Stretch & SqueezeLand: 19 Summary

August 12, 2012

Chapter Abstract

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

This journey began with a request to "help me explain my data."

It introduced a third tool for the analysis of chaotic data: geometric, dynamical, topological.

When applicable $(d = 3)$ it provides information about mechanism, is robust against noise, and carries its own rejection criterion.

It has stimulated an exchange of information between mathematicians and physicists.

It has opened many doors and has pointed to new directions in which to search.

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Summary

1 Question Answered $\Rightarrow$

2 Questions Raised

We must be on the right track !

Alice in Stretch & SqueezeLand: 19
Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Original Objectives Achieved

There is now a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Result

There is now a classification theory for low-dimensional strange attractors.

1. It is topological
2. It has a hierarchy of 4 levels
3. Each is discrete
4. There is rigidity and degrees of freedom
5. It is applicable to R^3 only — for now

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

The Classification Theory has 4 Levels of Structure

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

The Classification Theory has 4 Levels of Structure

1. Basis Sets of Orbits

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

The Classification Theory has 4 Levels of Structure

1. Basis Sets of Orbits
2. Branched Manifolds

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

The Classification Theory has 4 Levels of Structure

1. Basis Sets of Orbits
2. Branched Manifolds
3. Bounding Tori

Alice in Stretch & SqueezeLand: 19 Summary

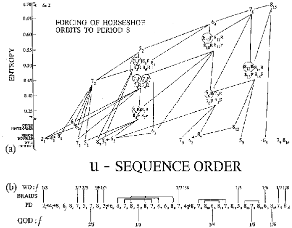
Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

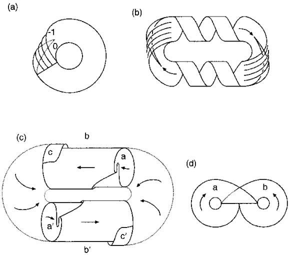
The Classification Theory has 4 Levels of Structure

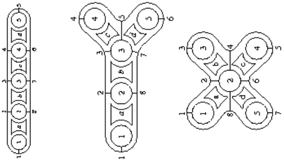
1. Basis Sets of Orbits
2. Branched Manifolds
3. Bounding Tori
4. Extrinsic Embeddings

Four Levels of Structure

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10







Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Poetic Organization

LINKS OF PERIODIC ORBITS

organize

BOUNDING TORI

organize

BRANCHED MANIFOLDS

organize

LINKS OF PERIODIC ORBITS

Answered Questions

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

There is a Representation Theory for Strange Attractors

There is a complete set of rerpesentation labels for strange attractors of any genus g.

The labels are complete and discrete.

Representations can become equivalent when immersed in higher dimension.

All representations (embeddings) of a 3-dimensional strange attractor become isotopic (equivalent) in R^5.

The *Universal Representation* of an attractor in R^5 identifies mechanism. No embedding artifacts are left.

The topological index in R^5 that identifies mechanism remains to be discovered.

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

Some Unexpected Results

- Perestroikas of orbits constrained by branched manifolds
- Routes to Chaos = Paths through orbit forcing diagram
- Perestroikas of branched manifolds constrained by bounding tori
- Global Poincaré section = union of $g-1$ disks
- Systematic methods for cover - image relations
- Existence of topological indices (cover/image)
- Universal image dynamical systems
- NLD version of Cartan's Theorem for Lie Groups
- Topological Continuation – Group Continuuation
- Cauchy-Riemann symmetries
- Quantizing Chaos

Alice in Stretch & SqueezeLand: 19 Summary

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

We hope to find:

- Robust topological invariants for R^N, $N > 3$
- A Birman-Williams type theorem for higher dimensions
- An algorithm for irreducible embeddings
- Embeddings: better methods and tests
- Analog of χ^2 test for NLD
- Better forcing results: Smale horseshoe, $D^2 \to D^2$, $n \times D^2 \to n \times D^2$ (e.g., Lorenz), $D^N \to D^N$, $N > 2$
- Representation theory: complete
- Singularity Theory: Branched manifolds, splitting points (0 dim.), branch lines (1 dim).
- Singularities as obstructions to isotopy

Chapter Summary-01
Summary-01
Summary-02
Summary-03
Summary-04
Summary-05
Summary-06
Summary-07
Summary-08
Summary-09
Summary-10

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