August 12, 2012
Chapter Abstract

The global torsion of an attractor inside a torus can be changed by integer values, $n$, in a simple way. This involves a simple global diffeomorphism.

Local diffeomorphisms are used to produce $q$-fold covers. Classical-like statistics (average energy, spin angular momentum) depend on the quantization indices $n, p/q$ in the expected intuitive way.
Creating New Attractors

Rotating the Attractor

\[
\begin{align*}
\frac{d}{dt}\begin{bmatrix} X \\ Y \end{bmatrix} & = \begin{bmatrix} F_1(X,Y) \\ F_2(X,Y) \end{bmatrix} + \begin{bmatrix} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{bmatrix} \\
\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} & = \begin{bmatrix} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} \\
\frac{d}{dt}\begin{bmatrix} u \\ v \end{bmatrix} & = RF(R^{-1}u) + Rt + \Omega \begin{bmatrix} -v \\ +u \end{bmatrix}
\end{align*}
\]

\[\Omega = n \omega_d \quad \text{Global Diffeomorphisms} \quad q \Omega = p \omega_d \quad \text{Local Diffeomorphisms (q-fold covers)}\]
Another Visualization

Cutting Open a Torus

\[ \phi = \omega t \]

\[ \dot{x} \]

\[ x \]

\[ \Phi \]

\[ 2\pi \]
Satisfying Boundary Conditions

Global Torsion

(a)

(b)
Two Phase Spaces: $R^3$ and $D^2 \times S^1$

Rossler Attractor: Two Representations

$R^3$

$D^2 \times S^1$

Rossler Attractor, Toroidal Representation

Index $(n_1,n_2) = (1,0)$
Rossler Attractor:

Two More Representations with \( n = \pm 1 \)

![Graphs showing Rossler Attractor with different indices (n_1, n_2) = (1,-1) and (1,1)].
Subharmonic, Locally Diffeomorphic Attractors

**Rossler Attractor:**

Two Two-Fold Covers with $p/q = \pm 1/2$

![Rossler Attractor, Toroidal Representation](image1.png)

![Rossler Attractor, Toroidal Representation](image2.png)
Rossler Attractor: Two Three-Fold Covers with $p/q = -2/3, -1/3$
Subharmonic, Locally Diffeomorphic Attractors

Rossler Attractor:

And Even More Covers (with $p/q = +1/3, +2/3$)
Representations of Duffing Attractor

Duffing Attractor, Toroidal Representation

Duffing Attractor
Harmonic Lift, $k=0$
Representations of Duffing Attractor

Duffing Attractor, Rotation by $\pm 1$

![Graph showing the Duffing Attractor with Harmonic Lift, $k = -1$ and $k = +1$.]
Representations of Duffing Attractor

Duffing Attractor, Rotation by ±2

![Graph showing Duffing Attractor with rotation by ±2](image-url)
New Measures

Angular Momentum and Energy

\[ L(0) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau XdY - YdX \]

\[ K(0) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \frac{1}{2} (\dot{X}^2 + \dot{Y}^2) dt \]

\[ L(\Omega) = \langle u \dot{v} - v \dot{u} \rangle \]

\[ K(\Omega) = \langle \frac{1}{2} (\dot{u}^2 + \dot{v}^2) \rangle \]

\[ = L(0) + \Omega \langle R^2 \rangle \]

\[ = K(0) + \Omega L(0) + \frac{1}{2} \Omega^2 \langle R^2 \rangle \]

\[ \langle R^2 \rangle = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau (X^2 + Y^2) dt = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau (u^2 + v^2) dt \]
New Measures, Diffeomorphic Attractors

Energy and Angular Momentum

Diffeomorphic, Quantum Number n
New Measures, Subharmonic Covering Attractors

Energy and Angular Momentum

Subharmonics, Quantum Numbers \( p/q \)

**Torsion Integral**

**Energy Integral**